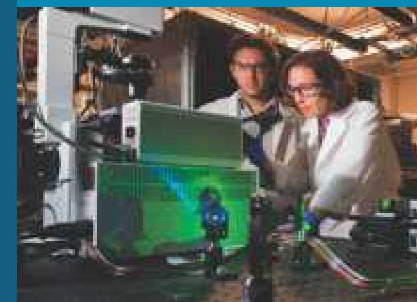
SAND2019-14679PE

Predicting System Response at Unmeasured Locations



Internal SNL Presentation Dec 2019

PRESENTED BY

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Motivation for improving system dynamic response estimates

- Components need to be qualified to survive dynamic environments in the system.
- Often one or more system level field/flight tests are conducted with some accelerometers at some convenient locations.
- These locations are usually not the attachment locations of the components.
- Ideally, one would like to know how the base moves and how the component moves with respect to the base. This is never provided.
- Typically there is a specification in each of the x y and z directions, preferably from some field measurement in each of those directions. Often the field measurement is some distance away from attachment point(s). Rotations are neglected. These limited measurements provide poor quantification of the environment.
- To address unit-to-unit variability, straight line envelopes of measured responses are employed. Much more energy is required to drive the enveloped responses and this is branded as conservatism, but since the original environment was poorly quantified, the amount of conservatism is unknown.
- **Too much conservatism often breaks components in qualification tests that would have survived the field environment. The failure modes may be different in the laboratory than in the field.**
- This work with AWE attempts to take a step toward providing the ideal estimate of all responses necessary to truly define a measured field/flight environment for a base mounted component.

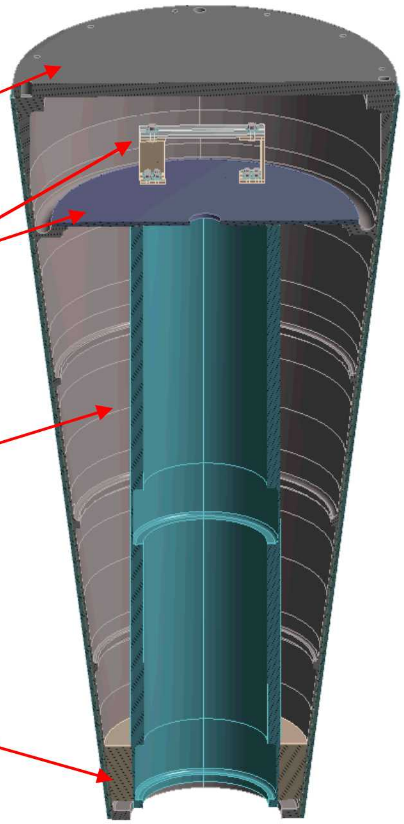
Approaches to estimate unmeasured system responses from a few measurements (the predictions here in acceleration spectral density)

- The desire was to measure the motion in a system with 30 accelerometers. From those 30 vibration measurements, an expansion to other unmeasured target DoF is to be performed which will define the component environment or base input to the component environment.
- The first approach utilizes a correlated finite element model (based on modal test) of the system to expand the responses through a modified version of the System Equivalent Reduction Expansion Process (SEREP) by O'Callahan and Avitabile to estimate unmeasured response DoF. I present results from the traditional SEREP with one set of mode shapes for the entire bandwidth
- The second approach uses a modified SEREP with a different partial set of the modes in four different bandwidths
- The third approach also utilizes a correlated finite element model and a multi-input, multi-output (MIMO) control algorithm to estimate pseudo-forces that will cause the system to be driven at each frequency line in the way the field environment drove the system.
- The last approach derives experimental basis vectors from a system level modal test. These vectors are utilized in much the same way as SEREP utilizes mode shapes to predict target responses that are measured in the modal test, but unmeasured in the field test.

Proof of concept Hardware for estimating unmeasured responses MATV

- The project proposed to prove the estimation of unmeasured responses concept using research hardware provided by AWE known as the Modal Analysis Test Vehicle (MATV) which would be tested in a field random acoustic environment.

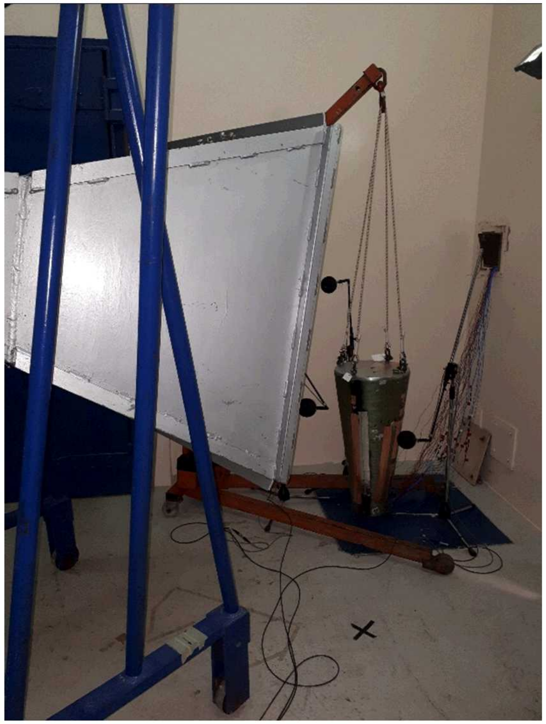
- MATV Description
  - A meter long
  - 47 kg
  - Composite wrapped on aluminum substrate cone
  - Large end aluminum cover plate
  - Aluminum internal flat component plate
  - Bracket called the Removable Component (RC) bolted to the internal component plate
  - Steel pipe bolted to the component plate
  - Foam support between pipe and cone at small end



Field Acoustic Test for MATV

- A field acoustic test was run to 147 dB at the Institute of Sound and Vibration Research at Southampton University in a reverberant chamber with horn.

- Place in corner of chamber
- Horn
- MATV suspended by bungees
- 67 total accelerometer channels recorded

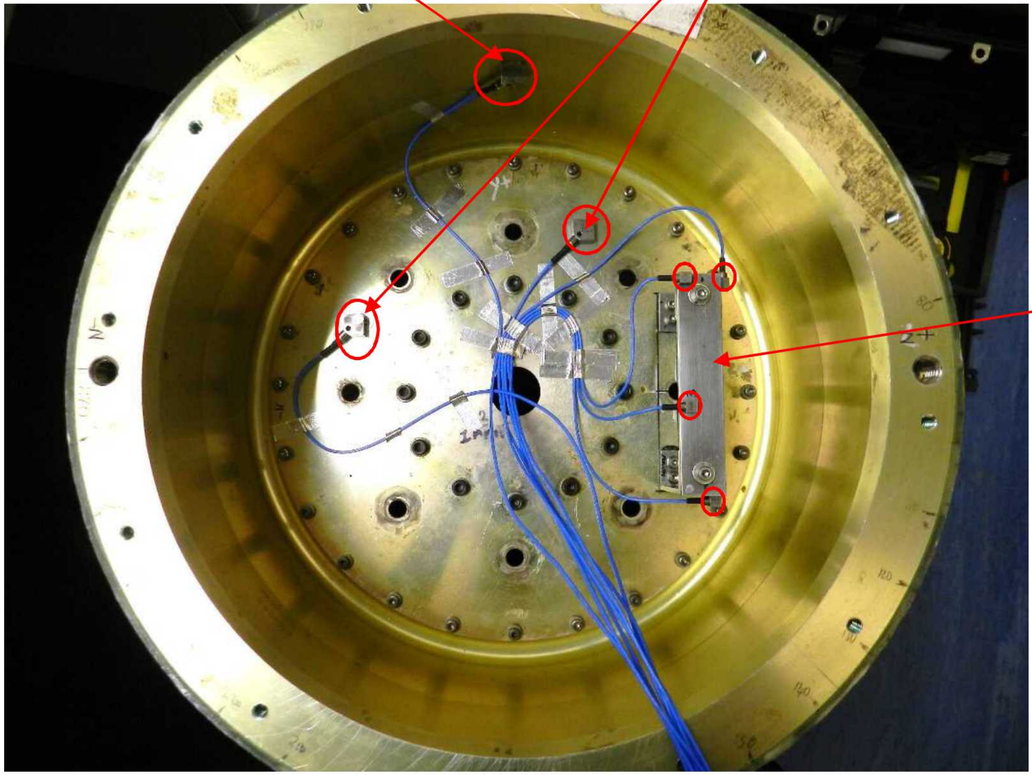


MATV Truth "Unmeasured" Accel Locations

- 14 truth accelerometer locations were chosen either on the RC or triaxial locations at typical mounting locations for a component. These were not used in the 30 accelerometer measurement set to predict the responses, but were targets for the estimates generated from the expansions.

