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ON THE APPLICATION OF MBPE FOR MITIGATING CORRUPTED DATA IN RADAR APPLICATIONS

MASTER'S THESIS DEFENSE

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OUTLINE

- Introduction
- Literature Review
- Theoretical Background
- Methods Used
- Simulation Results
- Summary of Results
- Conclusion

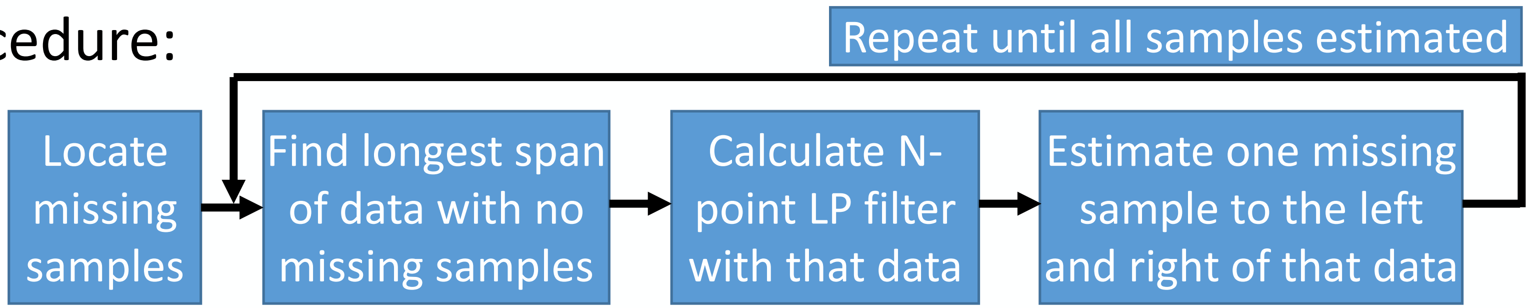
INTRODUCTION

- In radar transmission and receiving, missing data or gaps in the data can occur for many reasons, including [1]:
  - Hardware failures
    - Transmission or reception is incorrect
    - Transmits when it should be receiving
  - Signal Jamming
  - Many targets
  - Burst waveforms
  - Poor SNR
- If we do not account for the missing data, false frequencies can be introduced and correlation sidelobes are increased significantly
- We would like to investigate methods of data interpolation to mitigate these issues

REVIEW OF EXISTING METHODS

REVIEW OF EXISTING METHODS

- Linear Prediction Filter [1]
  - Algorithm for interpolating missing complex data with spectrally consistent estimates
  - Procedure:



  - Examples demonstrated show virtually no difference in the complex signal and its spectrum between the original data and the interpolated version
  - Limitations:
    - Data must be coherent
    - Data must be approximately stationary
    - Works best with pure tones

REVIEW OF EXISTING METHODS

- Adaptive Linear Prediction using AR Models [2]
  - Adaptive algorithm for restoration of lost sample values that can be locally described as AR processes
  - Restrictions:
    - Positions of missing data must be known
    - Missing data must be embedded in a reasonably large neighborhood of known samples
  - Uses a suboptimal prediction approach by:

Initialize estimate vector $\hat{x}(t)$ → Use $\hat{x}(t)$ to minimize NLS error function to get AR parameter estimates $\hat{a}$ → Use $\hat{a}$ to obtain final approximations of the missing data

REVIEW OF EXISTING METHODS

- Adaptive Linear Prediction using AR Models [2]
  - Results:
    - Used short audio signals to interpolate missing data; calculated interpolation errors and conducted listening tests

Short bursts (~16 samples) had nearly unnoticeable interpolation errors

Longer bursts (~50 samples) sometimes have noticeable errors

Very short signals (≤6 samples) with no out-of-band components obtain better results with standard band-limited interpolation method

REVIEW OF EXISTING METHODS

- Matched-Filter-based Bandwidth Extrapolation using AR Models [3]
  - "generate a signal that could improve the range resolution after extrapolation but does not suffer the problem of having the response of individual targets starting and ending at vastly different positions as in the case of the dechirped signal"
    - Traditional BWE uses stretch processing before conducting BWE
    - The authors of this paper propose using matched-filtering instead
  - Method extrapolates range-Doppler (RD) processed data using linear prediction

REVIEW OF EXISTING METHODS

- Matched-Filter-based Bandwidth Extrapolation using AR Models [3]
  - Remarks

    Assumes clutter is in different Doppler bins than moving targets

    Order of AR model must be greater than or equal to the number of targets and less than 1/3 the total number of samples

  - Results
    - RD maps present similar results to conventional RD processing at twice the bandwidth
    - MF-BWE also results in SNR improvement of around 1.5 dB in the shown situation

OUR IDEA

- Apply model-based parameter estimation (MBPE) in frequency-domain to interpolate missing data

THEORETICAL BACKGROUND

PRONY'S METHOD

- Model-Based Parameter Estimation (MBPE) was developed based on Prony's method (1795) [4,5]:

$$\hat{f}(x) = \frac{1}{2}\sum_{i=1}^{M} A_i e^{\lambda_i t} e^{\pm j\phi_i}$$

$\lambda_i = \sigma_i + j\omega_i$	σ_i	ω_i	ϕ_i	A	M
Eigenvalues	Damping components	Angular frequency components	Phase components	Amplitude components	Total number of exponentials to fit

- Which we can describe as

$$\hat{f}(\Delta_t n) = -\sum_{m=1}^{M} \hat{f}[\Delta_t(n-m)]P_m$$

$$\Rightarrow z^M - P_1 z^{M-1} - \cdots - P_M = \prod_{m=1}^{M}(z - e^{\lambda_m})$$

- These two representations are used in Prony's method to solve for the e^{λ_m} in the fitting model

INTRODUCTION TO MBPE

- Miller first presented a direct extension of Prony’s method as time-domain (TD) MBPE [7,8]
- Which was then converted into function and derivative sampling forms of FD MBPE [7,8]
- In a further paper, Miller extended the single sample forms of MBPE presented in [7,8] into a multiple-frequency, multiple-derivative form of MBPE, which is the formulation used in this research [9]

MATHEMATICAL BASIS FOR MULTI-FREQUENCY, MULTI-DERIVATIVE (MFMD) MBPE

- Frequency-domain Fitting Model (FM) [9]:

$$F(X) = F_p(X) + F_{np}(X) = \sum_{\alpha=1}^{W} \frac{R_\alpha}{(X - s_\alpha)} + \sum_{\beta=-Q}^{R} C_\beta X^\beta$$

$F_p(X)$	$F_{np}(X)$	R_α	s_α	α	W
Pole component	Non-pole component (accounts for driven response)	Residues	Poles	Dummy variable	Number of poles

β	Q	R
Dummy variable	Part of representation of non-pole term (typically 0 or 1)	Part of representation of non-pole term

MATHEMATICAL BASIS FOR MFMD MBPE

- Rather than use the $F_{np}(X) = \sum_{\beta=-Q}^{R} C_\beta X^\beta$ to represent the non-pole contribution, we can convert the FM to a rational function with $Q = R = 0$:

$$F(X) = \frac{N(X)}{D(X)} = \frac{\sum_{i=0}^{n} N_i X^i}{\sum_{i=0}^{d} D_i X^i}$$

  - Where we can change the order of the numerator and denominator terms to be unequal to approximate the $F_{np}(X)$ contribution, if desirable

MATHEMATICAL BASIS FOR MFMD MBPE

- Equation to find N_i and D_i coefficients:

$$\sum_{i=1}^{d} D_i \sum_{k=0}^{\min(i,t_j)} C_{t_j,t_j-k} \frac{i!}{(i-k)!} X_j^{i-k} F^{(t_j-k)}(X_j) - \sum_{i=t_j}^{n} N_i \frac{i!}{(i-t_j)!} X_j^{i-t_j} = -F^{(t_j)}(X_j) \quad (26)$$

- This equation has the constraint that $D_0 = 1$ in order to make it uniquely solvable
- $j = 0, \dots, M$; $t_j = 0, \dots, T_j$; C_{t_j,t_j-k} is the binomial coefficient; and $M \times (T_j + 1) \geq n + d + 1$

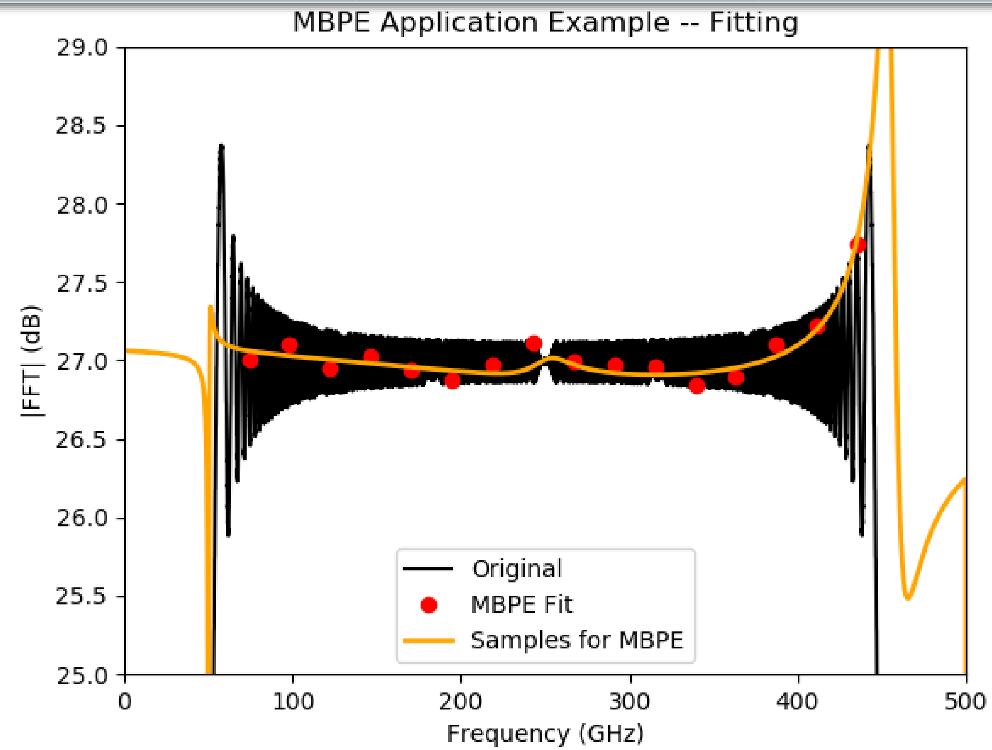
M	T_j
Total number of samples	Total number of derivatives

