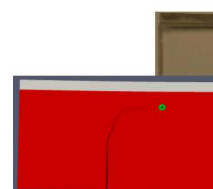
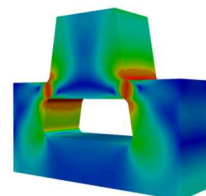
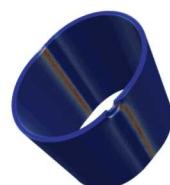
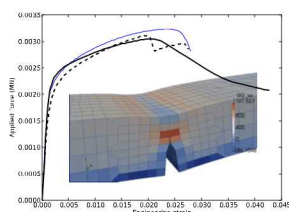
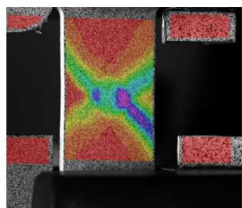


ATA/I558 Interchange

EOS for Shock



Presenter: David Kittell
[DRAFT] Anticipated UUR Content

ATA Site Visit

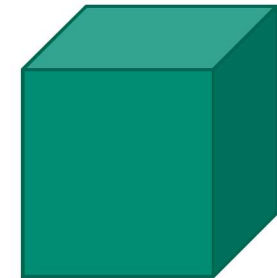
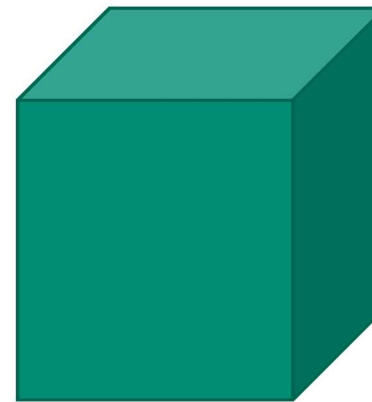
December 11, 2019

Extreme Material Behavior and Equation of State (EOS)

Problems that require the use of an EOS often coincide with and require simulation tools to handle the following

- Multidimensional domains (2D & 3D)
- Multiple materials in contact with each other
- Large deformations (not finite strains)
- **Shock waves**
- High strain rates (e.g., $> 1\text{e}5\text{-}1\text{e}7\text{ s}^{-1}$)
- Elastic-plastic behavior
- Thermal softening
- High explosive detonation
- Fracture (spall, especially)

Think of EOS as stress
due to volumetric
compressibility



304 Stainless Steel 7580 kg/m³

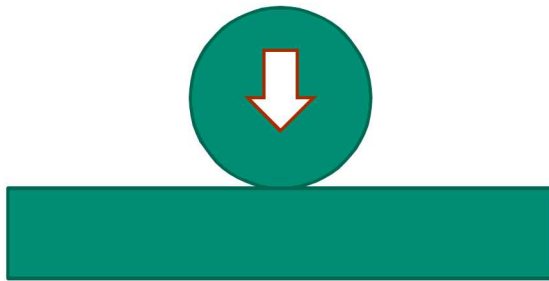
304 Stainless Steel 9000 kg/m³
(For a short $\sim\mu\text{s}$ timescale)

Demonstrative Example - copper penetrating a steel plate

OFHC Copper Sphere Projectile

- 4 mm radius
- Initial velocity 4.52 km/s
- Adaptive mesh refinement

That's Mach 13 !!!
(= 4520 / 344)

