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Optimization methods for hypersonic boundary layer transition

PRESENTED BY

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Outline

- Transition physics overview
- Issues with existing methods
- Introduction to "Optimization" methods
- Mathematical approach
- Examples of analysis
- Application to transition prediction
- Comparison with wind tunnel measurements
- Summary and future work

Transition Physics

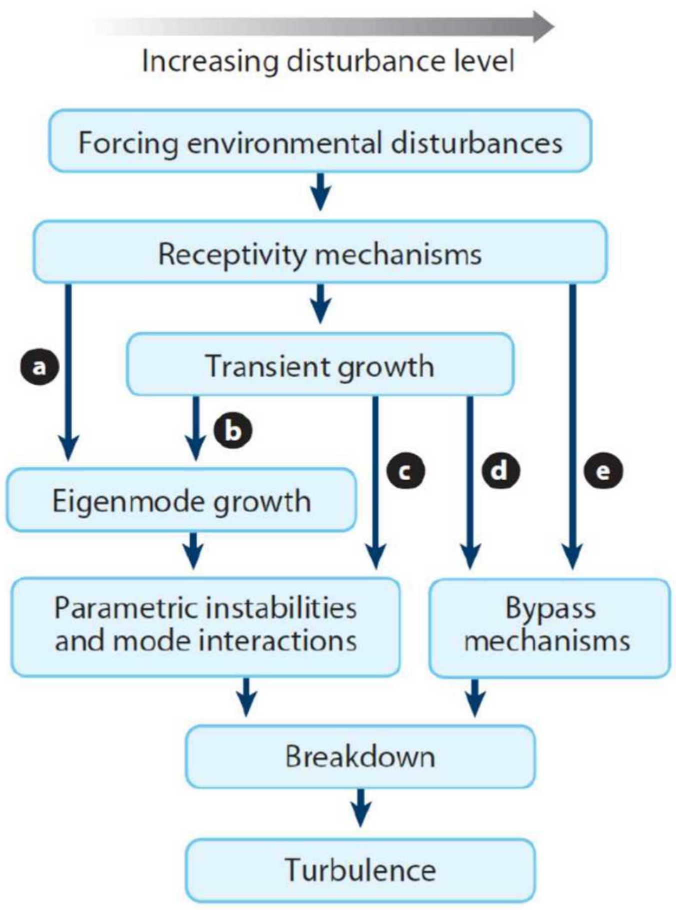
There are 3 main pathways for laminar-turbulent transition:

- **"Modal" instabilities:**
  Exponentially-growing hydrodynamic instabilities
- **"Transient Growth":**
  Algebraically-growing disturbances
- **Bypass transition:**
  Large perturbations causing immediate non-linearity and breakdown

Each of these pathways currently requires a different type of analysis

- Modal Instabilities: eigenvalue analysis, PSE with one (or a few) modes
- Transient growth: superposition of eigenmodes, input/output methods
- Bypass transition: correlations, DNS, ??

Transition prediction methods should handle all possible transition pathways.



(Fedorov 2011, Ann. Rev Fl. Mech)

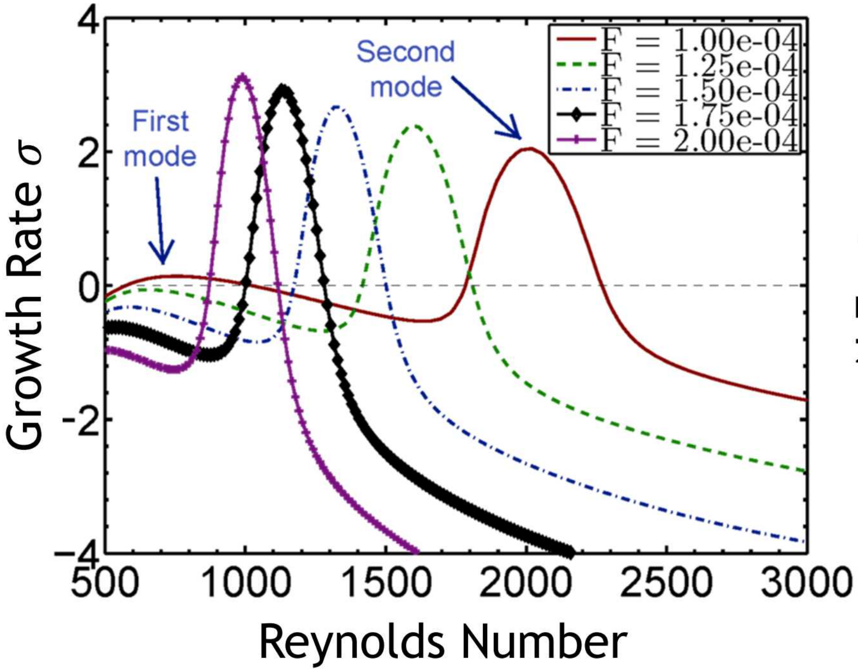
"Modal" Instabilities

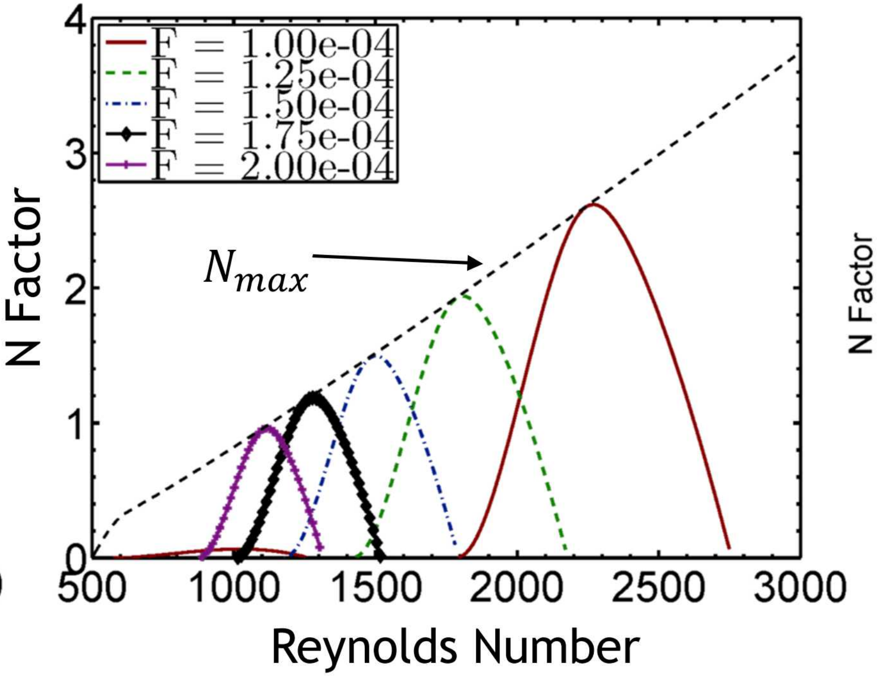
Conventional method for analyzing modal instabilities:

1. Determine hydrodynamic instabilities with positive growth rates ($\sigma > 0$)
2. Integrate growth of instabilities

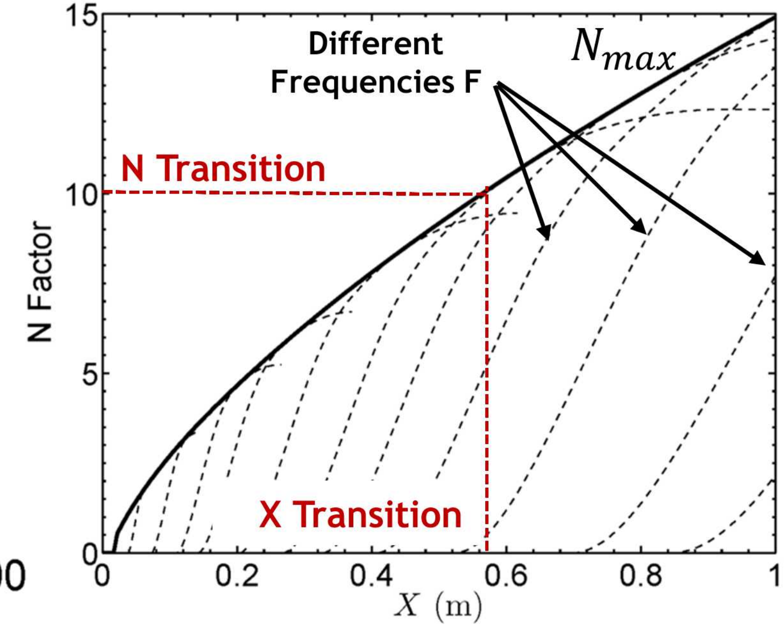
$$N = \int_{x_o}^{x} \sigma(\xi) d\xi \qquad \frac{A(x)}{A_o} = e^{N(x)}$$

3. Predict transition when $N = N_{tr}$





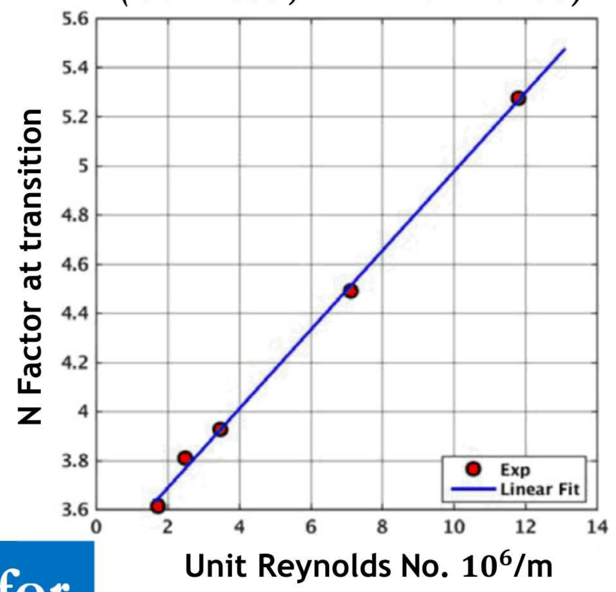
Flat plate BL
Mach 5



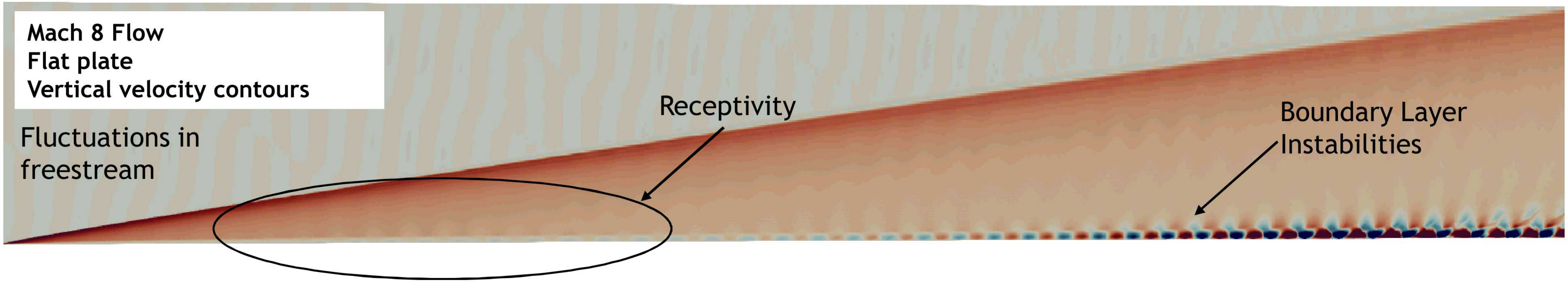
"Receptivity"

- Transition is sensitive to the type of perturbation that creates instability waves
- The generation of instability waves by perturbations is called "Receptivity"
- Larger perturbations cause earlier transition
- The "N Factor" method does not capture this effect
- Wind tunnel experiments show **the "N Factor of transition" is not a constant**

(Marineau, AIAA-2017-0766)



Transition prediction methods should account for disturbances sources and receptivity.