EXAMPLE APPLICATION OF MBPE

M	n	$d = n + 1$	T_j
17	4	5	0

N_i	22.022+j0.358	1.795-j32.887	-34.55-j2.799	-1.495+j20.839	
D_i	1	0.024-j1.501	-1.566-j0.043	-0.0047+j0.931	-0.0071-j0.02

$$F(X) = \frac{N(X)}{D(X)} = \frac{\sum_{i=0}^{n} N_i X^i}{\sum_{i=0}^{d} D_i X^i}$$



Simulation

SIMULATION SETUP

- $x(t) = \begin{cases} Ae^{j2\pi f_0 t} e^{j\pi\mu t^2}, 0 \le t \le T \\ 0, else \end{cases}$

A	f_0	B	T	μ	f_s	dt	τ_1	t
0.2	50 MHz	400 MHz	5 μs	B/T	1 GHz	1 ns	20 ns	0:dt:2T

- $rx(t) = \text{circshift}\left(x(t), \left(\frac{\tau_1}{dt}\right)\right)$
- Generate signal with missing gaps ($gap(t)$) by randomly selecting gap starting positions and setting those sections of the signal to 0
- Generate frequency-domain (FD) versions as $FFT\big(rx(t)\big) = Rx(X)$ and $FFT\big(gap(t)\big) = Gap(X)$
  - Where $X = e^{-1j2\pi f}$ and $0 \le f \le f_s$

ALGORITHM NOTES

- The algorithm applies the MFMD form of MBPE for a set of n, T_j, and $skip$ values
  - It then minimizes the error in FD magnitude between the provided signal and the MBPE fit

Iteration vectors:

- $n = 1:1:14$
- $T_j = 0:1:1$
- $skip = 101:10:301$

Additional Parameter Definitions:

- $M = len(known[0: skip: end])$
  - Where $known$ is the index vector representing samples between f_0 and $f_0 + B$ in FD
- $d = M - n - 1$

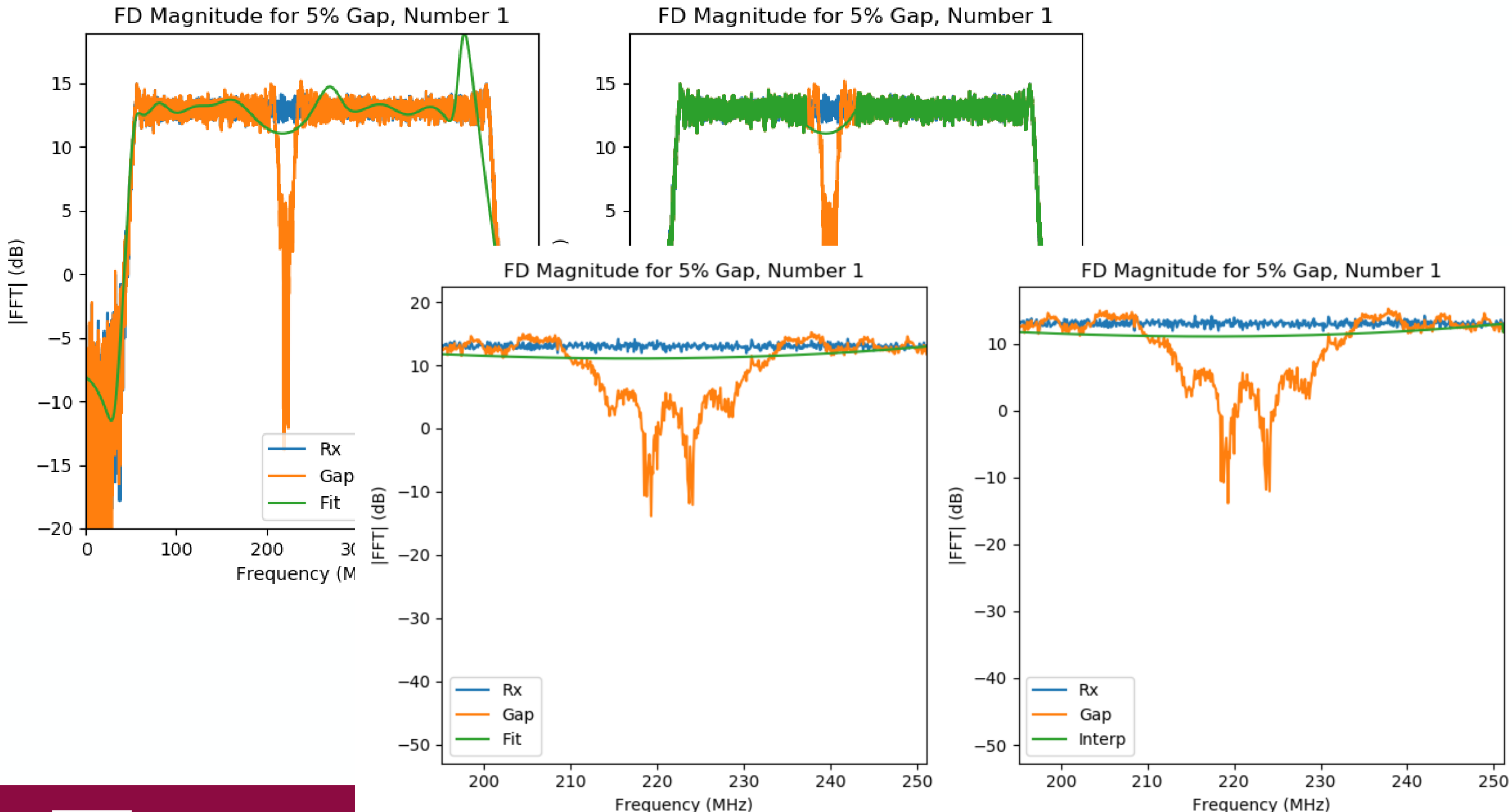
- Once it obtains the optimal values, it uses them to obtain a MBPE fit ($Fit(X)$)signal across the entire frequency band
- That data is then used to interpolate across the missing gap in the signal ($Interp(X)$)

FIT TO COMPLEX DATA

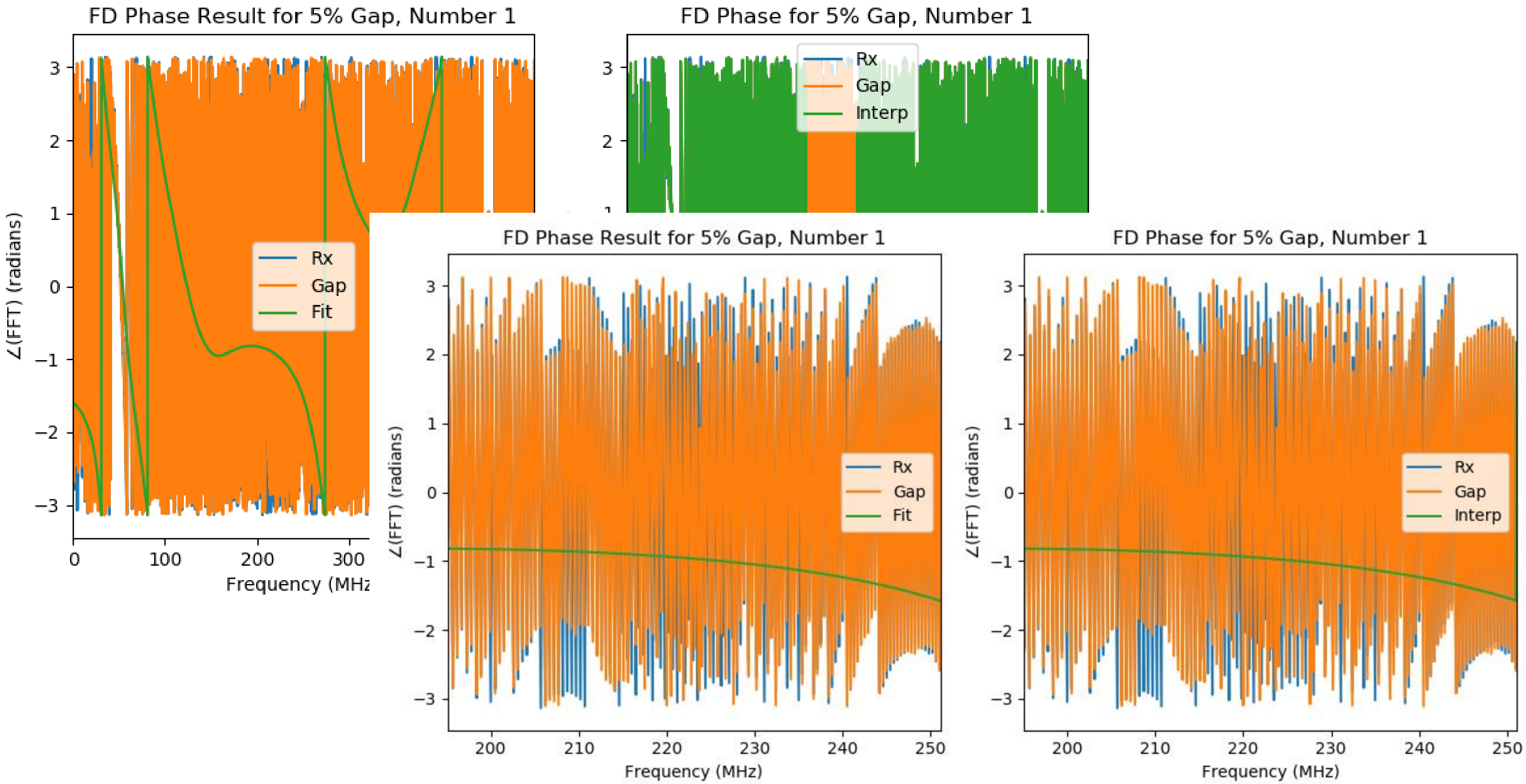
NOTES

- The MFMD algorithm was used to fit two examples (gap locations) of each of six gap sizes on complex FD data:
  - 1%, 2%, 5%, 10%, 15%, 20%
- The algorithm also does not include the data points in the 5% of the signal on either side of the gap, to minimize the effects of the Gibbs phenomena resulting from the time-domain gaps
- IPR/Correlation Functions are windowed using a Chebyshev window