1 Triax on Cone

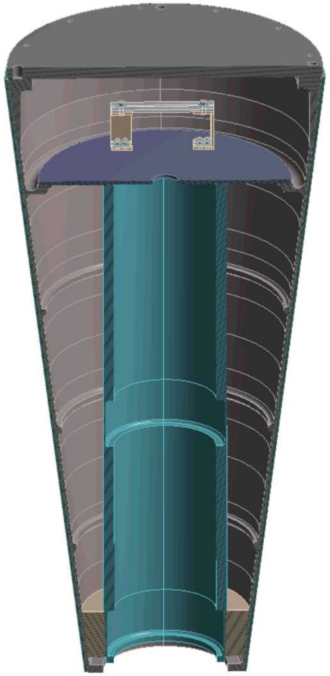
2 Triax on Component Plate



RC – 5 dof chosen on 4 Triaxes

Finite Element Model Updating

- 78 modes were used in the 2000 Hz bandwidth of interest in the FE model.
- A three shaker modal test was performed up to 2000 Hz.
- All identified test ovaling modes were within 5 percent of the model frequencies.
- The foam contact at small end was adjusted to bring the first axial mode and the first torsion mode within 5 percent of the corresponding modal test modes.
- 22 modes were well correlated in frequency up to 1329 Hz torsion mode.
- Higher bending modes were not addressed due to resource constraints.



Baseline SEREP approach with full modal filter

- Full modal filter means only one set of mode shapes was used in the entire frequency band
- The modal filter is the pseudo-inverse of the mode shape matrix
- More measurements than modes are required to get a least squares solution for the modal response that can be propagated through the entire system to the unmeasured DoF
- Low condition number for the mode shape matrix is better (<100 very rough no.)

$$\begin{Bmatrix} \ddot{\boldsymbol{x}}_m \\ \ddot{\boldsymbol{x}}_u \end{Bmatrix} \approx \begin{bmatrix} \boldsymbol{\Phi}_m \\ \boldsymbol{\Phi}_u \end{bmatrix} \{\ddot{\boldsymbol{q}}\} \tag{1}$$

Physical Cross Spectral Density Matrix

$$\{\ddot{\boldsymbol{q}}\} = \boldsymbol{\Phi}_m^{+}\{\ddot{\boldsymbol{x}}_m\} \tag{2}$$

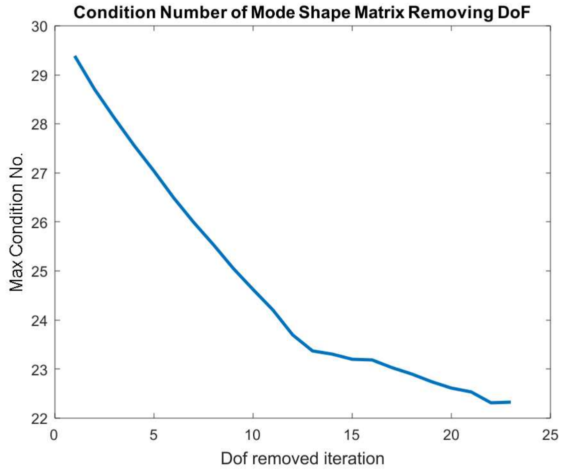
$$\{\ddot{\boldsymbol{q}}\}\{\ddot{\boldsymbol{q}}\}^T = \boldsymbol{\Phi}_m^{+}\{\ddot{\boldsymbol{x}}_m\}\{\ddot{\boldsymbol{x}}_m\}^T\boldsymbol{\Phi}_m^{+T} \tag{3}$$

$$\mathbf{S}_{qq} = \boldsymbol{\Phi}_m^{+}\mathbf{S}_{xx\_m}\boldsymbol{\Phi}_m^{+T} \tag{4}$$

$$\mathbf{S}_{xx\_u} \approx \boldsymbol{\Phi}_u\mathbf{S}_{qq}\boldsymbol{\Phi}_u^{T} \tag{5}$$

Baseline SEREP approach with full modal filter (pp 2)

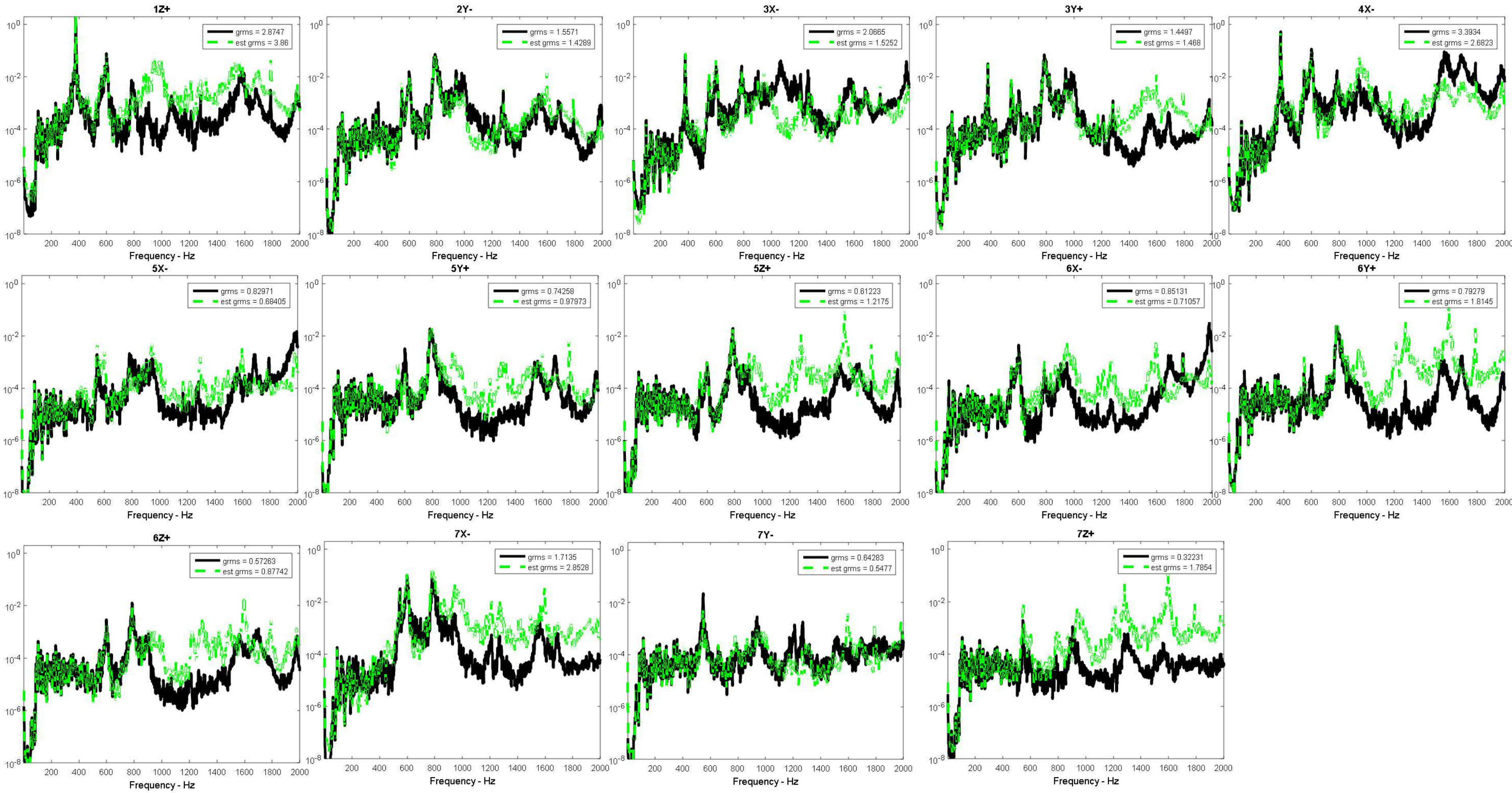
- A perfect mode shape for each mode is not required.
- A linear superposition of the chosen mode shapes must span the space of the actual motion for this technique to be accurate.
- Choose best 30 measurement gages from 53 available sensors
- With all 53 candidate gages, the condition number is 30 for first 20 modes, but jumps to 182 for 21 modes. Therefore we limit to 20 modes (920 Hz).
- Iterative sub-optimal process to reduce to 30 gages.
  - Remove 1 gage and calculate mode shape matrix condition no
  - Replace that gage and remove another and calculate mode shape matrix condition no
  - Repeat for all 53 gages
  - Discard the gage that produces the lowest condition number when removed
  - Repeat above 4 steps with the gages that are left, until the set is trimmed to 30 gages



$$\begin{Bmatrix} \ddot{x}_m \\ \ddot{x}_u \end{Bmatrix} \approx \begin{bmatrix} \boldsymbol{\Phi}_m \\ \boldsymbol{\Phi}_u \end{bmatrix} \{\ddot{\boldsymbol{q}}\} \tag{1}$$