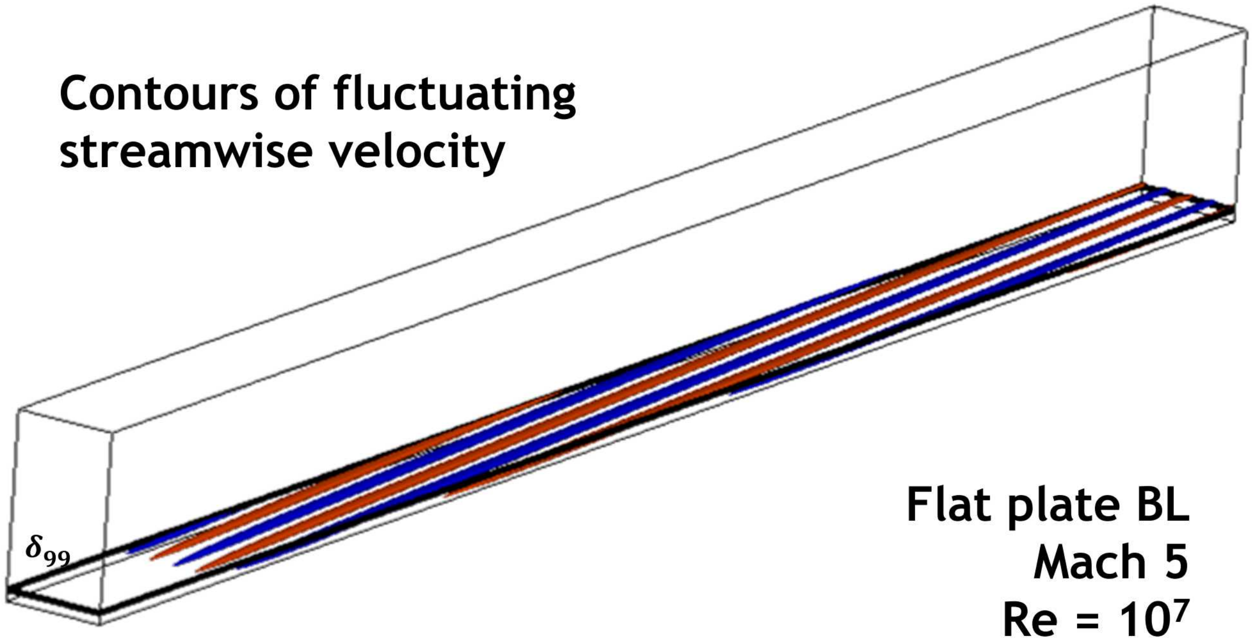
Transient Growth

Transient growth is qualitatively different from modal instabilities

- There may be no hydrodynamic instabilities in the flow field
- Perturbations are asymptotically stable
- Strong growth of streaky structures is caused by vortical disturbances
- During period of amplification, non-linear breakdown can be reached

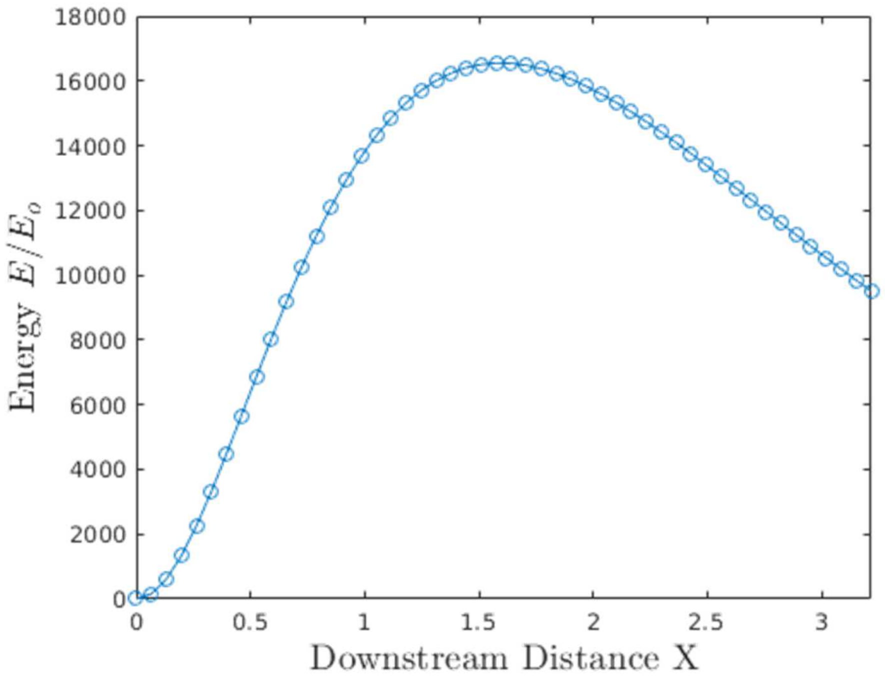
Transient Growth requires different analysis methods from modal instabilities

Contours of fluctuating streamwise velocity



Flat plate BL
Mach 5
Re = 10^7

Energy Growth



Complex Geometries

- Flight vehicles can produce 3D flow features
- Conventional methods of stability analysis do not apply to these

Transition prediction methods should be able to handle complex 3D flows.

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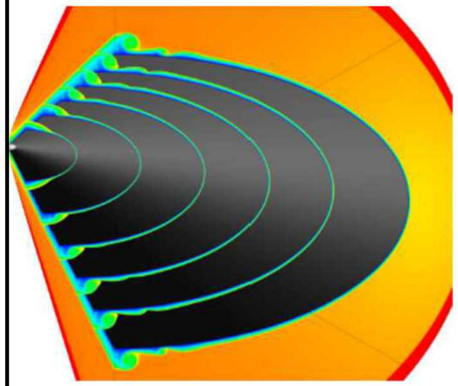
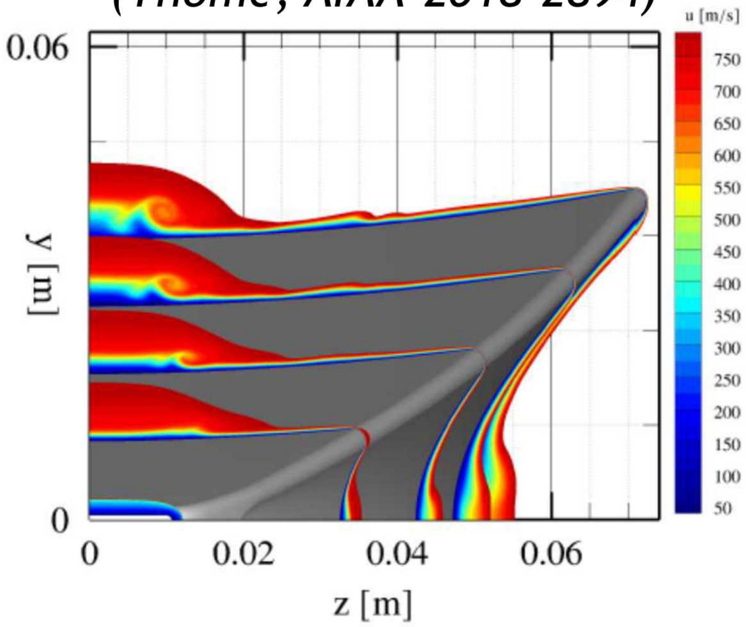


(Moyes, AIAA-2019-2972)

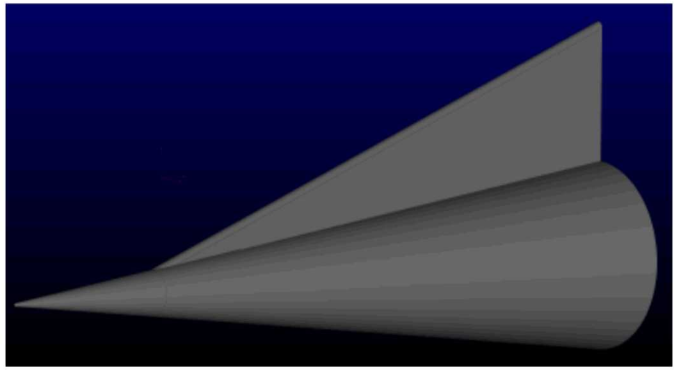
BoLT Flight Vehicle

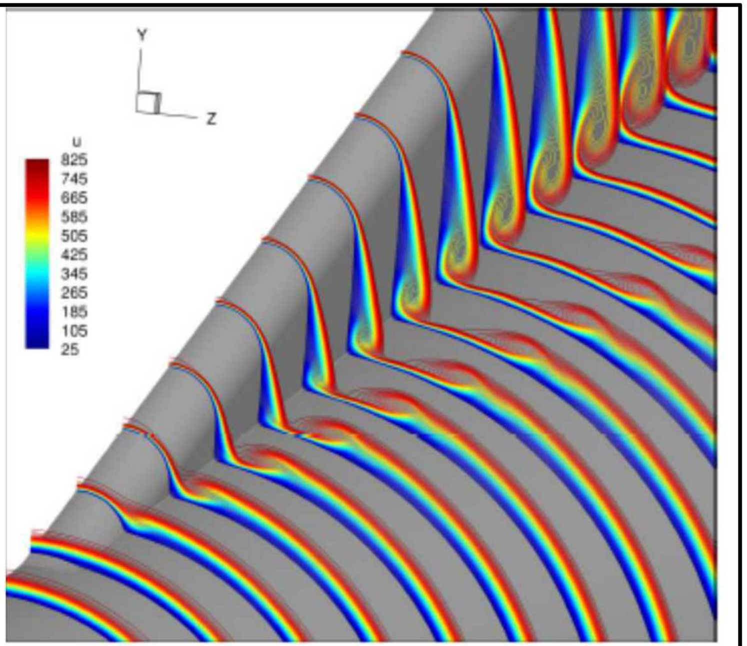
(Thome, AIAA-2018-2894)



Finned Cone Geometry

(Mullen, AIAA-2018-3072)





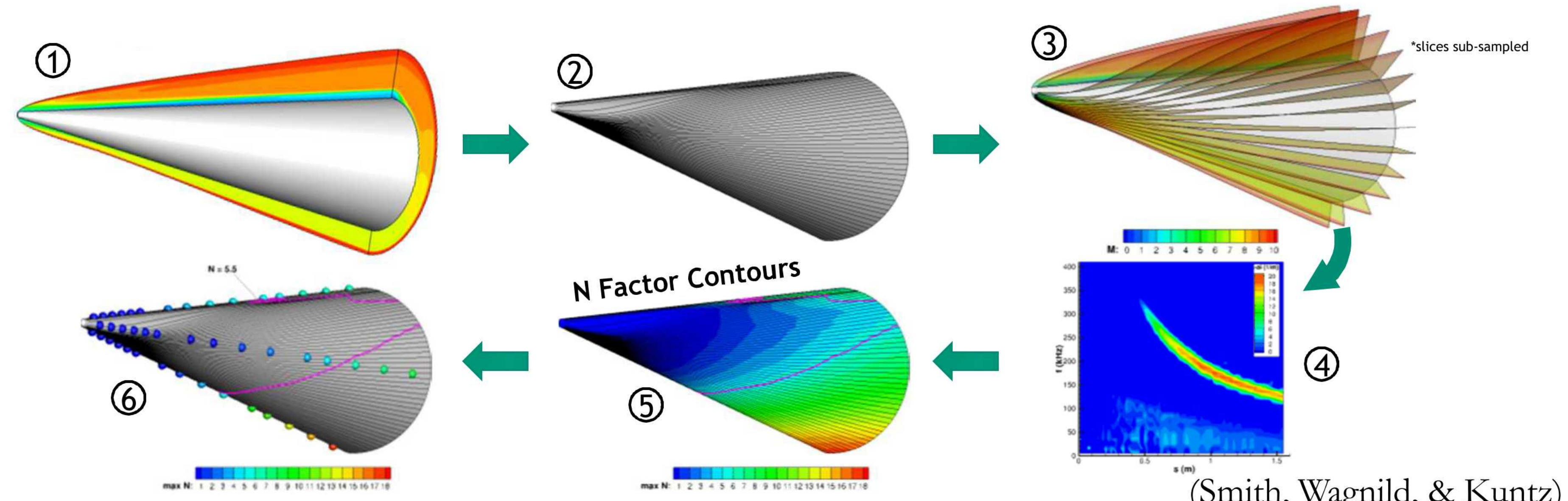
Typical Transition Analysis Procedure

Steps in typical transition analysis:

1. Conduct laminar CFD
2. Estimate the paths of propagation of instability waves
3. Extract 2D flow slices along disturbance propagation paths
4. Perform stability analysis on each slice
5. Integrate growth of instabilities on each slice (may use PSE)
6. Interpolate N factors and transition prediction onto geometry

Transition prediction needs to follow a simpler, less error-prone process

Each step requires user intervention and expertise and may introduce errors



(Smith, Wagnild, & Kuntz)

Summary: Issues with current methods

Existing "state of the art" transition prediction methods have the following deficiencies:

1. Separate methods are needed for each transition pathway
2. Receptivity is not accounted for
3. Few computational tools exist that can handle complex geometries
4. Analysis is cumbersome, error-prone, and requires much expert user intervention

A class of methods known as "Optimization" methods or "Input/Output" methods can address these deficiencies

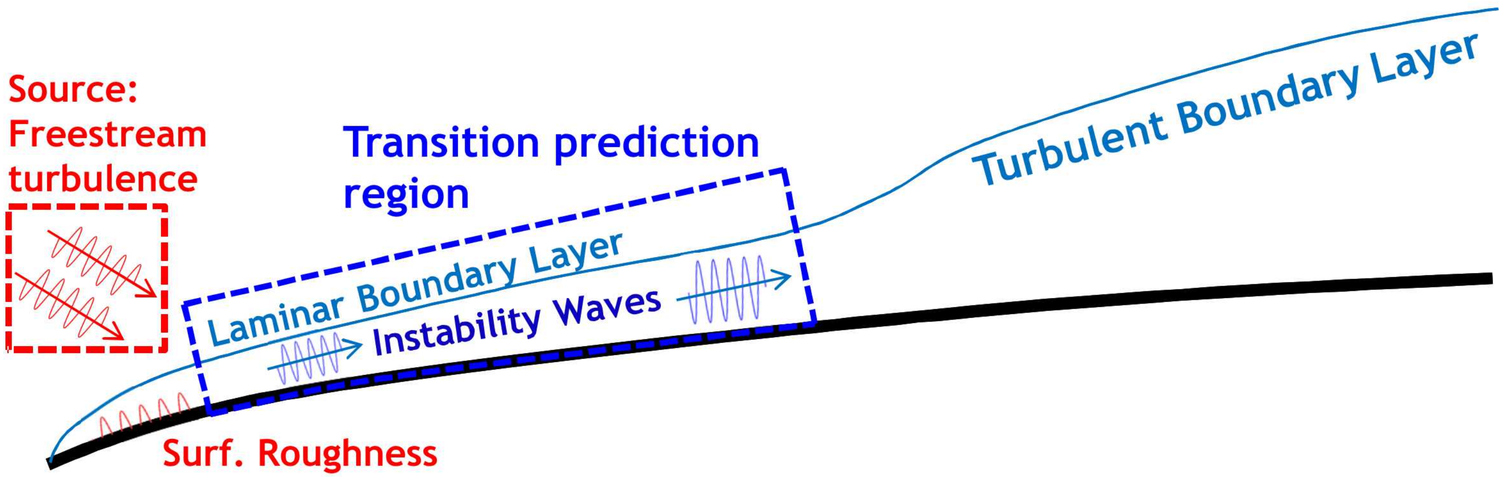
Optimization Methods

Optimization methods for transition prediction

The primary triggers for transition to turbulence are small imperfections (e.g., surface roughness, flow non-uniformities)

Transition prediction can be posed as an optimization problem: **For a given environment, what form of imperfection will cause transition first?**

1. Select a disturbance source region (e.g., vehicle surface, freestream flow)
2. Impose a known disturbance amplitude (e.g., roughness height, turb. kinetic energy)
3. Select a region where transition prediction is desired (e.g., laminar boundary layer)
4. Find the disturbance that experiences the largest possible amount of growth.
5. Predict transition when this disturbance exceeds a nonlinear breakdown threshold.



