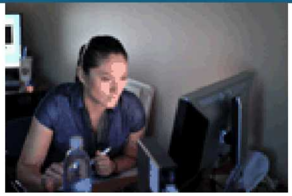
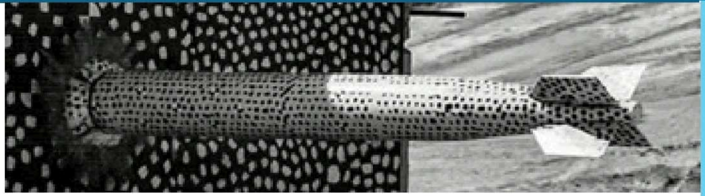
SAND2019-11050PE

A Comparison of Various Sensitivity Analysis Techniques Applied to SWiFT





PRESENTED BY

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Quick Bio

- Undergrad in Physics at Ohio State University
  - Research in Nuclear Energy, Cosmology

- Currently pursuing MS in Computational Science at University of California San Diego

- Goals: Build scientific computing skills that allow me to get involved in a variety of problems
  - HPC, Statistics, Data Visualization

Sensitivity Analysis

Wind turbine simulations are complex and often mask the relationship between input parameters and simulated outputs due to multi-scale nonlinear interactions across turbulent flow fields.

Sensitivity Analysis:

"The study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input" (Saltelli et al., 2004)

Motivations:

–Searching for errors in the model

–Research prioritization

–Model simplification

Input Parameters

ti : turbulence intensity $\sim N(11.12,\ 6.50)$

ws : wind speed $\sim N(6.49,\ 1.67)$

alpha : shear exponent $\sim N(0.249,\ 0.171)$

yaw : fixed nacelle angle relative to u−component wind $\sim N(7.65,\ 8.00)$

rho : air density $\sim N(1.057, 0.0281)$

tor : generator torque constant $\sim N(0.000442\ , 0.00005)$

spatial coherence parameters:

a_u, b_u : $\sim U(1.5,\ 26.0),\ \sim U(0,\ 0.08)$

a_v, b_v : $\sim U(1.7,\ 18.0),\ \sim U(0,\ 0.0045)$

a_w, b_w : $\sim U(2.0,\ 17.0),\ \sim U(0,\ 0.11)$

Quantities of Interest

–Generated Power (kW)

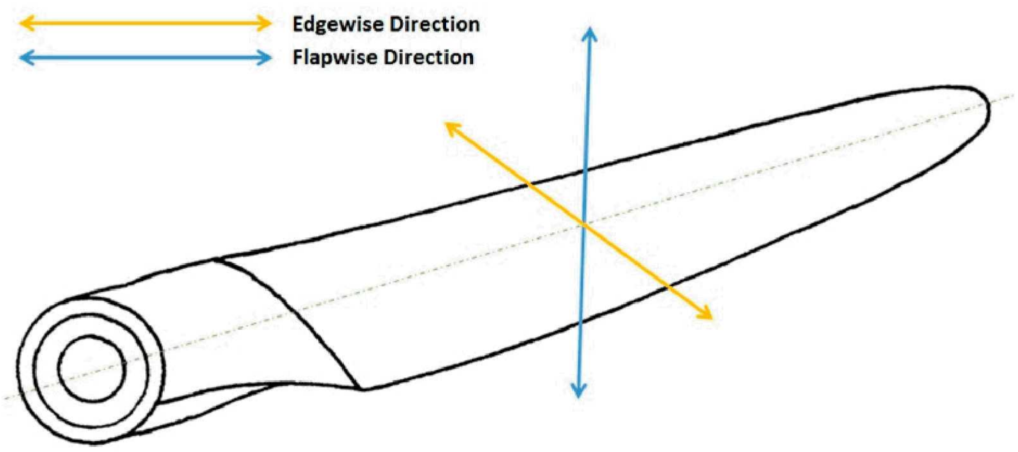
–RootMyb1 (kN·m)

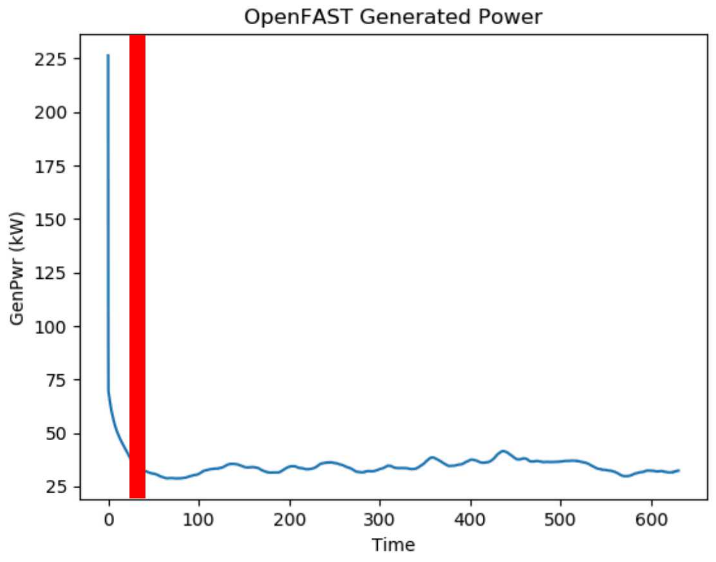
- Blade 1 flapwise moment at the blade root

–RootMxb1 (kN·m)

- Blade 1 edgewise moment at the blade root

Simulations run for 630s, QoIs all averaged over 10 minutes after removing initial 30 second transient





Simulation Workflow

TurbSim v2 is used to create turbulent wind time series

↓

OpenFAST: Aero–Servo–Elastic tool then simulates the wind turbine

For a 630 second real time simulation, ~1400 seconds of cpu time required

Each simulation is serial, no communication required between simulations –> embarrassingly parallel

```
srun="srun -N1 -n1 -c1 --exclusive"

parallel="parallel --delay 0.2 -j $SLURM_NTASKS --joblog EE_Final_10.log --resume"

$parallel "cd /nscratch/akesbha/Elementary_Effects/EE_Final_10/workdir_{1}_{2} && $srun ./ratnaworabhansfruit.sh" ::: {0..29} ::: {0..12}
```

Elementary Effects for Screening Parameters

$$Y = f(X_1, X_2, \dots, X_k)$$

$$EE_i = \frac{f(X_1, \dots, X_i + \Delta, \dots, X_k) - f(X_1, \dots, X_i, \dots, X_k)}{\Delta}$$

$$\mu_i = \frac{1}{r}\sum_{j=1}^{r} EE_i^j$$

Y: QoI

f: simulation + post-processing

$X_1, X_2, \dots, X_k$: k input parameters

Δ: perturbation

r : number of starting points

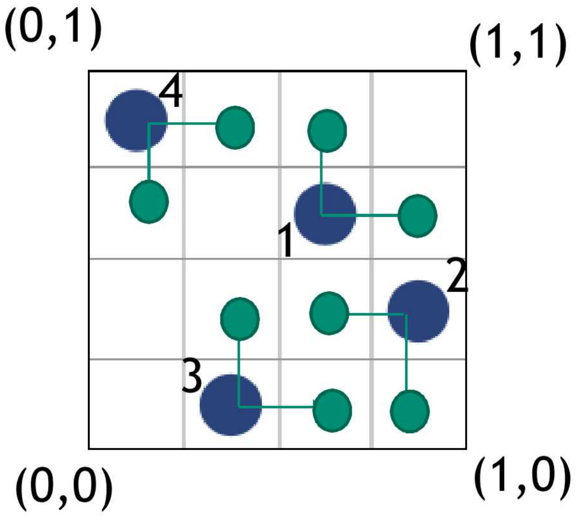
$$\boxed{\mu_i{}^* = \frac{1}{r}\sum_{j=1}^{r} |EE_i^j|}$$

Measure of sensitivity: Higher values highlight important effects and that the model is sensitive to input variation

$$\boxed{\sigma_i = \sqrt{\frac{1}{r}\sum_{j=1}^{r} (EE_i^j - \mu_i)}}$$

Measure of interactions and/or non-linearity.

Latin Hypercube Sampling + Perturbations



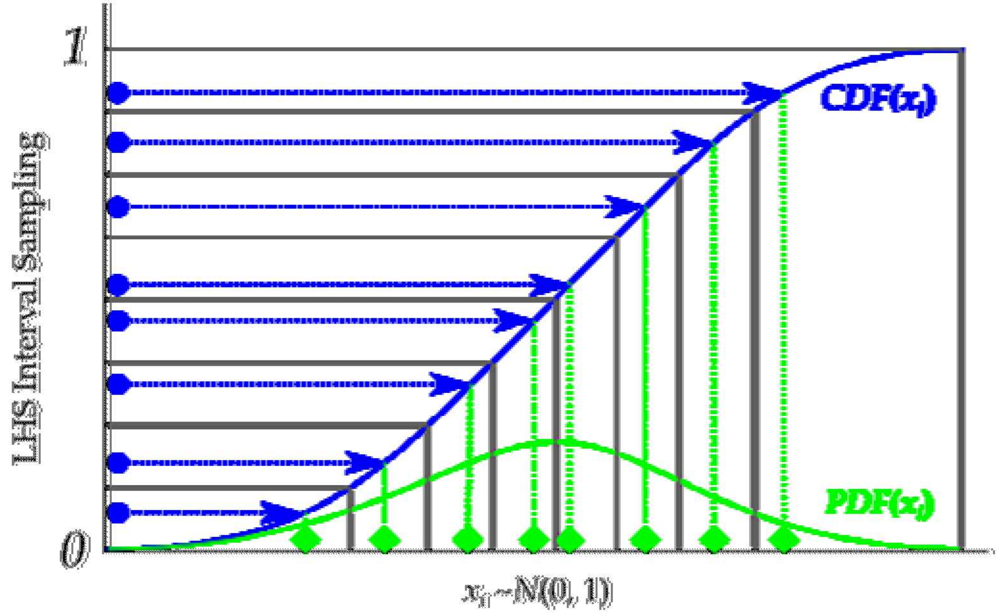
$$EE_i = \frac{f(X_1, \dots, X_i + \Delta, \dots, X_k) - f(X_1, \dots, X_i, \dots, X_k)}{\Delta}$$

$$\Delta = \pm 0.3$$

Alternated sign for each sample to prevent a bias toward the upper end of the distribution
If boundary reached, flipped sign again to stay in input domain.

