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Optimizing Shaker Locations for Multi-Shaker Environmental Testing

PRESENTED BY

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Motivation for a better ground test method

- Daborn, et al. at AWE showed that a single shaker or state-of-the-art twin shaker test could perfectly control one (or two) input accelerometer responses but totally failed to match the field environment at any other responses on the system.
- The failure to match any other responses with traditional shaker testing has led to many other control schemes using force or response limiting at other degrees of freedom to try to keep the laboratory test simulation from being too ridiculous.
- Single shaker tests require huge shakers/amplifiers/cooling/physical facilities/power/costs and time to shift axes.
- Daborn, et al. at AWE showed that vibrations induced on a scale model missile in a wind tunnel could be reproduced with an approximate boundary condition and three shakers and 13 control accelerometers with multi-input multi-output (MIMO) control.
- The technique was named Impedance-Matched Multi-Axis Testing (IMMAT)
- Similar results were first achieved at SNL simulating an acoustic test on a mock bomb with an open loop control scheme with 6 shakers and 37 control accels.
- Other demonstrations have provided useful results using the IMMAT approach.

What are some of the outstanding Multi-Shaker Testing Questions

- HOWEVER, we have not had technology to guide:
  - How many shakers are required for a particular test?
  - Where should the shakers be placed?
  - Can the shakers or amplifiers physically achieve the environment, i.e. will the test exceed the physical capabilities such as max output current / voltage / force / displacement?
- This work was focused on attempting to answer these questions.

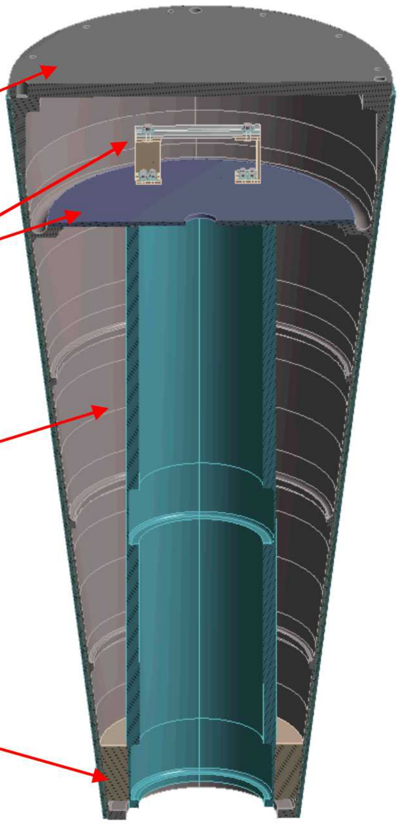
Approach to develop IMMAT test design

- A model resulting in acceleration to force frequency response functions can be derived from a finite element model of a test article (a modal model is very efficient). For a given number of shakers, the amount of mechanical power required to drive the test article to the target responses is a constant.
- The missing piece is the shaker and amplifier dynamics and physical limitations, which can require massively different current, voltage or power requirements to meet target accelerations depending on where it is attached to the test article.
- We characterized one shaker/amplifier pair to understand the physical limits and develop a four dof electro-mechanical model.
- Once this model is available, it can be attached through substructuring principles to the test article modal model at any desired dof.
- Then a standard control equation can back calculate the output voltage required for each amplifier to achieve the control accelerations.
- Other quantities, such as shaker force, output current, shaker displacement, control error can be checked to see if they are acceptable.
- This model is amenable to an optimization approach, and one simple idea was used for this work.

Proof of concept Hardware for IMMAT test design - MATV

- The project proposed to prove the IMMAT test design using research hardware provided by AWE known as the Modal Analysis Test Vehicle (MATV) which would be tested in a field random acoustic environment. Then a designed IMMAT test would be run with multiple shakers to attempt to simulate the field accelerations.

- MATV Description
  - A meter long
  - 47 kg
  - Composite wrapped on aluminum substrate cone
  - Large end aluminum cover plate
  - Aluminum internal flat component plate
  - Bracket called the Removable Component (RC) bolted to the internal component plate
  - Steel pipe bolted to the component plate
  - Foam support between pipe and cone at small end