Baseline SEREP approach with full modal filter (pp 3)

- Target ASDs from 20 mode (full) modal filter



- 20 modes doesn't span the motion space beyond about 900 Hz, no real surprise with 70 plus modes in the system

Modified SEREP approach with partial modal filter in 4 frequency bands

- Each frequency band must have less than 30 modes keep the calculation of the modal q's overdetermined if we utilize 30 well placed measurement gages.
- We attempt to "pick" the modes that will be active in each frequency band.
- As a minimum number of modes starting point, we pick the modes with natural frequencies contained in a calculation bandwidth with an attempt to keep the condition number of this initial set of modes in each frequency band to about 10. We know that if we add more modes later the condition number will go "up" and we are definitely trying to stay below 100. Our initial condition number utilizes all 53 candidate accelerometers.
- Sensor optimization is performed with the initial set of modes in each frequency band, optimizing on the sum of the condition numbers from the 4 mode shape matrices.

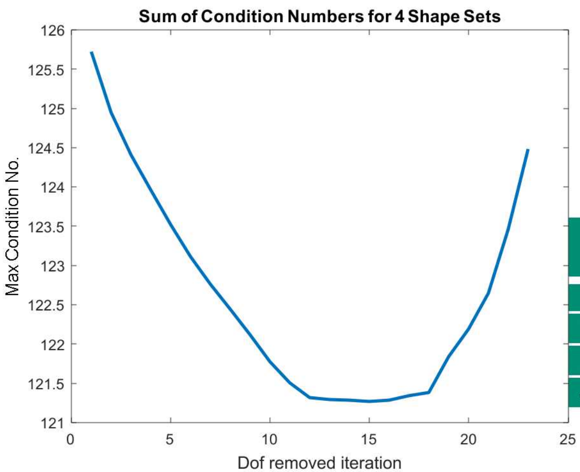


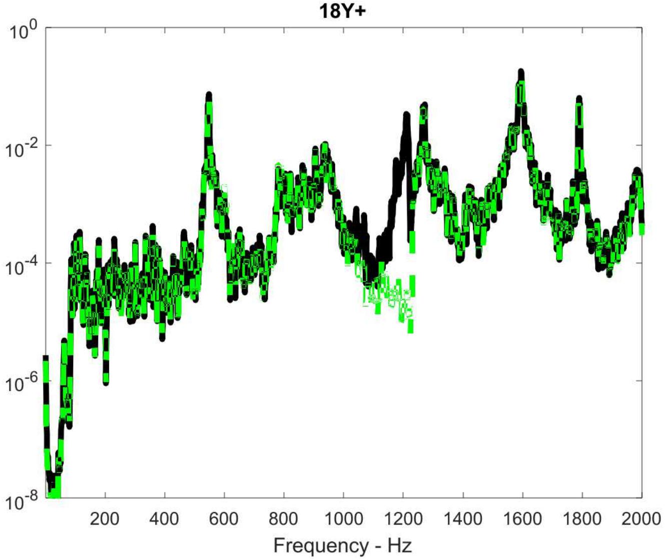
Table 1 – Bandwidth Selection/Initial Modes/Final Modes/Condition Number

Band Number	Frequency Band (Hz)	Initial Modes/Final Modes	Condition Number (Initial/Reduced Sensor/Final)
1	0-900	1-16 / 1-16	25/21/21
2	901.25-1230	17-31 / 17-31,33,34	50/45/46
3	1231.25-1624	32-50 / 26,32-50,54,57	26/27/31
4	1625.25-2000	51-70,73,78 /51-70,73,78	30/32/32

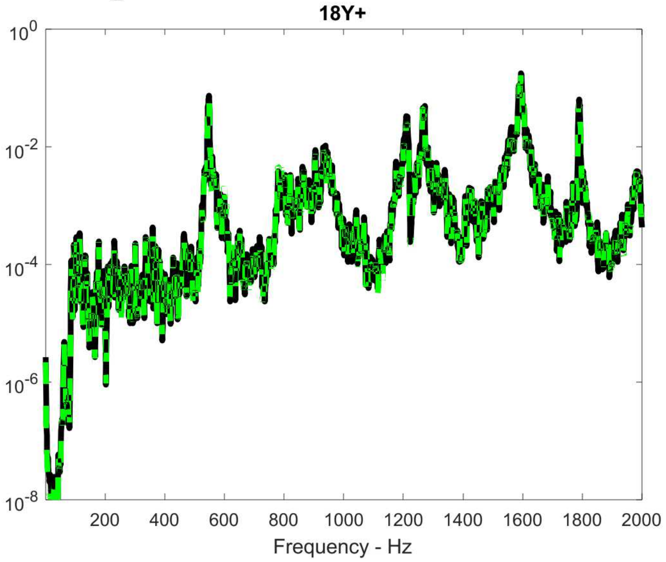
Modified SEREP approach with partial modal filter in 4 frequency bands pp. 2

- Mode augmentation in each band was accomplished after seeing how well the measurement gages fit the measured ASDs
- Modes were added one at a time to a band to attempt to reduce/remove the dropout
- The maximum number of added modes in a frequency band was 3