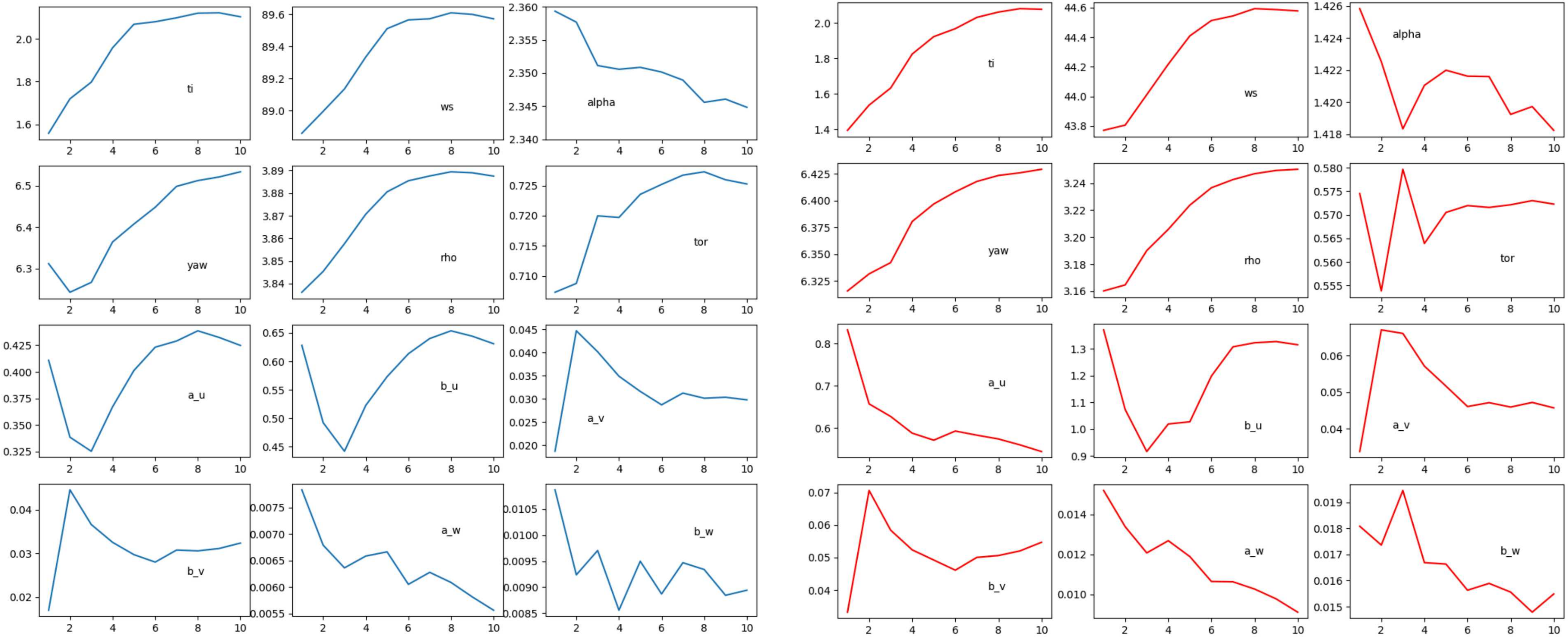
Example of k=2 parameters, r=4 samples

$r \cdot (k+1)$ = 12 simulation starting points



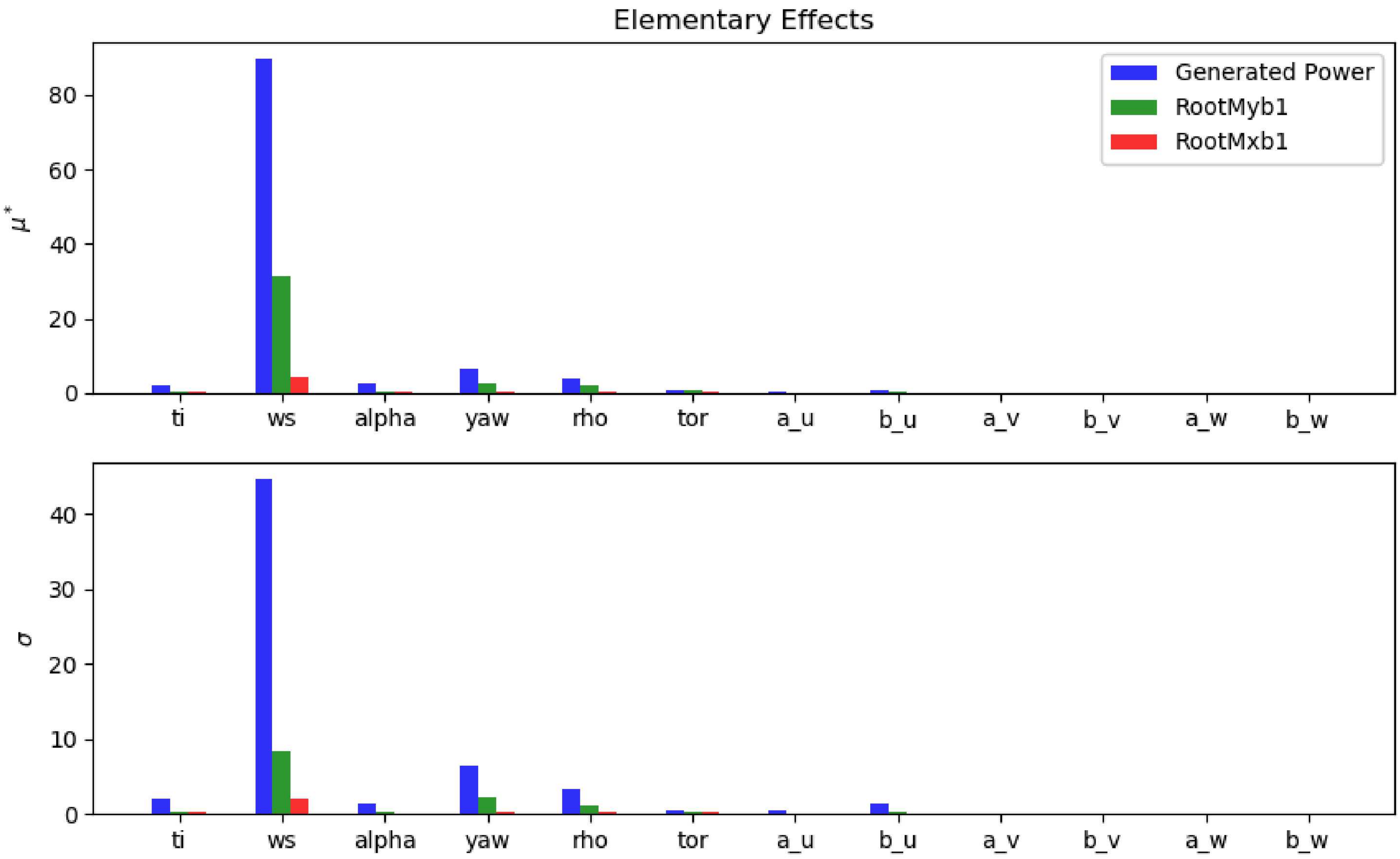
x−axis: number of turbulence seeds measure is averaged over
y−axis: measure value (μ_i^* or σ_i)

μ_i^* σ_i

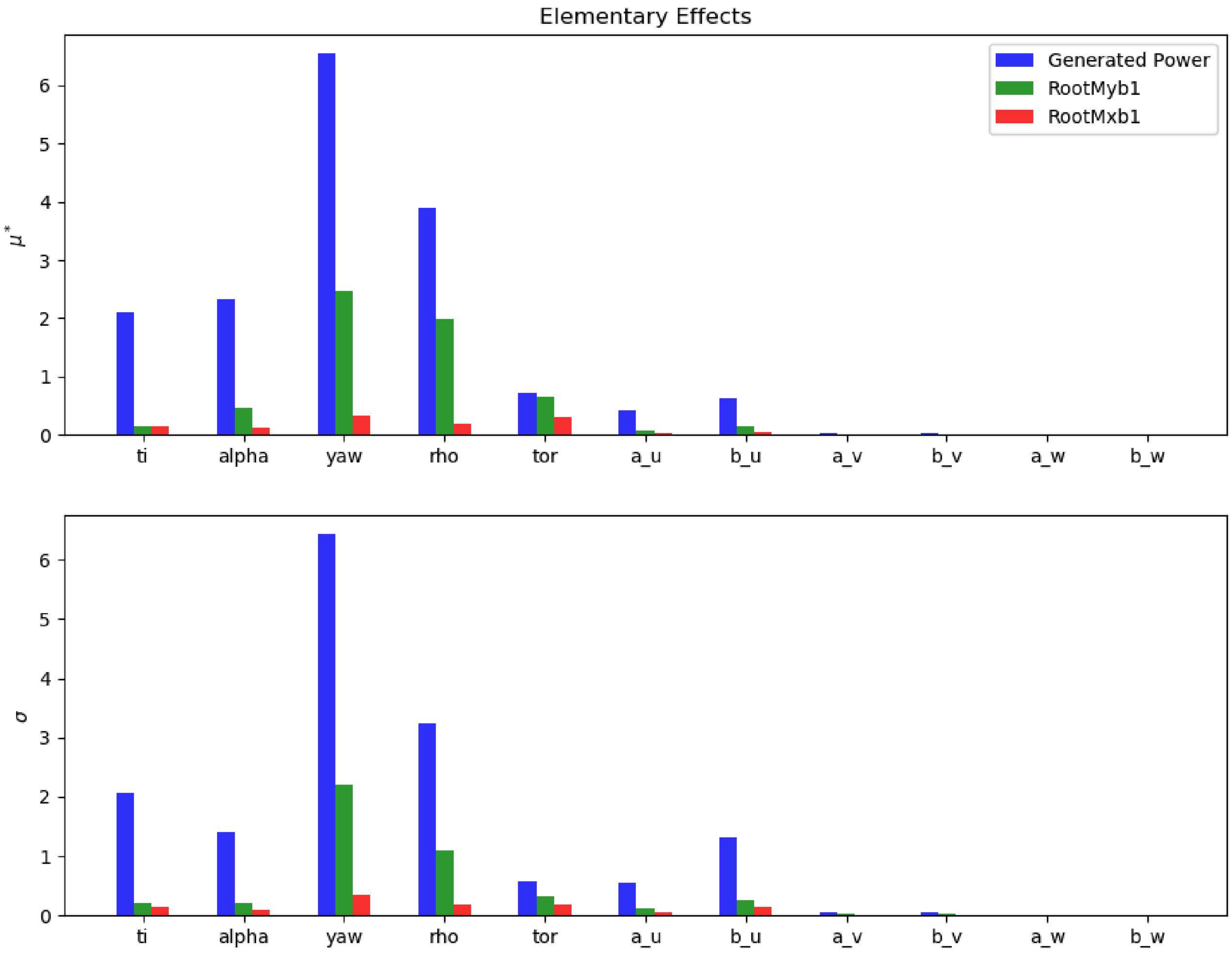


Convergence of EE measures for Generated Power

$$r \cdot (k+1) \cdot ts = 30 \cdot 13 \cdot 10 = 3900 \text{ simulations}$$



Wind speed is significantly more influential than other parameters can be taken out of consideration in future sensitivity analyses