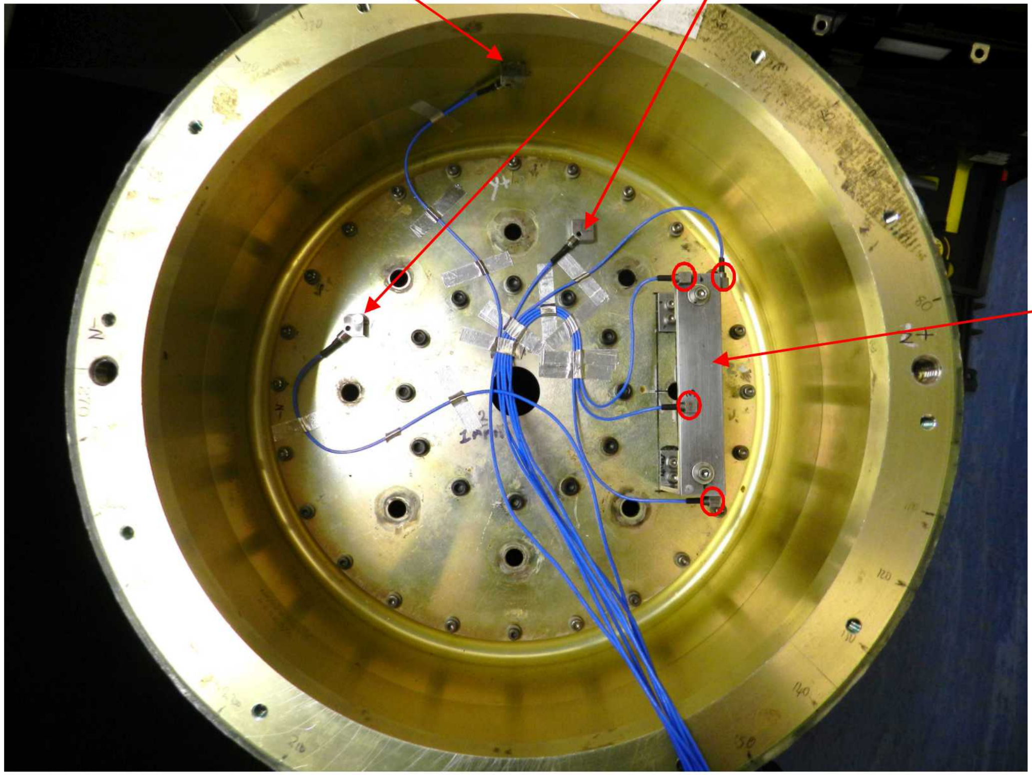


MATV Control Accel Locations

- 14 control accelerometer locations were chosen either on the RC or triaxial locations at typical mounting locations for a component

1 Triax on Cone

2 Triax on Component Plate

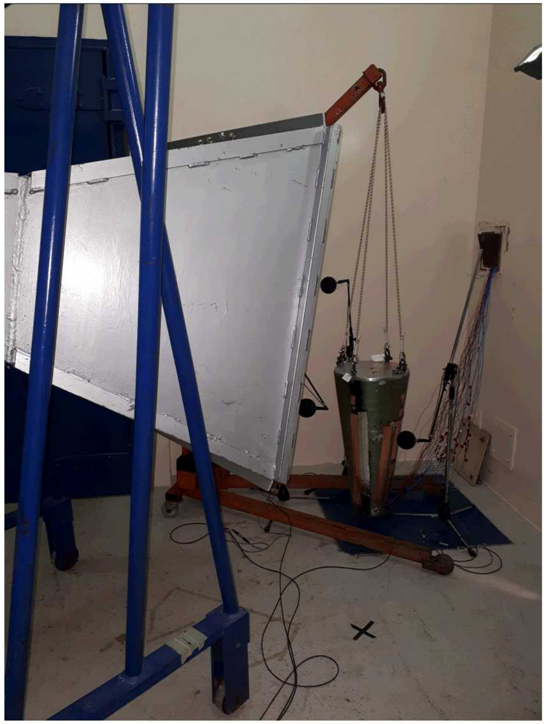


RC – 5 dof chosen on 4 Triaxes

Field Acoustic Test for MATV

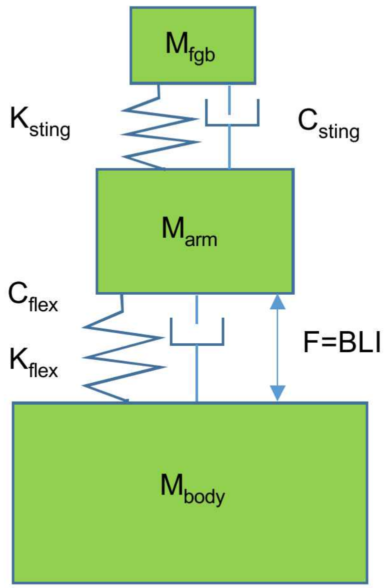
- A field acoustic test was run to 147 dB at the Institute of Sound and Vibration Research at Southampton University in a reverberant chamber with horn.

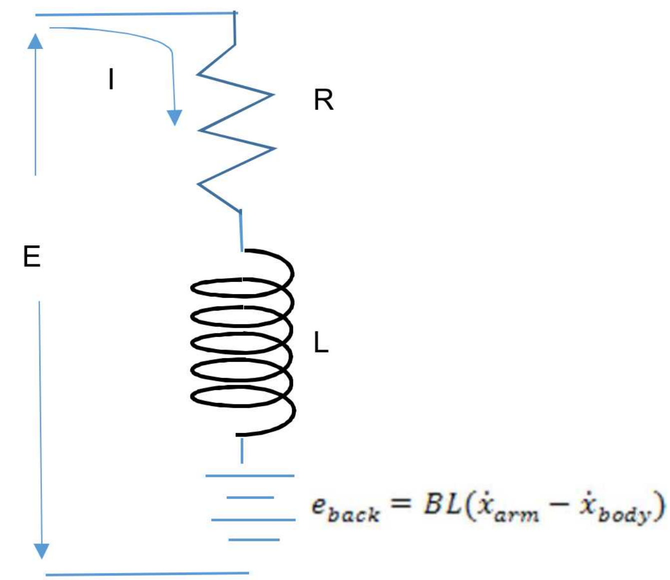
- Place in corner of chamber
- Horn
- MATV suspended by bungees
- 69 total accelerometer channels recorded