Spatial vs Temporal methods

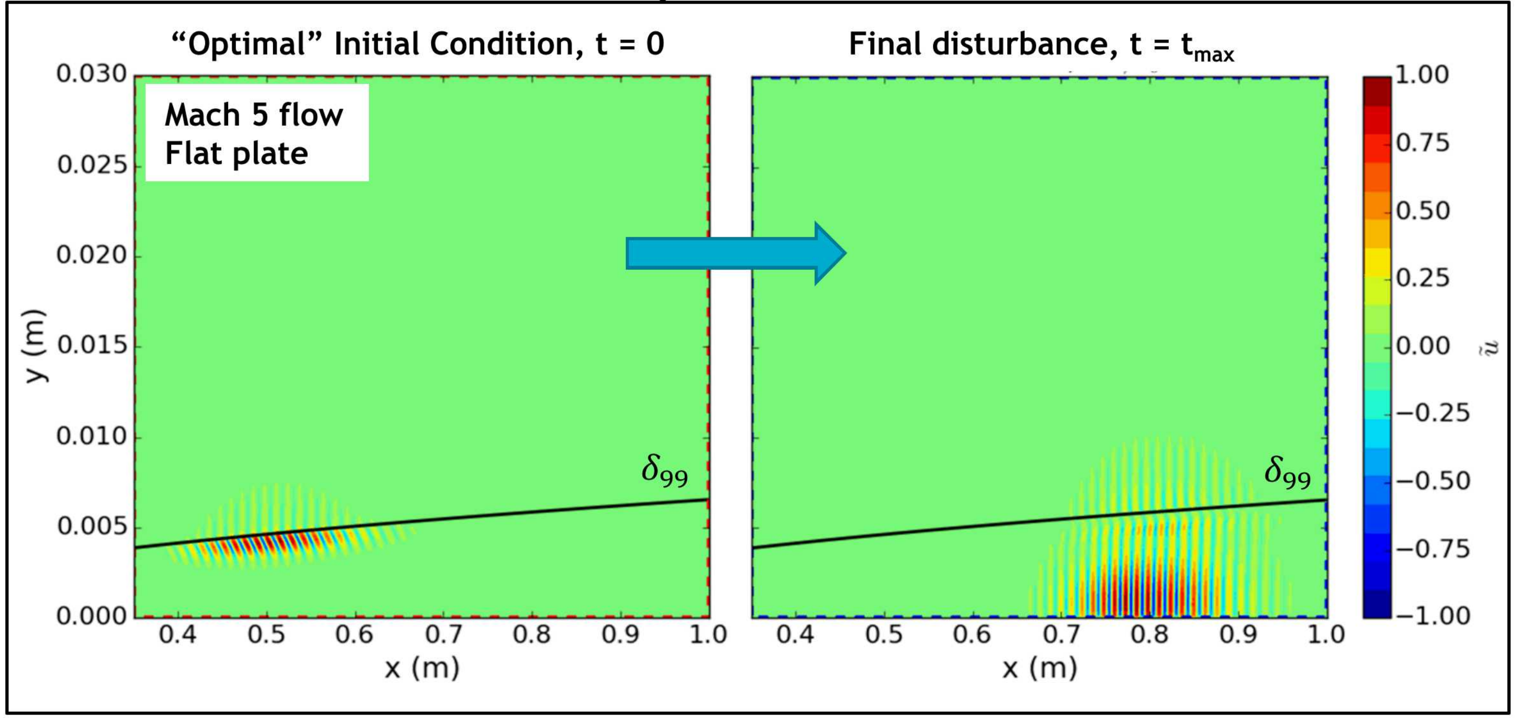
Temporal Method:

- Find the initial condition that grows the most in time
- Good for "absolute" instabilities (separated flows, re-circulation bubbles)

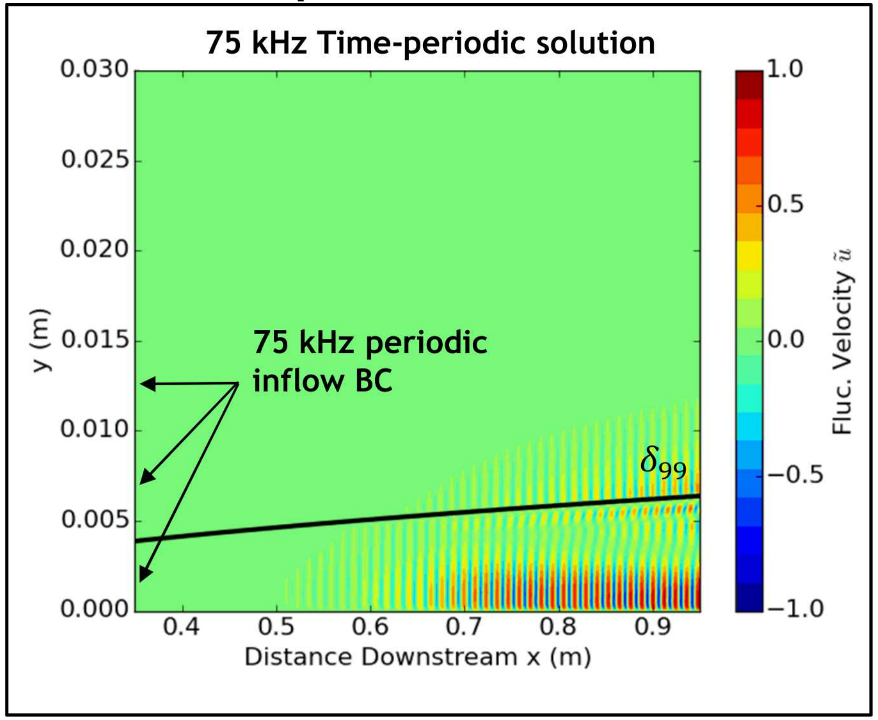
Spatial Method:

- Find the time-periodic inflow boundary condition that amplifies the most
- Good for "convective" instabilities (boundary layer instabilities)

Temporal Method



Spatial Method



Mathematics – Temporal Method

Linearized Navier-Stokes:

$$\frac{\partial \boldsymbol{q}}{\partial t} = \boldsymbol{\mathcal{L}q} \qquad \boldsymbol{q}(t) = e^{\mathcal{L}t}\boldsymbol{q}_o \qquad \boldsymbol{q} = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$$

Disturbance energy measure:

$$E = (\boldsymbol{q}, \boldsymbol{q}) \qquad (\boldsymbol{u}, \boldsymbol{v}) = \int_\Omega \boldsymbol{v}^H \boldsymbol{W u}\, dV$$

Maximize energy growth:

$$\frac{E(t)}{E_o} = \frac{(\boldsymbol{q},\boldsymbol{q})}{(\boldsymbol{q}_o,\boldsymbol{q}_o)} = \frac{(e^{\mathcal{L}t}\boldsymbol{q}_o, e^{\mathcal{L}t}\boldsymbol{q}_o)}{(\boldsymbol{q}_o,\boldsymbol{q}_o)} = \frac{(\boldsymbol{q}_o, e^{\mathcal{L}^\dagger t} e^{\mathcal{L}t}\boldsymbol{q}_o)}{(\boldsymbol{q}_o,\boldsymbol{q}_o)}$$

Solve Rayleigh Quotient with power iteration method:

1. Guess initial condition $\boldsymbol{q}_o$
2. Compute $\boldsymbol{e}^{\mathcal{L}t}\boldsymbol{q}_o$ (Forward problem)
3. Compute $\boldsymbol{e}^{\mathcal{L}^\dagger t}(\boldsymbol{e}^{Lt}\boldsymbol{q}_o)$ (Adjoint problem)
4. Use the result as a new guess for $\boldsymbol{q}_o$

$e^{\mathcal{L}t}$	Approximated as	$\frac{\partial q}{\partial t} = \boldsymbol{\mathcal{L}q}$
$e^{\mathcal{L}^\dagger t}$	Approximated as	$\frac{\partial q}{\partial t} = \boldsymbol{\mathcal{L}}^\dagger\boldsymbol{q}$

Procedure involves a sequence of direct/adjoint solves

Mathematics – Spatial Method

Time periodic linearized Navier-Stokes:

$$\boldsymbol{\mathcal{L}q} = \boldsymbol{B}\boldsymbol{q}_{bc} \qquad \boldsymbol{q} = \boldsymbol{\mathcal{L}}^{-1}\boldsymbol{B}\boldsymbol{q}_{bc}$$

$\boldsymbol{q}_{bc}$ = Vector of boundary values
$\boldsymbol{B}$ = Boundary condition map

Disturbance energy measure on interior and boundary:

$$E = (\boldsymbol{q},\boldsymbol{q}) \qquad (\boldsymbol{u},\boldsymbol{v}) = \int_{\Omega} \boldsymbol{v}^H \boldsymbol{W}\boldsymbol{u}\, dV$$

$$E_{bc} = (\boldsymbol{q},\boldsymbol{q})_{bc} \qquad (\boldsymbol{u},\boldsymbol{v})_{bc} = \int_{\partial\Omega} \boldsymbol{v}^H \boldsymbol{W}\boldsymbol{u}\, dV$$

Maximize disturbance energy:

$$\frac{E}{E_{bc}} = \frac{(\boldsymbol{q},\boldsymbol{q})}{(\boldsymbol{q}_{bc},\boldsymbol{q}_{bc})_{bc}} = \frac{(\boldsymbol{\mathcal{L}}^{-1}\boldsymbol{B}\boldsymbol{q}_{bc},\boldsymbol{\mathcal{L}}^{-1}\boldsymbol{B}\boldsymbol{q}_{bc})}{(\boldsymbol{q}_{bc},\boldsymbol{q}_{bc})_{bc}} = \frac{\left(\boldsymbol{q}_{bc},\boldsymbol{B}^{\dagger}\boldsymbol{\mathcal{L}}^{\dagger^{-1}}\boldsymbol{\mathcal{L}}^{-1}\boldsymbol{B}\boldsymbol{q}_{bc}\right)_{bc}}{(\boldsymbol{q}_{bc},\boldsymbol{q}_{bc})_{bc}}$$

Solve Rayleigh Quotient with power iteration method:

1. Guess boundary condition $\boldsymbol{q}_{bc}$
2. Compute $\boldsymbol{\mathcal{L}}^{-1}\boldsymbol{B}\boldsymbol{q}_{bc}$ (Forward problem)
3. Compute $\boldsymbol{B}^{\dagger}\boldsymbol{\mathcal{L}}^{\dagger^{-1}}\boldsymbol{\mathcal{L}}^{-1}\boldsymbol{B}\boldsymbol{q}_{bc}$ (Adjoint problem)
4. Use the result as a new guess for $\boldsymbol{q}_{bc}$

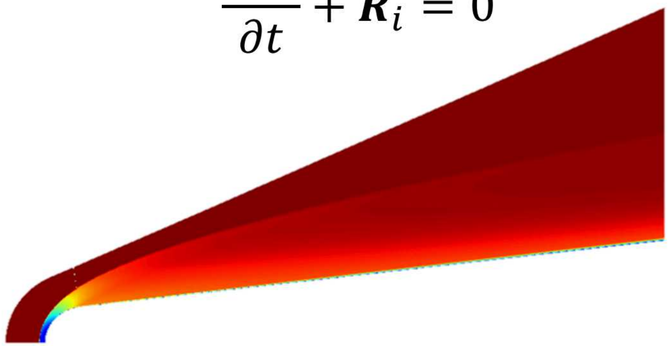
Procedure involves solving a sequence of direct/adjoint linear systems

Numerical Approach

- Method is implemented inside a standard finite-volume CFD code (SPARC)
  - Seamless interface with CFD solver (same grid, numerics, reacting gas physics).
  - Scalability (3D flows)
  - In-situ transition prediction
- Implementation:
  - Linearized NS operator obtained using automatic differentiation (overloaded real type)
  - Implementation is agnostic to CFD model. Works for any flux scheme, gas model, discretization, etc.
  - Jacobian matrix evaluated using "coloring" technique for computational efficiency
  - Optimization method boils down to building the Jacobian matrix + a sequence of linear solves.