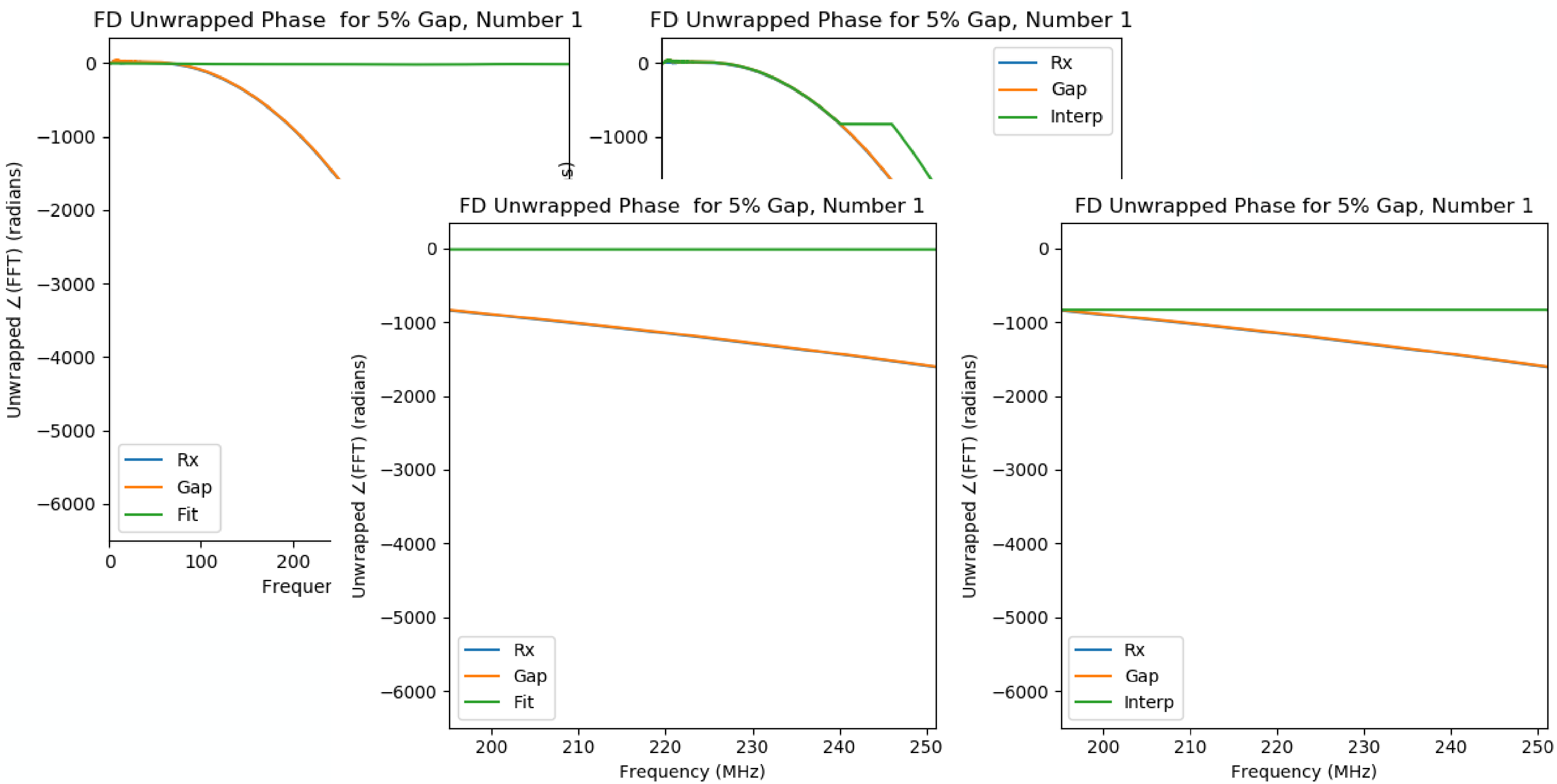
FREQUENCY-DOMAIN MAGNITUDE RESULT EXAMPLE USING COMPLEX SIGNAL



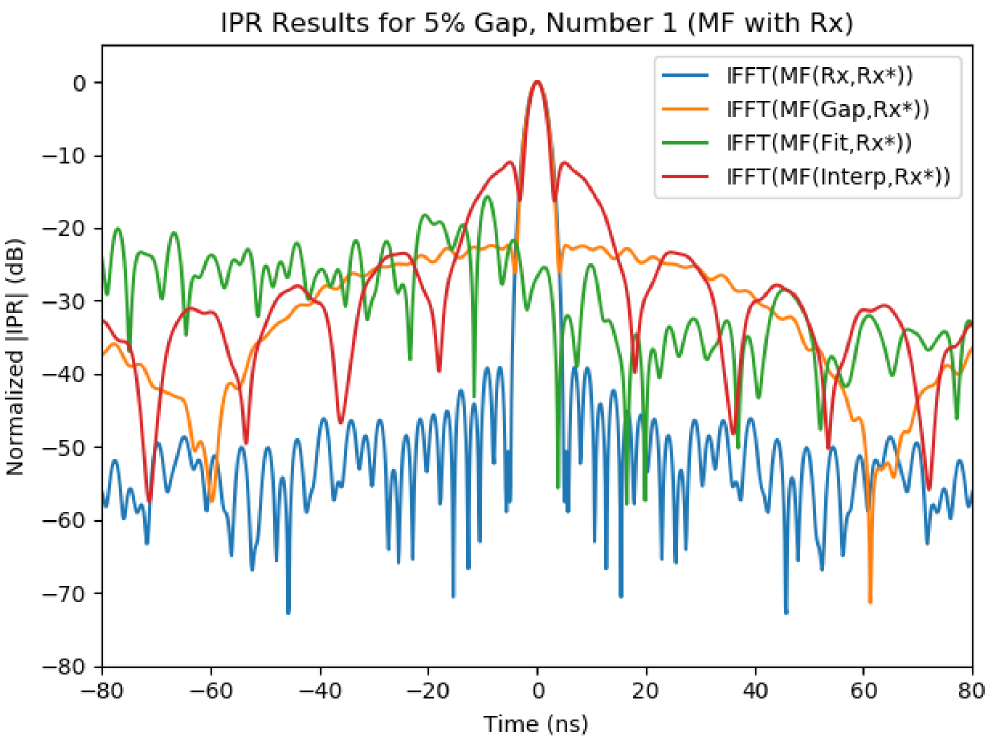
FREQUENCY-DOMAIN PHASE RESULT EXAMPLE USING COMPLEX SIGNAL



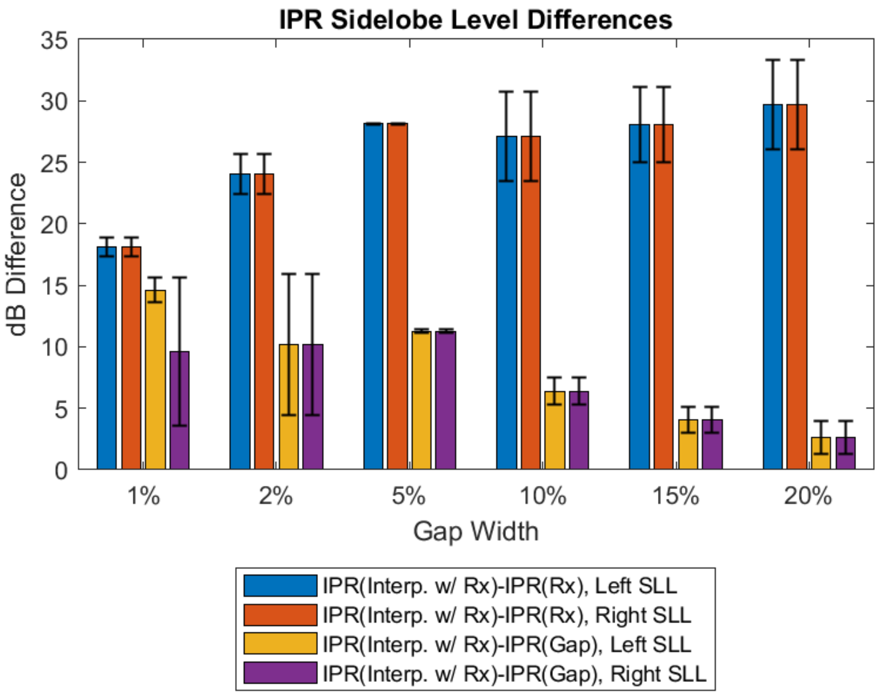
FREQUENCY-DOMAIN UNWRAPPED PHASE RESULT EXAMPLE USING COMPLEX SIGNAL



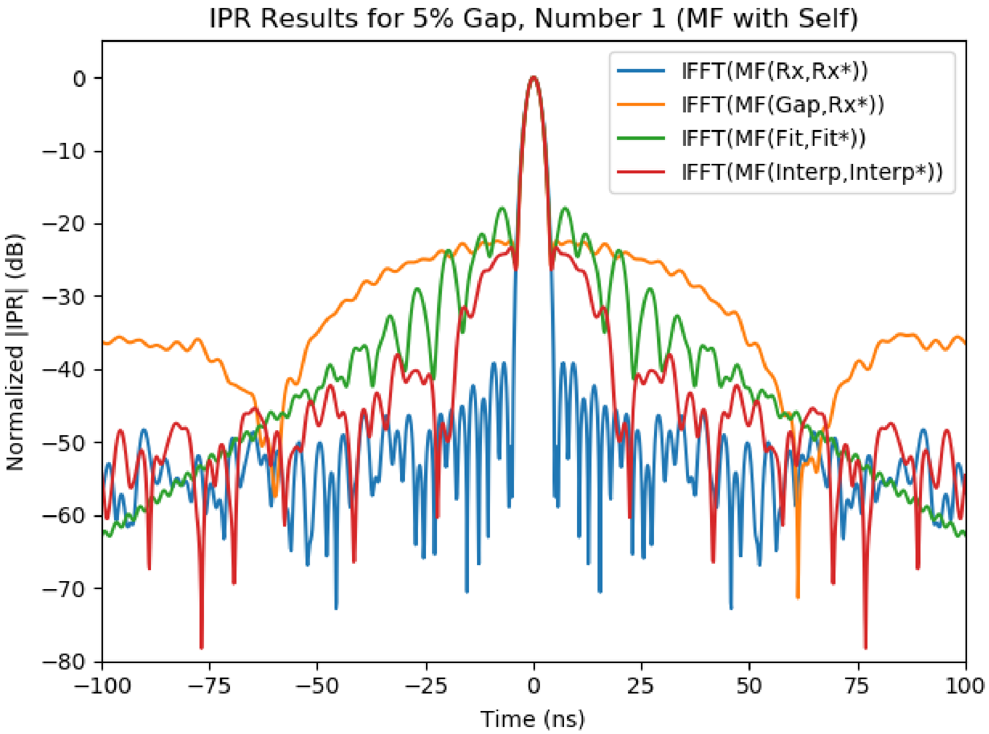
CORRELATION RESULT EXAMPLE USING COMPLEX SIGNAL (MF USING RX)



