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Implementation of Verification Tests in the Integrated TIGER Series

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ITS—Integrated TIGER Series

- Series of Coupled Electron/Photon Monte Carlo Transport Codes with CAD Geometry
- Group of multimaterial and multidimensional codes designed to provide state-of-the-art description of the production and transport of the electron/photon cascade

-ITS Version 6.0 Manual, Revision 4, SAND2008-3331

- Capable of doing both multigroup and continuous-energy calculations



View of Monte Carlo,
Monaco from the east

Monte Carlo Method

- **Monte Carlo methods describe a class of numerical methods which attempt to solve problems in engineering and science by statistical simulation.**
- **Originated in the Manhattan Project to solve problems of particle transport related to nuclear weapons design**
- **Current Applications: nuclear reactor analysis, radiation cancer therapy, oil well exploration, high energy physics experiments, turbulence, phase transitions in materials, fundamental particle physics, reliability, decision processes, economic modeling, transmission of infectious diseases, and a number of applications in medical/radiation physics, including instrumentation research, dosimetry development, radiation standardization, and radiation treatment prescription**

Monte Carlo Method: How does it work?

- **Summary of a Canonical Monte Carlo Simulation:**

- The physical or mathematical system can be defined by probability density functions
- Uniformly distributed random numbers ξ on $[0,1)$ are available
 - *These are usually provided by random number generators*
- Use ξ 's to sample outcomes
- Accumulate outcomes (tallies) which are estimates of the desired results
- Determine statistical error in these estimates
- Sample until error is acceptable

Monte Carlo Method: Estimating π by Throwing Darts

- Consider a unit circle inscribed in a square
- The ratio of the area of the circle to the area of the square is:

$$\frac{A_{circle}}{A_{square}} = \frac{\pi r^2}{(2r)^2} = \frac{\pi}{4}$$

- If we can find the ratio of the area of the circle to the area of the square, we can just multiply that ratio by 4 to obtain π
- This can be done by throwing darts randomly at the square—the ratio of darts that land inside the circle to the ratio of darts that land outside the circle, but inside the square, will be equal to the ratio of the area of the circle to the area of the square

Monte Carlo Method Example: Estimating π

- **How can we throw darts with a computer?**
 - We use a random number generator
- **Random number generators provide numbers which are uniformly distributed on $[0,1)$**
- **Thus, we can consider two random numbers at a time— ξ_x and ξ_y , for the x and y axes of the coordinate system**
- **Since the random numbers are between 0 and 1, we are really only considering $\frac{1}{4}$ of the circumscribed square**