Coherence parameters in the v and w components are orders less influential than other parameters and can be taken out of consideration in future sensitivity analyses

EE Sensitivity Rankings

Generated Power		Flapwise Bending Moment		Edgewise Bending Moment	
ws	89.6	ws	31.3	ws	4.50
yaw	6.53	yaw	2.47	yaw	0.33
rho	3.89	rho	2.00	tor	0.29
alpha	2.34	tor	0.64	rho	0.19
ti	2.10	alpha	0.47	ti	0.14
tor	0.73	ti	0.16	alpha	0.13
b_u	0.63	b_u	0.14	b_u	0.06
a_u	0.42	a_u	0.09	a_u	0.04
b_v	0.032	b_v	0.013	b_v	0.005
a_v	0.029	a_v	0.013	a_v	0.004
b_w	0.009	b_w	0.003	b_w	0.0017
a_w	0.006	a_w	0.002	a_w	0.0012

Comparison with Robertson Parameter Sensitivity Study

Parameters	BIN-1	
	Ultimate loads	Fatigue loads
α	8	5
β	1	2
L_u	-	-
L_v	-	-
L_w	-	-
σ_u	4	6
σ_v	-	-
σ_w	-	-
a_u	-	3
a_v	-	-
a_w	-	-
b_u	-	3
b_v	-	-
b_w	-	-
γ	-	-
PC_{uw}	-	-
PC_{uv}	-	-
PC_{vw}	-	-

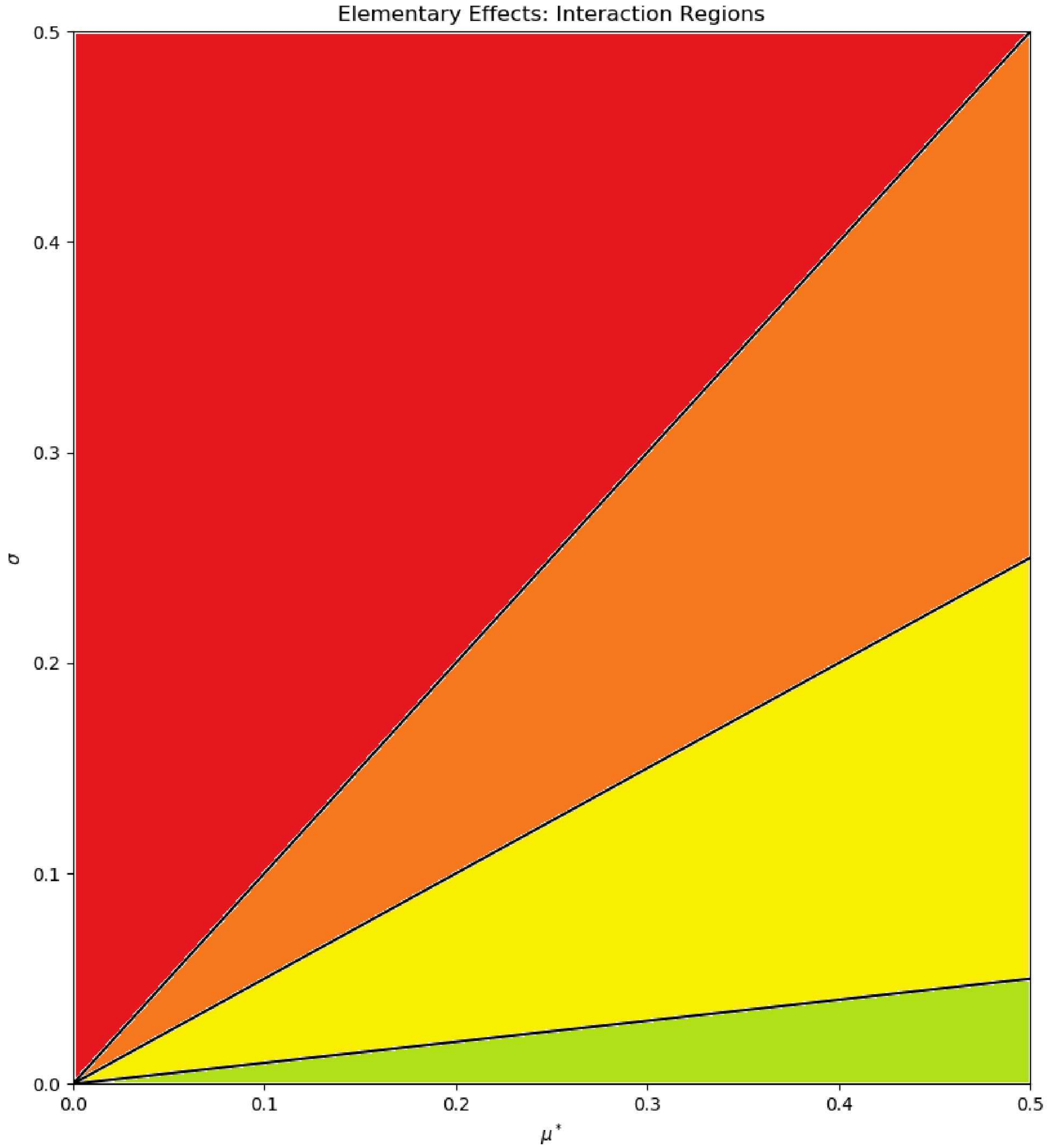
Flapwise Bending Moment

yaw	2.47
rho	2.00
tor	0.64
alpha	0.47
ti	0.16
b_u	0.14
a_u	0.09
b_v	0.013
a_v	0.013
b_w	0.003
a_w	0.002

Edgewise Bending Moment

yaw	0.33
tor	0.29
rho	0.19
ti	0.14
alpha	0.13
b_u	0.06
a_u	0.04
b_v	0.005
a_v	0.004
b_w	0.0017
a_w	0.0012

Robertson, Amy, Latha Sethuraman, Jason Jonkman, and Julian Quick. 2018. "Assessment of Wind Parameter Sensitivity on Ultimate and Fatigue Wind Turbine Loads: Preprint." Golden, CO: National Renewable Energy Laboratory. NREL/CP-5000-70445. www.nrel.gov/docs/fy18osti/70445.



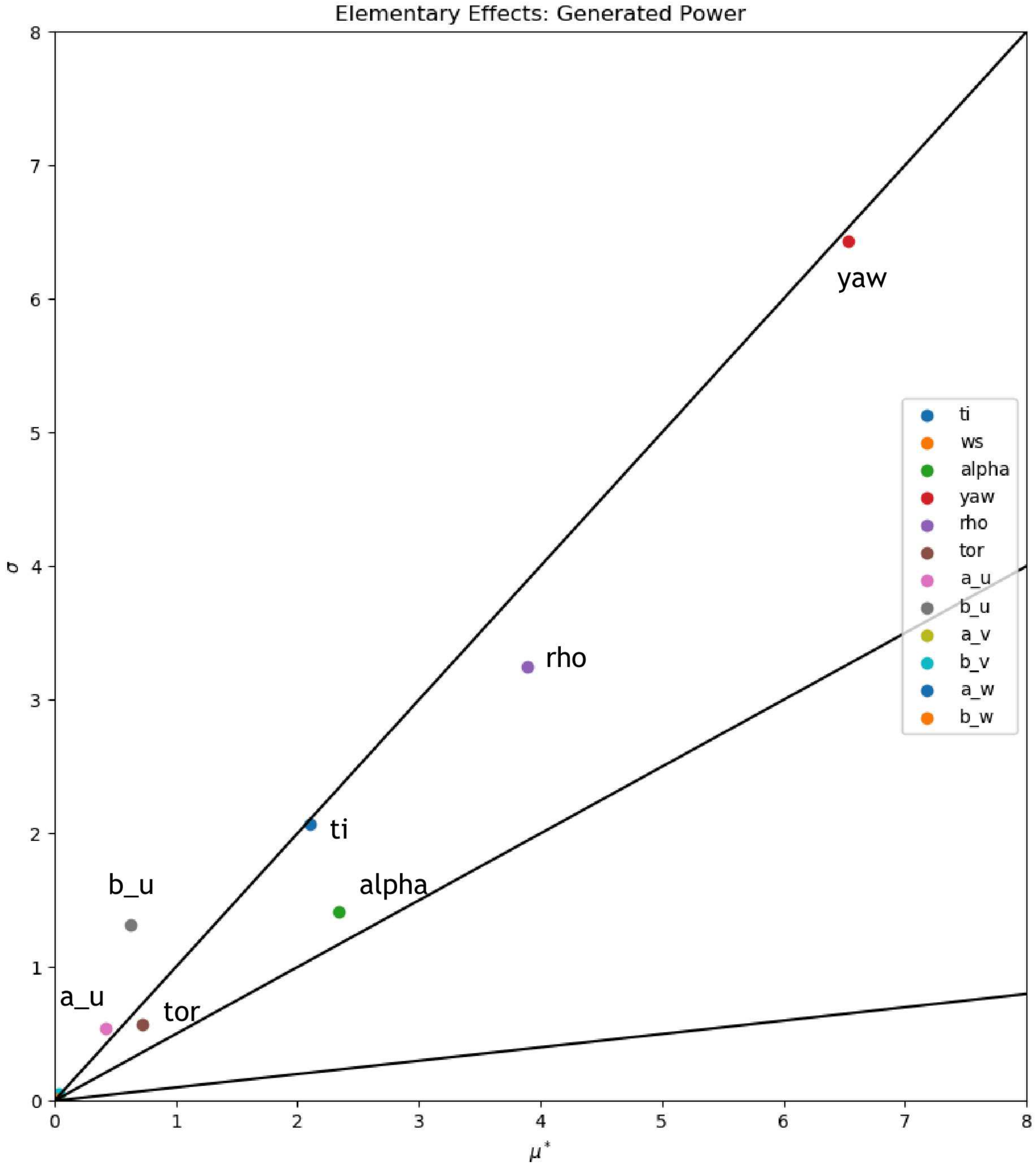
$$\rho_i = \frac{\sigma_i}{\mu_i^*}$$

$1.0 \le \rho_i$ i–th variable has non–linear and/or high interactions, non–monotonic effects