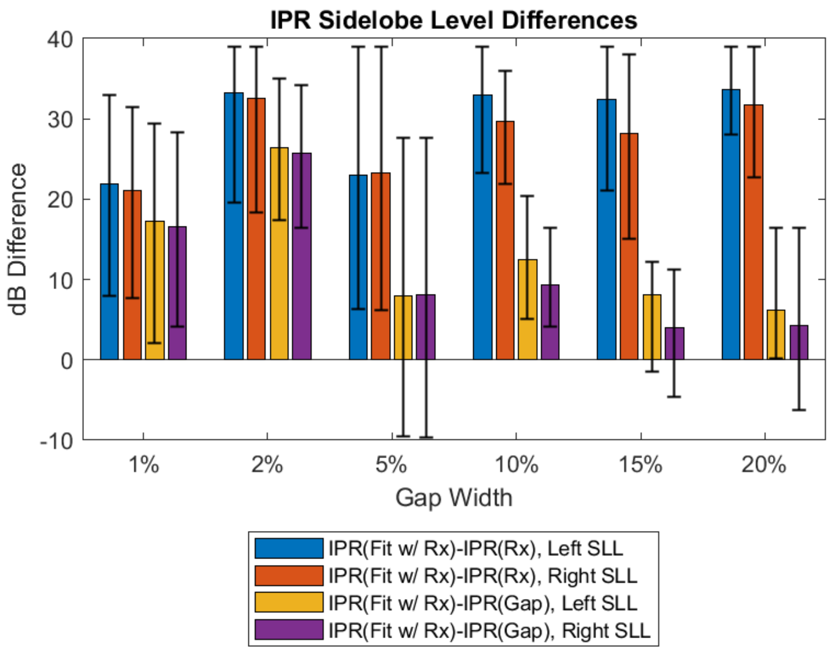
SIDELOBE LEVELS USING COMPLEX DATA

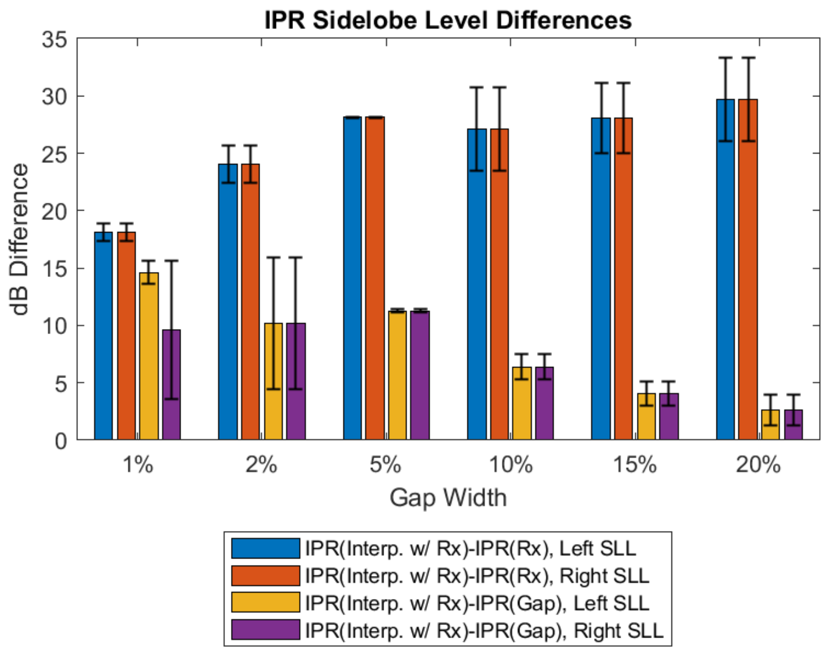


CORRELATION RESULT EXAMPLE USING COMPLEX SIGNAL (MF USING SELF)



SIDELOBE LEVELS USING COMPLEX DATA





SUMMARY OF RESULTS USING COMPLEX SIGNAL

- When using MFMD MBPE on the full complex FD signal, the fit in magnitude is good, but MBPE cannot handle the fast wrapping of the phase component
- This causes the sidelobe levels of the $Fit(X)$ and $Interp(X)$ IPRs to be higher than those for the $Gap(X)$ and $Rx(X)$ IPRs

NEXT STEP

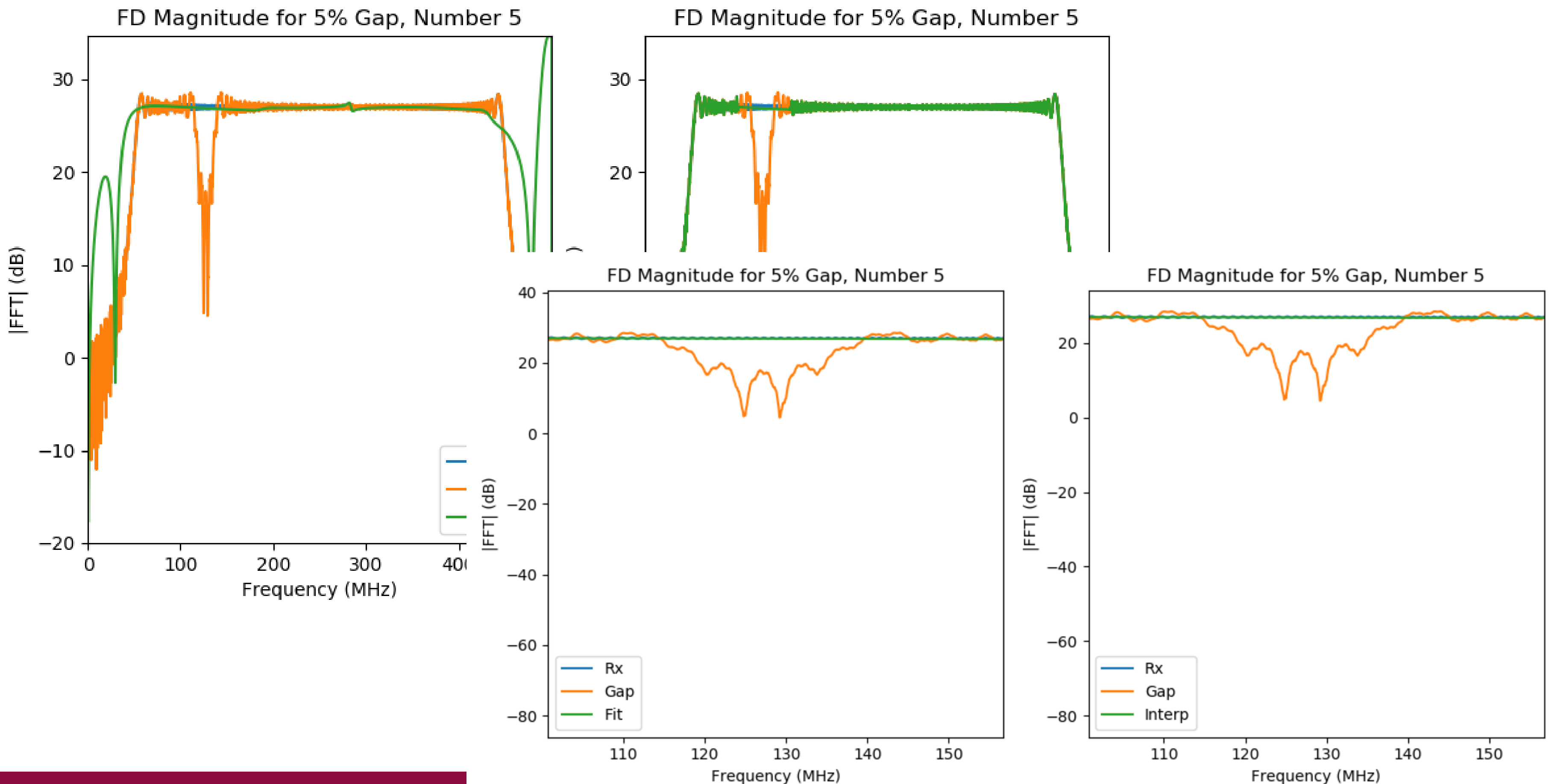
- To try to mitigate this issue, we will conduct MFMD MBPE on the magnitude and unwrapped phase of the signal separately
  - In other words, we will obtain parameters twice, once using $|Gap(X)|$ and once using $unwrap\left(\angle Gap(X)\right)$, which we will then rewrap and combine to obtain new versions of $Fit(X)$ and $Interp(X)$ that will hopefully lower the sidelobe level differences