Non-linear Compressible NS

$$\frac{\partial \boldsymbol{U}_i}{\partial t} + \boldsymbol{R}_i = 0$$

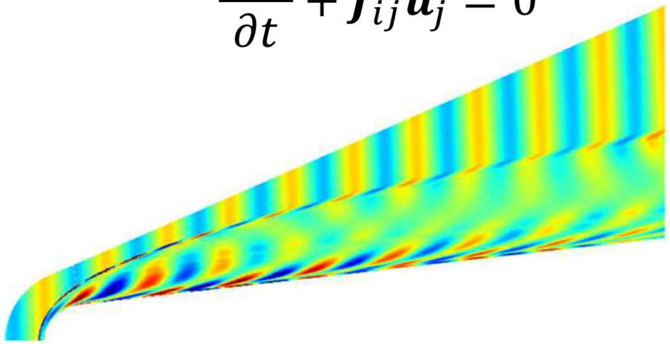


Automatic Differentiation

$$\boldsymbol{J}_{ij} = \frac{\partial \boldsymbol{R}_i}{\partial \boldsymbol{U}_j}$$

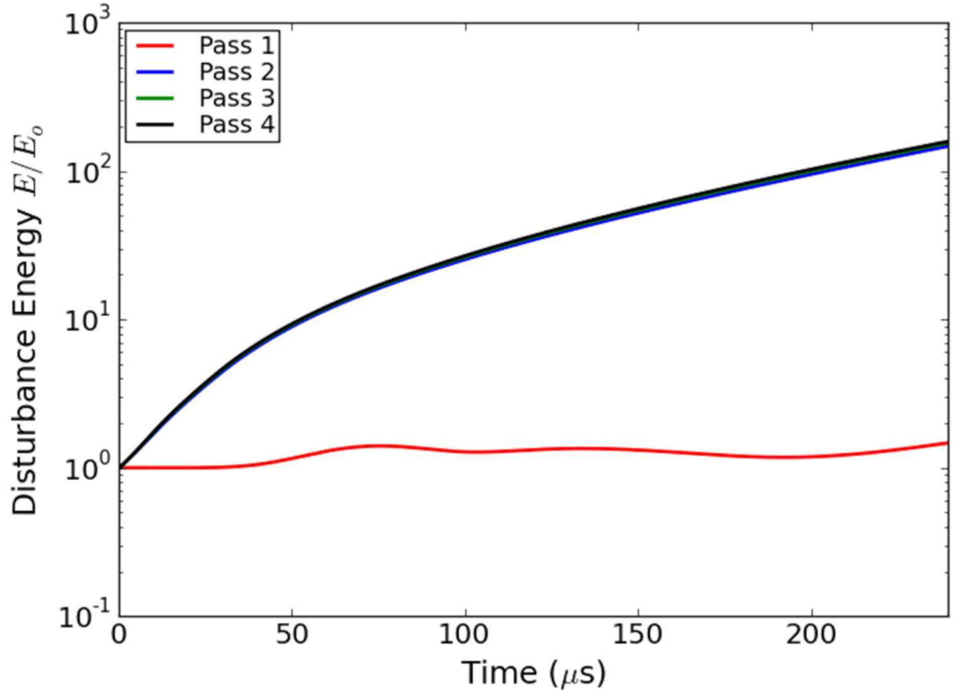
Linearized NS

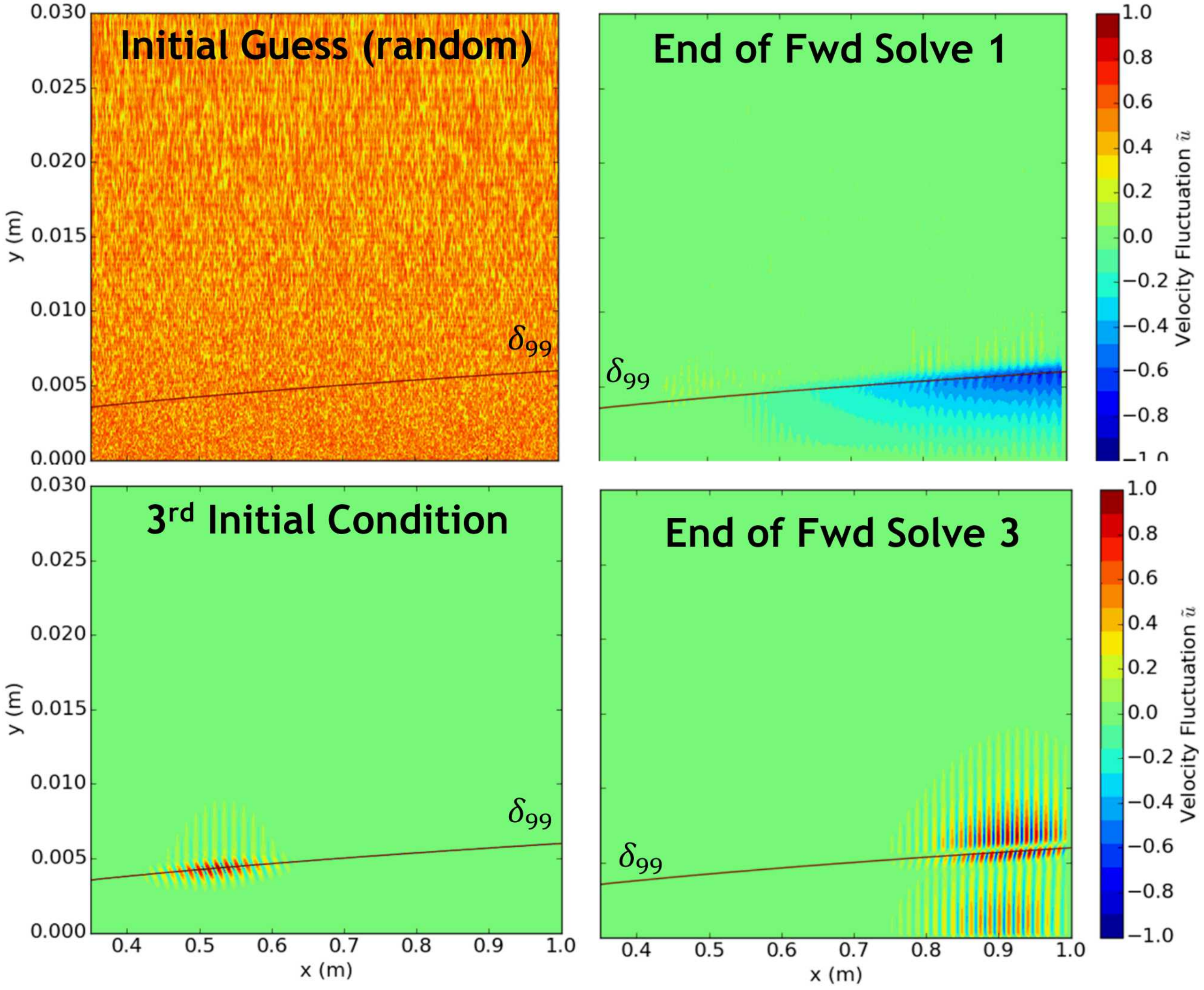
$$\frac{\partial \widetilde{\boldsymbol{u}_i}}{\partial t} + \boldsymbol{J}_{ij}\widetilde{\boldsymbol{u}_j} = 0$$



Example: Flat Plate, Temporal Analysis

- Solver rapidly converges on 2nd mode wave after 2-3 iterations
- Results are insensitive to initial guess
- Energy growth approaches exponential growth rate (in this case).





Constraints

We don't always want to find the "optimal" disturbance

If we have knowledge about the disturbance source, we can constrain the input

Examples:

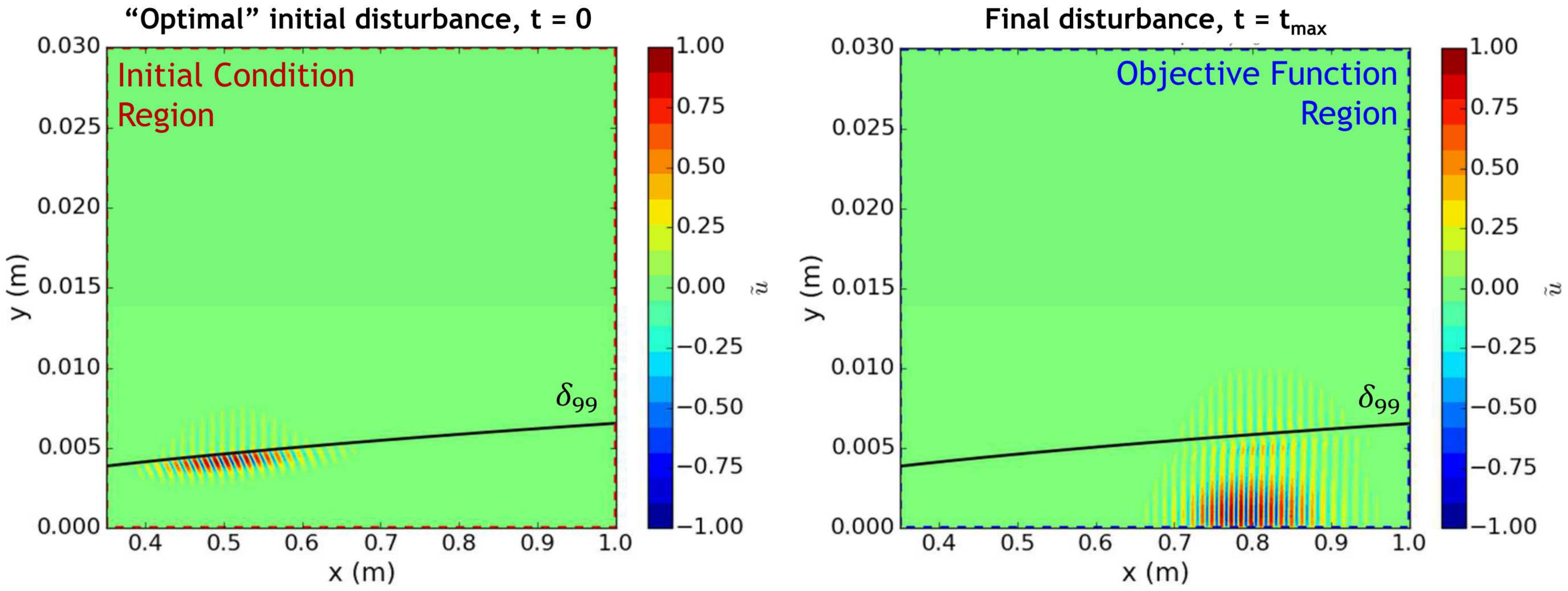
- Require freestream turbulence to originate outside the boundary layer
- Require freestream noise to consist of
  - Vorticity only (flight)
  - Vorticity and temperature spots only (flight)
  - Acoustic waves only (wind tunnel)
  - Acoustic waves at a prescribed angle (wind tunnel)
- Require surface roughness to be homogeneous or follow some pattern

The method always finds the earliest possible transition (conservative)

If we know more about the flight environment, the prediction gets less conservative

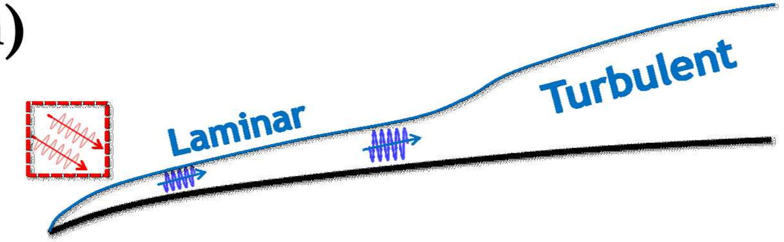
Example: Unconstrained Solutions

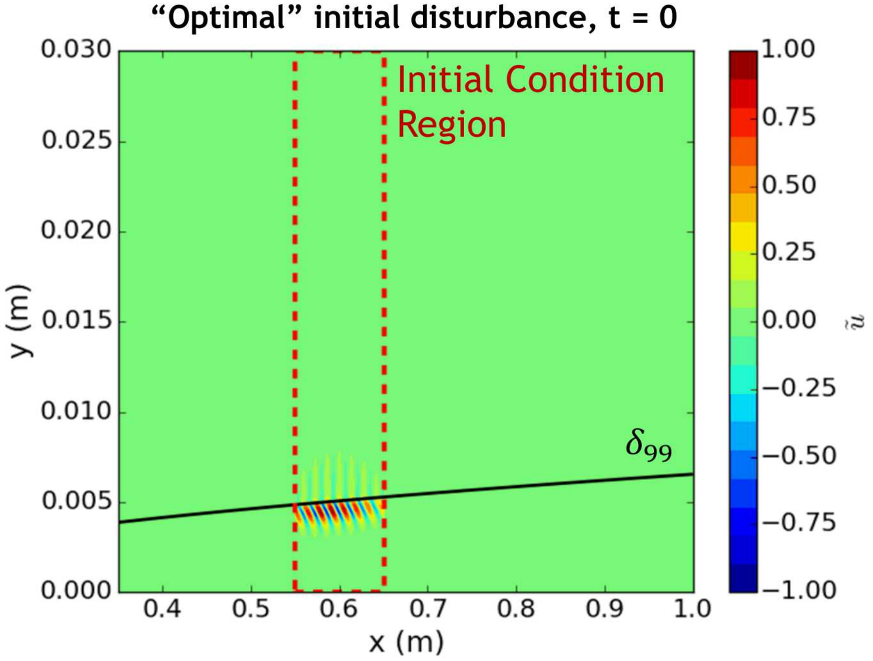
- Allow disturbances to begin anywhere in domain
- Objective function is evaluated over entire domain
- Result is a wave packet of classic "2nd mode" instability waves
  - Wave shape agrees with known 2nd mode behavior
  - Frequency and wavelength match known 2nd mode properties