Shaker / Amplifier 4 dof Model

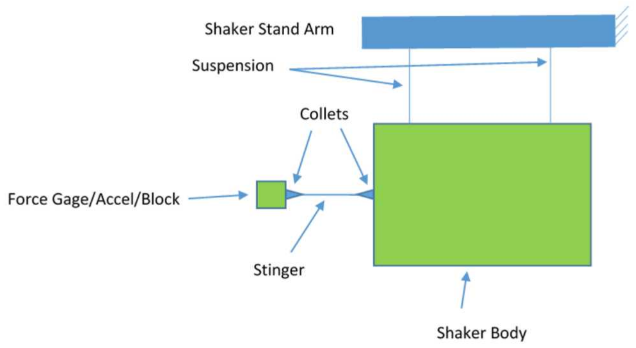
- As Tony Moulder and I began investigations to characterize a BEAK 1000 amplifier coupled with a LS-70 shaker, Phil Ind found a paper by Fox and Lang in the October 2001 Sound and Vibration magazine that modeled standard large laboratory shakers. With some small modifications, the 4 dof model is shown below.

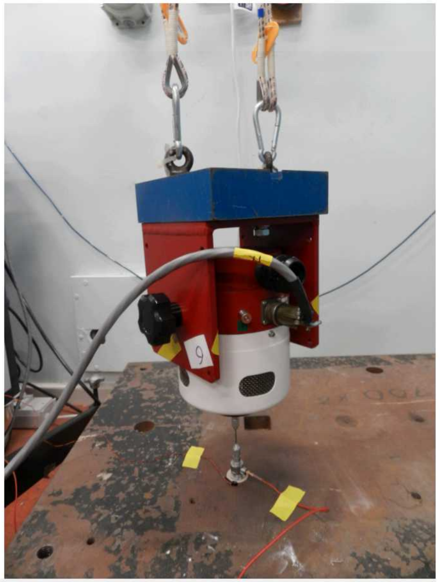


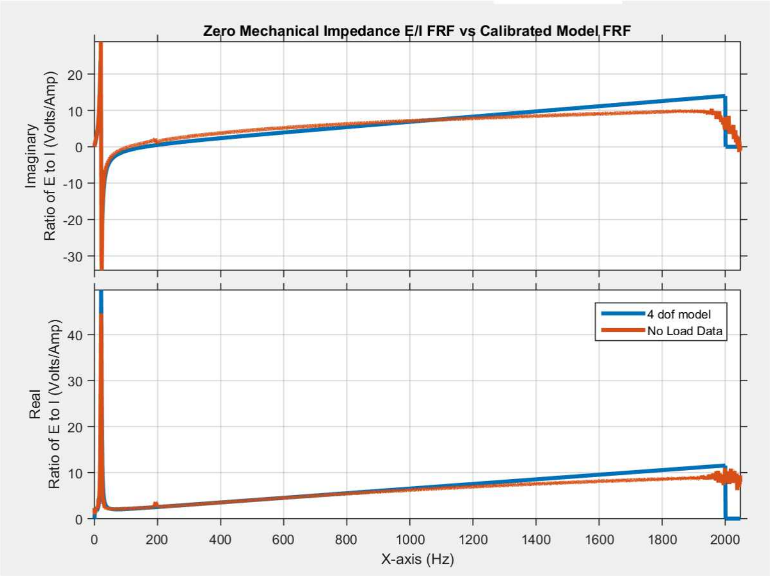


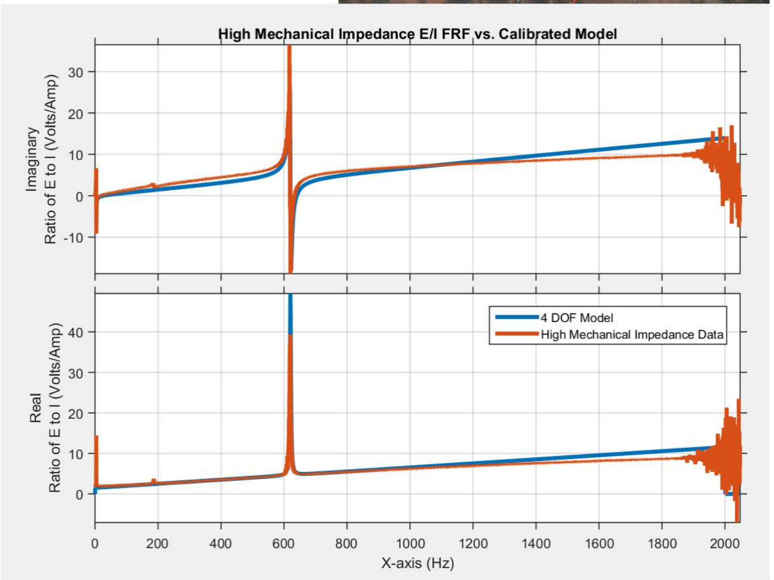
Shaker / Amplifier 4 dof Model

- The model was calibrated against two tests – zero and infinite mechanical impedance. M's were measured, R was published.
- Kflex and Ksting calibrated to achieve frequency match.
- B, L and Cflex calibrated to achieve amplitude match









Shaker / Amplifier 4 dof Model Equations

$$[K + j\omega C - \omega^2 M]\begin{Bmatrix} x_{fgb} \\ x_{arm} \\ x_{body} \\ I \end{Bmatrix} = \begin{Bmatrix} F_{fgb} \\ F_{arm} \\ F_{body} \\ E \end{Bmatrix}$$