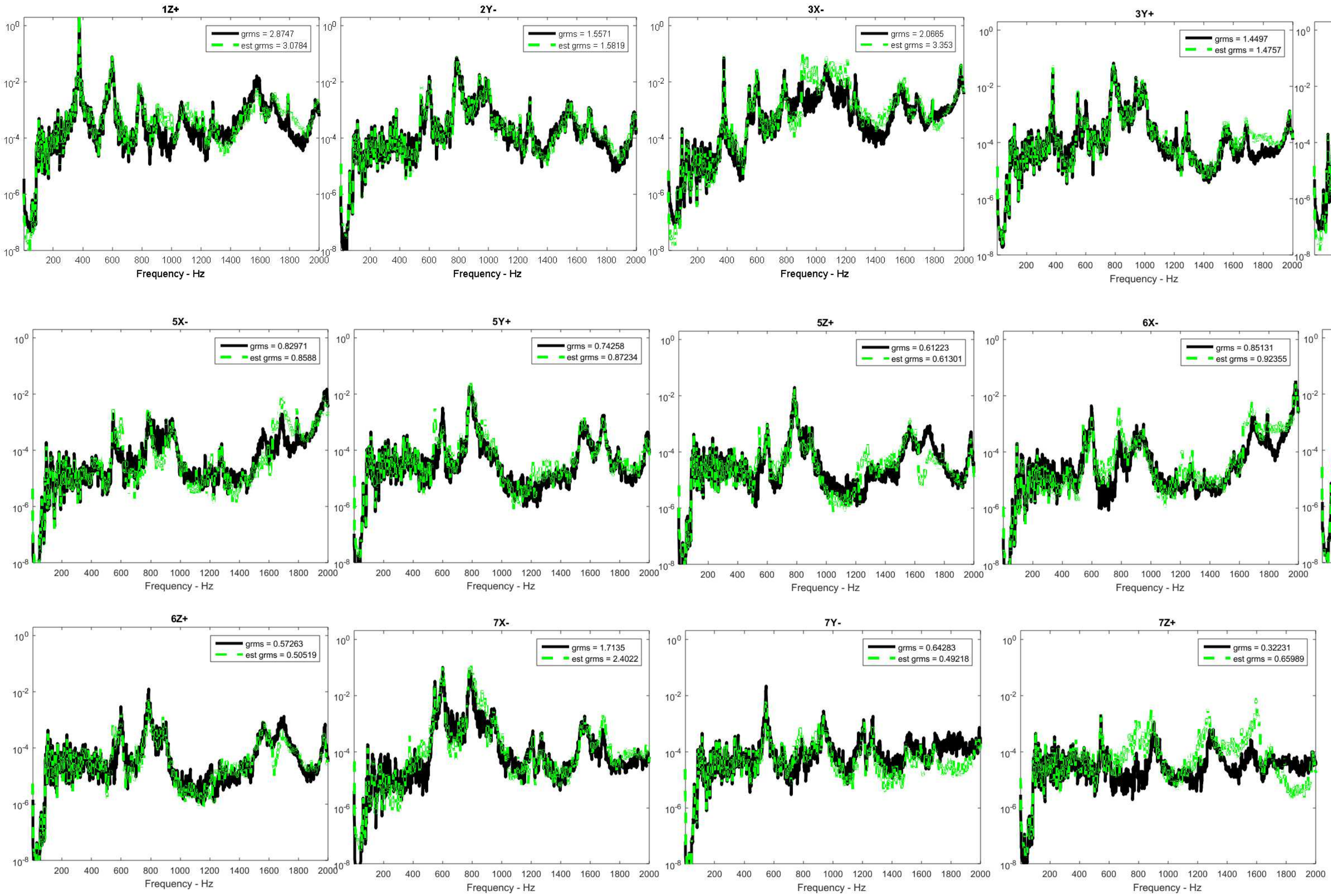
Dropout in Measurement Gage



Dropout Fixed with Added Modes



Modified SEREP approach with partial modal filter in 4 frequency bands pp. 2 - ASDs



- These results are better in high frequency than the previous full modal filter results with baseline SEREP and only 20 modes.

Control-Based Approach to Estimate

- We use the finite element model to develop FRFs from a set of 34 candidate input locations to all the measured DoF as well as the unmeasured DoF.
- Where the modal frequencies and damping were extracted in the modal test, the corresponding FE modes were set to those measured frequencies and damping.

$$\{\ddot{\mathbf{x}}(\omega)\} = \mathbf{H}_{xf}(\omega)\{f(\omega)\} \tag{6}$$

$$\mathbf{H}_{xf}(\omega) = \mathbf{\Phi}_x \begin{bmatrix} \ddots & 0 & 0 \\ 0 & \dfrac{-\omega^2}{\omega_r^2 + 2j\zeta_r\omega_r\omega - \omega^2} & 0 \\ 0 & 0 & \ddots \end{bmatrix} \mathbf{\Phi}_f^T \tag{7}$$

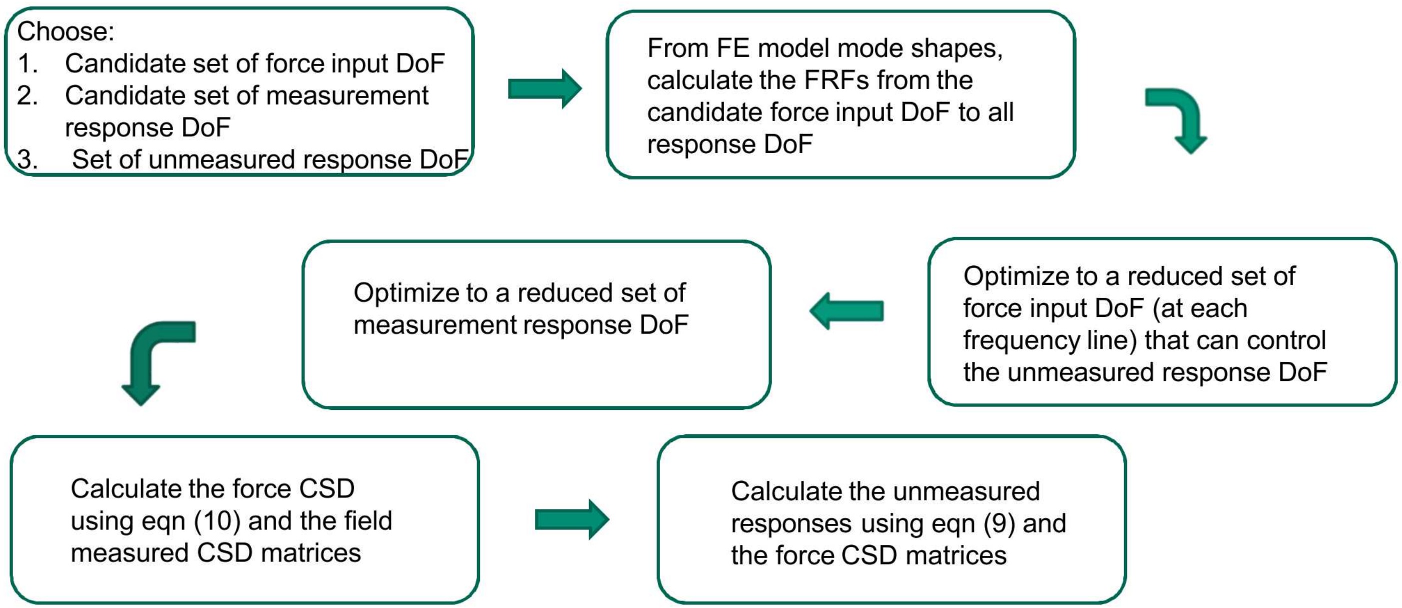
$$\{\ddot{x}(\omega)\}\{\ddot{x}(\omega)\}^{*T} = \mathbf{H}_{xf}(\omega)\{f(\omega)\}\{f(\omega)\}^{*T}\mathbf{H}_{xf}(\omega)^{*T} \tag{8}$$

$$\mathbf{S}_{xx}(\omega) = \mathbf{H}_{xf}(\omega)\mathbf{S}_{ff}(\omega)\mathbf{H}_{xf}(\omega)^{*T} \tag{9}$$

$$\mathbf{S}_{ff}(\omega) = \mathbf{H}_{xf}(\omega)^{+}\mathbf{S}_{xx_{meas}}(\omega)\mathbf{H}_{xf}(\omega)^{*T^{+}} \tag{10}$$

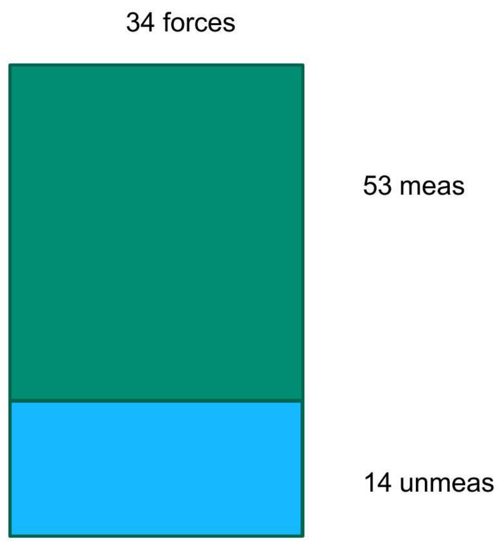
Control-Based Approach to Estimate pp2

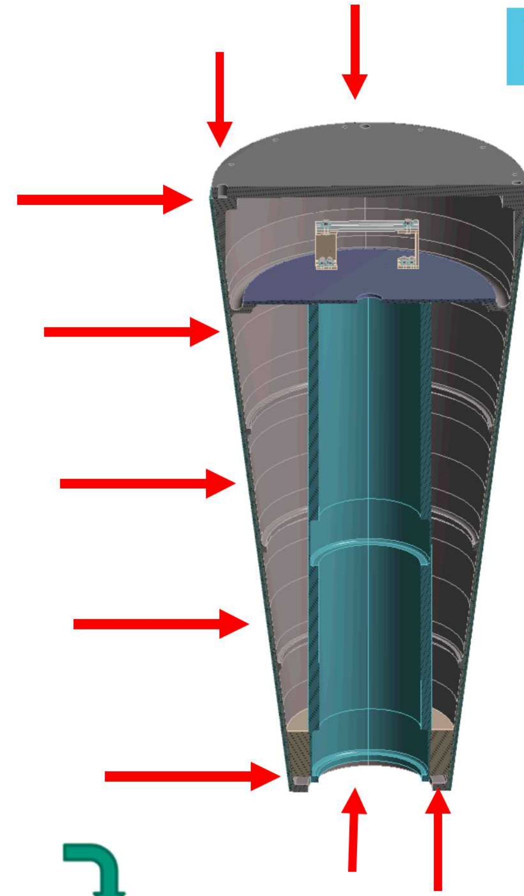
- Although it has several steps, the control-based approach is not nearly as subjective as the partial modal filter approach – the steps are much more defined.
- Here is the flow diagram outlining the steps at a high level.