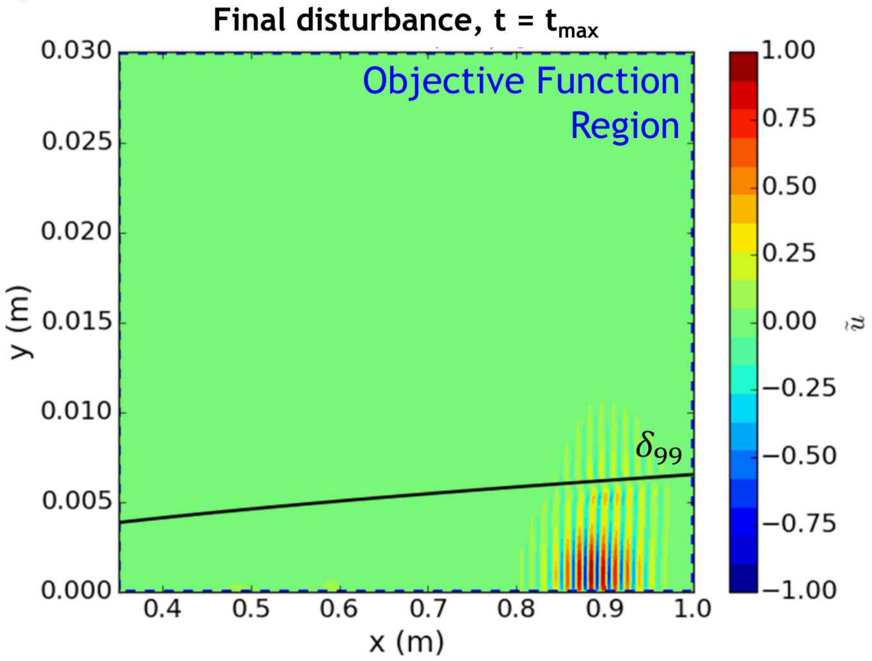


Example: Constrained Initial Conditions

- Disturbances can only begin in $0.55<x<0.65$ m
- Objective function is integrated over entire domain
- Energy is about 83% of unconstrained solution
- **This method allows us to force the disturbances to originate in the freestream (or wherever we want them)**

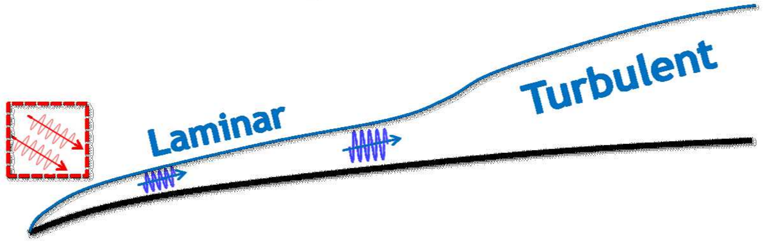




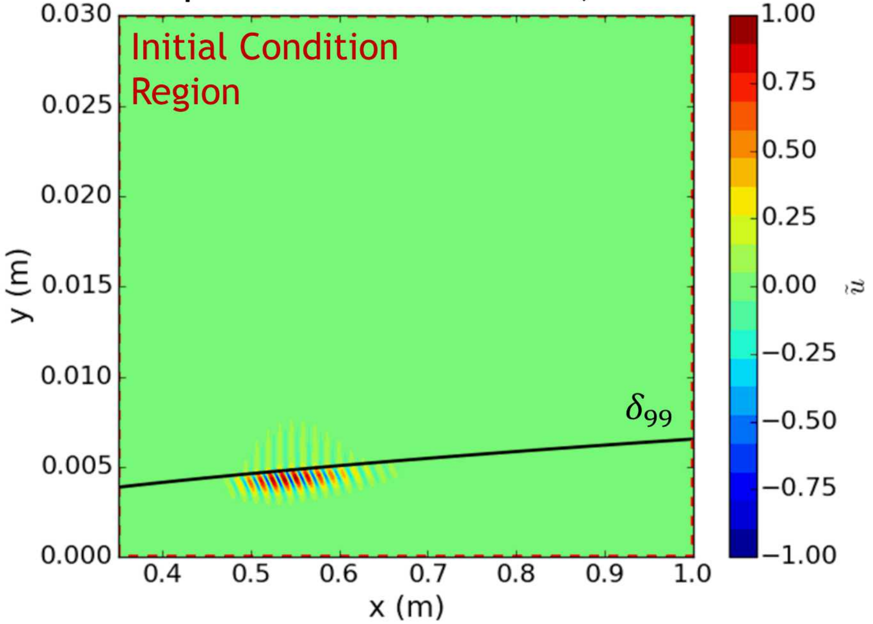


Example: Constrained Objective Function

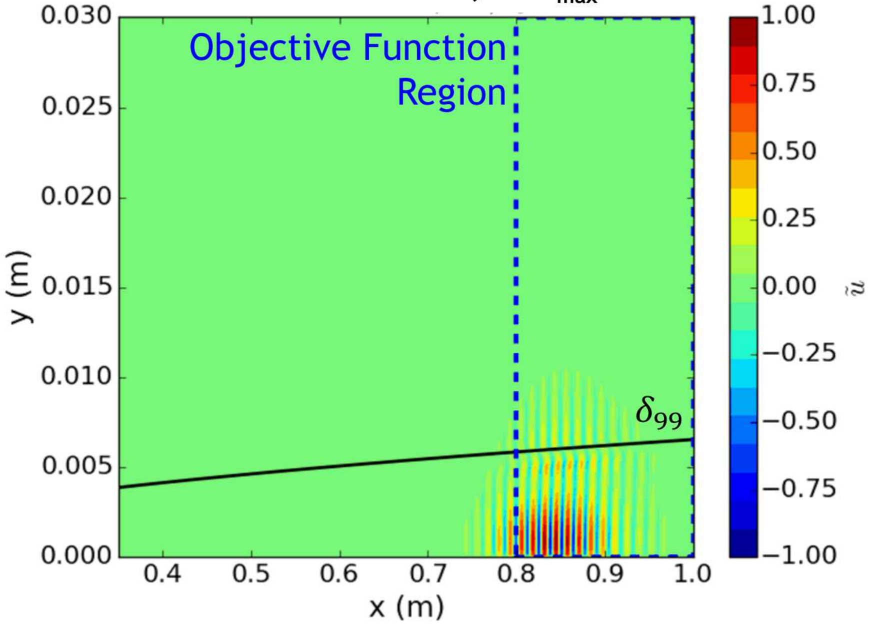
- Allow disturbances to begin anywhere in domain
- Objective function only evaluated for x>0.8 m
- Energy is 93% of unconstrained solution
- **With this approach, we can "turn off" turbulent regions in the transition analysis.**



"Optimal" initial disturbance, t = 0

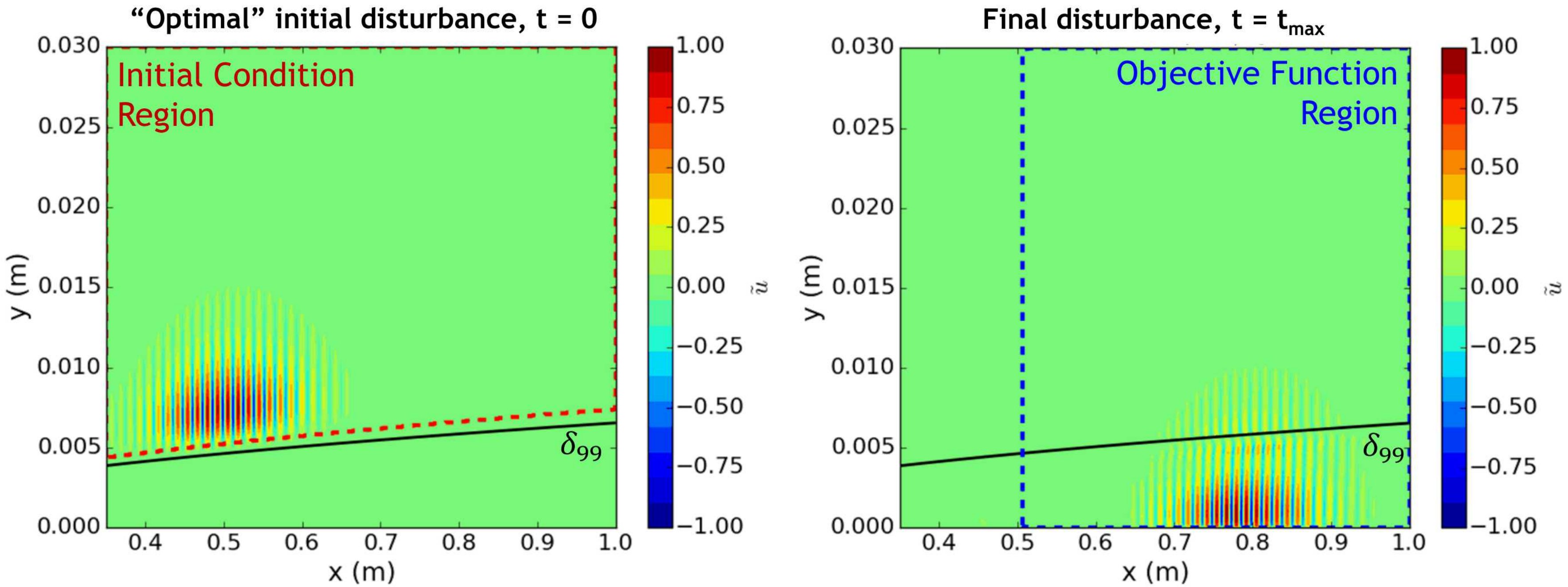


Final disturbance, t = t_{max}



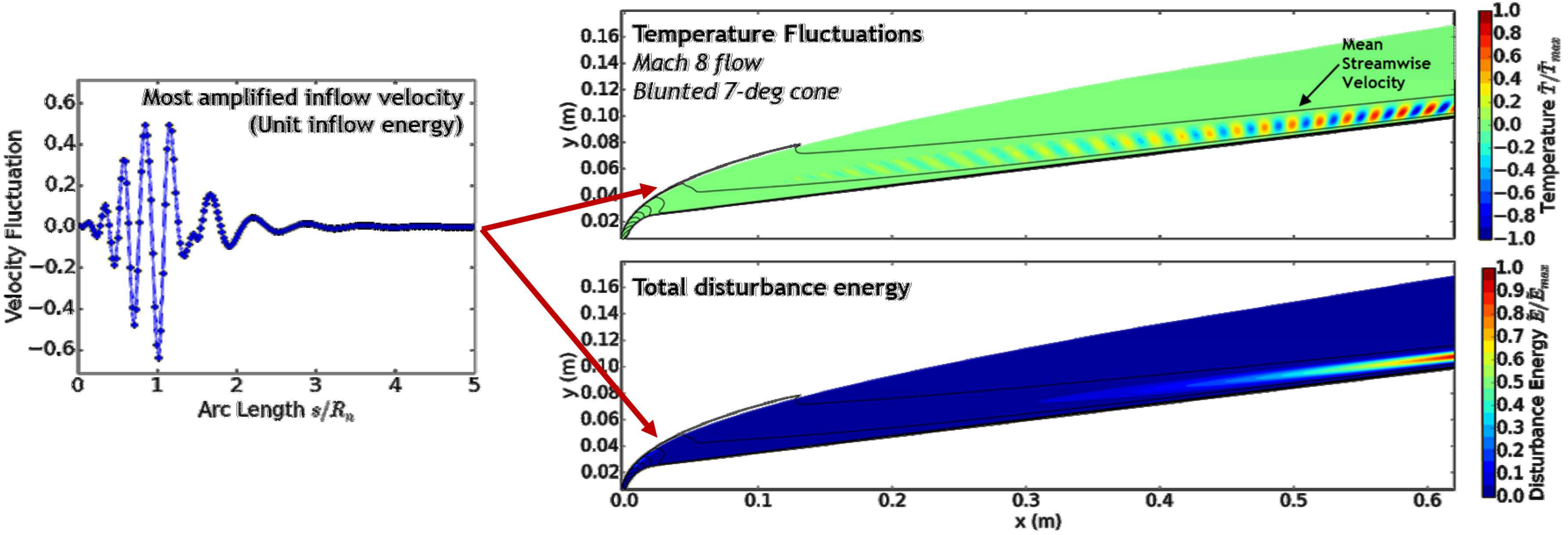
"Receptivity" problem example

- Only allow disturbances outside the BL
- Most dangerous disturbance is an acoustic wave parallel to the plate.
- Energy is only 19% of unconstrained solution
  - Receptivity is inefficient
  - This is why we can't just assume the instability waves are initiated at the freestream turbulence level.