$0.5 \le \rho_i \le 1.0$ i–th variable has quasi–monotonic effects

$0.1 \le \rho_i \le 0.5$ i–th variable has monotonic effects

$\rho_i \le 0.1$ i–th variable has almost linear effects

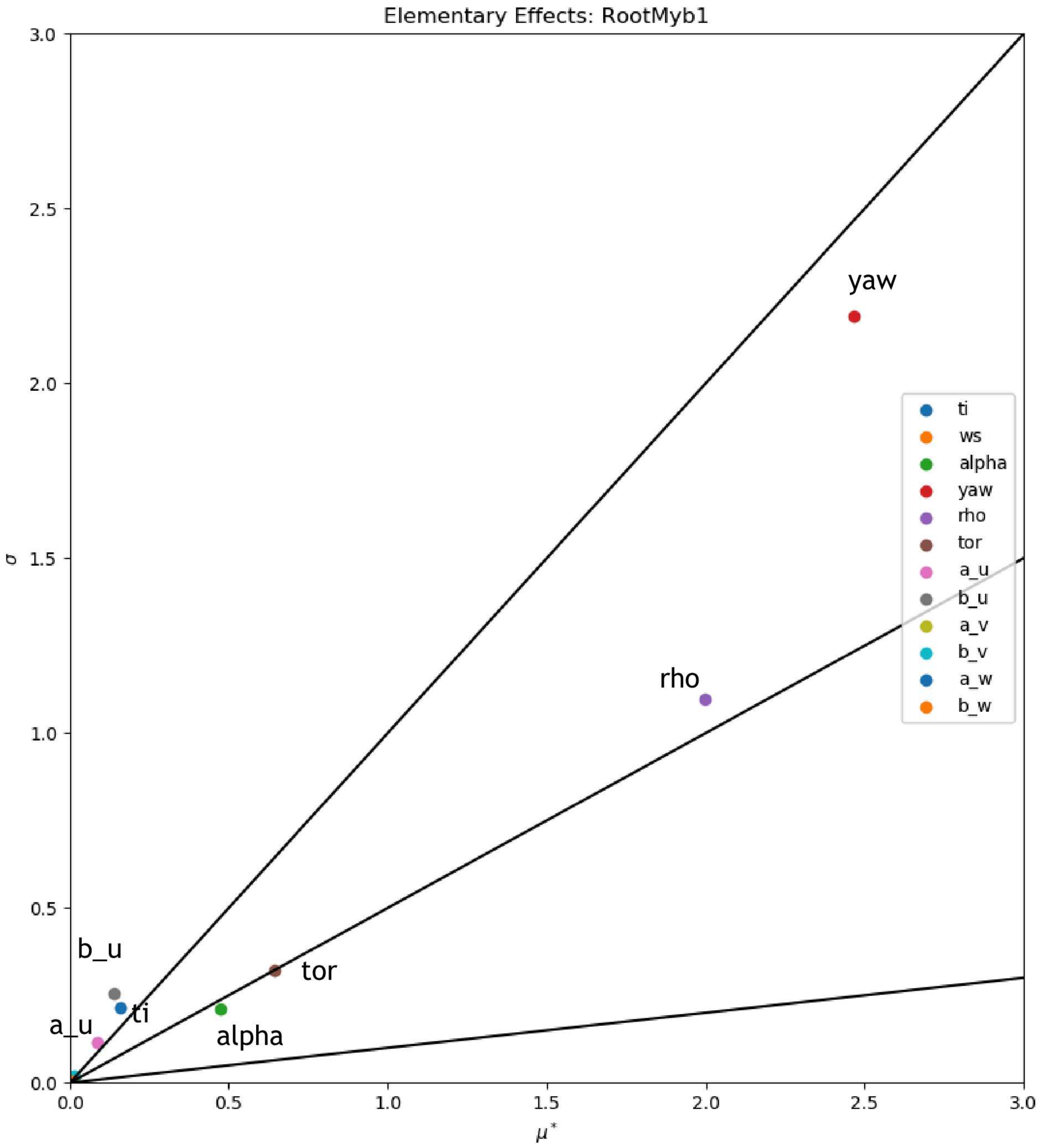


<u>non-linear and/or high interactions, non-monotonic effects:</u>

a_u, b_u

<u>quasi-monotonic:</u>

tor, alpha, ti, yaw, ws



<u>non-linear and/or high interactions, non-monotonic effects:</u>

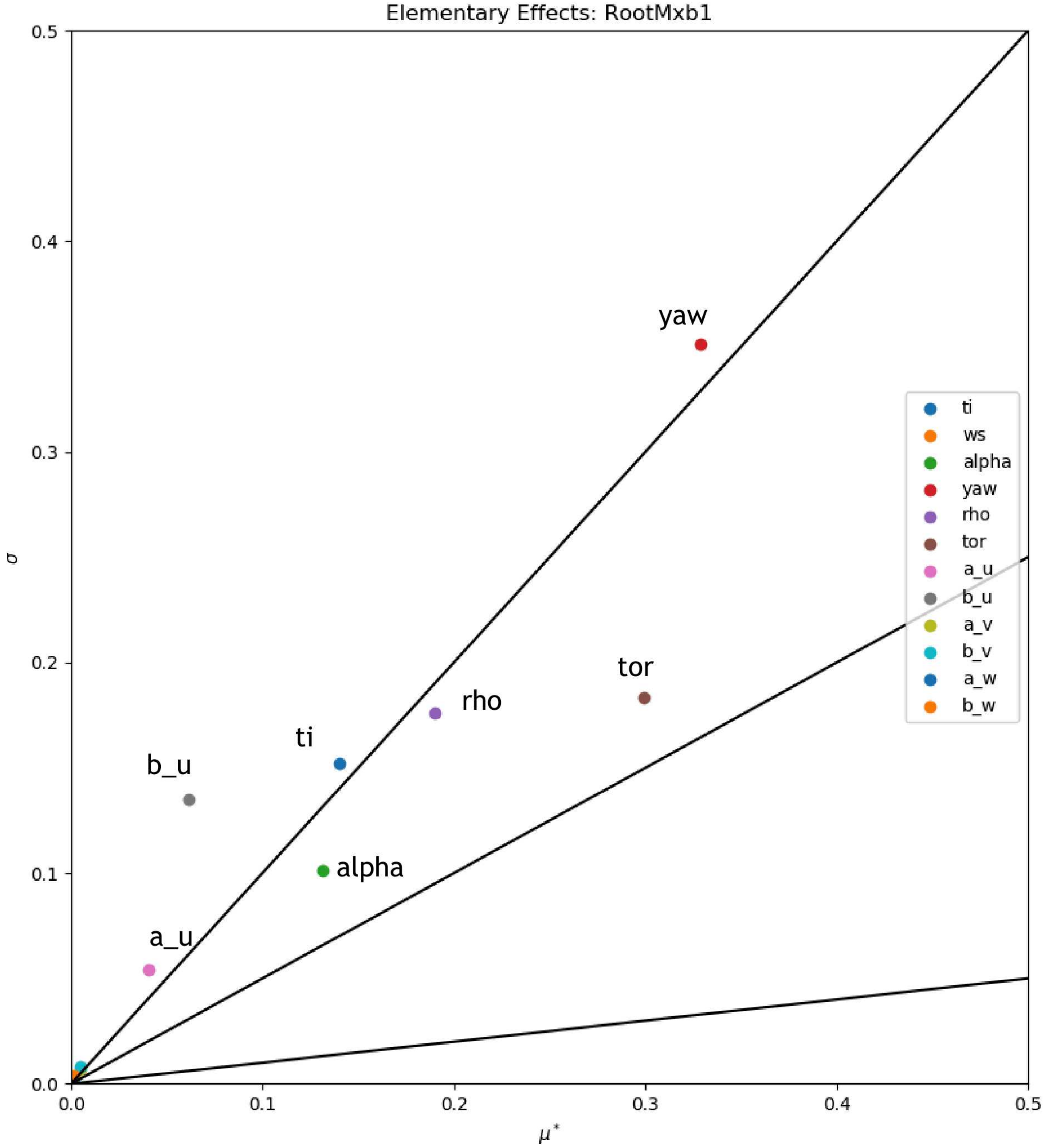
a_u, b_u, ti

<u>quasi-monotonic:</u>

yaw, rho

<u>monotonic:</u>

alpha, tor



<u>non-linear and/or high interactions, non-monotonic effects:</u>

a_u, b_u, yaw, ti

<u>quasi-monotonic:</u>

alpha, rho, tor

Variance-Based Sensitivity Analysis

High Dimensional Model Representation:

$$Y = f(\mathbf{X}) = f_0 + \sum_{i=1}^{k} f_i(X_i) + \sum_{i<j}^{k} f_{ij}(X_i, X_j) + \cdots + f_{12\ldots k}(X_1, X_2, \ldots, X_k)$$

All terms in decomposition are orthogonal:

$f_0 = E(Y)$

$f_i = E(Y\,|X_i) - f_0$

$f_{ij} = E(Y\,|X_i, X_j) - f_0 - f_i - f_j$

f_i is the effect of varying X_i alone

f_{ij} is the effect of varying X_i and X_j simulataneously (along with individual variations)

Variance-Based Sensitivity Analysis

$$f(\mathbf{X}) = f_0 + \sum_{i=1}^{k} f_i(X_i) + \sum_{i<j}^{k} f_{ij}(X_i, X_j) + \cdots + f_{12\ldots k}(X_1, X_2, \ldots, X_k)$$

Square and integrate both sides:

$$\mathrm{Var}(Y) = \sum_{i=1}^{k} V_i + \sum_{i<j}^{k} V_{ij} + \ldots + V_{12\ldots k}$$

The decomposition of the output is now in quantified terms for each input (and interactions)

Main Effect

$$S_i = \frac{V_i}{\mathrm{Var}(Y)} = \frac{\mathrm{Var}_i\left(\mathrm{E}_{\sim i}(Y \mid X_i)\right)}{\mathrm{Var}(Y)}$$ $\longrightarrow$ If the conditional expectation has a large variation across X_i values, then X_i is important

The main effect sensitivity index corresponds to the fraction of the uncertainty in the output, Y, that can be attributed to input X_i alone.

Total Effect

$$\mathrm{Var}(Y) = \mathrm{Var}\big(E(Y|X_{\sim i})\big) + E(\mathrm{Var}(Y|\ X_{\sim i}))$$

$$E(\mathrm{Var}(Y|\ X_{\sim i})) = \mathrm{Var}(Y) - \mathrm{Var}\big(E(Y|X_{\sim i})\big)$$

$$\downarrow$$

Remaining variance of Y that would be left, on average, if we could determine the true values of $X_{\sim i}$.
Average calculated over all possible combinations of $X_{\sim i}$.

$$S_{T_i} = \frac{E(\mathrm{Var}(Y|\ X_{\sim i}))}{\mathrm{Var}(Y)}$$

The total effects index corresponds to the fraction of the uncertainty in the output, Y , that can be attributed to input X_i AND its interactions with other variables.