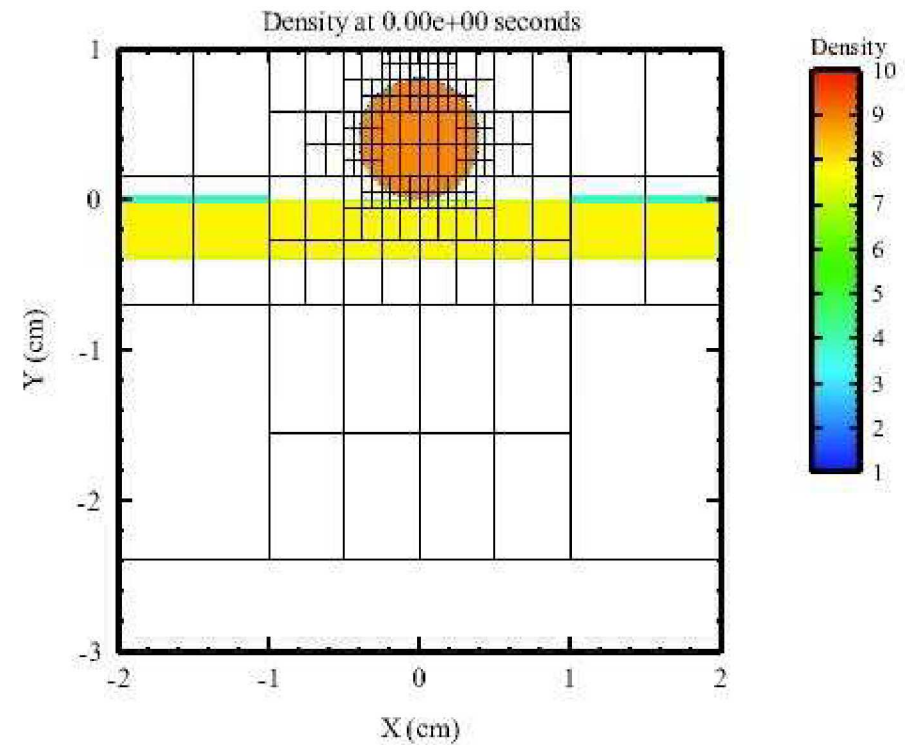


4340 Steel Target Plate

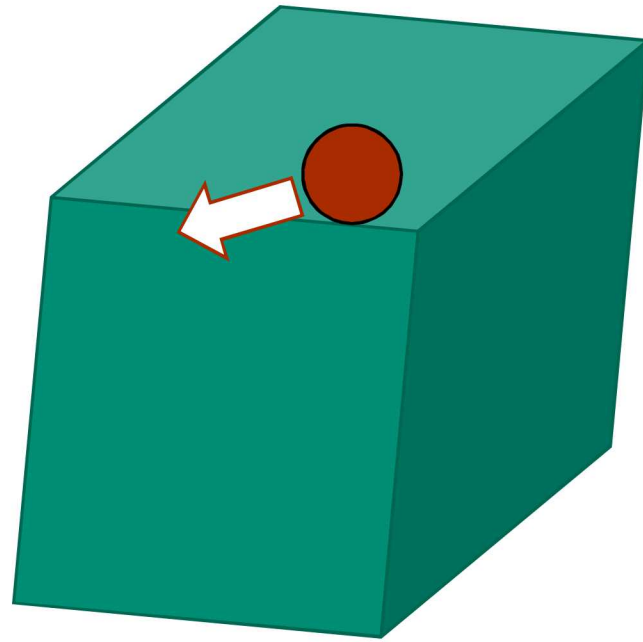
- 4 mm thick

Each Material...

- Has a user-defined **Mie-Grüneisen EOS** with 5 model parameters
- Assumes a constant isotropic yield surface (Elastic-Perfect Plasticity von Mises, EPPVM) when shocked, decreasing to zero with thermal softening
- Has 3 user-defined strength model parameters