FIT USING MAGNITUDE AND UNWRAPPED PHASE DATA SEPARATELY

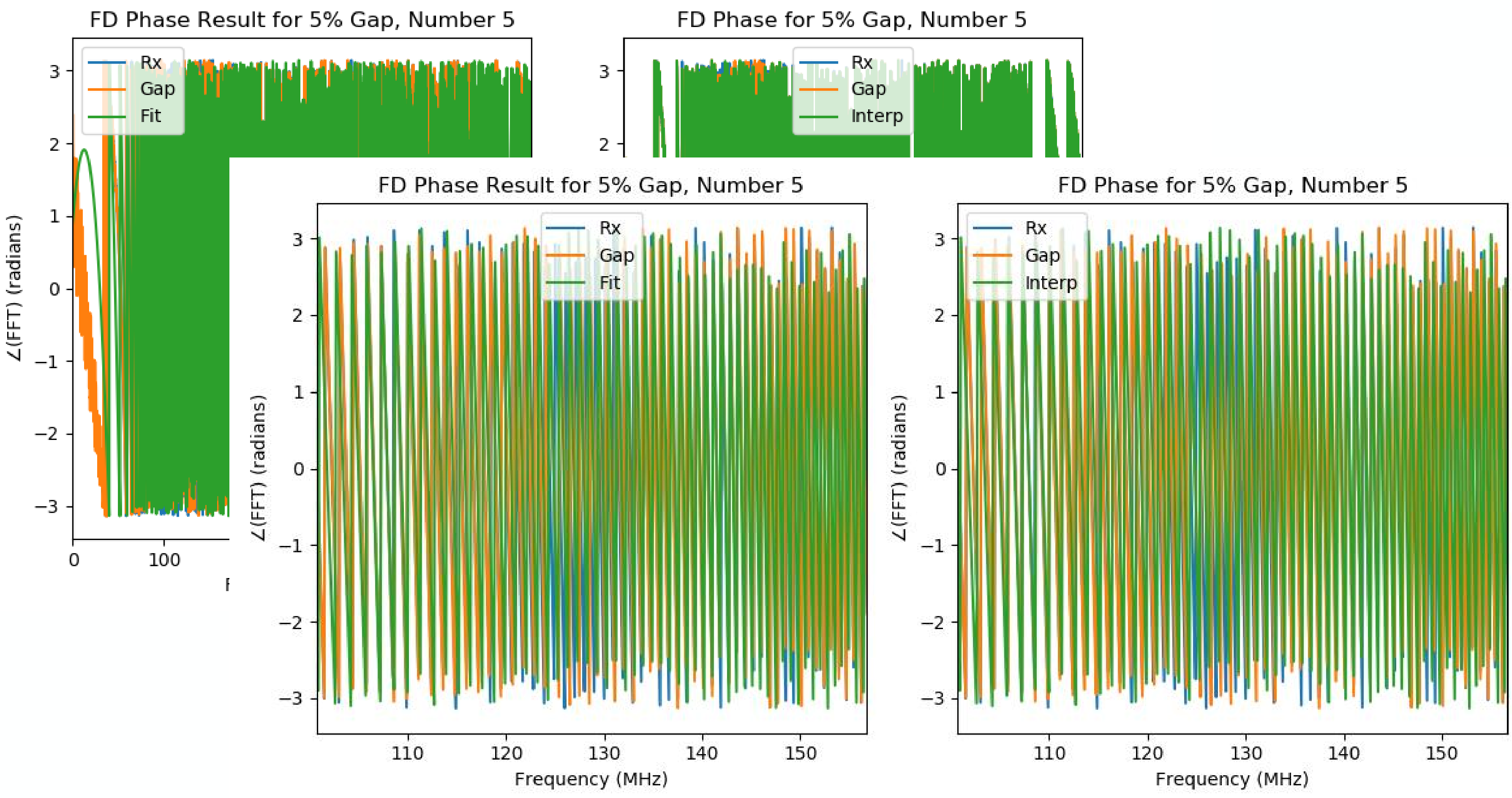
NOTES

- The MFMD algorithm was used to fit ten examples (gap locations) of each of six gap sizes on separate magnitude and unwrapped phase data:
  - 1%, 2%, 5%, 10%, 15%, 20%
- The algorithm also does not include the data points in the 5% of the signal on either side of the gap, to minimize the effects of the Gibbs phenomena resulting from the time-domain gaps
- IPR/Correlation Functions are windowed using a Chebyshev window