Quick Example

3 factor model: X_1, X_2, X_3

$$1 = \underbrace{S_1 + S_2 + S_3}_{\text{Main Effect}} + \underbrace{S_{12} + S_{13} + S_{23}}_{\text{2nd order interaction}} + \underbrace{S_{123}}_{\text{3rd order interaction}}$$

$$\underbrace{S_{T_1}}_{\text{Total Effect for } X_1} = S_1 + S_{12} + S_{13} + S_{123}$$

Sampling and Estimators

Each row is an input vector:

$$\text{Sob}(4,3) = \left[\underbrace{\begin{matrix} 0.500 & 0.500 & 0.500 \\ 0.250 & 0.750 & 0.250 \\ 0.750 & 0.250 & 0.750 \\ 0.125 & 0.625 & 0.875 \end{matrix}}_{\textbf{A}} \quad \underbrace{\begin{matrix} 0.500 & 0.500 & 0.500 \\ 0.750 & 0.250 & 0.750 \\ 0.250 & 0.750 & 0.250 \\ 0.875 & 0.625 & 0.125 \end{matrix}}_{\textbf{B}}\right]$$

$$\mathbf{A}_{\mathrm{B}}^{(1)} = \begin{matrix} 0.500 & 0.500 & 0.500 \\ 0.750 & 0.750 & 0.250 \\ 0.250 & 0.250 & 0.750 \\ 0.875 & 0.625 & 0.875 \end{matrix}$$

$$\mathbf{A}_{\mathrm{B}}^{(2)} = \begin{matrix} 0.500 & 0.500 & 0.500 \\ 0.250 & 0.250 & 0.250 \\ 0.750 & 0.750 & 0.750 \\ 0.125 & 0.625 & 0.875 \end{matrix}$$

$$\mathbf{A}_{\mathrm{B}}^{(3)} = \begin{matrix} 0.500 & 0.500 & 0.500 \\ 0.250 & 0.750 & 0.750 \\ 0.750 & 0.250 & 0.250 \\ 0.125 & 0.625 & 0.125 \end{matrix}$$

$N(\text{rows}) = 4,\ k(\text{columns}) = 3,$
$N(k+2) = 20$ simulation starting points

Janon et al estimator:

$$S_d = \frac{\int f^2(\mathbf{x})d\mathbf{x} - \int\int f(\mathbf{x}_{\sim d}, x'_d) f(\mathbf{x}_{\sim d}, x_d) d\mathbf{x}'_d d\mathbf{x}}{\int f(\mathbf{x})^2 d\mathbf{x} - \left(\int f(\mathbf{x}) d\mathbf{x}\right)^2}$$

$$= \frac{\frac{1}{N}\sum f(B)_n \cdot f(A_B^d)_n - \left(\frac{1}{2N}\sum(f(B)_n + f(A_B^d)_n)\right)^2}{\mathrm{Var}(Y)}$$

Jansen estimator:

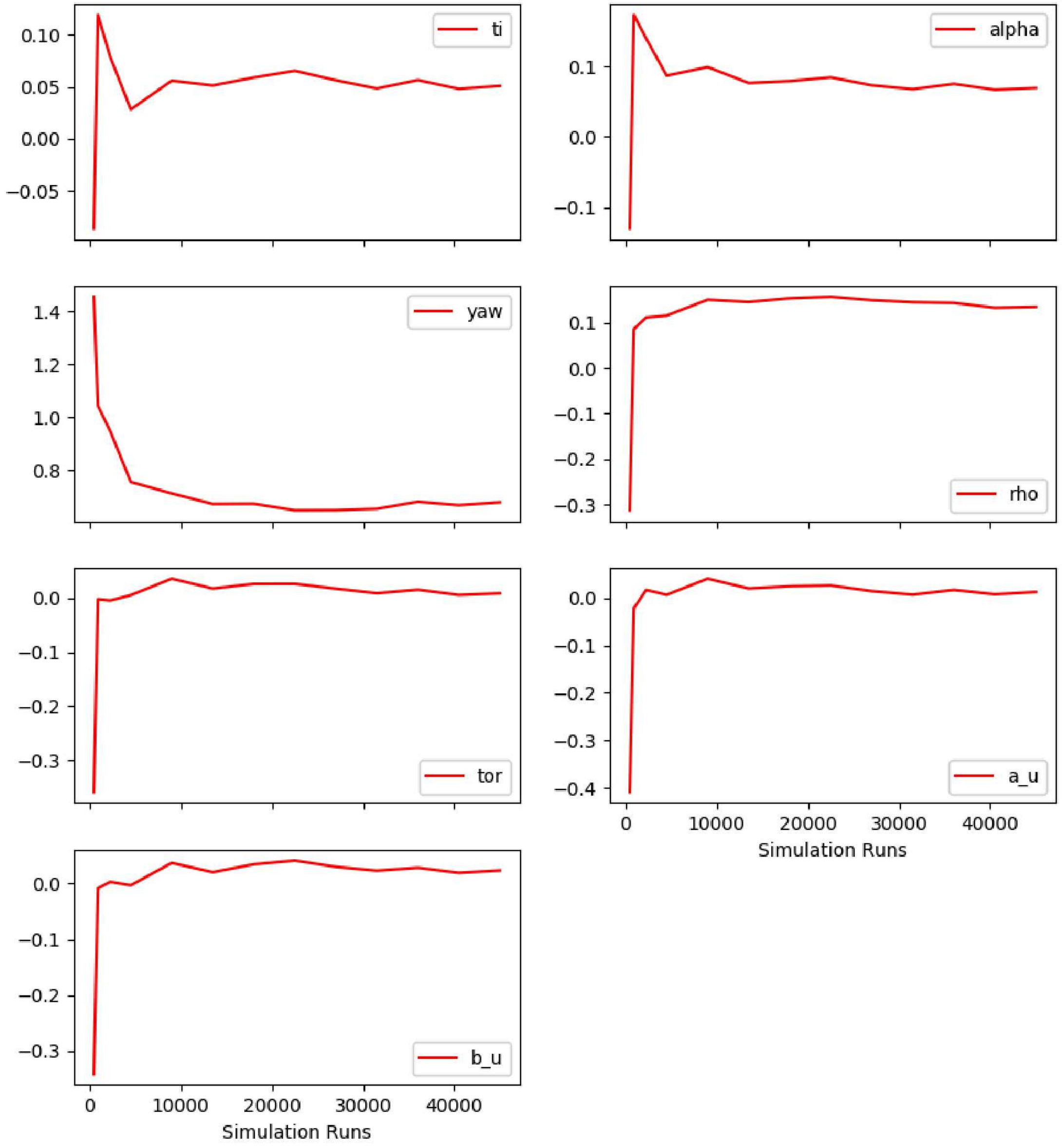
$$S_{T_d} = \frac{\int f^2(\mathbf{x})d\mathbf{x} - \int\int f(\mathbf{x}_{\sim d}, x'_d) f(\mathbf{x}_{\sim d}, x_d) d\mathbf{x}'_d d\mathbf{x}}{\int f(\mathbf{x})^2 d\mathbf{x} - \left(\int f(\mathbf{x}) d\mathbf{x}\right)^2}$$

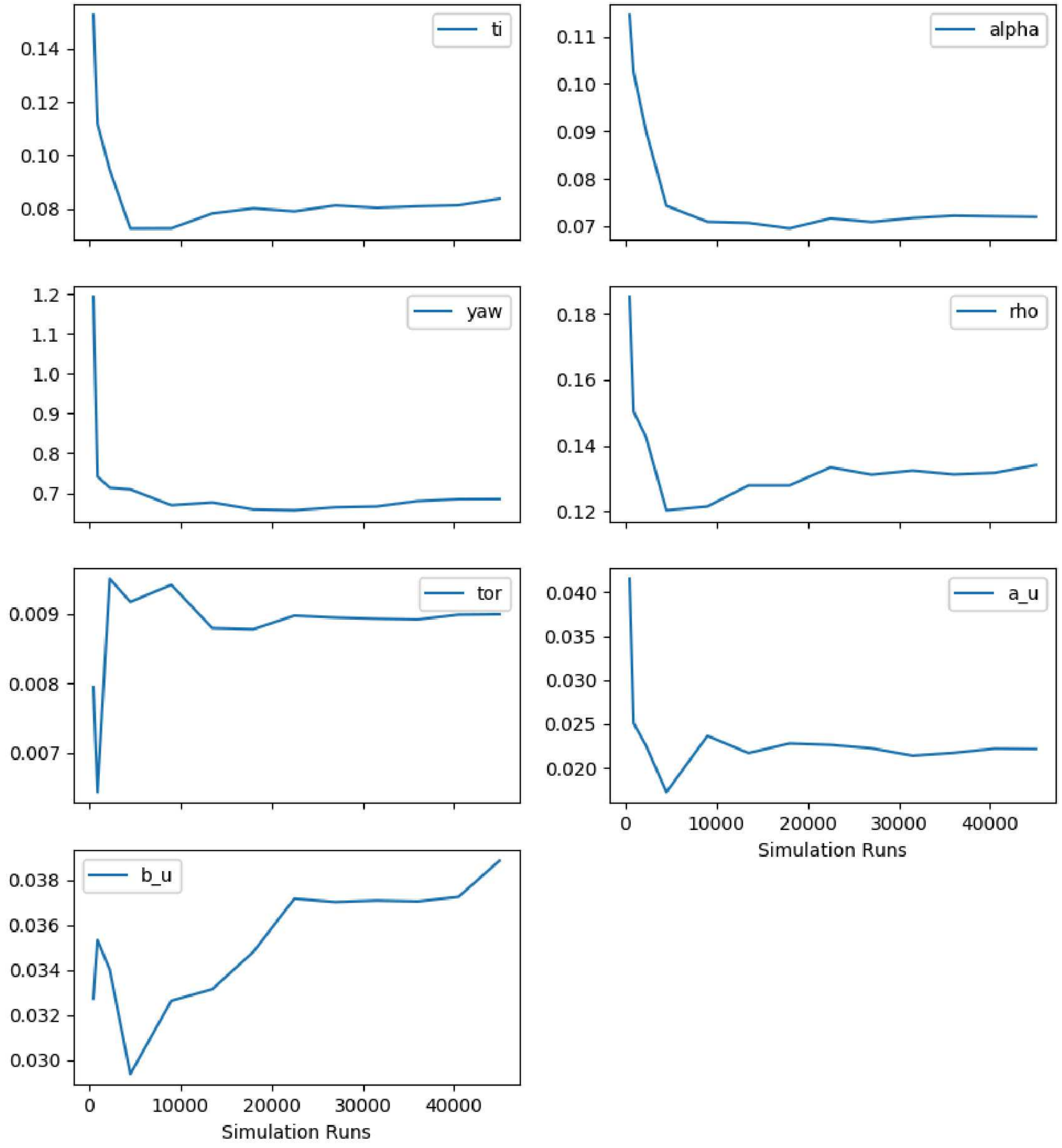
$$= \frac{\frac{1}{2N}\sum(f(A)_n - f(A_B^d)_n)^2}{\mathrm{Var}(Y)}$$