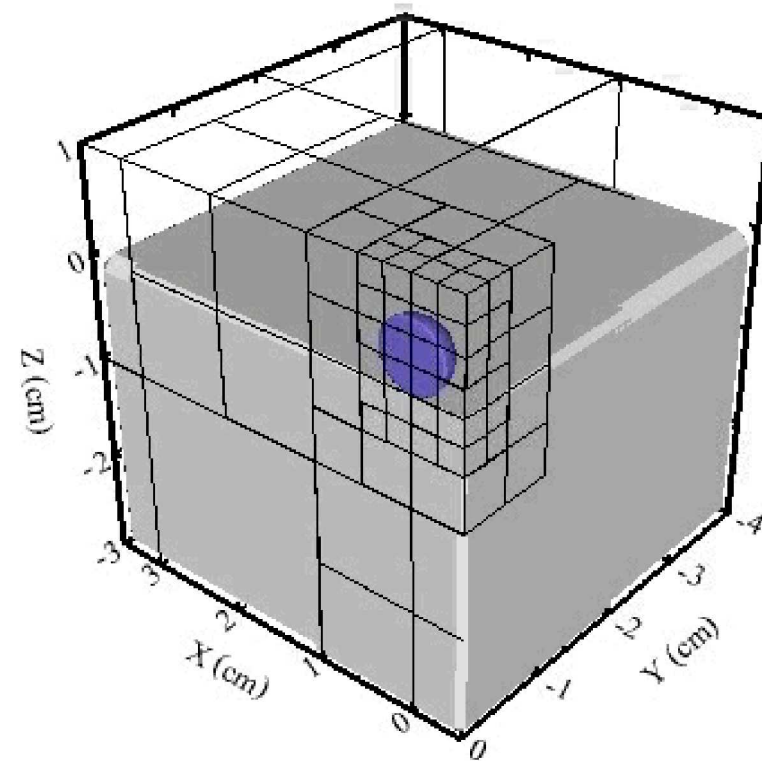
Demonstrative Example - oblique impact




14.95 degrees

-4.83 km/s in X
-1.29 km/s in Y

That's Mach 14.5 !!!

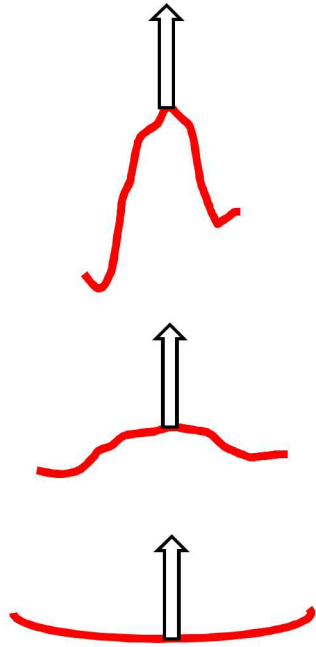


0.00e+00 secs.

Each Material...

- Is 2024-T4 Aluminum
- Uses a predefined **Mie-Grüneisen EOS** from the “EOS_data” file
- Uses a predefined Steinberg-Guinan-Lund viscoplastic strength model from the “VP_data” file

Demonstrative Example - explosively formed projectile (multi-mat)