Monte Carlo Method Example: Estimating π

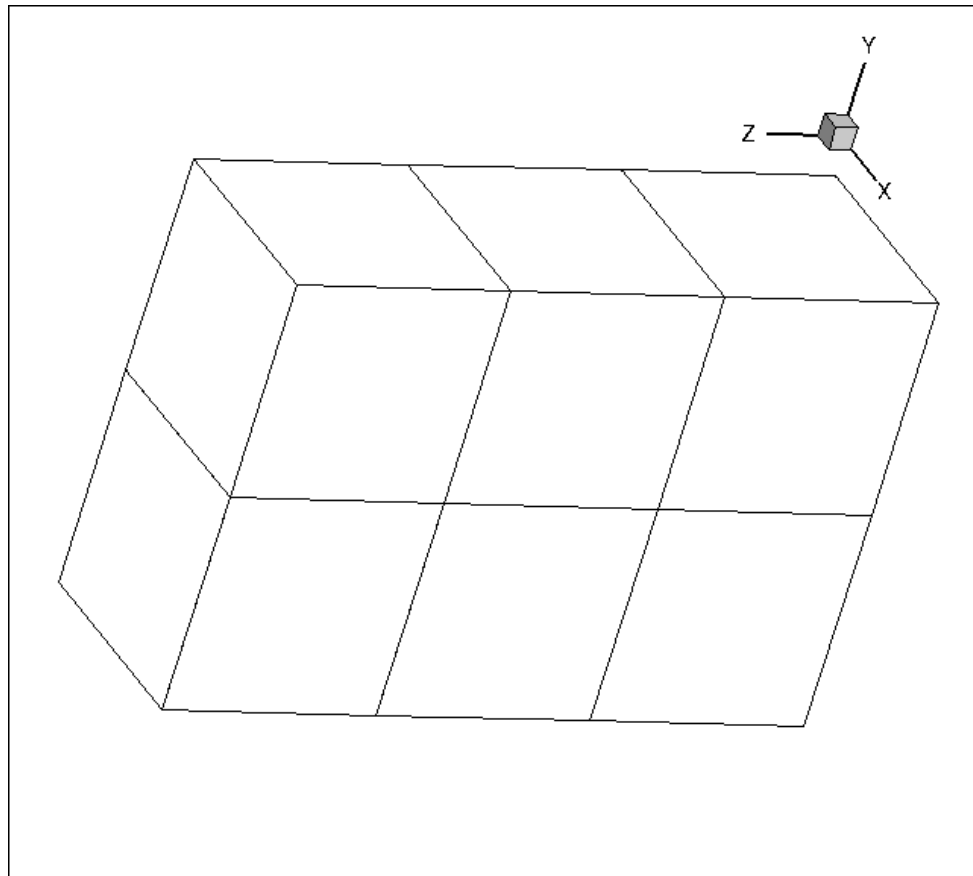
- If $\xi_x^2 + \xi_y^2 \leq 1$, that dart landed in the quarter-circle
- Otherwise, the dart was not inside the circle
- Keep track of how many darts landed inside the circle, and how many didn't
- When a sufficiently large number of darts have been thrown, estimate π by calculating the ratio of the number of darts that landed inside the circle to the number of darts that landed outside of the circle and multiply by 4

Verification

- **Comparison of code results for a specific problem to a known analytic solution**
- **Used to verify that the code yields acceptable mathematical result**
- **Verification tests can utilize a number of statistical tests to compare numeric results from the code to be verified to the analytic solution**
 - Statistical tests are necessary because, due to the random nature of a Monte Carlo code, there will be some difference between the numeric and analytic results
 - Statistical tests tell us whether this difference is statistically significant, which would indicate that the code does not yield an acceptable result when the result is compared to a known analytic solution
 - Anderson-Darling Test was used in this case to develop a verification test

Verification Test Design: Geometry

- **Voided right parallelepiped, with sides 1 cm, 2 cm, and 3 cm, with each face divided into 1 cm x 1 cm subsurfaces (shown below)**



Verification Test Design: Analytic Solution

- **Uniform Isotropic Flux**
- **Tally electron emission: all electrons entering parallelepiped will be tallied then terminated**
- **Electron emission tally is on subsurfaces (shown)—subsurfaces each have unit area, so that tally is in units of electrons/MeV-sr-cm²**
- **Since flux is uniform and isotropic, and each subsurface has the same area, the expected current per unit energy and solid angle (analytic solution) should be the same through each subsurface**

Statistical Test

- **Goodness-of-fit test: tests whether a sample of data fits a given distribution**
 - In this case, output from ITS was tested against the standard normal distribution
 - Central Limit Theorem: mean of a sufficiently large number of independent random variables is approximately normally distributed
 - To convert data sample to fit standard normal distribution (rather than any other normal distribution), use the following formula:

$$t = \frac{x - x_{analytic}}{u}$$

t: variable that is standard normally distributed

x: original purported normally-distributed variable (ITS output)