Example: Blunted cone, spatial analysis

- The transition mechanism on highly-blunted cones is not currently known
- No modal instabilities have been found that explain transition measurements
- Optimization method provides a possible solution
- Method finds highly-amplifying waves in the shock layer / boundary layer



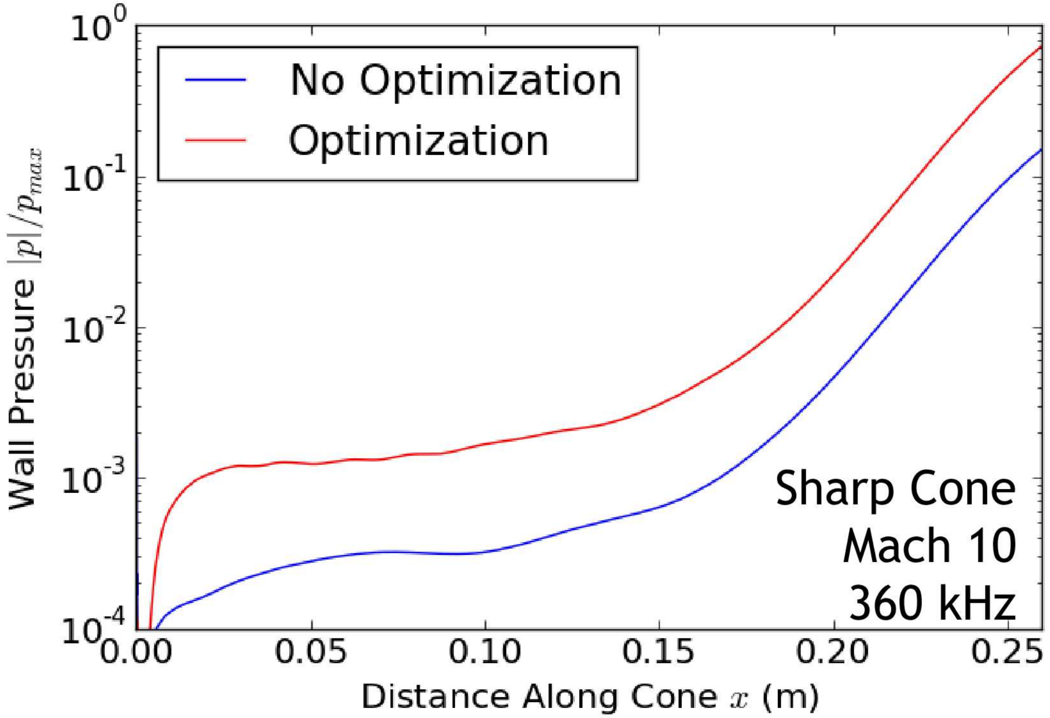
Importance of optimization

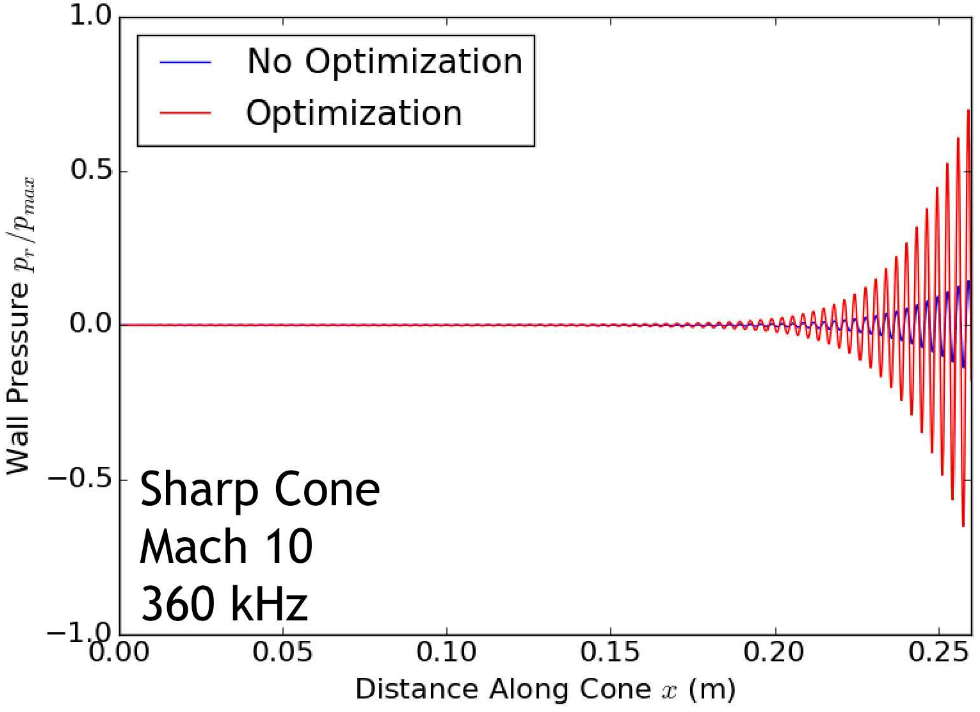
To evaluate the importance of optimization, compare:

- Planar, slow acoustic waves parallel to cone axis
- Optimized boundary condition

Performing optimization maximizes receptivity near leading edge of cone

Worst-case disturbance reaches an amplitude 5x greater than planar acoustic waves



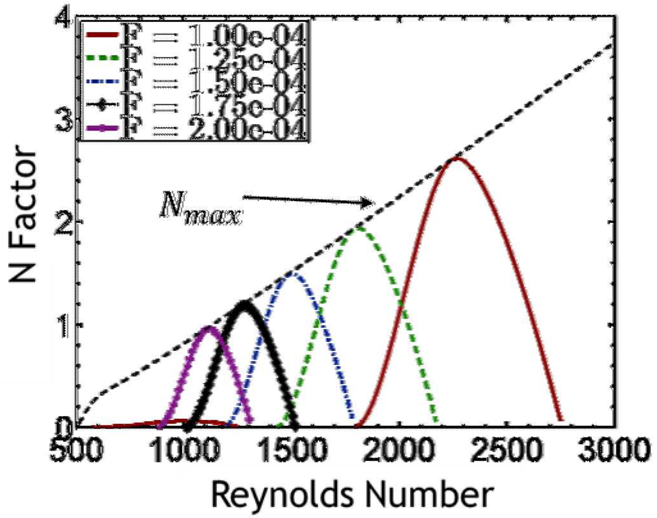


Application of Method to Transition Prediction

Transition Prediction with I/O Methods

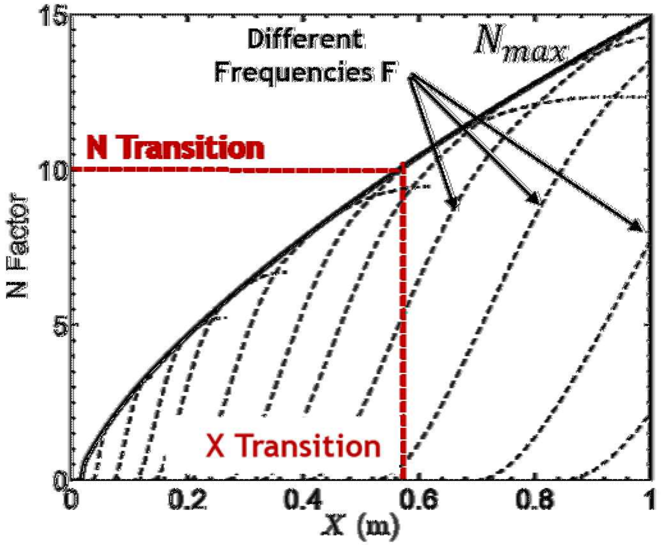
Existing methods:

- Most existing methods analyze each frequency independently.
- The growth of each frequency is monitored to compute an "N Factor"
- The envelope over all frequencies is the "Maximum N Factor"
- Transition is predicted at an "N Factor of transition"



Deficiencies:

- Does not account for amplitude of noise sources
- The "N Factor of transition" is highly variable
  - Can range from 3 (in some wind tunnels) to 10 or greater
  - Can depend on freestream Re by a factor of 2 or more ("Unit Re effect")
  - Can depend on angle of attack or other parameters by a factor of 2-4



Is N factor really the right quantity to consider?

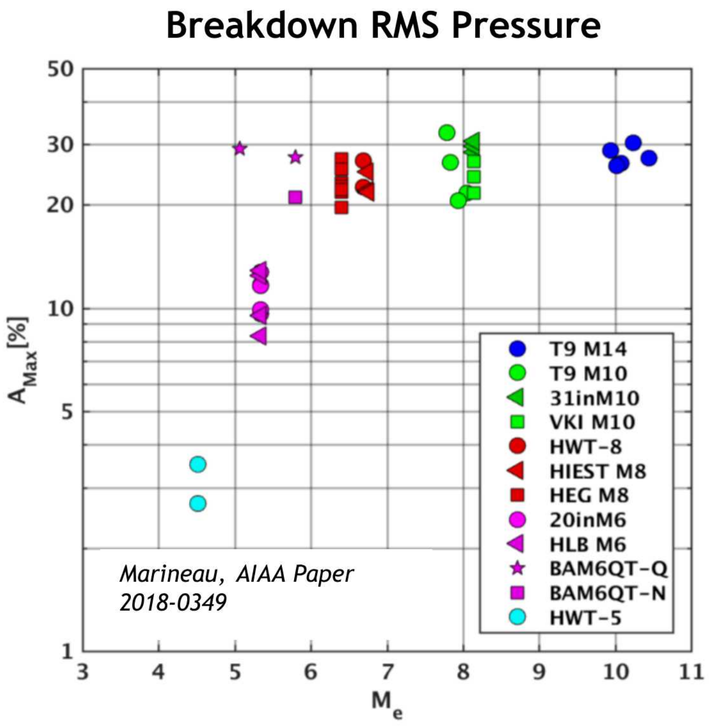
Proposed Approach

Transition prediction correlates better with RMS pressure amplitude

For 2nd mode transition, RMS fluctuations at transition are 25-30%

Proposed procedure:

1. Determine best estimate of spectrum of forcing (e.g., wind tunnel noise spectrum)
$$p'_\infty = f_{PSD}(\omega)$$
2. Run optimization analysis to determine amplification A for each frequency.
3. Scale the freestream PSD by the amplification A
$$p'(x,y,\omega) = f_{PSD}(\omega) A^2(x,y,\omega)$$
4. Integrate over frequency to compute the RMS pressure:
$$p_{rms}(x,y) = \sqrt{\int_0^\infty p'(x,y,\omega)d\omega}$$
5. Predict transition when RMS pressure exceeds breakdown amplitude



Example: Sharp cone at zero AOA

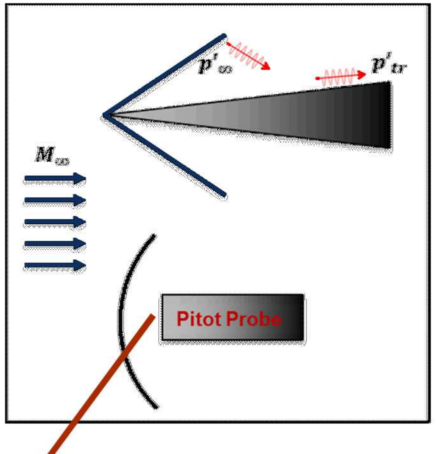
Step 1: Model the wind tunnel noise

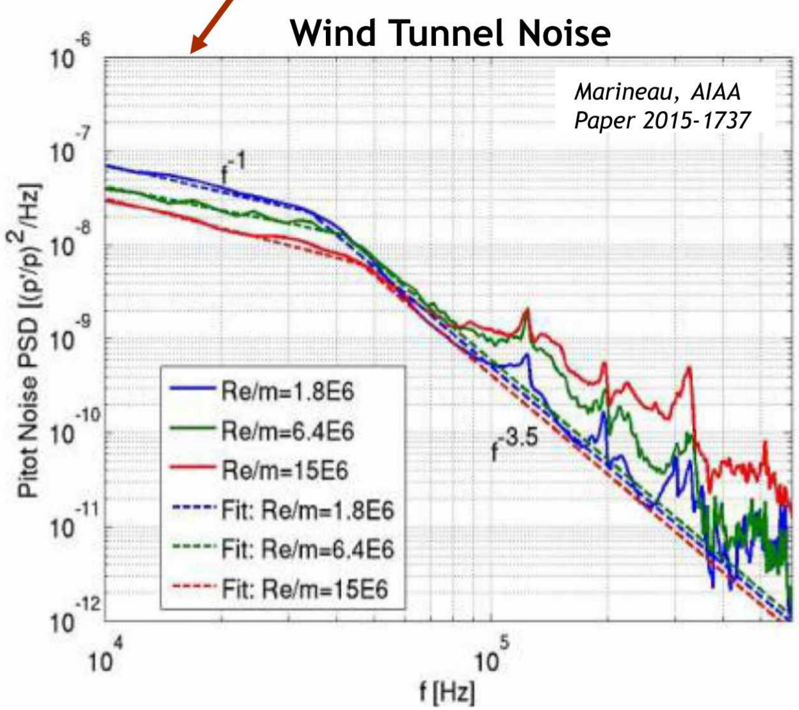
Piecewise Exponential model

$$p'_\infty = \begin{cases} f^{-m_1} & f < f_{cut} \\ f^{-m_2} & f \geq f_{cut} \end{cases}$$

Note:

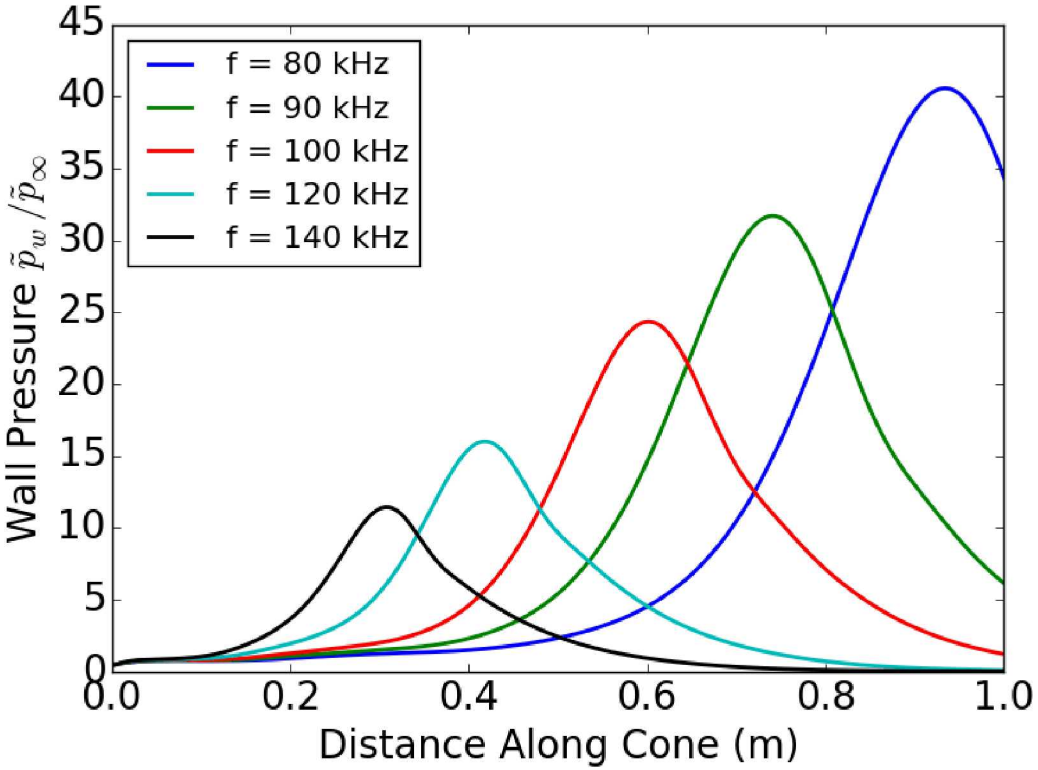
- Pitot measurements are post-shock
- Noise levels increase across shock
- But relative noise $(p'/\bar{p})$ is close to constant across shock



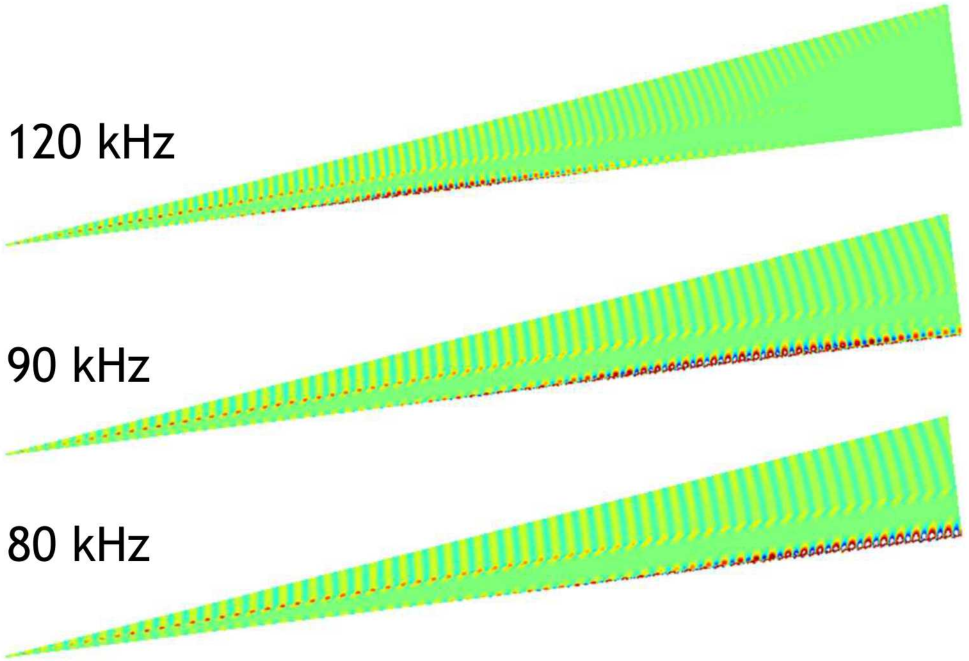


Example: Sharp cone at zero AOA

Step 2: Solve optimization problem for each frequency



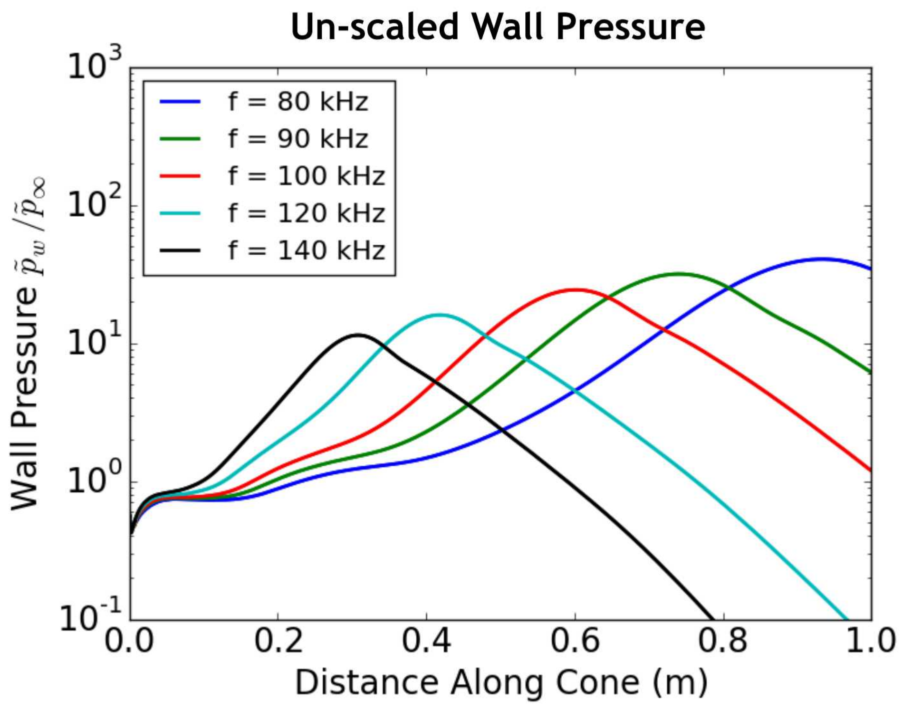
Pressure Contours

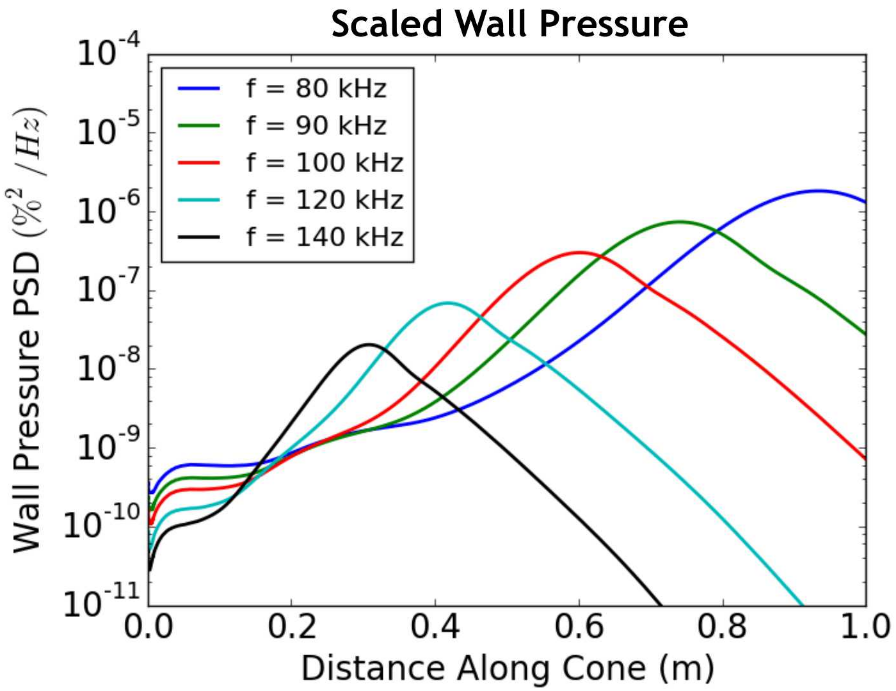


Example: Sharp cone at zero AOA

Step 3: Scale pressure by freestream PSD

- Low frequencies reach higher PSD amplitudes due to greater freestream noise

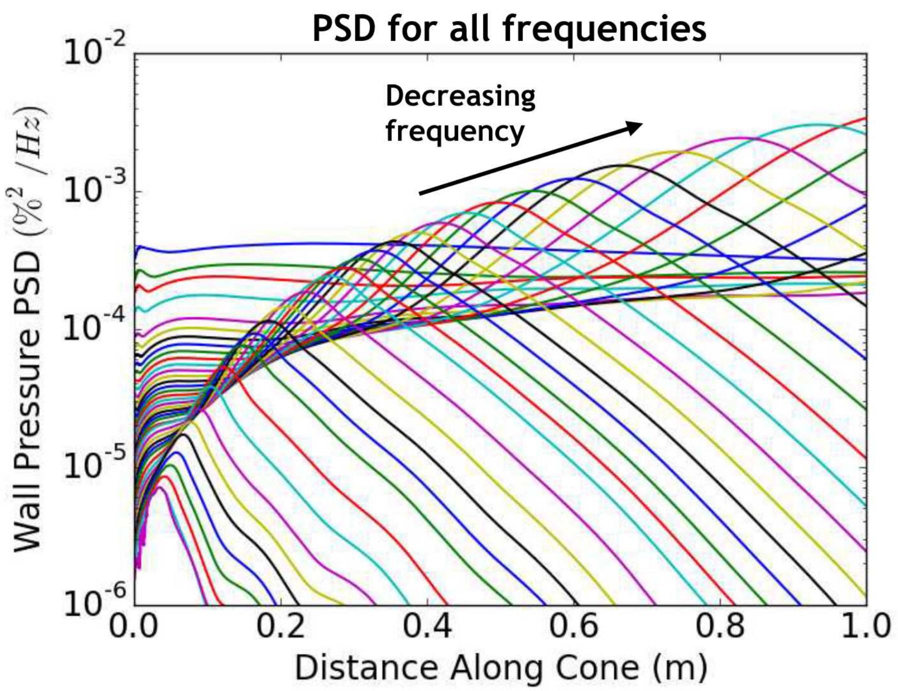


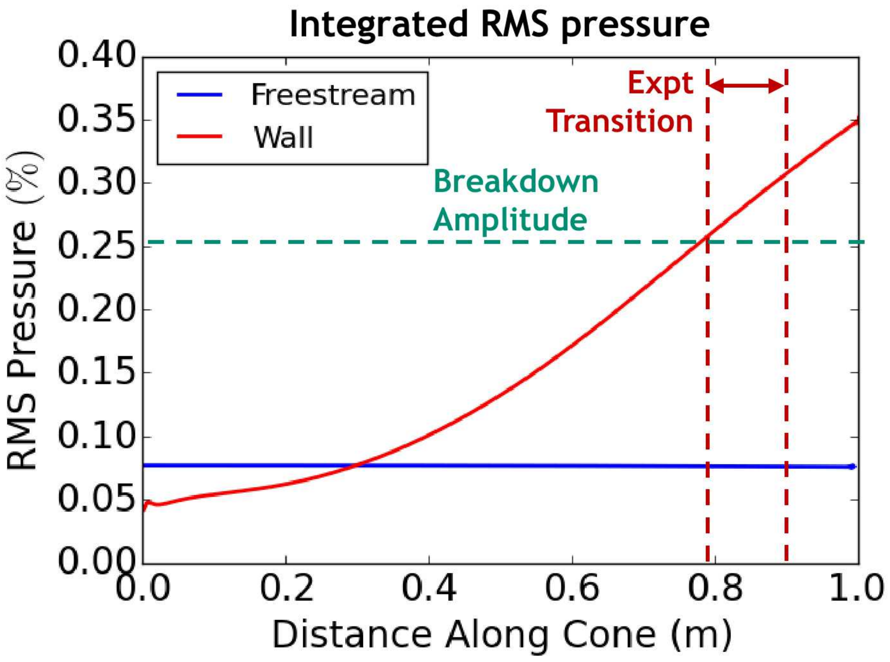


Example: Sharp cone at zero AOA

Steps 4-5: Integrate PSDs to get RMS pressure. Predict transition.