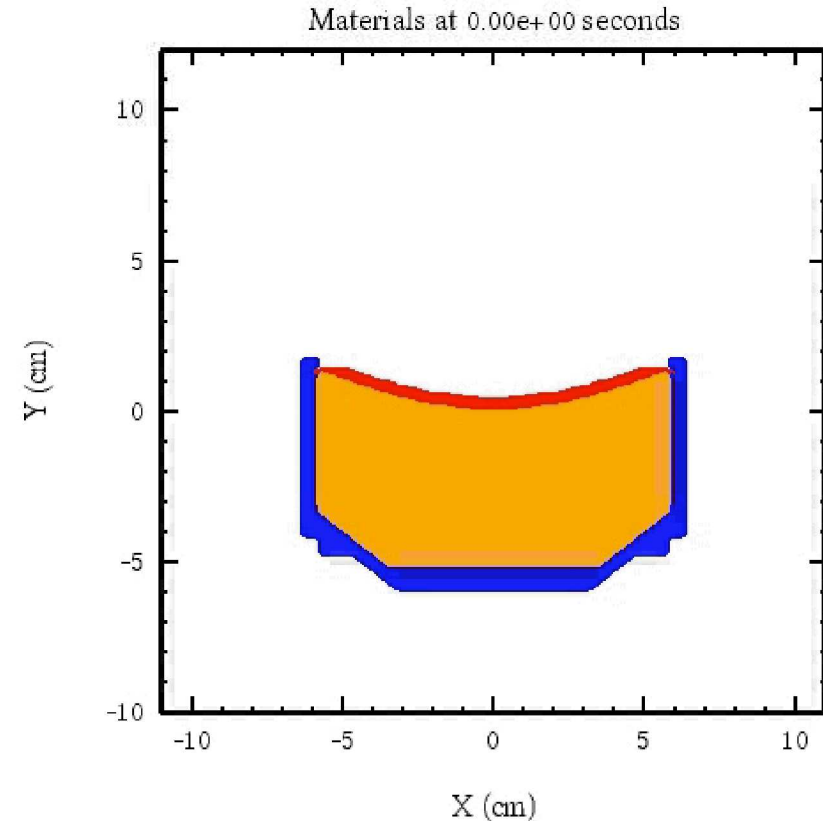
Red = OFHC Copper Lens

- Formed as the gas (i.e. detonation products) accelerates and then deforms the metal, thus launching it into flight
- Similar to how a shape charge works
- Metal stays in the solid phase
- Same **Mie-Grüneisen EOS** from Cu-impact example
- Same EPPVM strength model from Cu-impact example

HE Detonation
Products (High
T's and P's)

Orange = High Explosive

- Solid model assumes correct density at zero pressure
- Programmed burn converts HE to gas products
- Predefined **Jones-Wilkins-Lee EOS** from "EOS_data" file



6

Demonstrative Example - meteor strike on Earth (geological material)



Notes

- 20 km/s Earthstrike forms a ~8 km wide crater
- Asteroid and target are predefined **ANEOS models** for quartz (Analytic Equation of State Package)
- Strength model is predefined BDL (Brittle Damage with Localized Thermal Softening, by Dave Crawford)

Some challenges exist for EOS / shock at SNL

- Where can I find an EOS to use?
 - Internal SNL ASCII data files
 - Journal papers
 - Experiments may be needed = \$\$\$
- What is the reliability of “EOS_data” and “VP_data” files?
- Are the EOS parameters sufficient, or do I need the analytic equations as well?
- Can I write my own EOS and if so, how?
- Is the Mie-Grüneisen EOS form standardized? – **NO**
- Should the EOS be wide ranging (e.g., solid-liquid transition, critical point)?

The EOS_data file (SNL-CTH)