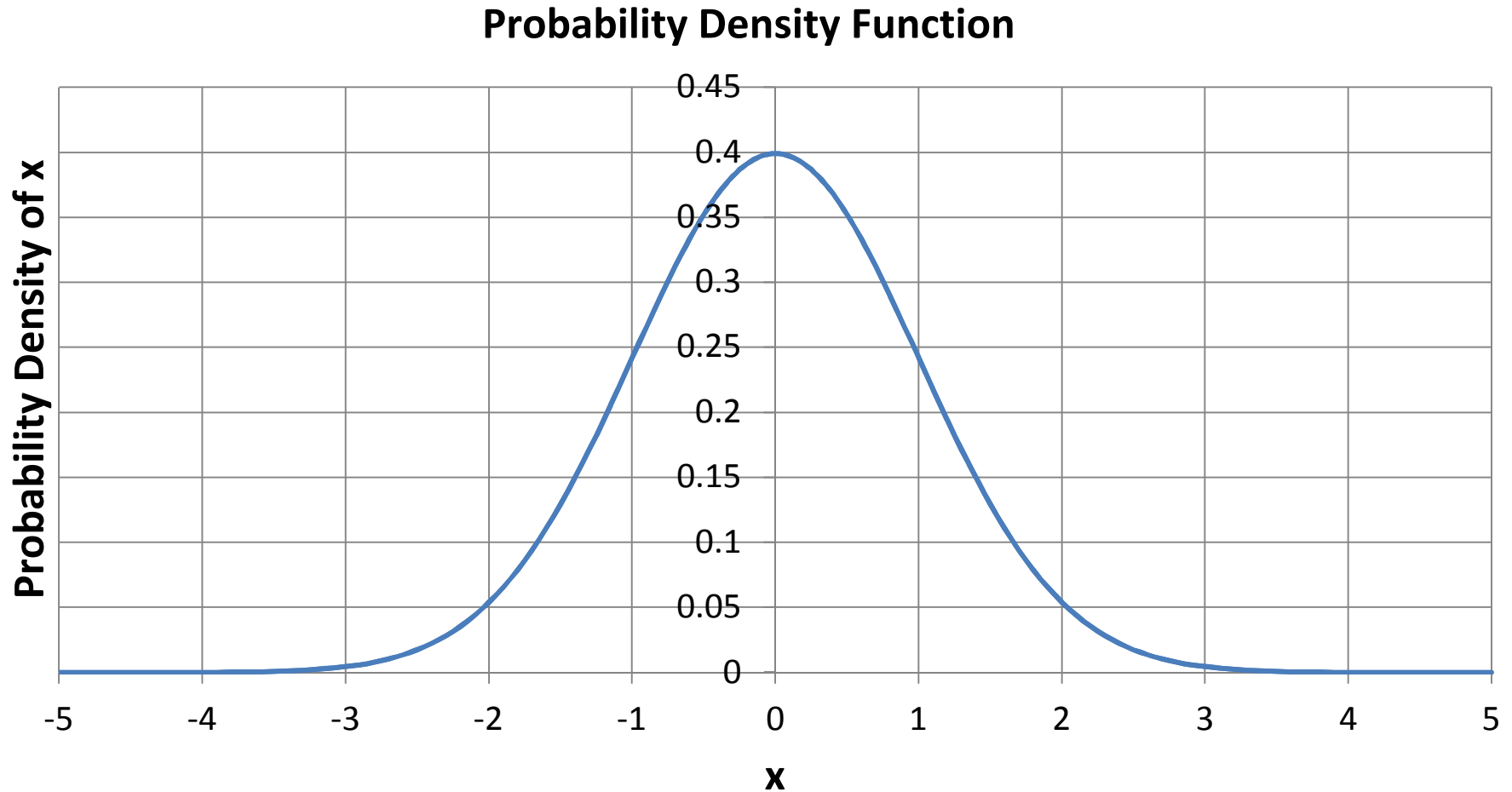
$x_{analytic}$: expected value of x (known analytic solution)