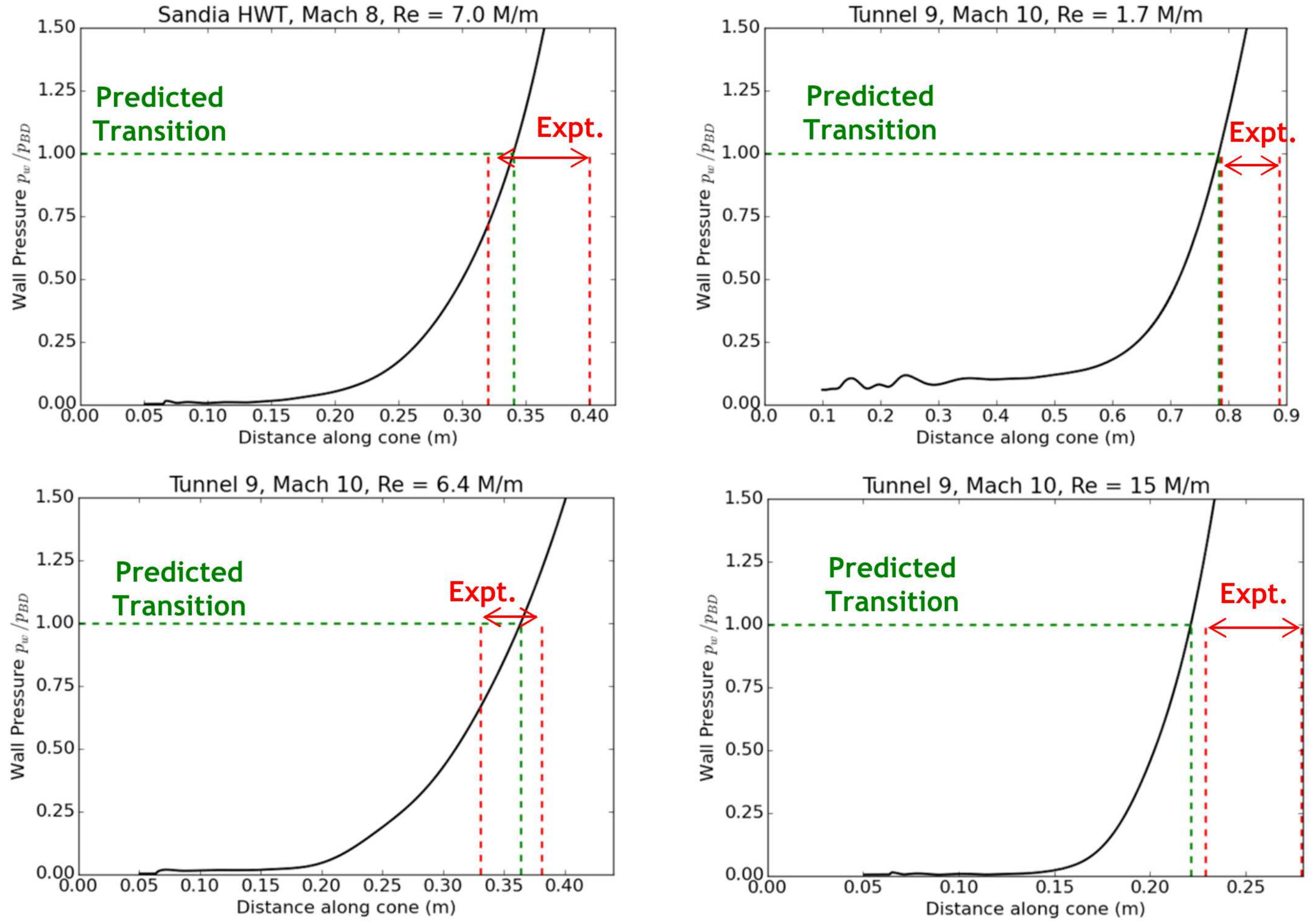
$$p_{rms}(x,y) = \sqrt{\int_0^{\infty} p'(x,y,\omega)d\omega}$$





Transition Prediction

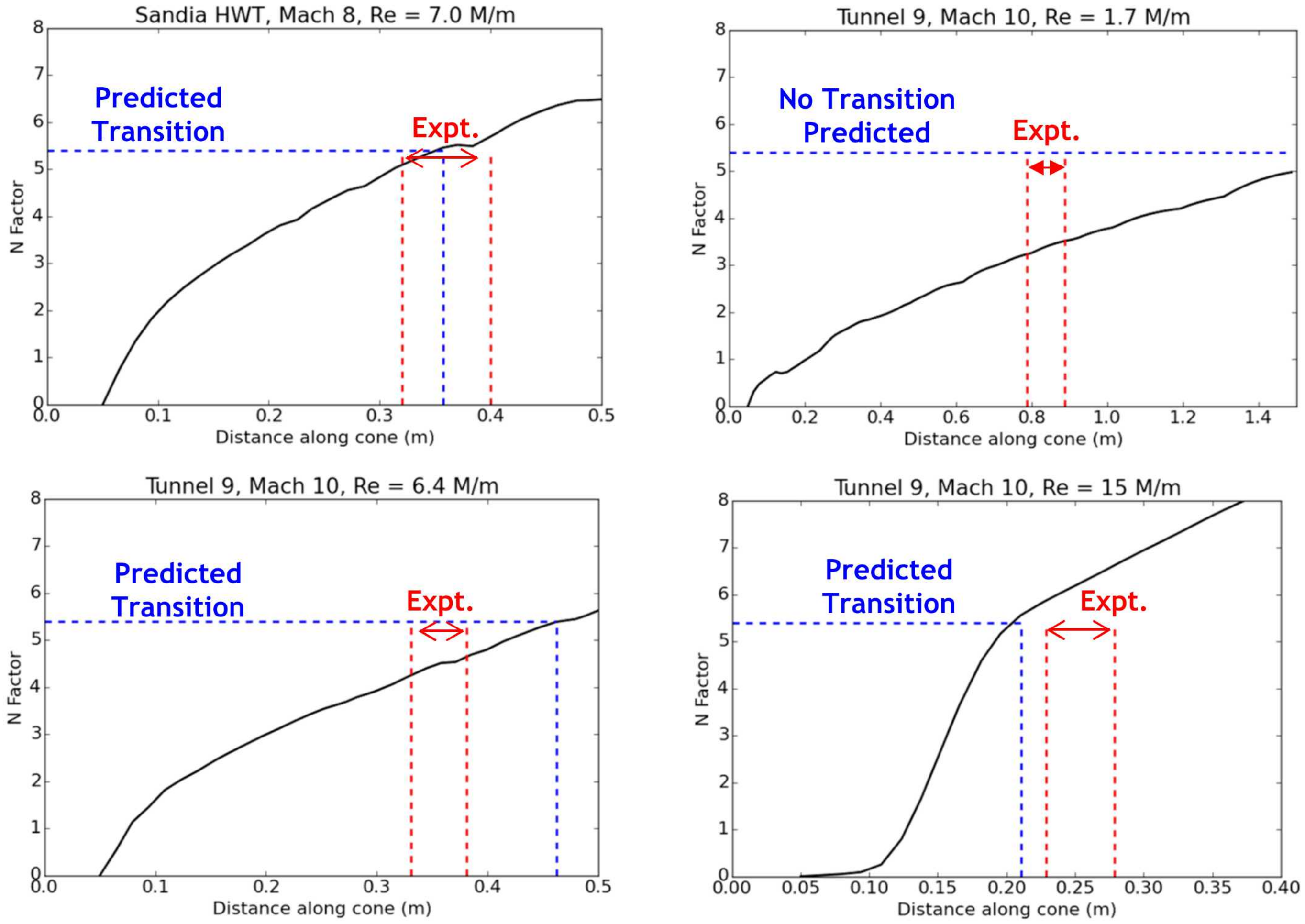
- Transition prediction method was applied to sharp cones (2^{nd} mode transition)
- Good agreement with experiment has been found for all cases tested so far
- Method captures the correct variation with freestream Reynolds number for Re = 1.7-15 million/m.



Tunnel	Mach	Re (M/m)	Xtr (m)
Sandia HWT8	7.9	7.0	0.36
AEDC T9	9.9	15	0.25
AEDC T9	9.6	6.4	0.36
AEDC T9	9.4	1.8	0.84

Comparison with conventional methods

- The same cases were analyzed with conventional STABL transition analysis software
- Transition was predicted using a transition N factor of 5.4 (chosen to match Sandia HWT results)
- Conventional analysis is not correct because it does not account for receptivity.



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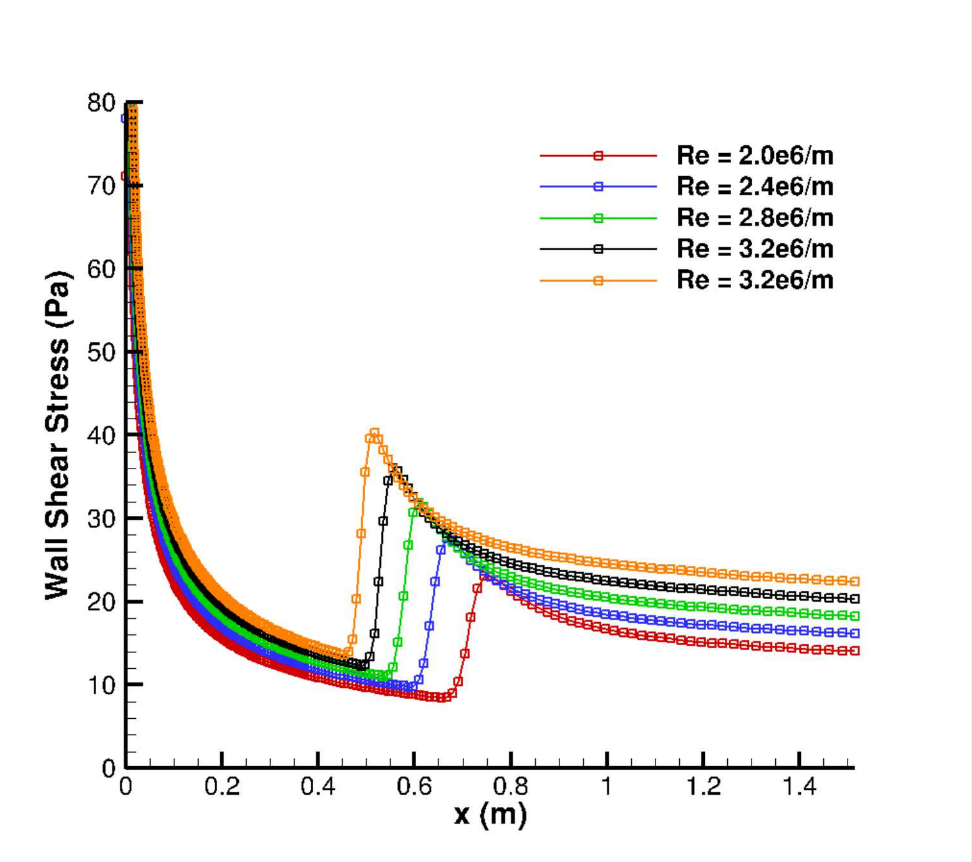
Coupled Transition Prediction + RANS

Transition prediction can now be loosely coupled with RANS CFD:

1. Run laminar CFD
2. Run optimization method for transition prediction
3. Turn on turbulence trips
4. Run tripped RANS CFD



Unlike other approaches:

- CFD and stability analysis use same mesh
- No extraction/interpolation of flow field is needed
- No user intervention is required

Summary & Ongoing Work

Summary:

- Optimization method shows promise for solving some of the problems with existing transition prediction methods.
- Optimization method captures transition in wind tunnels and accounts for changes in freestream noise with Reynolds number

Ongoing Efforts:

- Validation against more complex flows
  - Blunt Cones
  - Cones at AOA
  - BOLT
  - Cone with fin
  - Re-entry F
  - Past RV flights

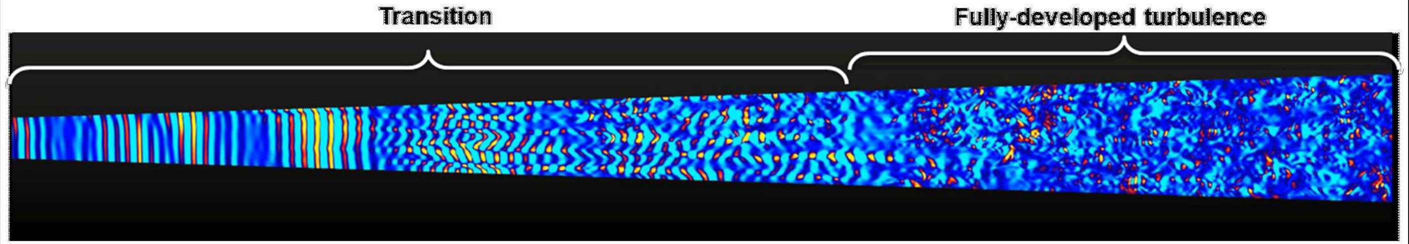
Outlook: Create a coupled CFD solver that combines in-situ transition prediction with transitional RANS modeling.

Questions?

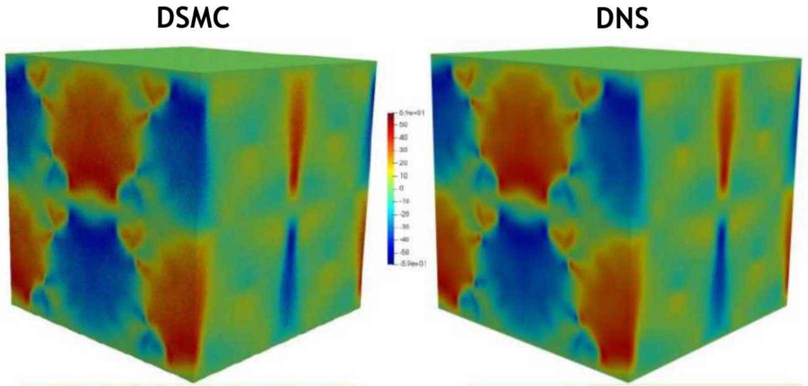
Backup

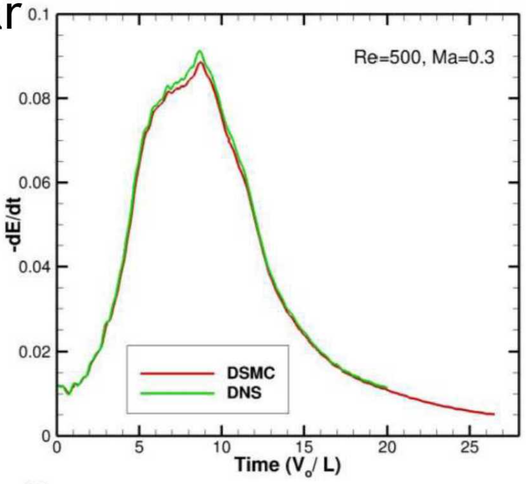
Direct numerical simulations of high Mach, high Re transition and turbulence.

Sharp Cone
Mach 8 Flow



Comparison of continuum and molecular simulations of compressible turbulence





Stability and transition analysis of complex flow fields