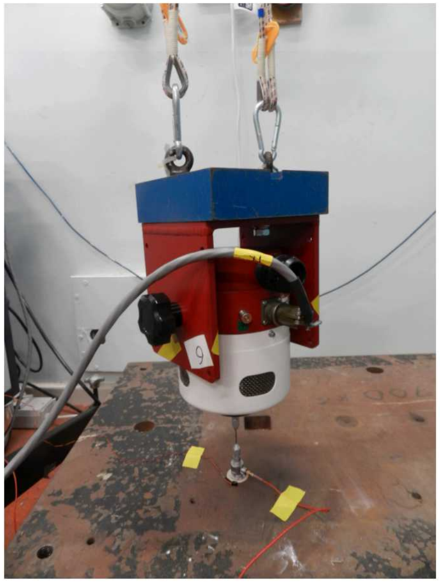
$$K = \begin{bmatrix} Ksting & -Ksting & 0 & 0 \\ -Ksting & Ksting + Kflex & -Kflex & -BL \\ 0 & -Kflex & Kflex & BL \\ 0 & 0 & 0 & R \end{bmatrix}$$

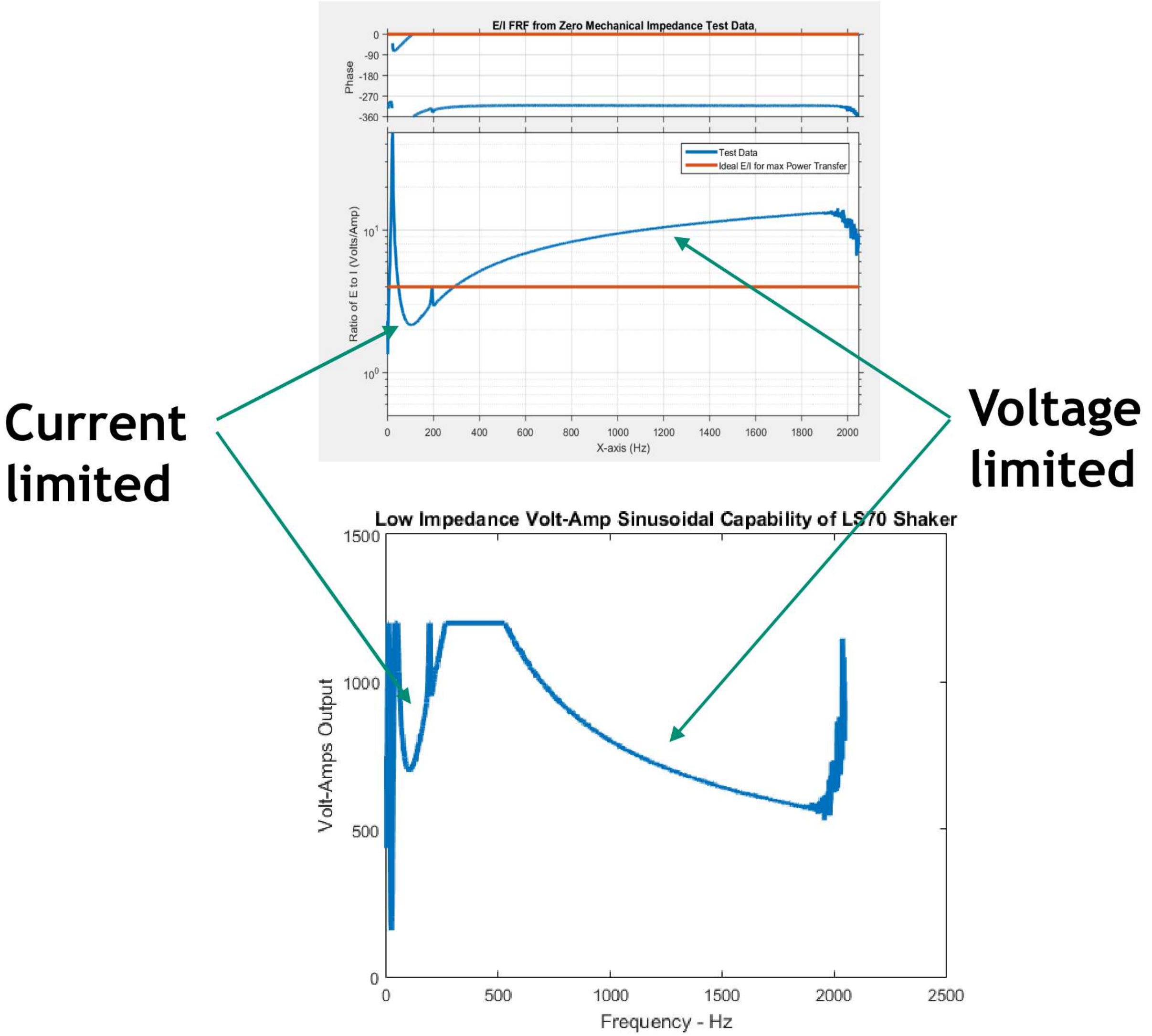
$$C = \begin{bmatrix} Csting & -Csting & 0 & 0 \\ -Csting & Csting + Cflex & -Cflex & 0 \\ 0 & -Cflex & Cflex & 0 \\ 0 & BL & -BL & L \end{bmatrix}$$

$$M = \begin{bmatrix} Mfgb & 0 & 0 & 0 \\ 0 & Marm & 0 & 0 \\ 0 & 0 & Mbody & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Shaker / Amplifier Limits measured for 1296 VA capable amp