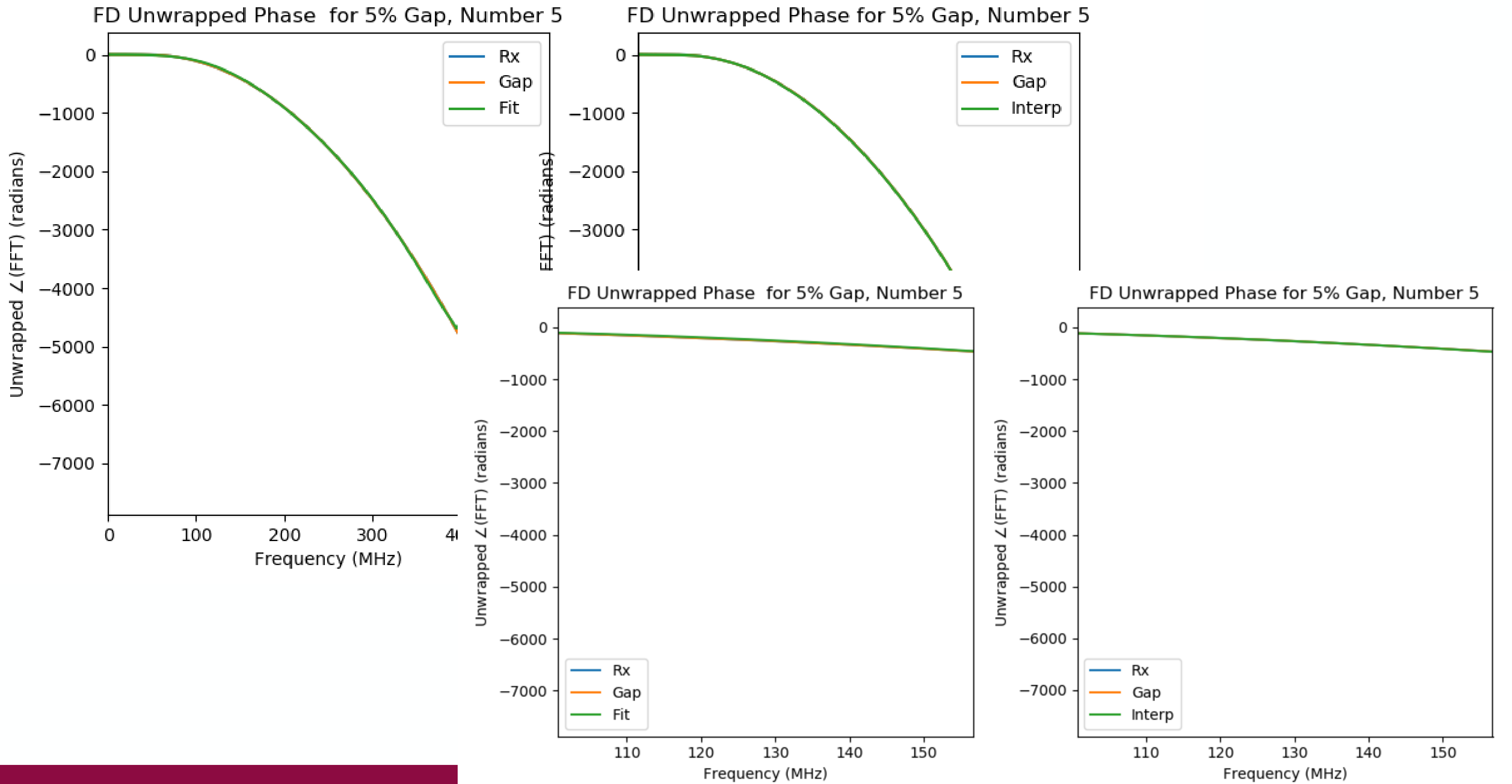
FREQUENCY-DOMAIN MAGNITUDE RESULT EXAMPLE USING MAG/PHASE SEPARATELY



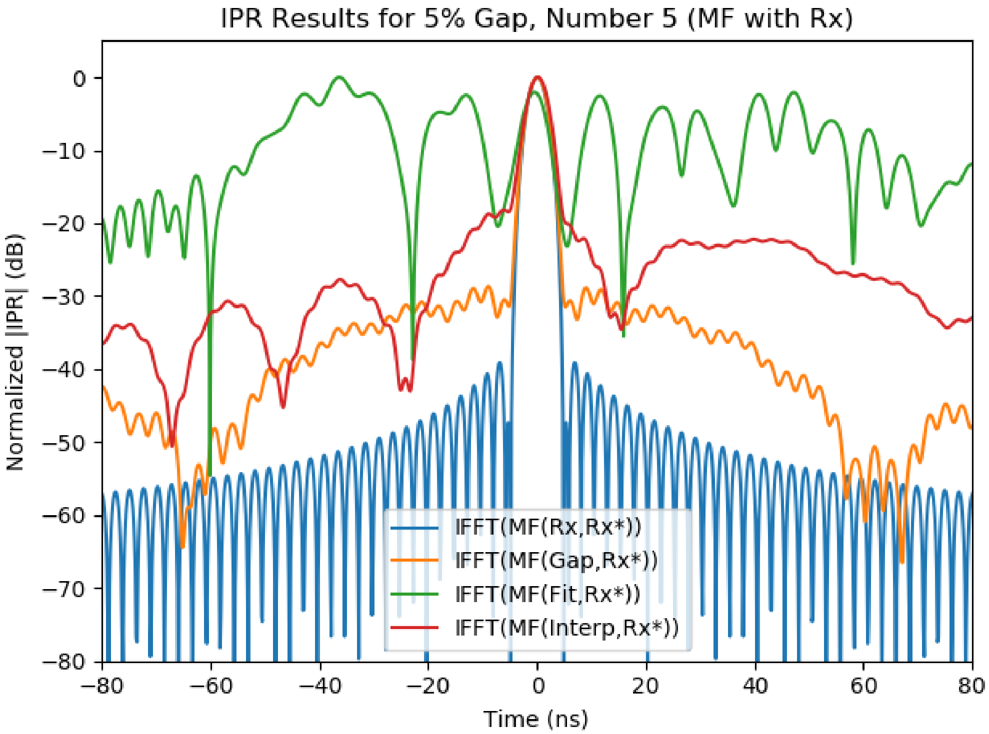
FREQUENCY-DOMAIN PHASE RESULT EXAMPLE USING MAG/PHASE SEPARATELY



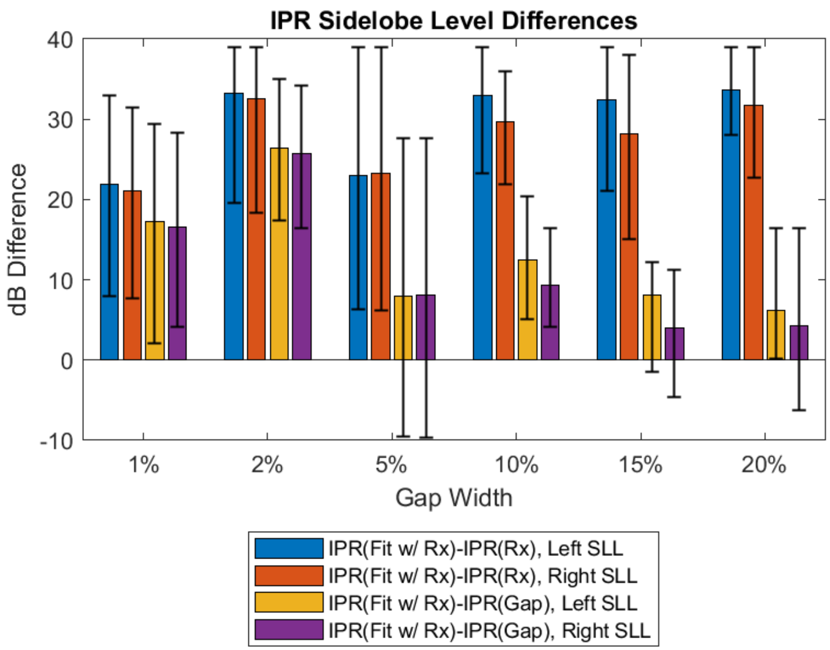
FREQUENCY-DOMAIN MAGNITUDE RESULT EXAMPLE USING MAG/PHASE SEPARATELY

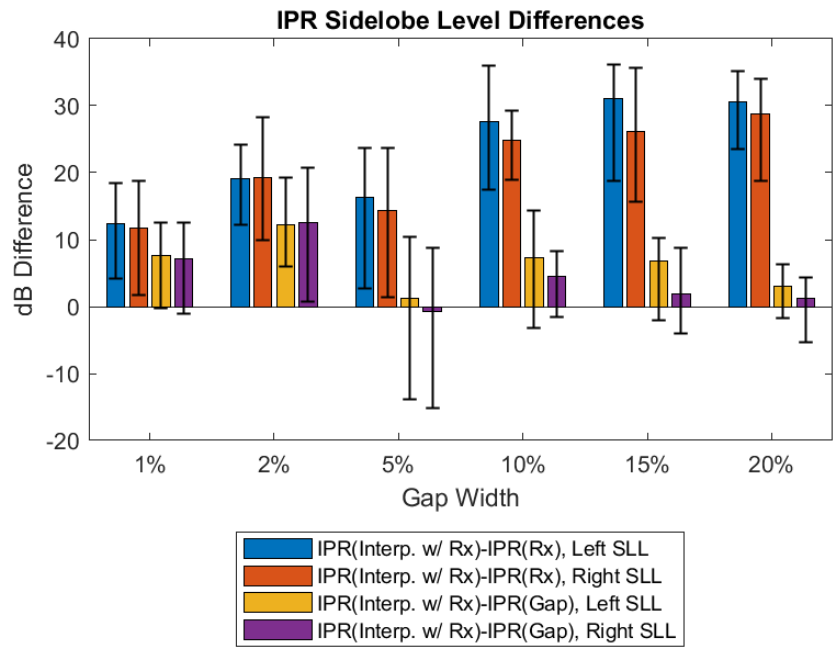


CORRELATION RESULT EXAMPLE USING MAG/PHASE SEPARATELY(MF USING RX)

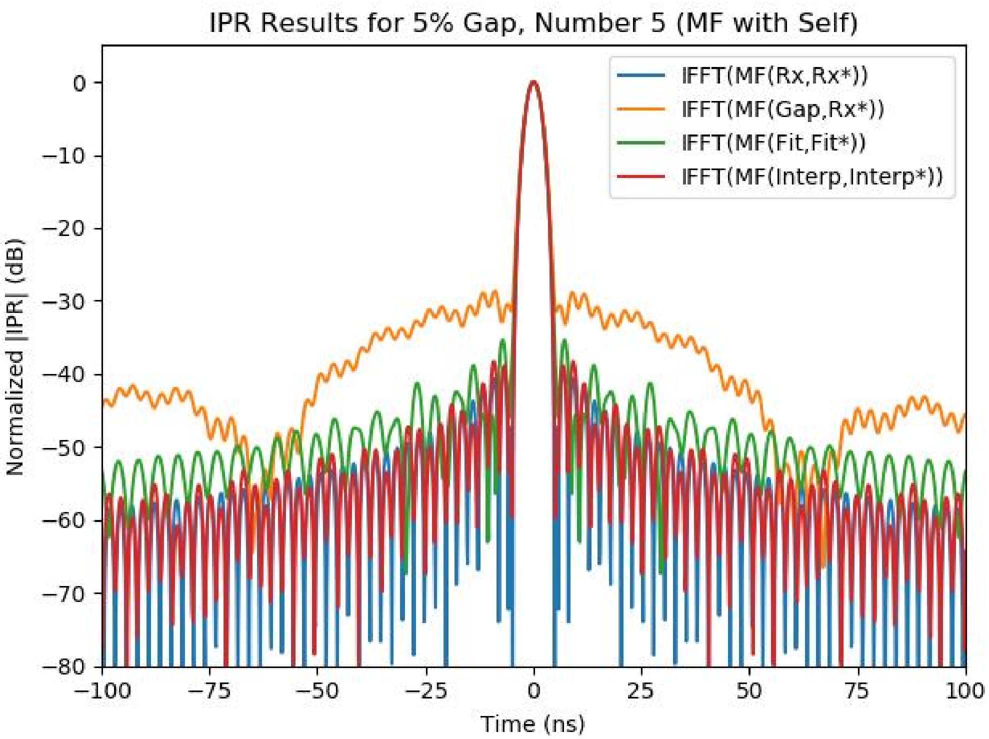


SIDELOBE LEVELS USING MAG/PHASE SEPARATELY

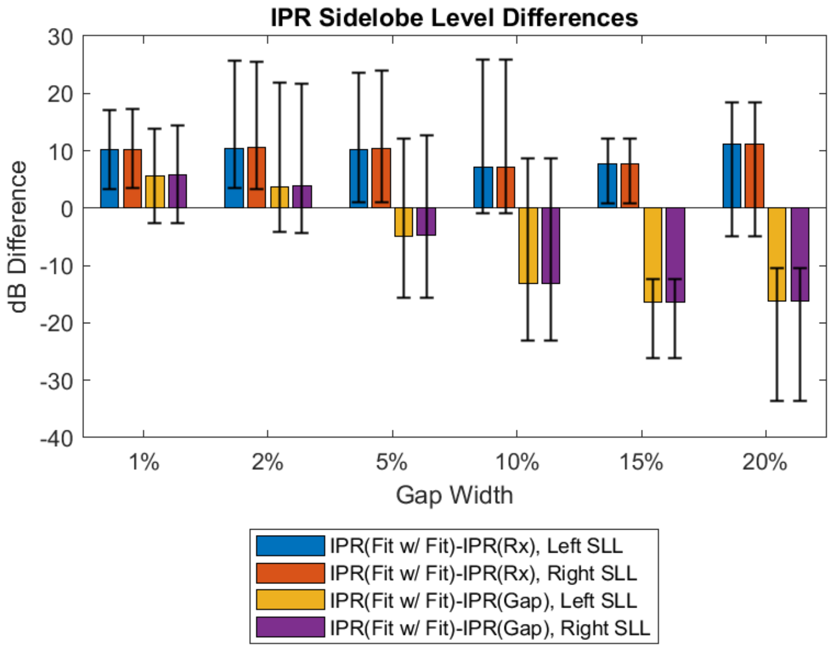


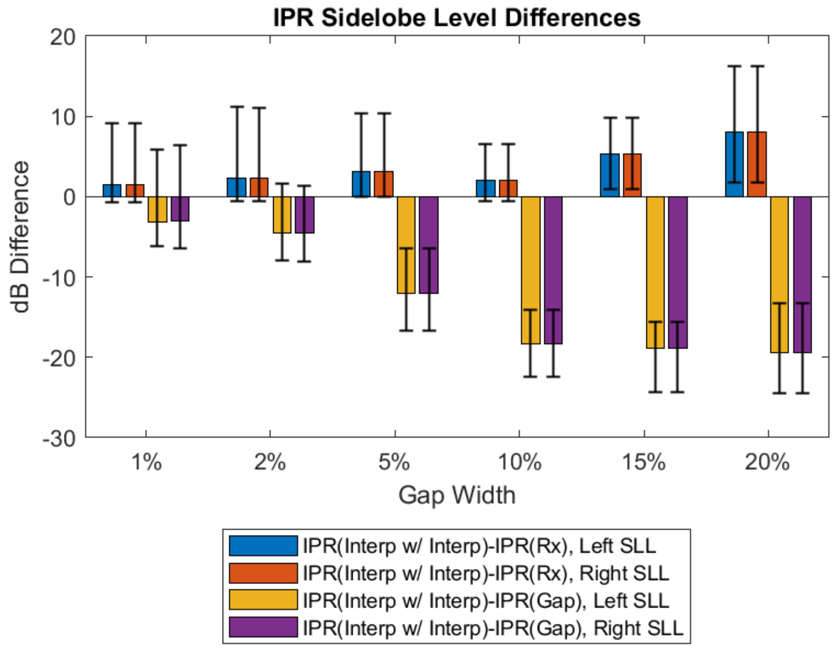


CORRELATION RESULT EXAMPLE USING MAG/PHASE SEPARATELY (MF USING SELF)



SIDELOBE LEVELS USING MAG/PHASE SEPARATELY





SUMMARY OF RESULTS USING MAG/UNWRAPPED PHASE SEPARATELY

- When using MFMD MBPE on the magnitude and unwrapped phase of the FD response separately, we obtain good fit in both
- When calculating IPR based upon $Rx(X)$, we still have problems with the $Fit(X)$ and $Interp(X)$ sidelobe levels being higher than the $Gap(X)$ and $Rx(X)$ sidelobe levels