Control-Based Approach to Estimate pp2

$$H(\omega_i) =$$





Choose:
1. Candidate set of force input DoF
2. Candidate set of measurement response DoF
3. Set of unmeasured response DoF

→ From FE model mode shapes, calculate the FRFs from the candidate force input DoF to all response DoF

→ Optimize to a reduced set of force input DoF (at each frequency line) that can control the unmeasured response DoF

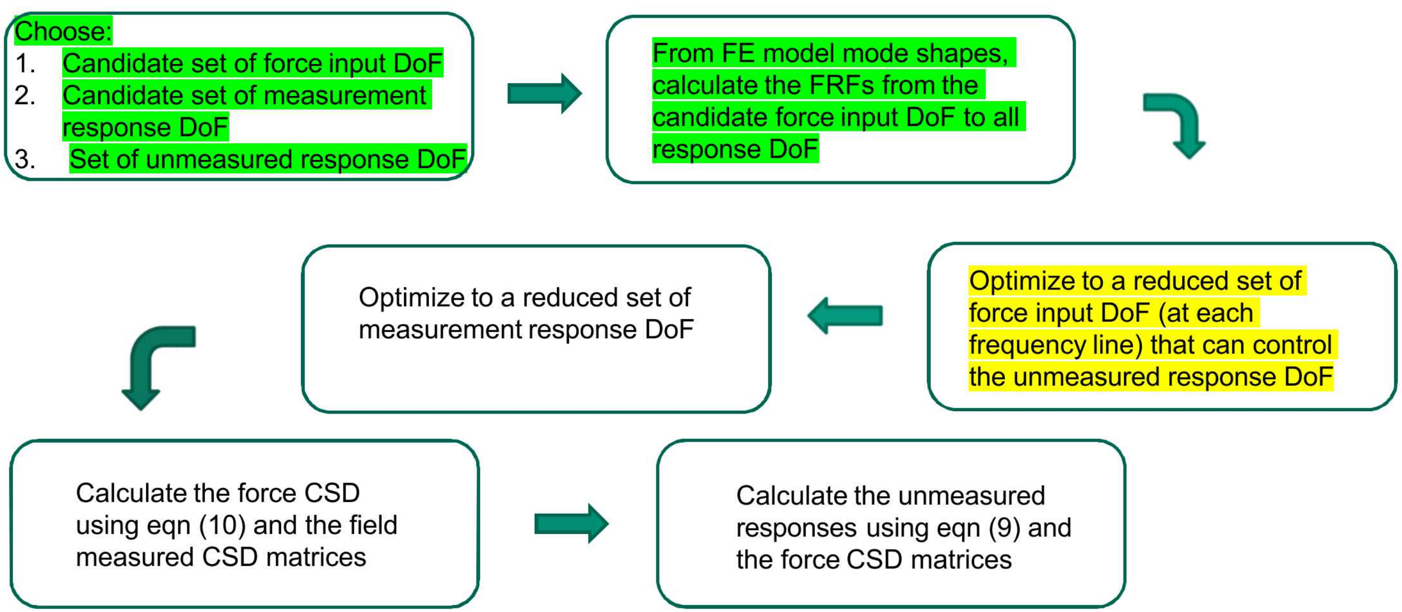
→ Optimize to a reduced set of measurement response DoF

→ Calculate the force CSD using eqn (10) and the field measured CSD matrices

→ Calculate the unmeasured responses using eqn (9) and the force CSD matrices

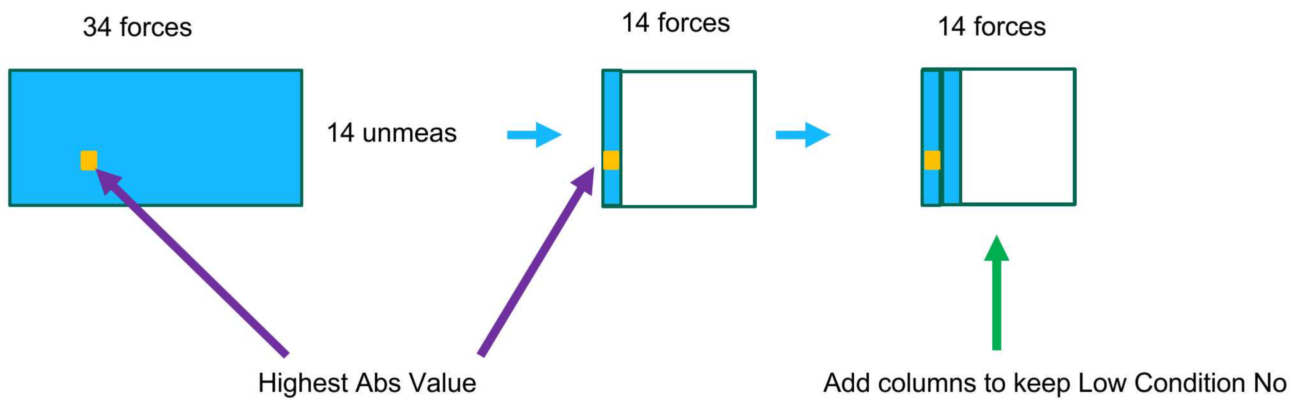
Control-Based Approach to Estimate – Reduced set of Force DoF

- Assuming the unmeasured response DoF have been chosen (here 14 DoF), choose the pseudo forcing DoF at every frequency line from the candidate force set (here 34 DoF) with the following steps:
  1. At the chosen frequency line, find the largest absolute value in the 14 x 34 FRF matrix and choose the associated forcing DoF as the first kept forcing DoF (results in the first 14 x 1 kept column of the FRF matrix at that frequency)
  2. Add 1 unkept column of the FRF matrix to the kept FRF matrix and calculate condition no. Remove the unkept column from the FRF matrix.
  3. Repeat step 2 for each unkept column of the FRF matrix.
  4. Select the force DoF which produced the lowest condition no. from step 2-3.
  5. Repeat steps 2-4 until a 14 x14 FRF matrix is obtained.
  6. Move to the next frequency line and repeat steps 1-6.



Control-Based Approach to Estimate – Reduced set of Force DoF

$$H(\omega_i) =$$



Choose:

1. Candidate set of force input DoF
2. Candidate set of measurement response DoF
3. Set of unmeasured response DoF

From FE model mode shapes, calculate the FRFs from the candidate force input DoF to all response DoF

Optimize to a reduced set of force input DoF (at each frequency line) that can control the unmeasured response DoF

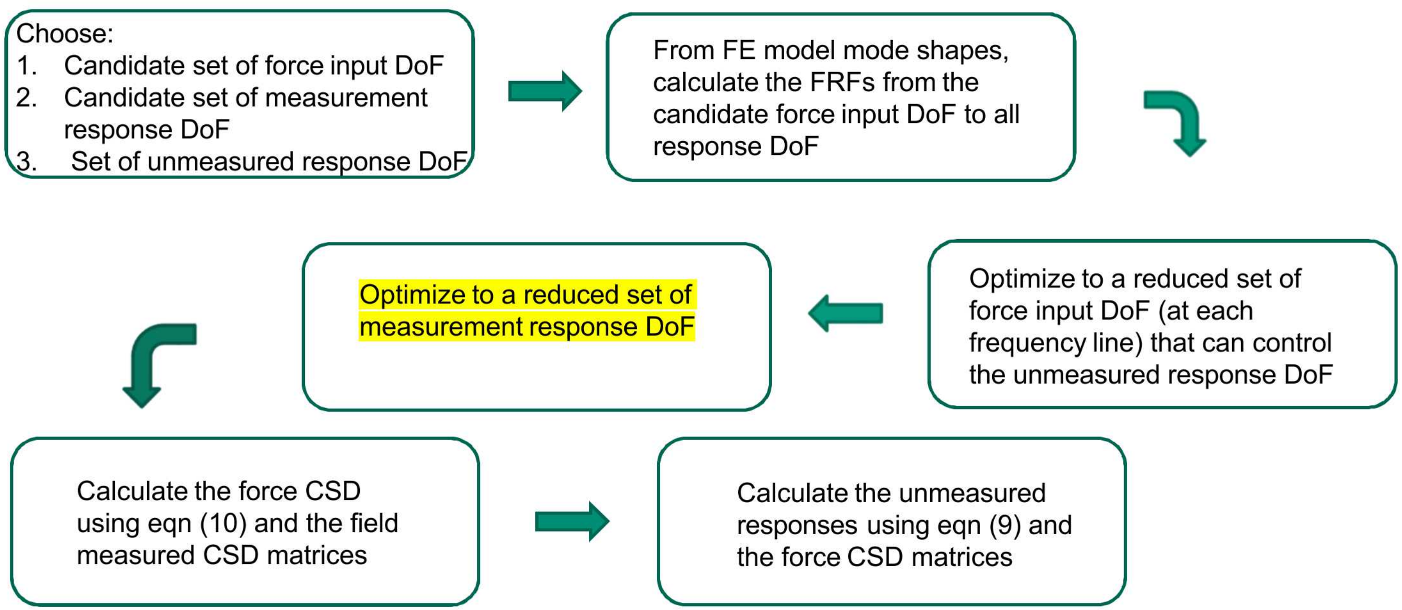
Optimize to a reduced set of measurement response DoF