Generated Power: Main Effect Index

Generated Power: Total Effect Index

Convergence of Indices for Generated Power





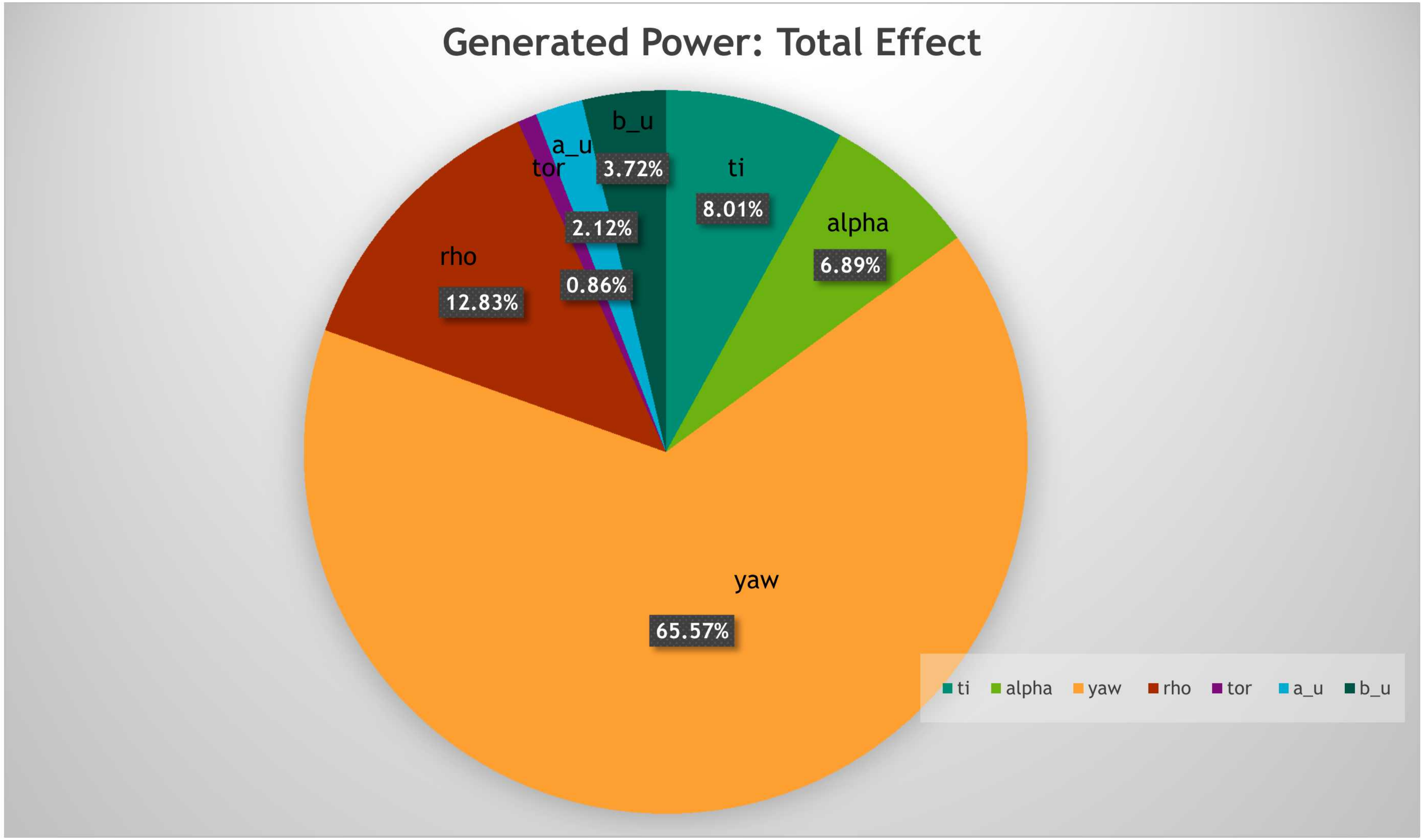
$$N \cdot (k+2) = 5000 \cdot 9 = 45000 \text{ simulations}$$

Main Effect

ti	alpha	yaw	rho	tor	a_u	b_u
0.0510	0.0691	0.678	0.134	0.00874	0.0125	0.0235

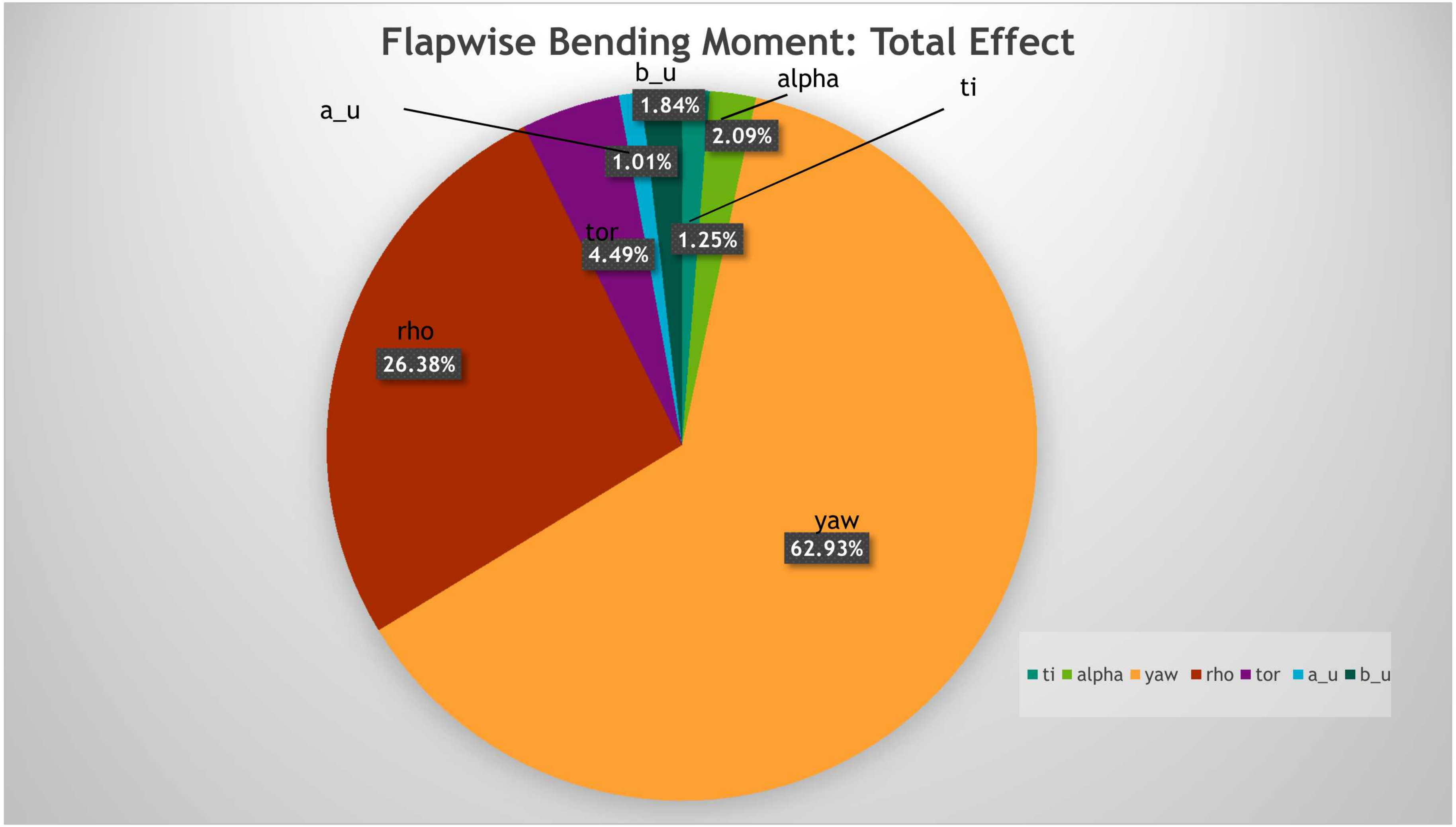
Total Effect

ti	alpha	yaw	rho	tor	a_u	b_u
0.0837	0.0720	0.685	0.134	0.00900	0.0221	0.0389



Main Effect	ti	alpha	yaw	rho	tor	a_u	b_u
	-0.00745	0.0119	0.636	0.271	0.0448	-0.00121	0.00593

Total Effect	ti	alpha	yaw	rho	tor	a_u	b_u
	0.0129	0.0215	0.647	0.271	0.0462	0.0104	0.0189