- The “go-to” EOS library for users of CTH
- Hundreds of materials to choose from in one single ASCII file
- Little to no documentation remains of the model pedigree – it’s only the parameters now
- Uses CTH’s material model interface only

```

Terminal
File Edit View Search Terminal Help

in single quotes, ending with a nonnegative integer (except for last
material).
--If NUK>0, the next input set lists a pair of character parameters for
each keyword. The first parameter is the model name (MGR, SES, etc.),
and the second parameter is the material name (IRON, HMX, etc.). This
list must also be terminated with a nonnegative integer (except for
the last material).
--The first predefined material should be 'USER'. Input parameters should
be zero except for any default values to be specified.
--Input for a model is terminated by giving a negative value to the last
integer on the last line of input for the last predefined material.

MGR -----Mie-Gruneisen EOS-----
22 0 0 keywords:
'R0' 'T0' 'CS' 'S1' 'G0' 'CV' 'ESFT' 'RP' 'PS' 'PE' 'CE' 'NSUB' 'S2' 'TYP'
'R0' 'T0' 'S' 'G0' 'B' 'XB' 'NB' 'PWR'

'USER'
0 2.567981E-2 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 2.0 0 !

'ANTIMONY'
6.700 2.567981E-2 1.983E5 1.652 0.60 2.38E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters

'BARIUM'
3.705 2.567981E-2 1.30E5 1.30 0.757 2.11E10 0 0 0 0 0 0
0.2 1 0 0 0 0 0 0 0 2.0 1 !
modified GMX-6 parameters

'BERYLLIUM'
1.850 2.567981E-2 7.920E5 1.124 1.19 3.21E11 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
modified Steinberg parameters

'CADIUM'
8.639 2.567981E-2 2.434E5 1.73 2.27 2.58E10 0 0 0 0 0 0
-0.09 1 0 0 0 0 0 0 0 2.0 1 !
modified GMX-6 parameters

'CALCIUM'
1.547 2.567981E-2 3.602E5 0.948 1.20 7.22E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters

'CESIUM'
1.826 2.567981E-2 0.650E5 1.400 0.866 2.63E10 0 0 0 0 0 0
-0.01 1 0 0 0 0 0 0 0 2.0 1 !
new US-UP fit, G0, CV

'CHROMIUM'