Calculate the force CSD using eqn (10) and the field measured CSD matrices

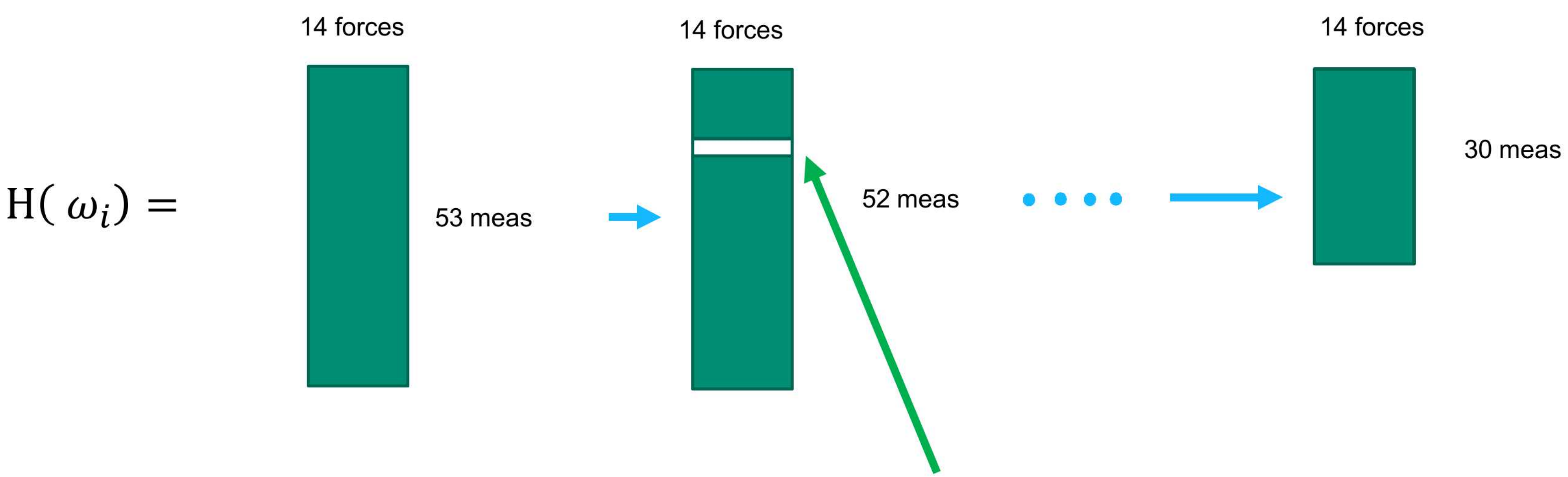
Calculate the unmeasured responses using eqn (9) and the force CSD matrices

Control-Based Approach to Estimate – Reduced set of Meas DoF

- Optimize from 53 candidate measurement DoF to 30 with the following steps:
  1. For the current candidate field response DoF set, remove one candidate DoF and calculate the condition number of the FRF matrix at every frequency line. Take the maximum condition number for all frequency lines and save it. Replace the candidate DoF that was removed.
  2. Repeat step 1 for all the rest of the candidate field response DoF.
  3. Discard the DoF from the candidate set whose removal produced the smallest maximum condition number.
  4. Repeat steps 1-4 until one has obtained the desired number of field sensors (30 for this work).
- Do not use low frequency lines since FRF matrix with more forces than rigid body modes have very high condition numbers (Here we start with lowest frequency at 240 Hz – If one looks at near zero frequencies with 14 forces condition no is 100,000+)



Control-Based Approach to Estimate – Reduced set of Meas DoF



Find sensor, when removed that produces lowest maximum Conditon No for any frequency line

Choose:
1. Candidate set of force input DoF
2. Candidate set of measurement response DoF
3. Set of unmeasured response DoF

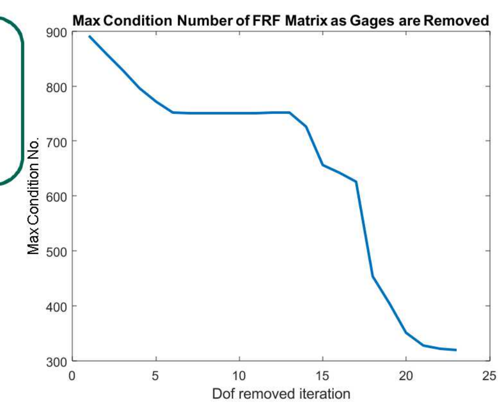
From FE model mode shapes, calculate the FRFs from the candidate force input DoF to all response DoF

Optimize to a reduced set of force input DoF (at each frequency line) that can control the unmeasured response DoF

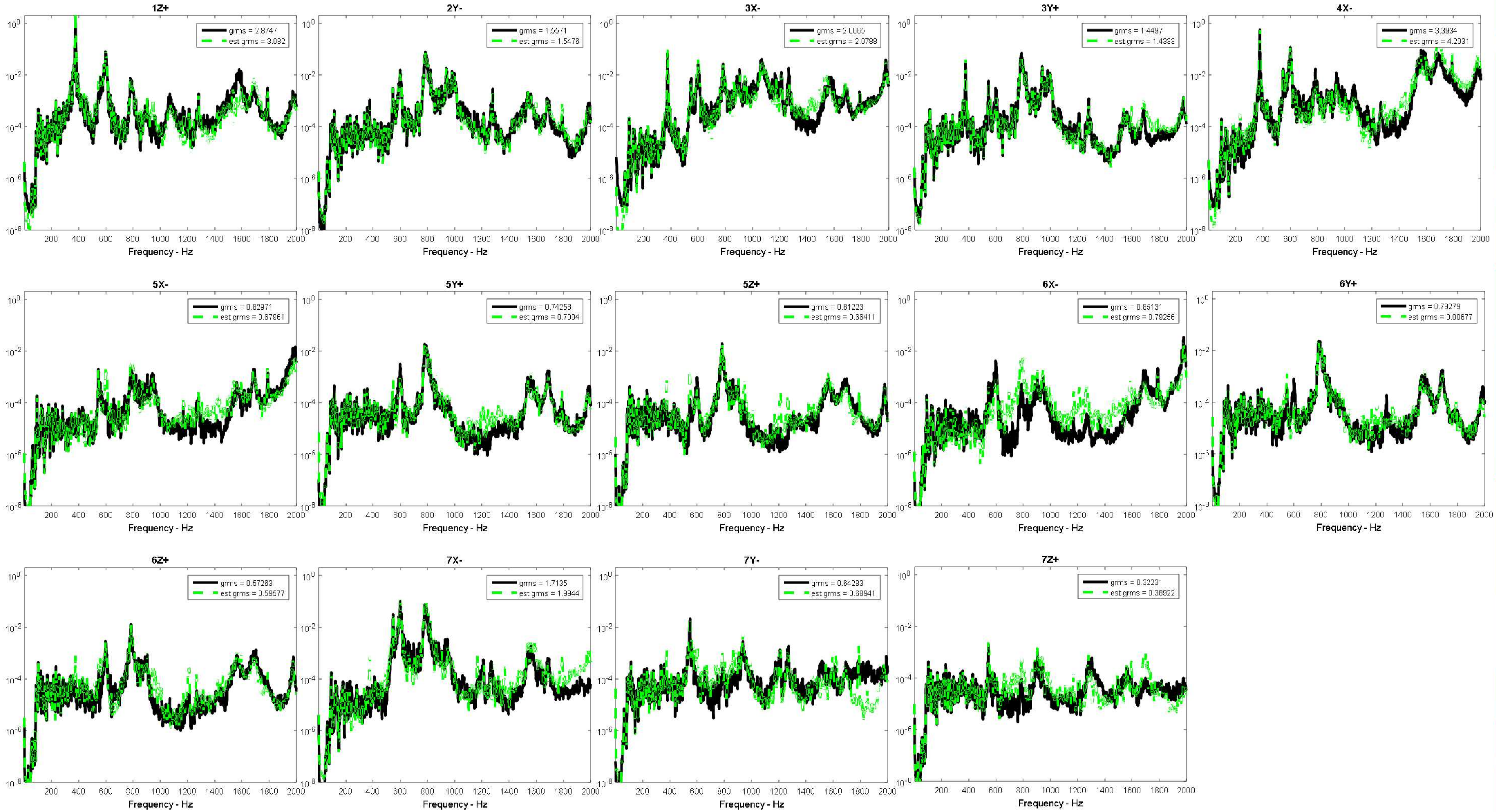
Optimize to a reduced set of measurement response DoF

Calculate the force CSD using eqn (10) and the field measured CSD matrices

Calculate the unmeasured responses using eqn (9) and the force CSD matrices



Control-Based Approach – ASD predictions, pp5



- These results are better and easier to implement than the SEREP methods.