Variance-Based Sensitivity Analysis

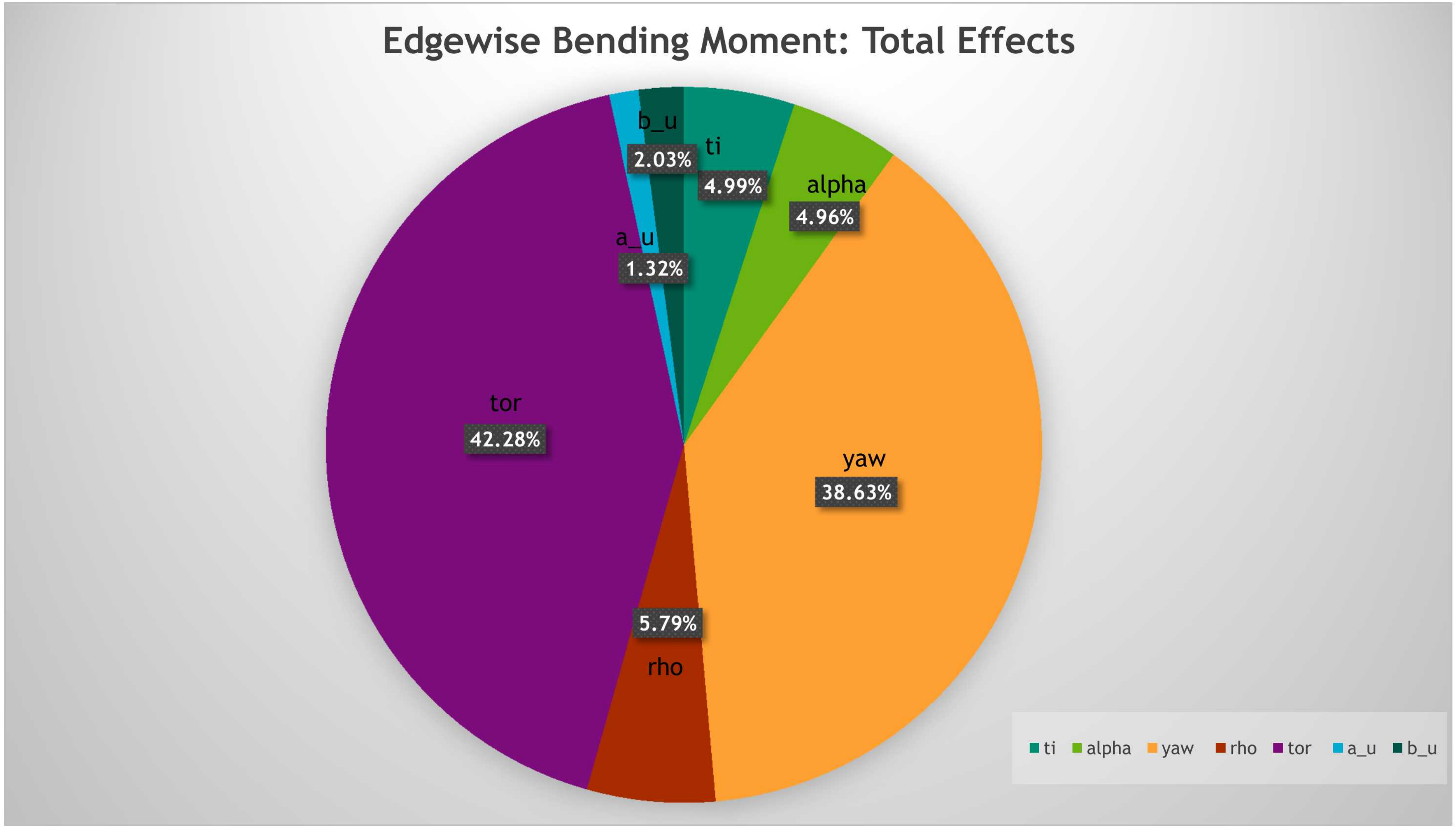
$$f = \sin(2\pi x_1 - \pi) + 7\sin^2(2\pi x_2 - \pi) + 0.1(2\pi x_3 - \pi)^4 \sin(2\pi x_1 - \pi)$$

Approach	Func Eval	Grid Type	S_1	S_2	S_3	S_{T_1}	S_{T_2}	S_{T_3}
LHS	5000	-	2.88446e-01	4.42871e-01	5.52915e-03	5.93029e-01	4.00977e-01	2.32240e-01
LHS	25000	-	3.05843e-01	4.37050e-01	-1.87194e-02	5.66739e-01	4.30240e-01	2.48935e-01
LHS	50000	-	3.28016e-01	4.34040e-01	-8.27517e-03	5.61866e-01	4.45603e-01	2.41798e-01
LHS	250000	-	3.15380e-01	4.42334e-01	5.15642e-03	5.55366e-01	4.43332e-01	2.46431e-01
LHS	500000	-	3.14020e-01	4.42135e-01	-9.13024e-04	5.60501e-01	4.33634e-01	2.36443e-01

Tang, G., Iaccarino, G., and Eldred, M.S., "Global Sensitivity Analysis for Stochastic Collocation Expansion, paper AIAA-2010-2922 in *Proceedings of the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference (12th AIAA Non-Deterministic Approaches Conference)*, Orlando, FL, Apr 12-15, 2010.

Main Effect	ti	alpha	yaw	rho	tor	a_u	b_u
	0.0388	0.0390	0.400	0.0551	0.424	0.00945	0.0143

Total Effect	ti	alpha	yaw	rho	tor	a_u	b_u
	0.0521	0.0518	0.403	0.0604	0.441	0.0137	0.0212



Generated Power: EE and VBD

EE Sensitivity:

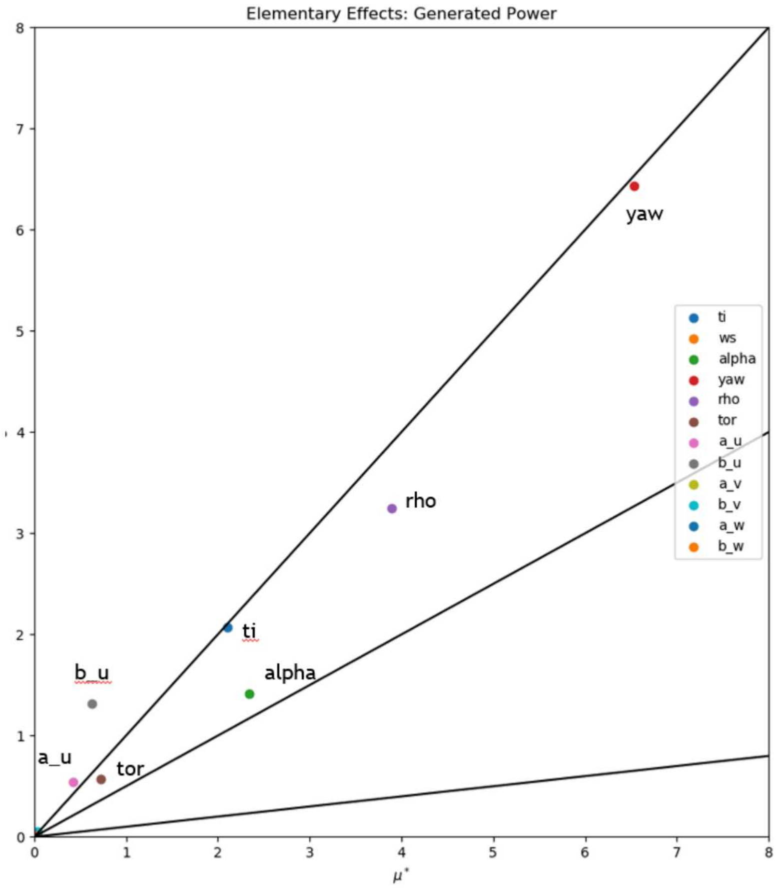
yaw	6.53
rho	3.89
alpha	2.34
ti	2.10
tor	0.73
b_u	0.63
a_u	0.42

VBD Total Sensitivity:

yaw	0.685
rho	0.134
ti	0.0837
alpha	0.072
b_u	0.0389
a_u	0.0221
tor	0.00900

Different rankings!

Generated Power: EE and VBD



a_u, b_u predicted as non-linear and/or highly interactive:

	ti	alpha	yaw	rho	tor	a_u	b_u
Main Effect	0.0510	0.0691	0.678	0.134	0.00874	0.0125	0.0235
Total Effect	0.0837	0.0720	0.685	0.134	0.00900	0.0221	0.0389
$\frac{T-M}{T}$	0.391	0.0404	0.01	0	0.028	0.436	0.399

=> EE somewhat predicts interactions

Flapwise Bending Moments: EE and VBD

EE Sensitivity	
yaw	2.47
rho	2.00
tor	0.64
alpha	0.47
ti	0.16
b_u	0.14
a_u	0.09

VBD Total Sensitivity	
yaw	0.647
rho	0.271
tor	0.0462
alpha	0.0215
b_u	0.0189
ti	0.0129
a_u	0.0104

Different rankings!

Edgewise Bending Moments: EE and VBD

EE Sensitivity		VBD Total Sensitivity	
yaw	0.33	tor	0.441
tor	0.29	yaw	0.403
rho	0.19	rho	0.0604
ti	0.14	ti	0.0521
alpha	0.13	alpha	0.0518
b_u	0.06	b_u	0.0212
a_u	0.04	a_u	0.0137

Different rankings!

Polynomial Chaos Expansion

Polynomial chaos: spectral projection using orthogonal polynomial basis fns

$$\boxed{R = \sum_{j=0}^{\infty} \alpha_j \Psi_j(\boldsymbol{\xi})}$$

using

$$\begin{aligned}
\Psi_0(\boldsymbol{\xi}) &= \psi_0(\xi_1)\,\psi_0(\xi_2) = 1 \\
\Psi_1(\boldsymbol{\xi}) &= \psi_1(\xi_1)\,\psi_0(\xi_2) = \xi_1 \\
\Psi_2(\boldsymbol{\xi}) &= \psi_0(\xi_1)\,\psi_1(\xi_2) = \xi_2 \\
\Psi_3(\boldsymbol{\xi}) &= \psi_2(\xi_1)\,\psi_0(\xi_2) = \xi_1^2 - 1 \\
\Psi_4(\boldsymbol{\xi}) &= \psi_1(\xi_1)\,\psi_1(\xi_2) = \xi_1\xi_2 \\
\Psi_5(\boldsymbol{\xi}) &= \psi_0(\xi_1)\,\psi_2(\xi_2) = \xi_2^2 - 1
\end{aligned}$$

Distribution	Density function	Polynomial	Weight function	Support range
Normal	$\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$	Hermite $He_n(x)$	$e^{\frac{-x^2}{2}}$	$[-\infty, \infty]$
Uniform	$\frac{1}{2}$	Legendre $P_n(x)$	1	$[-1, 1]$
Beta	$\frac{(1-x)^\alpha (1+x)^\beta}{2^{\alpha+\beta+1} B(\alpha+1,\beta+1)}$	Jacobi $P_n^{(\alpha,\beta)}(x)$	$(1-x)^\alpha (1+x)^\beta$	$[-1, 1]$
Exponential	e^{-x}	Laguerre $L_n(x)$	e^{-x}	$[0, \infty]$
Gamma	$\frac{x^\alpha e^{-x}}{\Gamma(\alpha+1)}$	Generalized Laguerre $L_n^{(\alpha)}(x)$	$x^\alpha e^{-x}$	$[0, \infty]$

- Estimate α_j using regression or numerical integration: sampling, tensor quadrature, sparse grids, or cubature

$$\boxed{\alpha_j = \frac{\langle R, \Psi_j \rangle}{\langle \Psi_j^2 \rangle} = \frac{1}{\langle \Psi_j^2 \rangle} \int_\Omega R\, \Psi_j\, \varrho(\boldsymbol{\xi})\, d\boldsymbol{\xi}}$$