7.117 2.567981E-2 5.173E5 1.473 0.88 5.57E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters, new G0

'COBALT'
8.820 2.567981E-2 4.752E5 1.350 1.90 4.78E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
modified GMX-6 parameters

'COPPER'
8.930 2.567981E-2 3.940E5 1.489 1.99 4.56E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters

'GOLD'
19.240 2.567981E-2 3.056E5 1.572 2.97 1.47E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters

'HAFNIUM1'
12.885 2.567981E-2 2.954E5 1.121 0.98 1.62E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters, low P

'HAFNIUM2'
12.885 2.567981E-2 2.453E5 1.353 0.98 1.62E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters, high P

'INDIUM'
7.279 2.567981E-2 2.419E5 1.536 2.38 2.52E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters, new G0

'IRIDIUM'
22.484 2.567981E-2 3.916E5 1.457 1.97 1.51E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters

'IRON_ALP'
7.850 2.567981E-2 4.605E5 1.456 1.65 5.12E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
new EOS for PTRAN, alpha phase

'IRON_EPS'
8.219 2.567981E-2 4.587E5 1.494 2.40 5.12E10 7.27E8 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
new EOS for PTRAN, epsilon phase

'LEAD'
11.350 2.567981E-2 1.980E5 1.580 2.77 1.40E10 0 0 0 0 0 0
-0.08 1 0 0 0 0 0 0 0 2.0 1 !
modified GMX-6 parameters

'LITHIUM'
0.530 2.567981E-2 4.645E5 1.133 0.81 4.17E11 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters

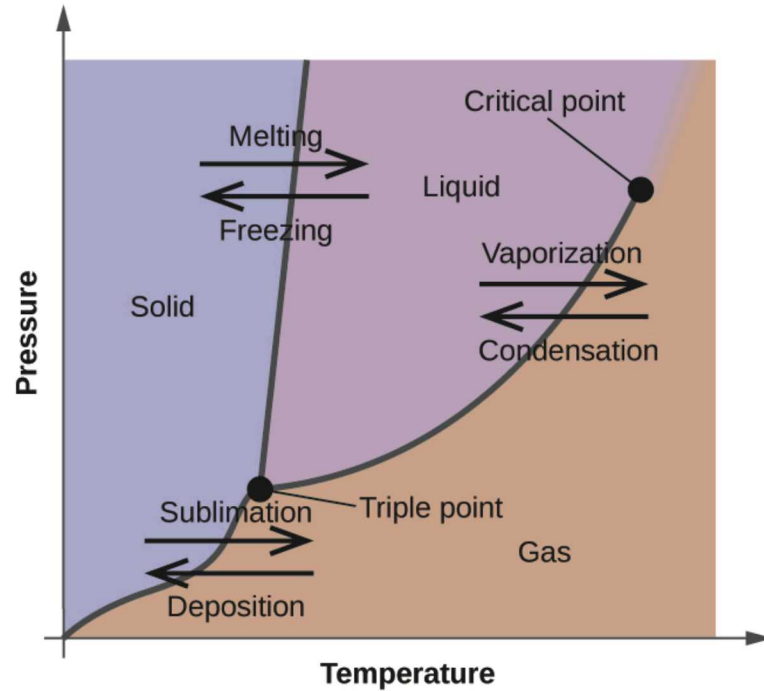
'MAGNESIUM'
1.740 2.567981E-2 4.492E5 1.263 1.42 1.19E11 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
GMX-6 parameters

'MERCURY'
13.540 2.567981E-2 1.449E5 2.150 2.715 1.44E10 0 0 0 0 0 0
-0.18 1 0 0 0 0 0 0 0 2.0 1 !
new EOS parameters

'MOLYBDENUM'
10.210 2.567981E-2 5.030E5 1.265 1.49 2.89E10 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 2.0 1 !
modified GMX-6 parameters
24,1
0%

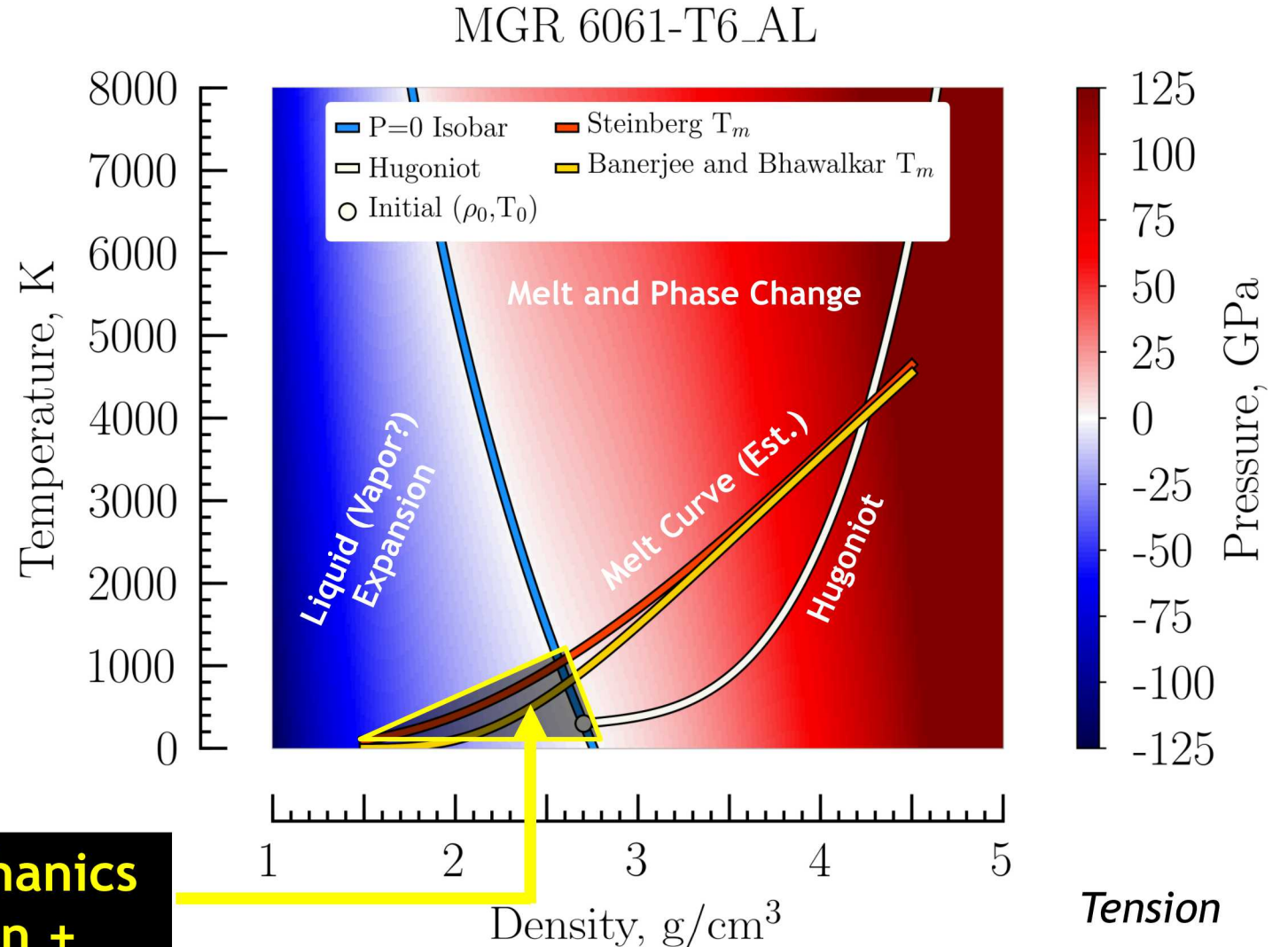
```