- Sine waves were input at single frequencies for both the high and low impedance arrangements.
- Published current limit of 18 amps RMS appeared accurate
- Published voltage limit of 72 V RMS appeared conservative, with amplifier capable of around 85 V RMS before distortion appeared





Substructuring Theory

The uncoupled equations of motion in the frequency domain using mass and stiffness matrices are shown below for MATV and one shaker/amplifier (Additional shaker/amplifiers would just be added as an additional block on the diagonal).

$$\left[\begin{bmatrix} \omega_{MATV}^2 & 0 \\ 0 & K_{S/A} \end{bmatrix} - \omega^2 \begin{bmatrix} I_{MATV} & 0 \\ 0 & M_{S/A} \end{bmatrix}\right] \begin{Bmatrix} q \\ x \end{Bmatrix} = \begin{Bmatrix} F \\ V \end{Bmatrix} \qquad (1)$$

The upper partition is diagonal and based on the free modal model of MATV hardware with generalized modal dof q and modal forces F. This partition was derived from the natural frequencies and mass normalized mode shapes up to the frequency of interest. The lower partition matrices are not diagonal and are based on the 4 dof shaker/amplifier that will be described later. A constraint is written as

$$x_{attachment}^{MATV} - x_{attachment}^{ForceGage} = 0 \qquad (2)$$

The constraint provides continuity at the connection dof between MATV and the shaker force gage. It can be written as a Boolean matrix with 1,-1,0 values at appropriate physical attachment dof

$$B\{x_{physical}\} = \{0\} \qquad (2a)$$

Substructuring Theory Continued

Since x is equal to Φq for the MATV using the modal substitution, then

$$\Phi_{attachment}^{MATV} q - x_{attachment}^{ForceGage} = 0 \tag{3}$$

The constraint with the modal substitution becomes

$$B \begin{Bmatrix} \Phi q \\ x \end{Bmatrix} = B \begin{bmatrix} \Phi & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} q \\ x \end{Bmatrix} = \{0\} \tag{4}$$

Combining the first two terms as $\tilde{B}$ yields

$$\tilde{B} \begin{Bmatrix} q \\ x \end{Bmatrix} = \{0\} \tag{5}$$

Using deKlerk's primal form to reduce the redundant dof requires a substitution

$$\begin{Bmatrix} q \\ x \end{Bmatrix} = \tilde{L}\{\eta\} \tag{6}$$

Substituting eqn (6) into (5) requires $\tilde{L}$ to be in the null space of $\tilde{B}$

$$\tilde{B}\tilde{L}\{\eta\} = \{0\} \tag{7}$$

$$\tilde{L} = null(\tilde{B}) \tag{8}$$

Substructuring Theory Completed

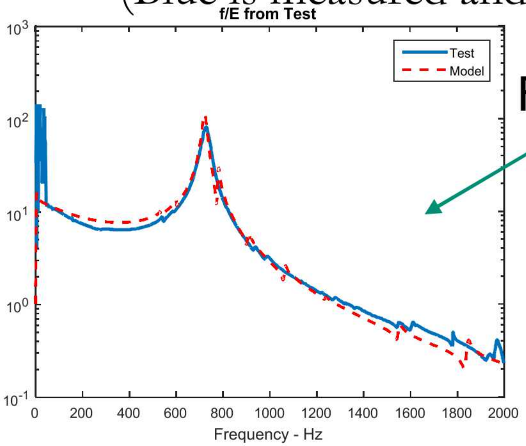
Substitute eqn (6) into (1) and premultiply by $\tilde{L}^T$ to get

$$\tilde{L}^T\left[\begin{bmatrix}\omega_{MATV}^2 & 0\\ 0 & K_{S/A}\end{bmatrix}-\omega^2\begin{bmatrix}I_{MATV} & 0\\ 0 & M_{S/A}\end{bmatrix}\right]\tilde{L}\{\eta\}=\tilde{L}^T\begin{Bmatrix}F\\ V\end{Bmatrix} \qquad (9)$$

This provides the new dynamic matrix (K,C,M) on the left side which can be inverted to extract the FRF matrix (H(ω)) to the output voltage of the amplifier. (Damping matrix is left out of above for convenience, but diagonal modal damping is included in the normal way).

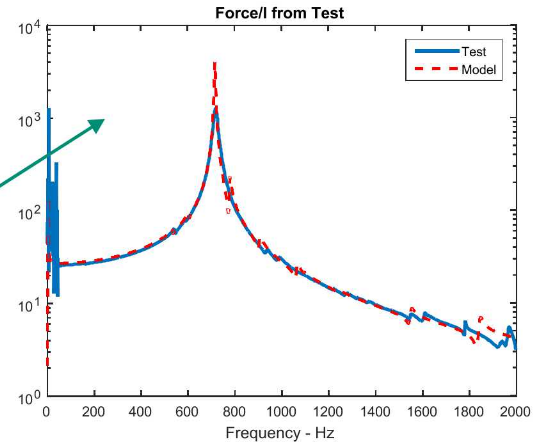
Single Shaker / MATV Validation of Substructure Model