Experiment-Based Approach to Estimate Unmeasured Responses

- A laboratory pre-test of the system with all the unmeasured and candidate measurement DoF instrumented can provide a purely experimental approach
- The theory is analogous to SEREP using singular vectors derived from a laboratory test (same as used for a MIMO modal test)

$$\begin{Bmatrix} \ddot{\boldsymbol{x}}_m \\ \ddot{\boldsymbol{x}}_u \end{Bmatrix} \approx \begin{bmatrix} \mathbf{U}_m \\ \mathbf{U}_u \end{bmatrix} \{\ddot{\boldsymbol{p}}\} \tag{11}$$

$$\{\ddot{\boldsymbol{p}}\} = \mathbf{U}_m^{+}\{\ddot{\boldsymbol{x}}_m\} \tag{12}$$

$$\{\ddot{\boldsymbol{p}}\}\{\ddot{\boldsymbol{p}}\}^T = \mathbf{U}_m^{+}\{\ddot{\boldsymbol{x}}_m\}\{\ddot{\boldsymbol{x}}_m\}^T\mathbf{U}_m^{+T} \tag{13}$$

$$\mathbf{S}_{\boldsymbol{pp}} = \mathbf{U}_{\boldsymbol{m}}^{+}\mathbf{S}_{\boldsymbol{xx\_m}}\mathbf{U}_{\boldsymbol{m}}^{+\boldsymbol{T}} \tag{14}$$

$$\mathbf{S}_{xx\_u} \approx \mathbf{U}_u\mathbf{S}_{pp}\mathbf{U}_u^T \tag{15}$$

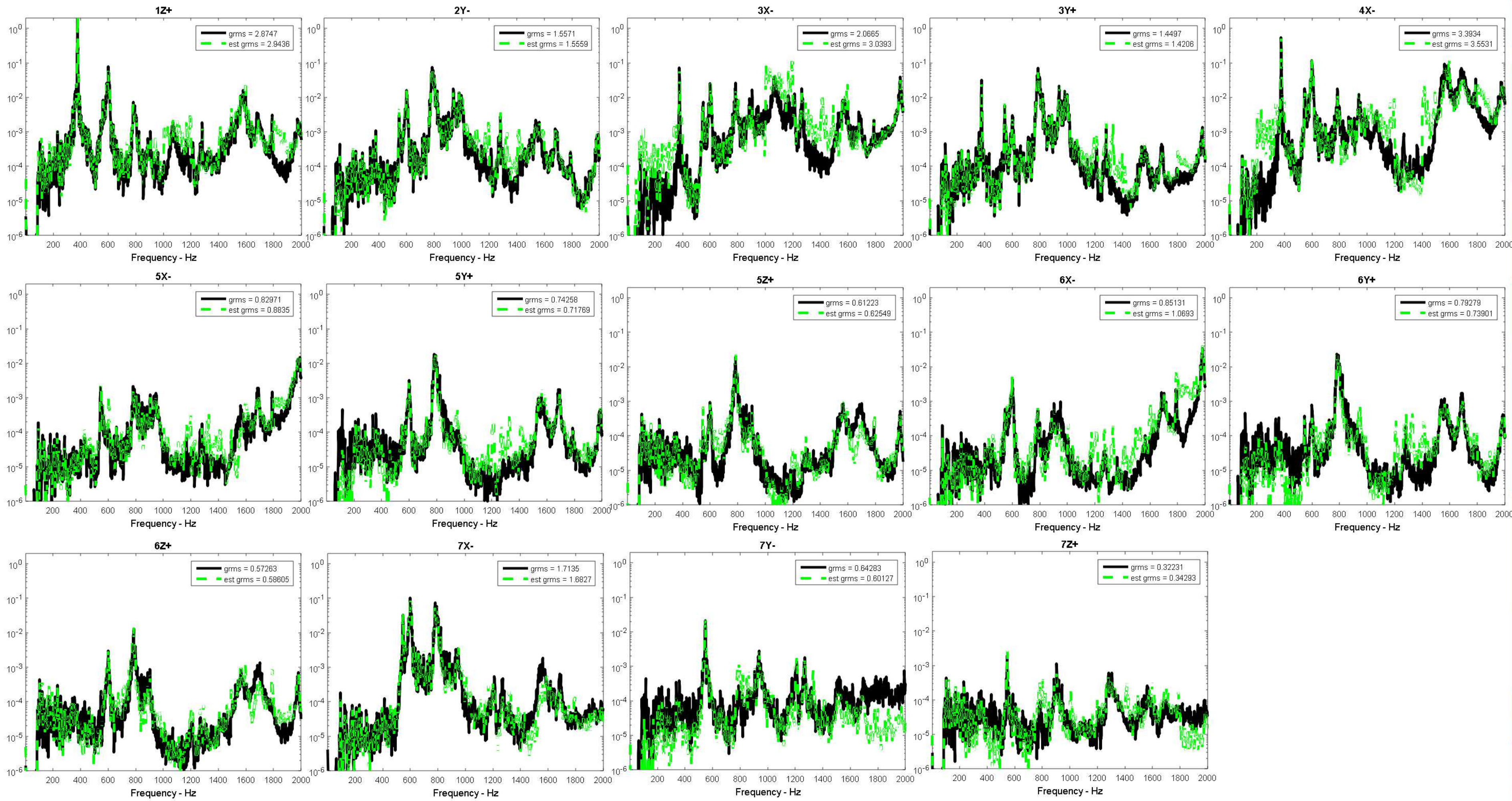
Experiment-Based Approach pp. 2

- The FRF data were taken from the 3-shaker modal test used for model correlation.
- This approach was an "afterthought".
- For the future, more input locations would be utilized to help ensure every mode is well excited.
- Best results using real and imaginary part of FRFs. Using real part only was almost as good. Using imaginary part was worst.
- Divided into ten 200 Hz wide bandwidths with first 15 U vectors obtained from equation 17.
- Optimized down to 30 gages on sum of the condition no of 10 $\mathbf{U}_m$ matrices similar to previous work.

$$\begin{bmatrix}\mathbf{HMAT}_m \\ \mathbf{HMAT}_u\end{bmatrix} = \begin{bmatrix} real(\mathbf{H}_{xm\_f1}) & real(\mathbf{H}_{xm\_f2}) & real(\mathbf{H}_{xm\_f3}) & imag(\mathbf{H}_{xm\_f1}) & imag(\mathbf{H}_{xm\_f2}) & imag(\mathbf{H}_{xm\_f3}) \\ real(\mathbf{H}_{xu\_f1}) & real(\mathbf{H}_{xu\_f2}) & real(\mathbf{H}_{xu\_f3}) & imag(\mathbf{H}_{xu\_f1}) & imag(\mathbf{H}_{xu\_f2}) & imag(\mathbf{H}_{xu\_f3}) \end{bmatrix} \quad (16)$$

$$\left[\begin{matrix}\mathbf{U}_m \\ \mathbf{U}_u\end{matrix}, \mathbf{S}, \mathbf{V}\right] = svd\left(\begin{bmatrix}\mathbf{HMAT}_m \\ \mathbf{HMAT}_u\end{bmatrix}\right) \quad (17)$$