Tabular EOS with Phase Regions



Generic phase diagram from
<https://opentextbc.ca/chemistry/chapter/10-4-phase-diagrams/>

Solid Mechanics
“Tension + Fracture”



10 EOS from the Literature

This is my preferred method as an analyst...



Paper, e.g. Sewell and Menikoff, *AIP Conf. Proc.* **706** (2004)

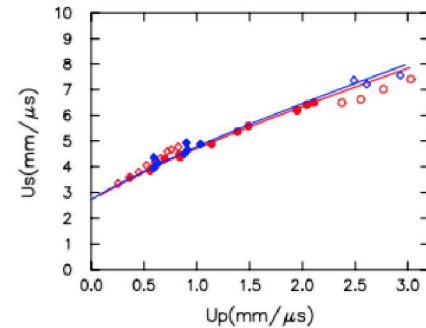


FIGURE 1. Mechanical response of HMX. Red and blue curves are isotherm and Hugoniot loci, respectively, from EOS model. Blue symbols are Hugoniot data [6, pp. 595–596] and red symbols are isothermal data [1, 2]. Open circles are above the pressure induced phase transition reported in [1] and are not used in the calibration of the EOS model.

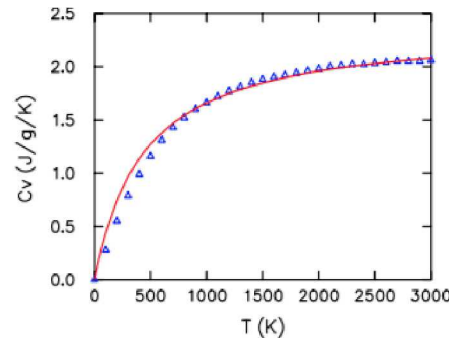


FIGURE 2. Specific heat. Symbols are from molecular dynamics calculations [3, p. 96] and red curve is fit, Eq. (3), used for the EOS model.

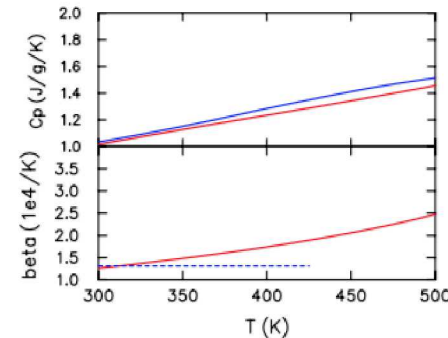
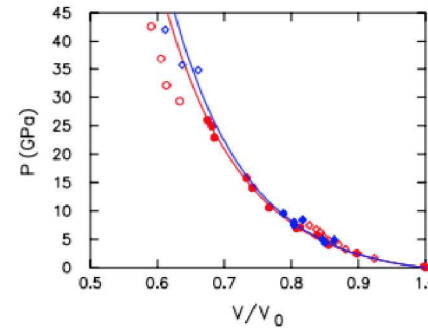


FIGURE 3. Specific heat [7] and the volumetric coefficient of thermal expansion [8] at atmospheric pressure. Blue curve is experimental data and red curve is model EOS.

Implicit,
Non-linear analytic Eqns.