NEXT STEP

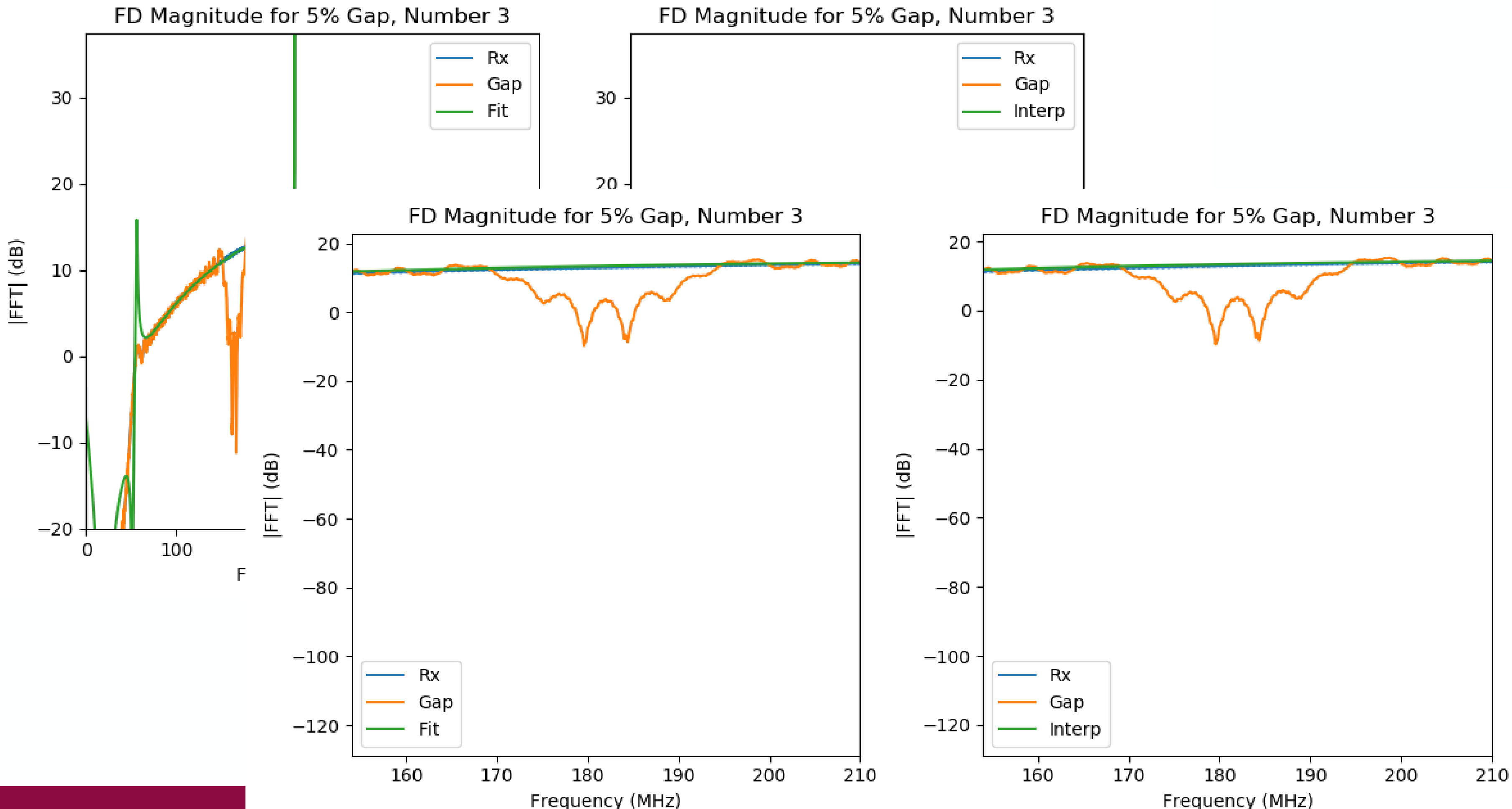
- A potential solution to minimize this issue is to window the magnitude response of the signal prior to fitting MFMD MBPE to it, to smooth out the Gibbs phenomena and the signal in general

FIT USING WINDOWED MAGNITUDE AND UNWRAPPED PHASE DATA SEPARATELY

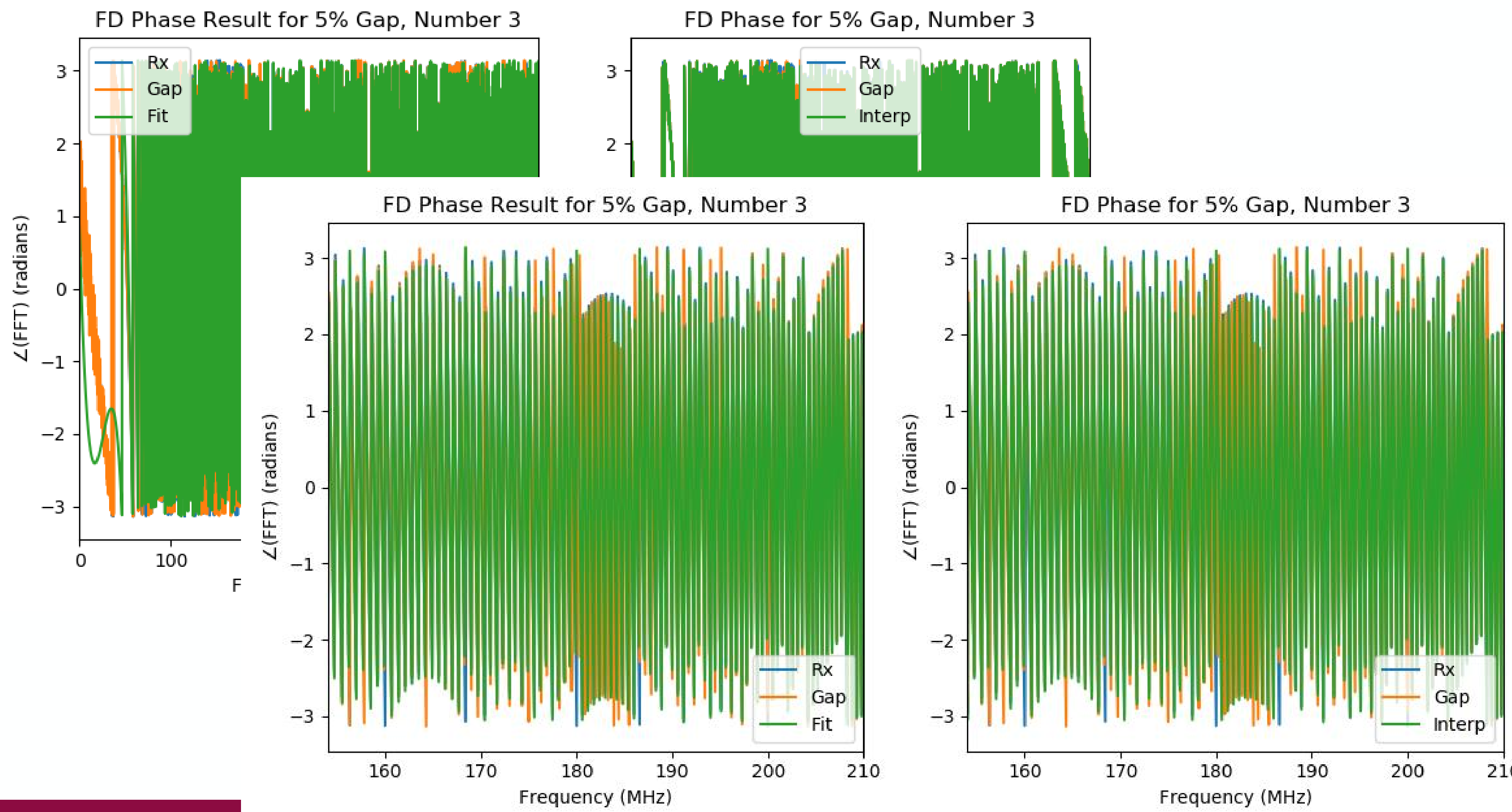
NOTES

- The MFMD algorithm was used to fit five examples (gap locations) of each of six gap sizes on separate magnitude and unwrapped phase data:
  - 1%, 2%, 5%, 10%, 15%, 20%
- The algorithm also does not include the data points in the 5% of the signal on either side of the gap, to minimize the effects of the Gibbs phenomena resulting from the time-domain gaps
- The magnitude response of each is windowed using a Chebyshev window before being fit