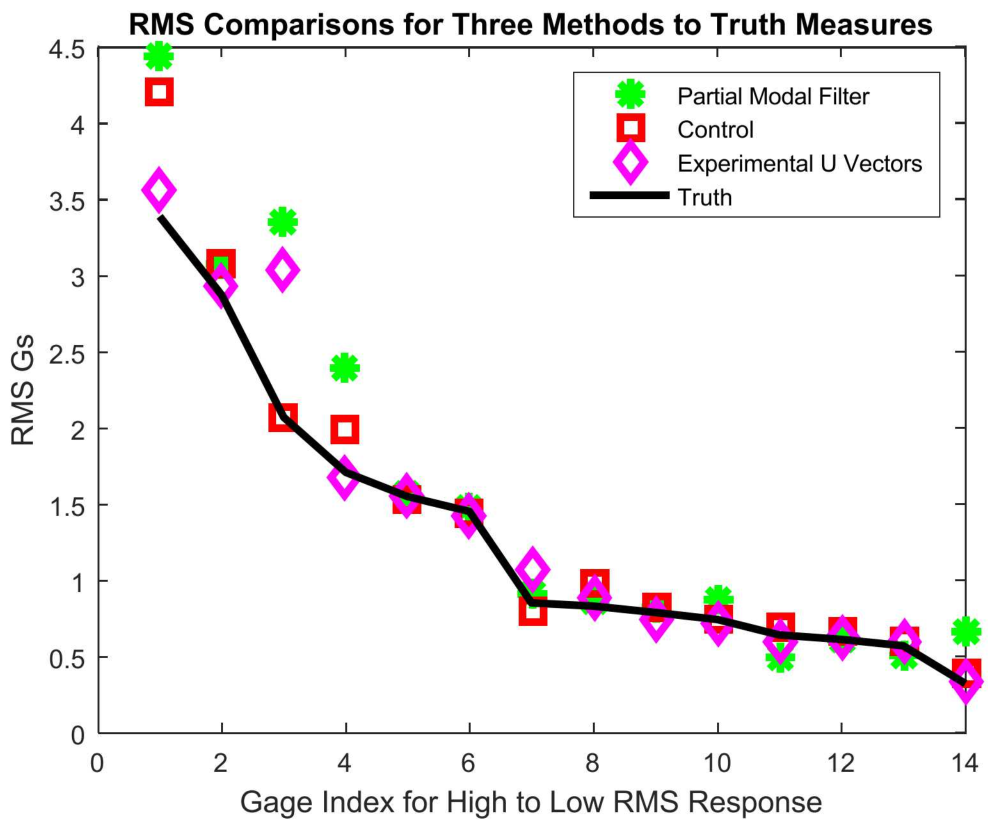
Experiment-Based Approach ASDs, pp. 3



- These results are slightly worse, line by line, than the control-based method.

Compare approaches

- RMS acceleration plot for 3 methods

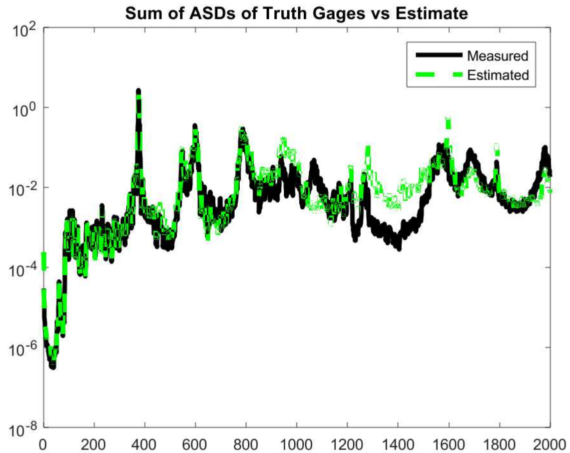


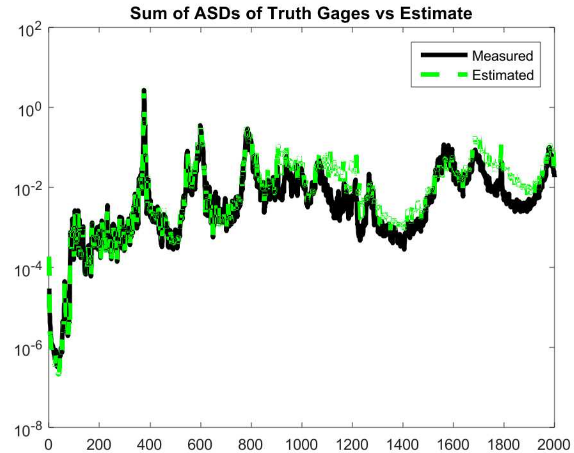
Compare approaches

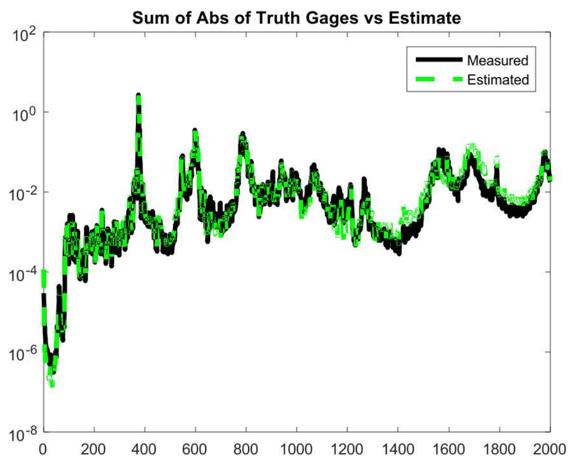
- Sum of ASD plots for all 4 estimates

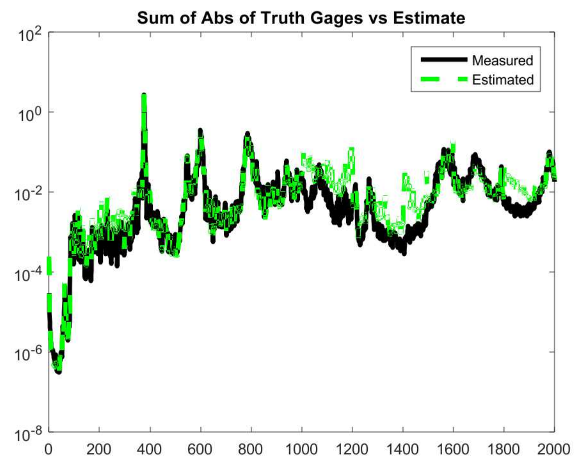
Full Modal Filter

Partial Modal Filter









Control-Based

Experiment Based

Compare approaches

- Full modal filter would have required many more gages (70?) – very good up to 900 Hz
- Partial modal filter provided a great improvement in higher frequencies, but requires a lot of user interaction and expert judgment in this implementation.
- Control-based algorithm was best overall. Pseudo-forces can make some adjustment for errors in FRFs to improve unmeasured predictions. More straightforward implementation than Partial modal filter approach.
- All of the above are performed with a FE model and can make predictions at any DoF in the FE model.
- The experiment-based approach was 2nd best to the control-based approach. However, providing more inputs in the MIMO FRF matrix might cause it to rival the control-based approach. (Recall this approach was an afterthought and utilized the 3 shaker modal test).
- The experiment-based approach can only make predictions at DoF instrumented in the laboratory pre-test. However, if there is no FE model, this may provide predictions if hardware is available for the non-destructive laboratory pre-test.

Conclusions

- We began with the SEREP approach using FE mode shapes which works well when you have more measurements than active modes (70+) in the bandwidth. In this case, restricting ourselves to 30 measurements, we could get good estimates of the unmeasured responses through about half of the frequency bandwidth. To obtain reproductions of the time domain responses, this approach, and the larger number of appropriately placed sensors would be required.
- By converting to the frequency domain, we were able to reduce the measurement set down to 30 sensors, since not all shapes are very active in specific frequency regions.
- The partial modal filter and control-based methods utilize the correlated FE model to appropriately interpolate the measured response to other unmeasured FE DoF. The control-based approach was best and had much less subjective (expert opinion) decisions to be made in the selection of frequency bandwidths and mode shapes.
- The experiment-based approach does not use a FE model but develops from a laboratory pre-test. It can make predictions at any of the unmeasured DoF that are included in the laboratory pre-test. It was second best in this work, but might rival the control based FE method with more shaker inputs in the MIMO laboratory pre-test.
- Expansion to unmeasured responses is now within the realm of possibility since the number of channels and bandwidth available in telemetry for field tests is increasing. The possibility of defining relatively accurate component motion is a driver to greatly improve the specification and qualification process as well as modeling and testing technology.