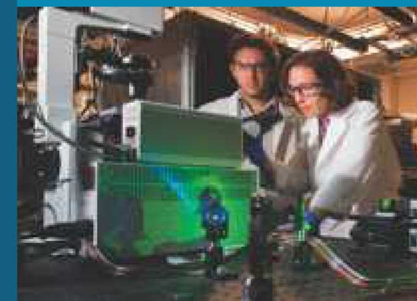
SAND2019-10289PE

Aubertin (SUVIC) Model



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Outline

1. Motivation
    a. Rational Thermodynamic Framework
    b. Kinematic Hardening
    c. Constant Strain Rate Tests
2. Model Formulation
3. First Calibration Attempt
4. Summary

Motivation

Framework of Rational Thermodynamics

1. First law of thermodynamics

$$\rho \dot{e} = \rho r - \nabla \cdot \boldsymbol{q} + \boldsymbol{\sigma} : \dot{\varepsilon}$$

2. Second law of thermodynamics

$$\rho \dot{s} - \rho \frac{r}{T} + \nabla \cdot \left(\frac{\boldsymbol{q}}{T} \right) \geq 0$$

3. Helmholtz free energy

$$\psi = \psi\left(T, \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{ie}}, \boldsymbol{\xi}\right) \qquad \boldsymbol{x} = \rho \frac{\partial \psi}{\partial \boldsymbol{\xi}}$$

4. Combine with several standard assumptions to obtain

$$s = -\frac{\partial \psi}{\partial T} \qquad \boldsymbol{\sigma} = \rho \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}}$$

$$\mathcal{D}^{\text{th}} = -\frac{1}{T} \boldsymbol{q} \cdot \nabla T \geq 0$$

$$\mathcal{D}^{\text{me}} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{\text{ie}} - \boldsymbol{x} \cdot \dot{\boldsymbol{\xi}} \geq 0$$

Framework of Rational Thermodynamics

5. Many ways to enforce dissipation inequality

$$\mathcal{D}^{\text{me}} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{\text{ie}} - \boldsymbol{x} \cdot \dot{\boldsymbol{\xi}} \geq 0$$

   a. Maximize mechanical dissipation

$$\min(-\mathcal{D}^{\text{me}} + \lambda f) \quad \rightarrow \quad \dot{\boldsymbol{\varepsilon}}^{\text{ie}} = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} \quad \text{and} \quad \dot{\boldsymbol{\xi}} = -\lambda \frac{\partial f}{\partial \boldsymbol{x}}$$

   b. Construct a dissipation potential, *g*
      i. *g* must be convex or a homogeneous function of degree *n*

$$\dot{\boldsymbol{\varepsilon}}^{\text{ie}} = w \frac{\partial g}{\partial \boldsymbol{\sigma}} \quad \text{and} \quad \dot{\boldsymbol{\xi}} = -w \frac{\partial g}{\partial \boldsymbol{x}}$$

6. Everything is derived from a Helmholtz free energy and a yield function (or a dissipation potential).
   a. Forces you to stay organized.
   b. Every internal state variable has a conjugate driving force.

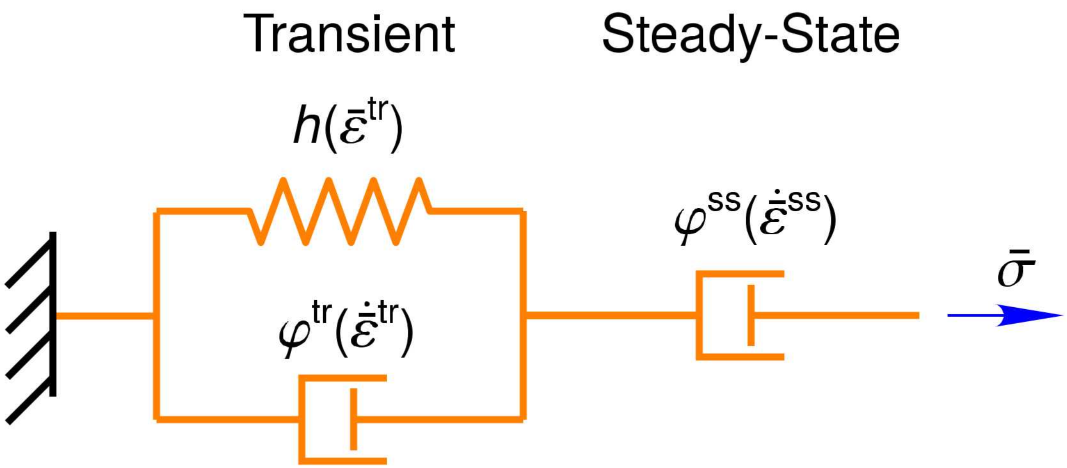
Attempt to Fit MD Model into Framework

1. Helmholtz free energy and dynamic yield functions

$$\psi = \frac{1}{2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{vp}}) : \boldsymbol{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{vp}}) + \psi^{\mathrm{hd}}(\bar{\varepsilon}^{\mathrm{tr}})$$

$$f^{\mathrm{ss}} = \bar{\sigma} - \varphi^{\mathrm{ss}}(\dot{\bar{\varepsilon}}^{\mathrm{ss}}) = 0$$

$$f^{\mathrm{tr}} = \bar{\sigma} - h(\bar{\varepsilon}^{\mathrm{tr}}) - \varphi^{\mathrm{tr}}(\dot{\bar{\varepsilon}}^{\mathrm{tr}}) = 0$$