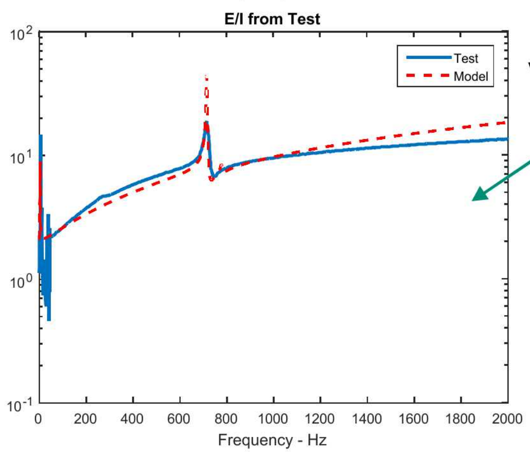
- To validate the combined shaker/amp + MATV model, AWE hooked up one shaker as shown in the picture.
- After raising the published shaker resistance, R, from 1.5 to 2 ohms and stiffening the stinger spring slightly to achieve a closer frequency match, some of the FRF quantities from the model and test are shown below. (Blue is measured and dashed red is model)



Force/Voltage

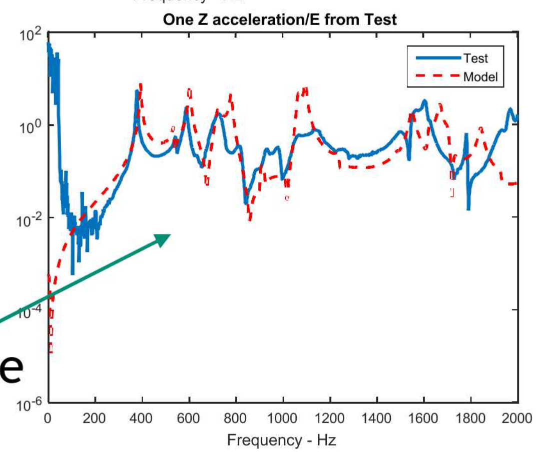
Force/Current





Voltage/Current

Response on RC Acceleration/Voltage



Shaker Control Equations

At an particular frequency line, ω , the response acceleration of the vector of target responses is (where H is the acceleration to voltage FRF and V is the vector of output voltages from each amplifier)

$$\{\ddot{x}\}=H\{V\} \quad (1)$$

Post multiplying by the transpose conjugate of the vector gives acceleration cross spectra due to amplifier voltage cross spectra

$$S_{xx}= H\{V\}\{V\}^T H^T = H S_{VV} H^T \quad (2)$$

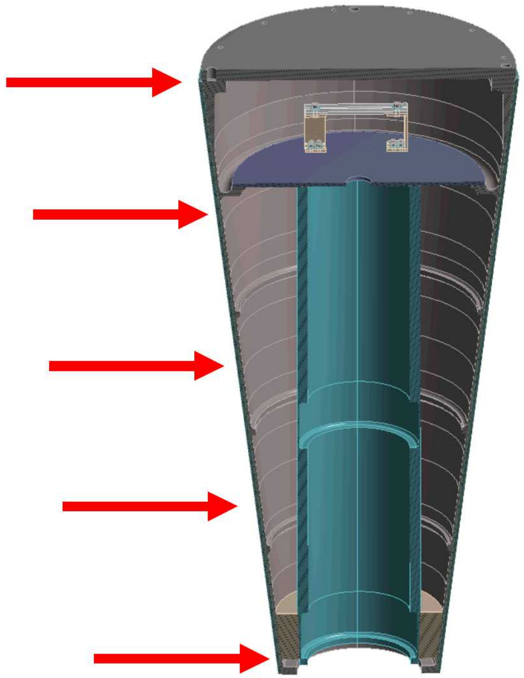
The required amplifier voltage cross spectra is then

$$H^+ S_{xx} H^{T+} = S_{VV} \quad (3)$$

For any particular shaker attachment, a model is substructured together to produce H and the S_{xx} was derived from the 14 target accelerations in the acoustic test. Once S_{VV} was calculated it could be substituted into eqn (2) using the appropriate H to calculate PSD's of amplifier current or shaker forces, etc.

Shaker Candidate Locations

- 34 candidate shaker locations were chosen to optimize to achieve an achievable IMMAT test to match the target cross spectra.
- Input normal to the cone, 5 axial stations, 0,15,30,45,75,90 degrees at each station (The 0-90 degree constraint was chosen so one could place all shakers on the floor if the axial direction of the MATV was made horizontal).
- 2 axial inputs at each end.
- All were logistically feasible individually, but two adjacent candidate points could cause interference between shakers.