u: Monte Carlo uncertainty

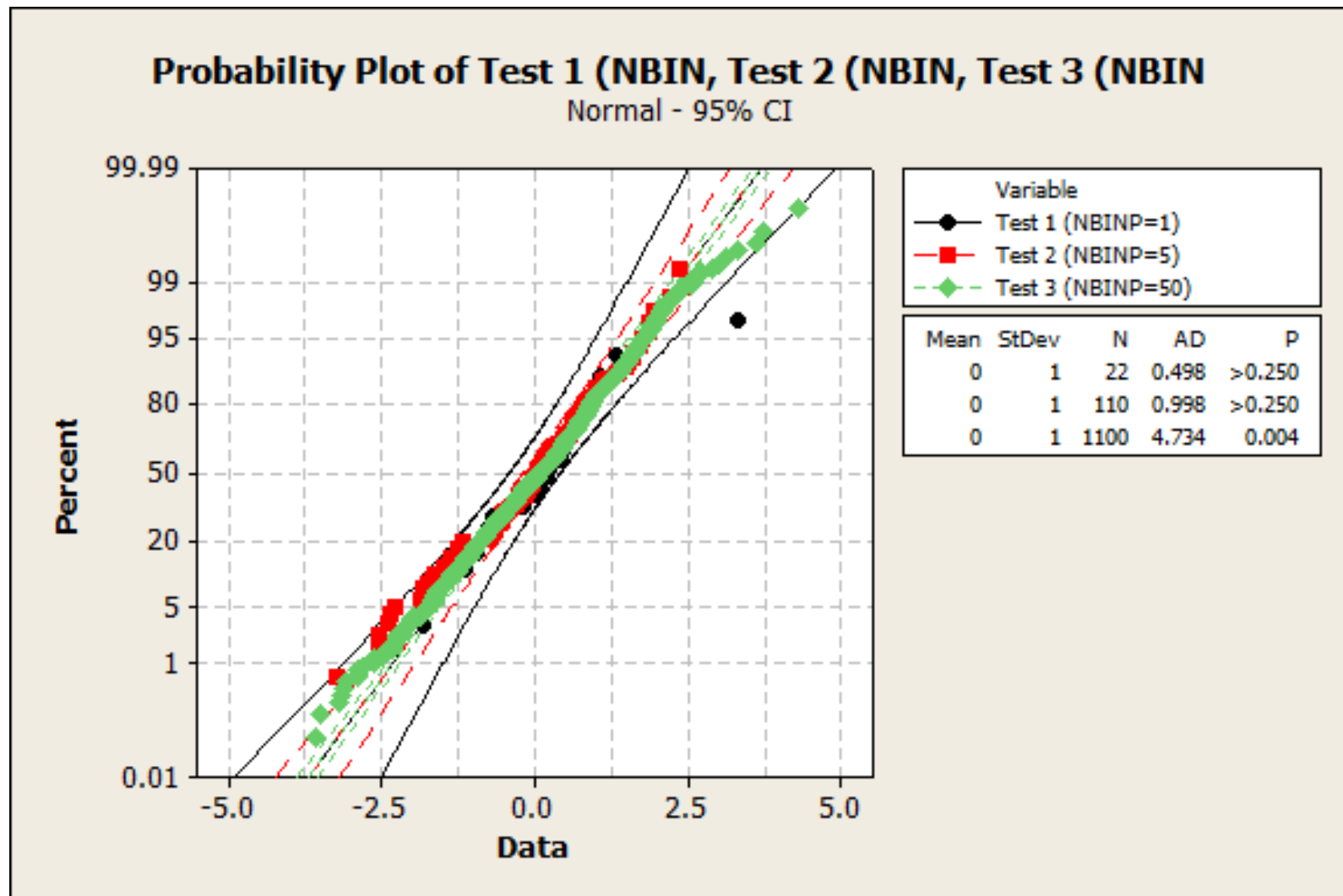
Anderson-Darling Test for Verification of ITS

- **Given no intrinsic FORTRAN function to evaluate the Anderson-Darling Distribution, a separate subroutine had to be written to do this**
- **Evaluating the Anderson-Darling Distribution—paper by George and John Marsaglia—outlines step-by-step how to evaluate the Anderson-Darling Distribution for a given number of independent samples**
- **After subroutine was written, results from subroutine were compared against Anderson-Darling Test results from Minitab statistics package for three different sets of independent random variables to verify accuracy**
- **Output from Anderson-Darling Test is p-value, which should be uniformly distributed on $[0,1)$ if a given verification test is run on ITS several times**

Standard Normal Distribution



Minitab Result



Future Work

- **Write a script that does the following:**
 - Create a loop to run ITS several times in a row for a given problem to verify that p-value is uniformly distributed between 0 and 1
 - Post-processes ITS output and run Anderson-Darling test on it
 - Results will be available after these steps are completed
- **Design more verification tests**

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Questions