A Birch-Murnaghan form is used for the cold curve

$$P_0(V) = \frac{3}{2} K_0 \left[\left(\frac{V}{V_0} \right)^{-7/3} - \left(\frac{V}{V_0} \right)^{-5/3} \right] \times \left(1 + \frac{3}{4} [K'_0 - 4] \left[\left(\frac{V}{V_0} \right)^{-2/3} - 1 \right] \right). \quad (2)$$

The specific heat is assumed to have the form

$$\hat{C}_V(\tilde{T}) = \frac{\tilde{T}^3}{c_0 + c_1 \tilde{T}^2 + c_2 \tilde{T}^2 + c_3 \tilde{T}^3}, \quad (3)$$

where $\tilde{T} = T/\theta(V)$ is a scaled temperature. This form has the asymptotically correct limits: $\hat{C}_V(T) \propto T^3$ for small T and $\hat{C}_V(T) \rightarrow \text{const}$ for large T . The Debye temperature is assumed to have the form

$$\theta(V) = \theta_0 \left(\frac{V_0}{V} \right)^a \exp[b(V_0 - V)/V]. \quad (4)$$

Consequently, the Grüneisen coefficient is given by

$$\Gamma(V) = - \frac{V}{\theta} \frac{d\theta}{dV} = a + b \frac{V}{V_0}. \quad (5)$$

The pressure and internal energy are determined by the standard thermodynamic relations $P = -\partial_V F$ and $e = F - T \partial_T F$.



Parameter Values

TABLE 1. Model EOS parameters for β -HMX.

ρ_0	1.9	g/cm ³
K_0	16.5	GPa
K'_0	8.7	—
a	1.1	—
b	-0.2	—
c_0	5.265×10^{-1}	K·kg/MJ
c_1	3.073×10^2	K·kg/MJ
c_2	1.831×10^5	K·kg/MJ
c_3	4.194×10^2	K·kg/MJ

A few dozen EOS data points (if lucky)

Worked Example for the Mie-Grüneisen EOS

The density of 6061-T6 aluminum is increased from 2.703 g/cm³ to 2.9 g/cm³ and the temperature is increased from 298 K to 350 K, what is the pressure?

Hugoniot Parameters

$$\rho_0 = 2.703 \text{ g/cm}^3$$

$$C_S = 5.22 \text{ km/s}$$

$$S_1 = 1.37$$

Grüneisen Parameter

$$\Gamma_0 = 1.97$$

Specific Heat

$$C_V = 922 \text{ J/kg}\cdot\text{K}$$

Pressure EOS

$$P(\rho, T) = \boxed{P_H}(\rho) + \rho_0 \Gamma_0 C_V [T - \boxed{T_H}(\rho)]$$

Hugoniot Shock-Particle Velocity Curve

$$U_S = C_S + S_1 u_p$$

...Doesn't look too bad, right?

Worked Example for the Mie-Grüneisen EOS

The equation will change for a different U_s -up and the integral requires numerical integration techniques. The answer is ~6.1 Gigapascals or 61 kilobars

$$P(\rho, T) = \frac{\rho_0 C_S^2 \left(1 - \frac{\rho_0}{\rho}\right) \left[1 - \frac{\Gamma_0}{2} \left(1 - \frac{\rho_0}{\rho}\right)\right]}{\left[1 - S_1 \left(1 - \frac{\rho_0}{\rho}\right)\right]^2} + \rho_0 \Gamma_0 C_V T$$

?

$$- \rho_0 \Gamma_0 C_V e^{\Gamma_0 \mu} \left[T_0 + C_V^{-1} \int_0^\mu e^{-\Gamma_0 \mu} \mu^2 U_S \frac{dU_S}{d\mu} d\mu \right]$$

where, $\mu = 1 - \frac{\rho_0}{\rho}$

Few analytic EOS are explicit, partly because the EOS data that is generated is not a direct measurement of the pressure or temperature. To arrive at this kind of explicit answer, one has to use ***thermodynamic identities***. For these reasons, each EOS is usually implemented as its own model.