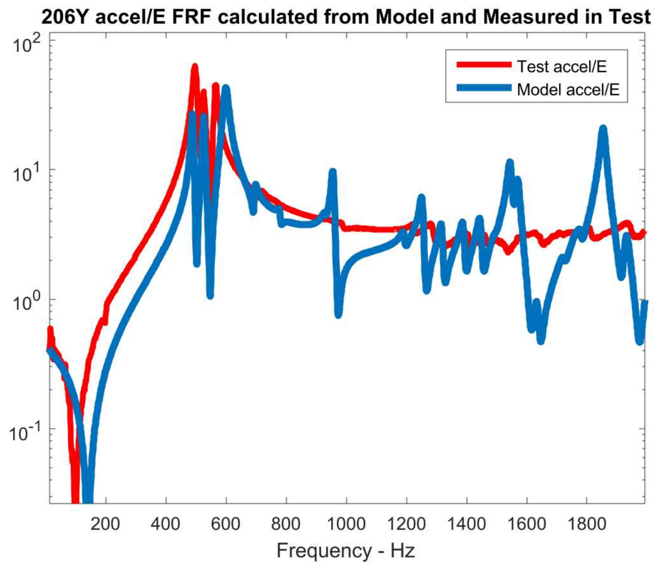


Shaker Optimization to minimize sum of Amplifier Output Voltages

- Optimize the best shaker to add to whatever is the current set
- Can minimize whatever quantitiy you like (force, voltage, current, control error, etc.)
- Physical Limits are 85V, 18A, 315N
- Here I choose to minimize the sum of Amplifier Output Voltages
- First shaker
  - Worst shaker 301Y- with 265 Volts
  - Best shaker 601X+ with 37 Volts, I=4 Amps, force=102N, dBerr = 18,
- Second shaker
  - Best shaker 506Y- with [30 33 Volts], [3 4 Amps], [79 99 N], dBerr = 8.7
- Third shaker
  - Best shaker 503Y- with [32 27 31Volts], [4 3 4 Amps], [83 74 108N], dBerr = 6.7
- Fourth shaker
  - Worst shaker 505Y- with 1835 Volts
  - Best shaker 204Y- with [33 24 29 50 V], [3 3 3 5Amps], [66 67 69 81N], dBerr = 4.7
- Fifth shaker
  - Best shaker 106Y- with [26 21 28 55 72V], [3 2 2 5 5Amps], [60 40 56 85 68N], dBerr = 3.9
- Sixth shaker (Blows Up!)
  - Best shaker 302Y- with [1073 6456 12300 27200 10900 30900V], [513 3087 5869 12980 5205 14761 A], [870 5900 13300 35300 13800 40050], dBerr = 3.1

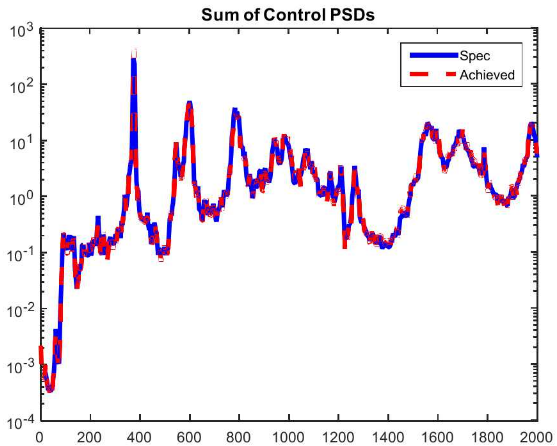
Final Optimization Shaker Locations

- Logistics were better to put axial shaker on floor instead of hanging as 601X+ required. Also two of the 500 series were very close to interfering at only 30 degrees apart.
- Final shaker were 501Y- 603X- 506Y- 206 Y-
- [26 34 25 74 V] [4 3 2 6 Amps] [102 102 52 102 N] 5.5 dBerr
- Actual voltage to run test was [31 22 21 29]
- The BIG MISS was on 206Y- : On further examination, high frequency modes above 2000Hz had tails that added a lot of response in the actual system. We did not include the high frequency modes but stopped close to 2000 Hz. MORAL OF THE STORY – INCLUDE HIGHER FREQUENCY MODES (maybe twice the bandwidth)

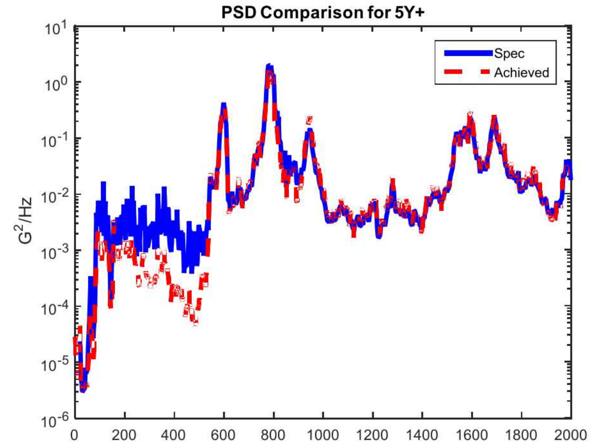


IMMAT Test Results

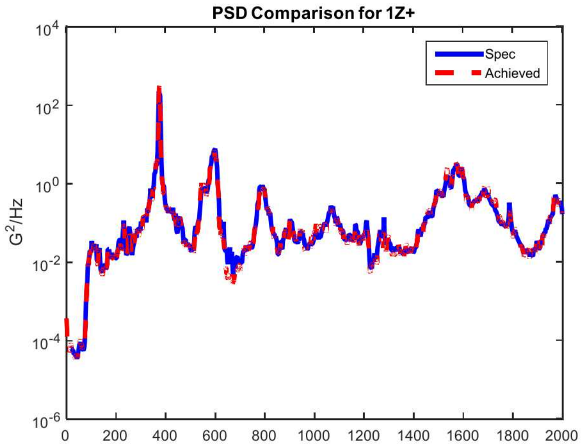
- The LMS control system easily matched the 14 control locations at -6, 0 and +3dB