Attempt to Fit MD Model into Framework

2. Turn crank to obtain

$$\dot{\boldsymbol{\varepsilon}}^{\text{vp}} = \left(\dot{\bar{\varepsilon}}^{\text{tr}} + \dot{\bar{\varepsilon}}^{\text{ss}}\right) \frac{\partial \bar{\sigma}}{\partial \boldsymbol{\sigma}}$$

$$\dot{\bar{\varepsilon}}^{\text{ss}} = \varphi^{\text{ss}-1}(\bar{\sigma})$$

$$\dot{\bar{\varepsilon}}^{\text{tr}} = \varphi^{\text{tr}-1}\left(\bar{\sigma} - h(\bar{\varepsilon}^{\text{tr}})\right)$$

2. Compare to existing MD equations

$$\dot{\bar{\varepsilon}}^{\text{ss}} = \sum_{i=0}^{3} \dot{\bar{\varepsilon}}_i^{\text{ss}}(\bar{\sigma})$$

$$\dot{\bar{\varepsilon}}^{\text{tr}} = \left\{\exp\left[\kappa(\bar{\sigma})\left(1 - \frac{\bar{\varepsilon}^{\text{tr}}}{\bar{\varepsilon}^{\text{tr}*}(\bar{\sigma})}\right)^2\right] - 1\right\} \dot{\bar{\varepsilon}}^{\text{ss}}$$

3. What is the hardening conjugate to $\bar{\varepsilon}^{\text{tr}}$?

$$h(\bar{\varepsilon}^{\text{tr}}) = \rho \frac{\partial \psi}{\partial \bar{\varepsilon}^{\text{tr}}}$$