–Michael Eldred, DAKOTA Manual

–Truncate polynomial at some finite level, P

–Uses output value in estimation of coefficient

–Once coefficients are estimated, can calculate Main Effect and Total Effect Indices

- –A fitted polynomial can be integrated analytically. No need for MC
- –Won't have same numerical issues as MC Integration resulting in nonsensical negative indices

Polynomial Chaos Expansion

Sparse-grid level	Simulations required
2	173
3	1179 X
4	6355 X
5	28678 X

X: could not figure out reason DAKOTA crashed after initial cycle of simulations.(Support suggested hardware failure)

**Take PCE results lightly, just wanted some comparison

```
Global sensitivity indices for each response function:
gen_power Sobol' indices:
                                  Main                 Total
                      2.9331160476e-02  3.0660248745e-02 ti
                      8.0576063926e-02  8.0919909162e-02 alpha
                      7.3580736873e-01  7.3668587984e-01 yaw
                      1.4194062398e-01  1.4262999050e-01 rho
                      9.2825377000e-03  9.4714668253e-03 tor
                      2.0990698390e-04  1.0743794651e-03 a_u
                      2.4514856909e-04  1.1653150981e-03 b_u
                           Interaction
                      2.2109268092e-06 ti alpha
                      1.2469802388e-04 ti yaw
                      2.1545884425e-04 alpha yaw
                      3.4792718827e-05 ti rho
                      8.5342087774e-05 alpha rho
                      5.1282493397e-04 yaw rho
                      5.8799500231e-05 ti tor
                      3.7962620713e-05 alpha tor
                      1.6727944086e-05 yaw tor
                      5.3785351752e-05 rho tor
                      5.2769464342e-04 ti a_u
                      6.8356299788e-07 alpha a_u
                      2.7361247141e-06 yaw a_u
                      1.5660111842e-06 rho a_u
                      1.1739813541e-05 tor a_u
                      5.8089245583e-04 ti b_u
                      2.1871931780e-06 alpha b_u
                      6.0652410972e-06 yaw b_u
                      1.0554185999e-06 rho b_u
                      9.9138949848e-06 tor b_u
                      3.2005232535e-04 a_u b_u
```

(Unconverged) PCE Total Sensitivity:

yaw	0.737
rho	0.143
alpha	0.0809
ti	0.0307
tor	0.00947
b_u	0.00117
a_u	0.00107

VBD Total Sensitivity:

yaw	0.685
rho	0.134
ti	0.0837
alpha	0.072
b_u	0.0389
a_u	0.0221
tor	0.00900

EE Sensitivity:

yaw	6.53
rho	3.89
alpha	2.34
ti	2.10
tor	0.73
b_u	0.63
a_u	0.42

```
rootmybl Sobol' indices:
                               Main                Total
                   1.1234331224e-03  2.4026740583e-03 ti
                   2.3518945871e-02  2.4294130328e-02 alpha
                   6.3417383047e-01  6.3530841011e-01 yaw
                   2.8478609594e-01  2.8530717214e-01 rho
                   4.8117560725e-02  4.8210668736e-02 tor
                   1.8593941077e-03  2.7143513169e-03 a_u
                   3.4590324129e-03  4.7243006583e-03 b_u
                        Interaction
                   4.6785280164e-06 ti alpha
                   4.3609657035e-06 ti yaw
                   6.6385889831e-04 alpha yaw
                   3.3337381955e-06 ti rho
                   4.8075040235e-05 alpha rho
                   4.3220455694e-04 yaw rho
                   1.7518421323e-05 ti tor
                   3.9229723924e-05 alpha tor
                   8.6437774619e-06 yaw tor
                   1.5065468060e-05 rho tor
                   4.1099369136e-04 ti a_u
                   1.1781973449e-05 alpha a_u
                   1.2306542571e-05 yaw a_u
                   1.7257621891e-05 rho a_u
                   7.1301567012e-06 tor a_u
                   8.3835559126e-04 ti b_u
                   7.5602927787e-06 alpha b_u
                   1.3204900618e-05 yaw b_u
                   5.1397746440e-06 rho b_u
                   5.5204627576e-06 tor b_u
                   3.9548722326e-04 a_u b_u
```

(Unconverged) VBD Total Sensitivity

yaw	0.635
rho	0.285
tor	0.0482
alpha	0.0243
b_u	0.00472
a_u	0.00271
ti	0.00240

VBD Total Sensitivity

yaw	0.647
rho	0.271
tor	0.0462
alpha	0.0215
b_u	0.0189
ti	0.0129
a_u	0.0104

EE Sensitivity

yaw	2.47
rho	2.00
tor	0.64
alpha	0.47
ti	0.16
b_u	0.14
a_u	0.09

```
rootmxbl Sobol' indices:
                               Main              Total
                      3.0786852812e-02  3.8397310291e-02 ti
                      1.0392098983e-01  1.0861644332e-01 alpha
                      3.4083098560e-01  3.4931849898e-01 yaw
                      1.1783983932e-01  1.2438331670e-01 rho
                      3.3836008464e-01  3.4250445718e-01 tor
                      2.2141588587e-02  3.1877975529e-02 a_u
                      1.9856979761e-02  3.1164677455e-02 b_u
                           Interaction
                      1.7951548682e-03 ti alpha
                      9.1584235448e-04 ti yaw
                      3.4179483468e-04 alpha yaw
                      1.7297187919e-03 ti rho
                      8.2237587735e-04 alpha rho
                      1.3959150581e-03 yaw rho
                      1.0145133249e-03 ti tor
                      1.3136461050e-04 alpha tor
                      1.6085143867e-03 yaw tor
                      1.5329469654e-04 rho tor
                      1.0191824232e-03 ti a_u
                      6.9038005233e-04 alpha a_u
                      1.5701500432e-03 yaw a_u
                      1.1918906752e-03 rho a_u
                      5.7488976042e-04 tor a_u
                      1.1360457161e-03 ti b_u
                      9.1438324016e-04 alpha b_u
                      2.6552967025e-03 yaw b_u
                      1.2502822832e-03 rho b_u
                      6.6179576474e-04 tor b_u
                      4.6898939879e-03 a_u b_u
```

(Unconverged) PCE Total Sensitivity

yaw	0.349
tor	0.343
rho	0.124
alpha	0.109
ti	0.0384
a_u	0.0319
b_u	0.0312

VBD Total Sensitivity

tor	0.441
yaw	0.403
rho	0.0604
ti	0.0521
alpha	0.0518
b_u	0.0212
a_u	0.0137

EE Sensitivity

yaw	0.33
tor	0.29
rho	0.19
ti	0.14
alpha	0.13
b_u	0.06
a_u	0.04

Table 7: Sobol' Indices for the Ishigami Function

$$f = \sin(2\pi x_1 - \pi) + 7\sin^2(2\pi x_2 - \pi) + 0.1(2\pi x_3 - \pi)^4 \sin(2\pi x_1 - \pi)$$

Approach	Func Eval	Grid Type	S_1	S_2	S_3	S_{T_1}	S_{T_2}	S_{T_3}
LHS	5000	-	2.88446e-01	4.42871e-01	5.52915e-03	5.93029e-01	4.00977e-01	2.32240e-01
LHS	25000	-	3.05843e-01	4.37050e-01	-1.87194e-02	5.66739e-01	4.30240e-01	2.48935e-01
LHS	50000	-	3.28016e-01	4.34040e-01	-8.27517e-03	5.61866e-01	4.45603e-01	2.41798e-01
LHS	250000	-	3.15380e-01	4.42334e-01	5.15642e-03	5.55366e-01	4.43332e-01	2.46431e-01
LHS	500000	-	3.14020e-01	4.42135e-01	-9.13024e-04	5.60501e-01	4.33634e-01	2.36443e-01
PCE	8	tr2	1.00000e-00	3.62082e-32	3.62082e-32	1.00000e-00	7.01534e-32	2.12723e-31
PCE	27	tr3	4.15063e-01	4.39941e-01	2.69333e-32	5.60059e-01	4.39941e-01	1.44995e-01
PCE	64	tr4	3.99679e-01	3.15380e-01	2.17138e-32	6.84620e-01	3.15380e-01	2.84941e-01
PCE	216	tr6	4.05169e-01	2.80300e-01	1.56108e-30	7.19700e-01	2.80300e-01	3.14531e-01
PCE	25	sp2	1.00000e-00	1.36887e-31	0.00000e-00	1.00000e-00	1.36887e-31	0.00000e-00
PCE	177	sp4	6.71159e-01	4.44492e-02	1.83743e-07	9.55551e-01	9.59348e-02	2.32906e-01
PCE	1073	sp6	3.69705e-01	3.54341e-01	3.14446e-19	6.45659e-01	3.54790e-01	2.75516e-01
PCE	6017	sp8	3.16646e-01	4.37727e-01	1.91006e-31	5.62273e-01	4.37727e-01	2.45627e-01
PCE	32001	sp10	3.13906e-01	4.42411e-01	2.04501e-30	5.57589e-01	4.42411e-01	2.43684e-01

Tang, G., Iaccarino, G., and Eldred, M.S., "Global Sensitivity Analysis for Stochastic Collocation Expansion, paper AIAA-2010-2922 in *Proceedings of the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference (12th AIAA Non-Deterministic Approaches Conference)*, Orlando, FL, Apr 12-15, 2010.

Parameters Conclusions

- Wind speed is orders of magnitude more important than any other parameter
  - SA purpose: research prioritization. Allocate resources to measure uncertainty, physics more.
- Across the studied QoIs, yaw had high sensitivity
  - SA purpose: research prioritization. Allocate resources to measure uncertainty, physics more.
- Some parameters could be insignificant for one QoI, but very significant for another QoI
  - tor had highest sensitivity for edgewise moment
  - SA purpose: searching for errors in the model
- Spatial coherence parameters consistently bottom of rankings
  - SA purpose: model simplification. Fix these values and simplify simulation.

Methods Conclusion

–Elementary Effects

- Computationally cheap
- Filters out extreme parameters for future studies
- Good proxy for total sensitivity
- Qualitative

–Variance Based QMC

- Computationally expensive
- Numerical issues with subtraction
- Slow convergence rate
- Quantitative: Main and Total Indices

–PCE

- Exponential convergence rate
- Provides quantitative indices via polynomial coefficients
- Provides functional relationships between a set of output response metrics and set of input variables
- DAKOTA parallelization issues and frequent crashing
  - Can fix this with more time
- Tough math behind the scenes, difficult to code

Thanks!

–Access to top tier supercomputing resources

–Improved Linux, shell–scripting skills, scaling code, managing large amounts of data

–Global Sensitivity Analysis techniques

–Learned about wind energy and the importance of simulations in field

- Got to climb SWiFT turbine!