Sum of PSDs

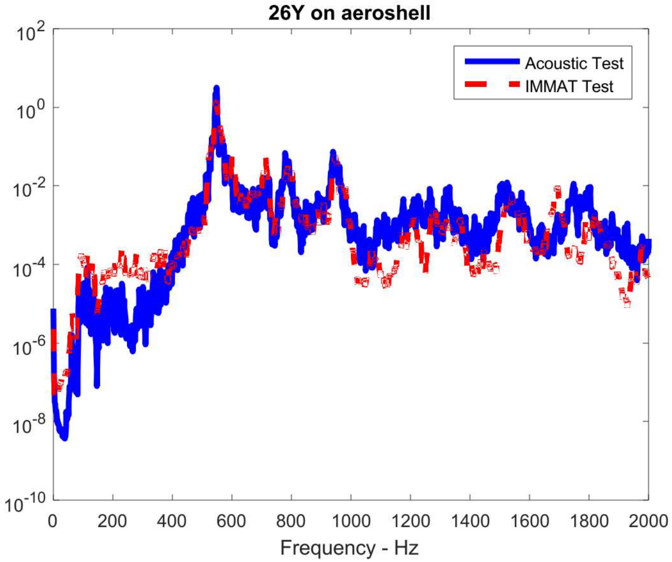


Worst PSD

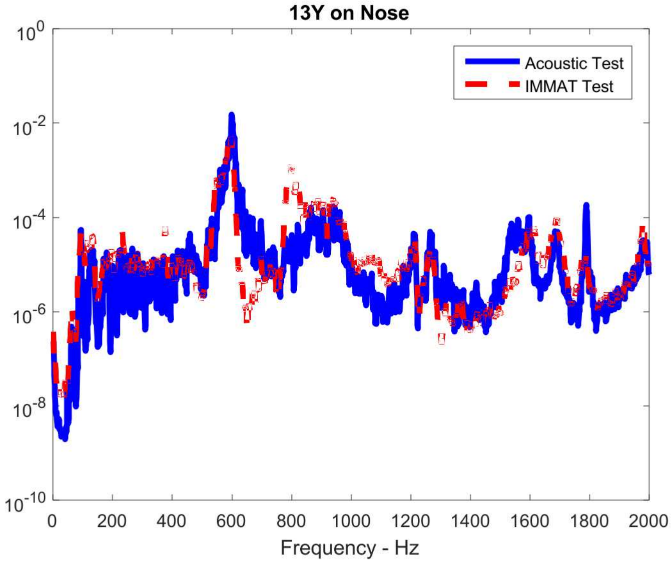


Best PSD

IMMAT Test Results PSD Match at Non Target DOF – 0dB case



26Y at middle of aeroshell



13Y at Forward End

Optimization Shaker Locations to minimize sum of stinger forces

- First shaker
  - Worst shaker 403Y- with 433 N
  - Best shaker 106Y- with 88 N, E=78, I=5.3 Amps, dBerr=22
- Second shaker
  - Worst shaker 105Y- with 415 N
  - Best shaker 601X+ with [66 61 N], E=[77 39 V], I=[5 3 Amps], dBerr=8.3
- Third shaker
  - Worst shaker 105Y- with 456 N
  - Best shaker 506Y- with [72 60 45N], E=[78 38 27V], I=[5 3 3 Amps], dBerr=7.3
- Fourth shaker
  - Worst shaker 406Y- with 326,000 N
  - Best shaker 102Y- with [56 67 37 65 N], E=[56 39 22 87 V], I=[4 3 2 6Amps], dBerr=4.7
- Fifth shaker
  - Worst shaker 406Y- with 329,000 N
  - Best shaker 301Y- with [56 67 37 65 N], E=[56 39 22 87 V], I=[4 3 2 6Amps], dBerr=4.0
- Sixth shaker (Blows Up!)
  - Worst shaker 105Y- with 1,450,000 N
  - Best shaker 305Y- with [34200 1375 32000 112 17000 N], E=[27000 1400 26000 660 13500 V], I=[12800 707 12400 313 6460 A], dBerr=3.1

Optimization Shaker Locations to minimize control error

- First shaker
  - Worst shaker 106Y- with 22 dBerr
  - Best shaker 303Y- with 130 N, E=184, I=16 Amps, dBerr=11
- Second shaker
  - Best shaker 104Y- with [190 110 N], E=[150 70 V], I=[16 6 Amps], dBerr=6.6
- Third shaker
  - Best shaker 601X with [157 91 62 N], E=[142 87 54 V], I=[13 6 4 Amps], dBerr=5.0
- Fourth shaker
  - Best shaker 304Y- with [232 57 58 178 N], E=[208 85 49 200V], I=[17 5 4 17 Amps], dBerr=3.9
- Fifth shaker
  - Best shaker 106Y- with [132 65 57 129 50N], E=[155 118 34 144 90 V], I=[14 7 3 13 6Amps], dBerr=3.4
- Sixth shaker
  - Best shaker 602X- with [79,200 19,800 32,800 77,100 28,800 33,800 N], E=[60,400 15,200 25,100 58,800 22,100 26,100 V], I=[28,800 7,300 12,000 28,100 10,600 12,500 Amps], dBerr=2.9

Conclusions

- Excellent control produced from optimization of 4 shakers to minimize amplifier voltage output
- Non-target PSDs are not as close as target, but still fair
- Fewer shakers required than I thought would be needed
- More shakers are not always better
- As shakers get "close together" they fight each other and require more voltage than amplifier may be able to provide
- Optimizing on control error produced requirements for shaker voltage that were physically unrealizable
- Minimizing force, voltage or control error produced nearly same control error estimates within 1 dB with 4 shakers
- Prediction for required voltage for one shaker (at most compliant location) was more than factor of 2 too high due to modal truncation
- We had enough voltage headroom to go to +3dB on target acceleration response PSDs