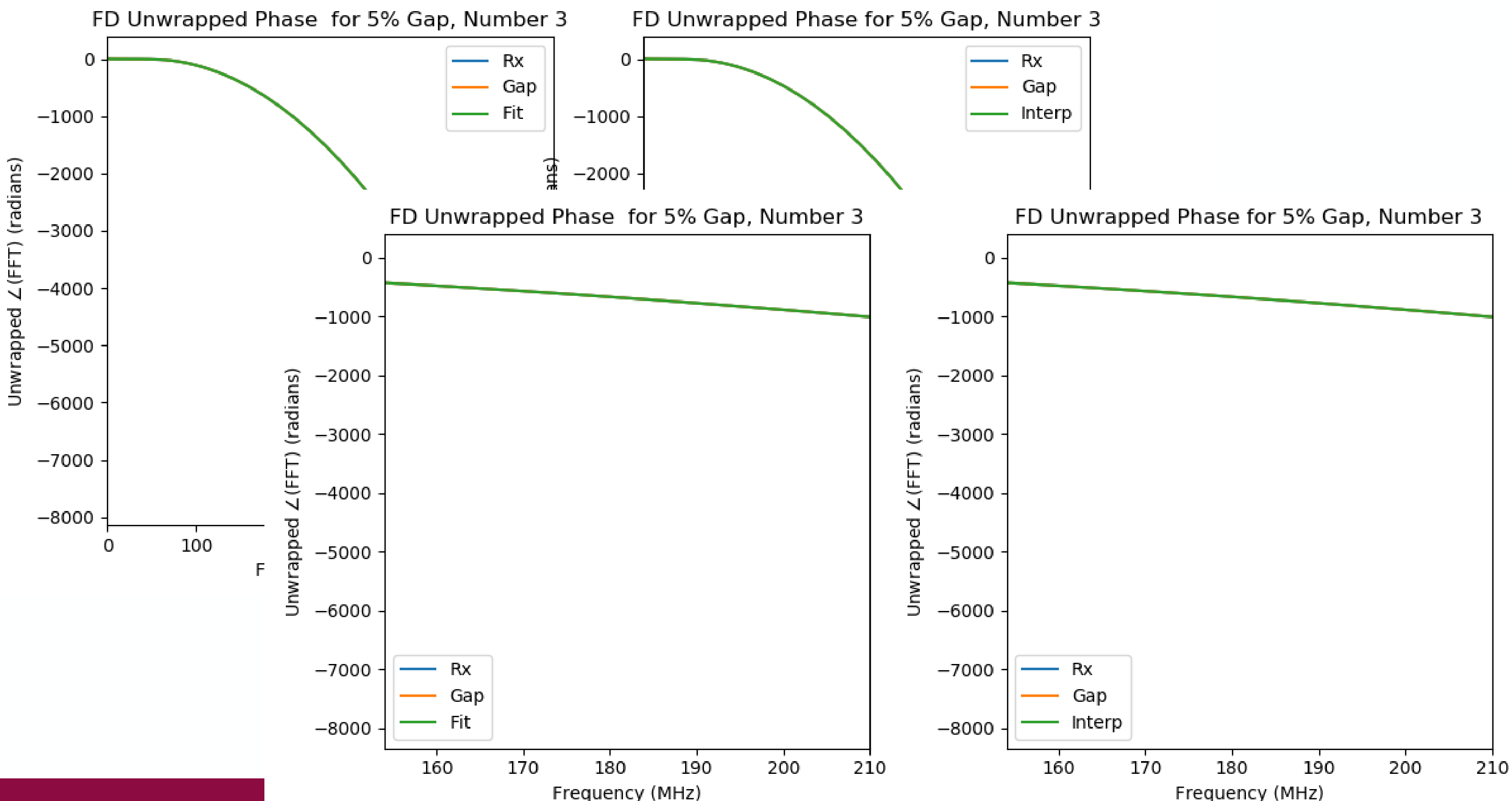
FREQUENCY-DOMAIN MAGNITUDE RESULT EXAMPLE USING WINDOWED MAG/PHASE SEPARATELY



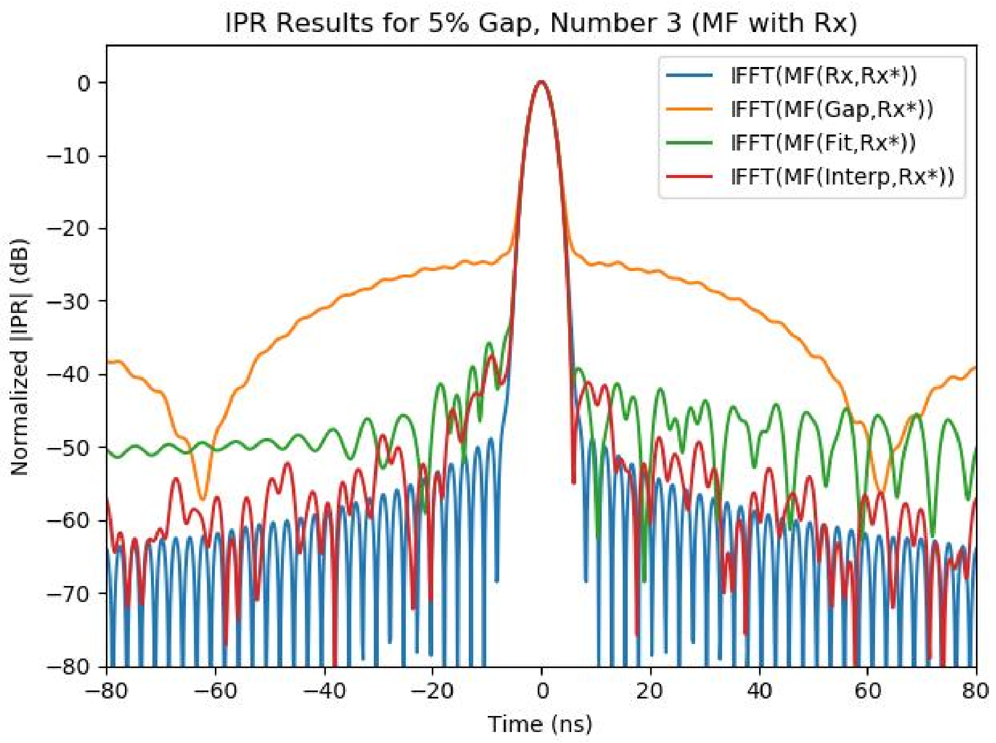
FREQUENCY-DOMAIN PHASE RESULT EXAMPLE USING WINDOWED MAG/PHASE SEPARATELY



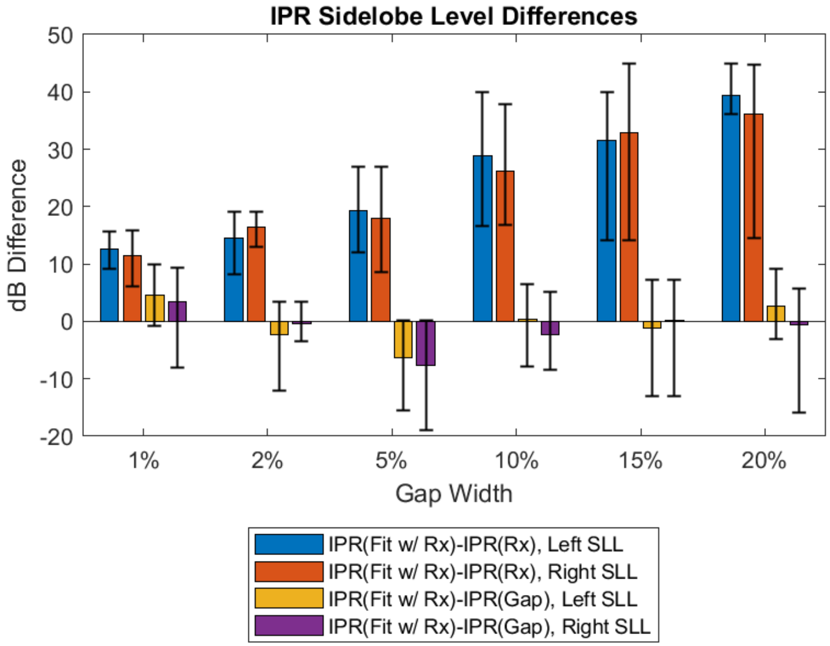
FREQUENCY-DOMAIN MAGNITUDE RESULT EXAMPLE USING WINDOWED MAG/PHASE SEPARATELY

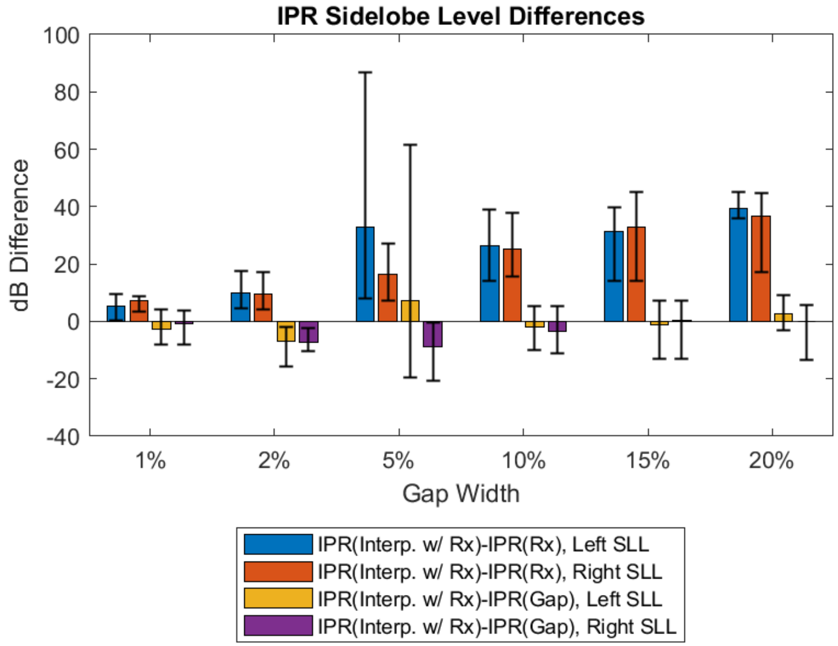


CORRELATION RESULT EXAMPLE USING WINDOWED MAG/PHASE SEPARATELY(MF USING RX)



SIDELOBE LEVELS USING WINDOWED MAG/PHASE SEPARATELY





SUMMARY

SUMMARY OF ALL RESULTS

- Complex Fit
  - If we fit using the complex FD responses, we can obtain reasonably good results in magnitude, but MBPE cannot fit the fast wrapping phase
  - This results in very high sidelobes relative to the $Gap(X)$ and $Rx(X)$ IPRs

SUMMARY OF ALL RESULTS

- Mag/Phase Separate Fit
  - If we fit the magnitude and unwrapped phase separately and then recombine them into a final $Fit(X)$ signal, the magnitude and phase fits are significantly better than in the previous case
  - When IPR is calculated relative to the $Rx(X)$ signal, however, the sidelobe levels of the $Fit(X)$ and $Interp(X)$ IPRs are higher than the $Gap(X)$ and $Rx(X)$ IPRs
  - If we instead try calculating the IPR of $Fit(X)$ and $Interp(X)$ relative to themselves, the sidelobe levels are lower than those for the $Gap(X)$ IPR, though still higher than those for the $Rx(X)$ IPR

SUMMARY OF ALL RESULTS

- Windowed Mag/Phase Separate Fit
  - The FD response results for $Fit(X)$ and $Interp(X)$ in this scenario are very similar to the ones for the Mag/Phase Separate Fit in that they fit very well to the desired signal $Rx(X)$
  - When calculating IPR with $Rx(X)$, $Interp(X)$ cases with gaps of less than 10% have sidelobes lower than $Gap(X)$, though still higher than $Rx(X)$
    - At gap widths greater than or equal to 10%, the sidelobe results are not as predictable and vary significantly depending on the location of the gap itself

CONCLUSION

CONCLUSION

- Applied MFMD MBPE to the problem of interpolating missing data in a radar signal in three ways
- Found that we can obtain IPR results with sidelobe levels lower than $Gap(X)$, but higher than $Rx(X)$, when using MFMD MBPE on the windowed magnitude response and unwrapped phase response separately for small gap sizes (<10%)
- This indicates that there is a potential for MFMD MBPE to be a viable solution for data interpolation for radar applications with some further work

FUTURE WORK

- Additional preprocessing
  - Try to smooth out some of the rapid changes in magnitude and phase
- Multiple gaps
- Multiple targets
- Investigate effects of noise on fit and sidelobe levels
- Investigate effects of non-uniform sampling for fit

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