Stress Drop Experiments

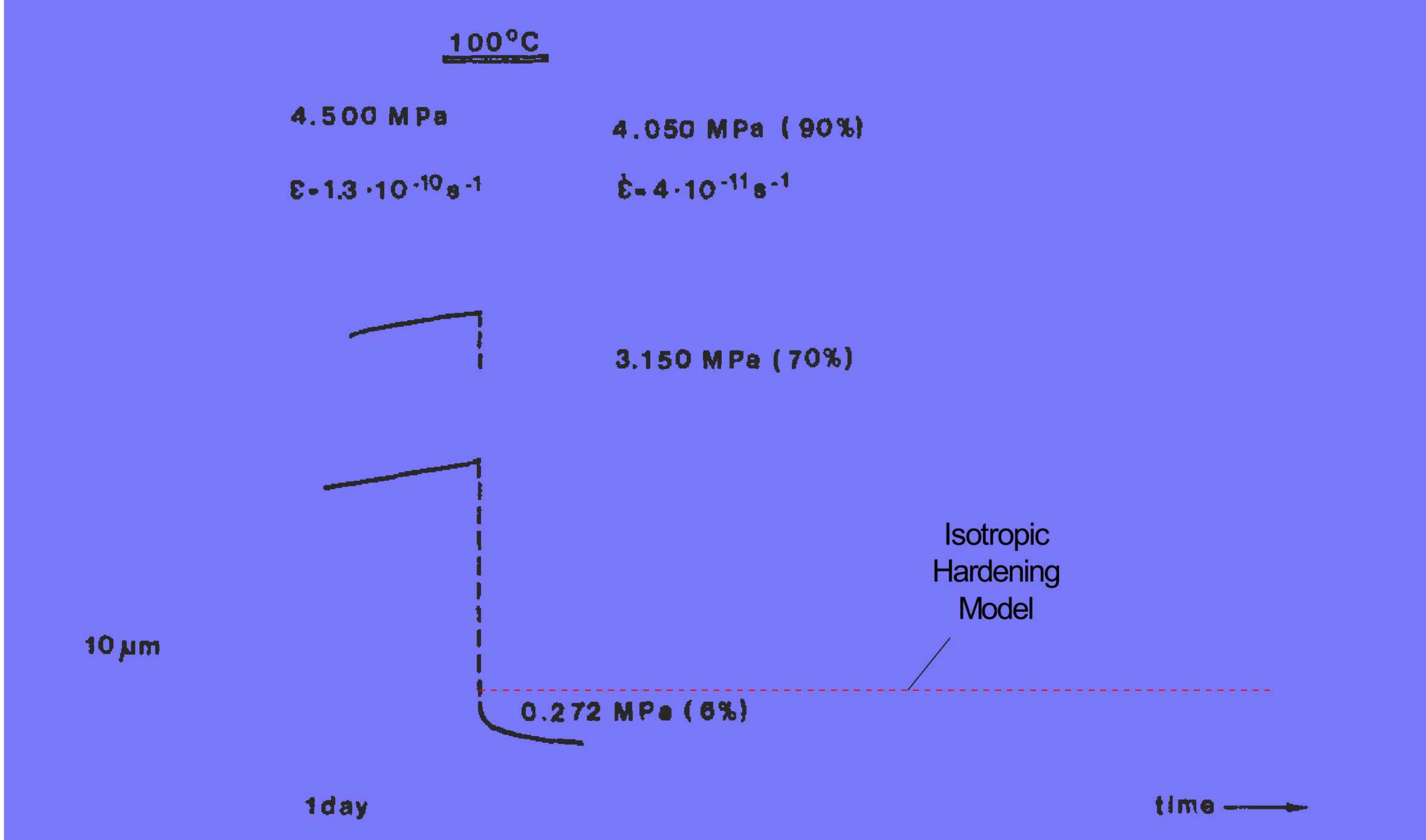
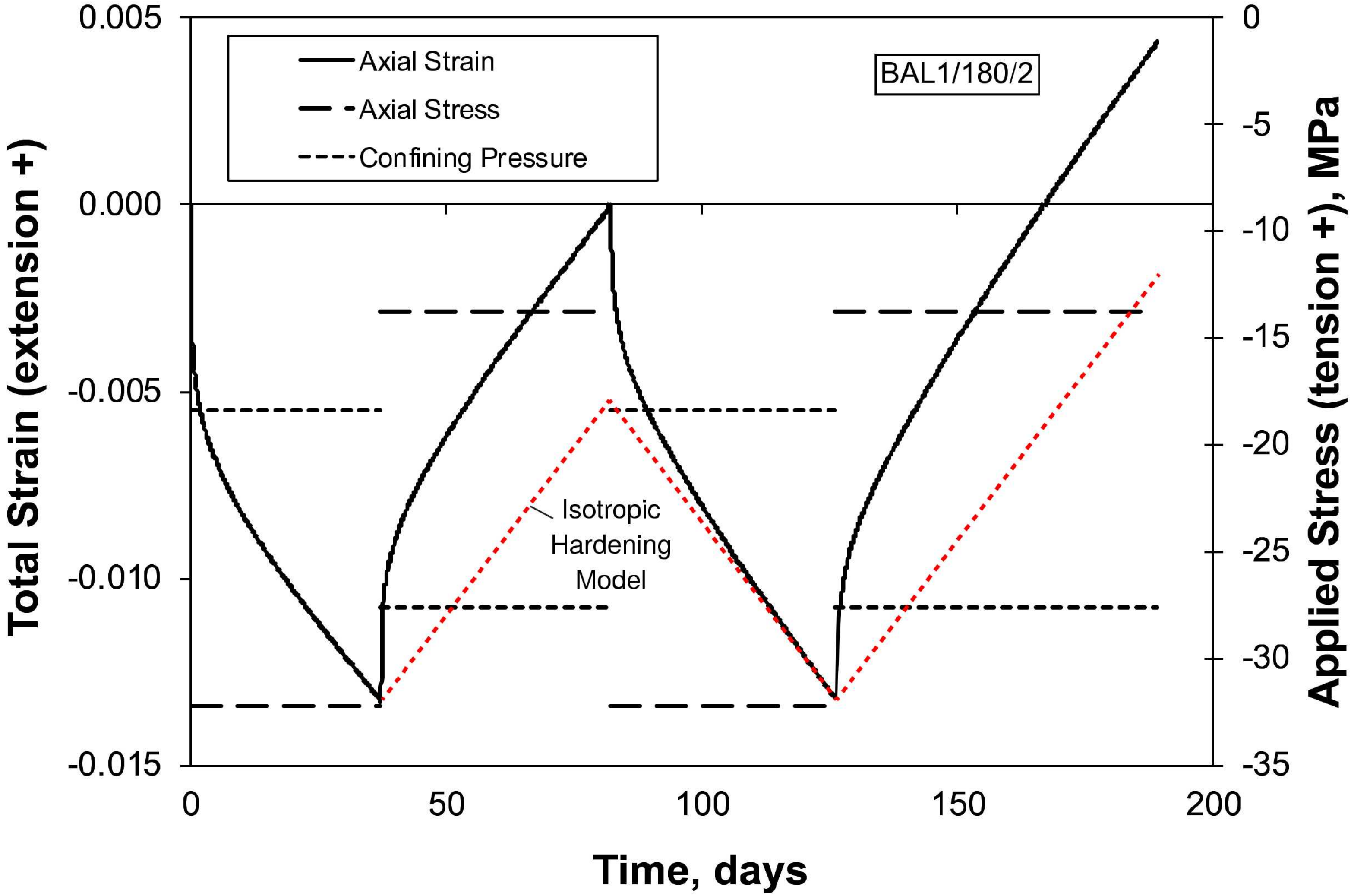


Figure 3 – Creep Experiment at 100 °C with Different Stress Drops. Sample ASSE 8/6.1.

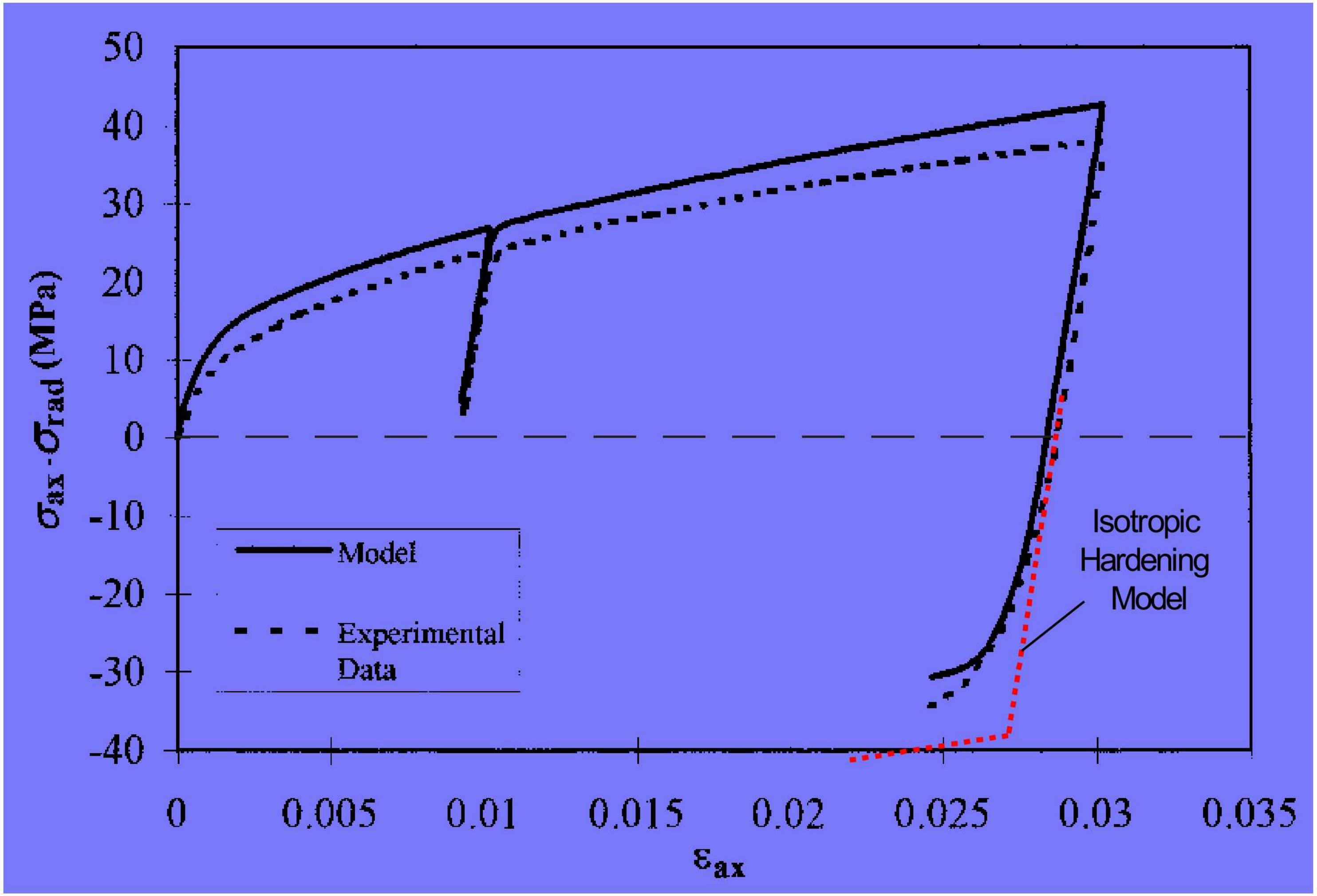
Hunsche, U. 1988. Measurement of creep in rock salt at small strain rates. Proceedings of the 2nd Conference on the Mechanical Behavior of Salt. Pg. 187-196 (Modified)

Triaxial Compression / Extension Cycling



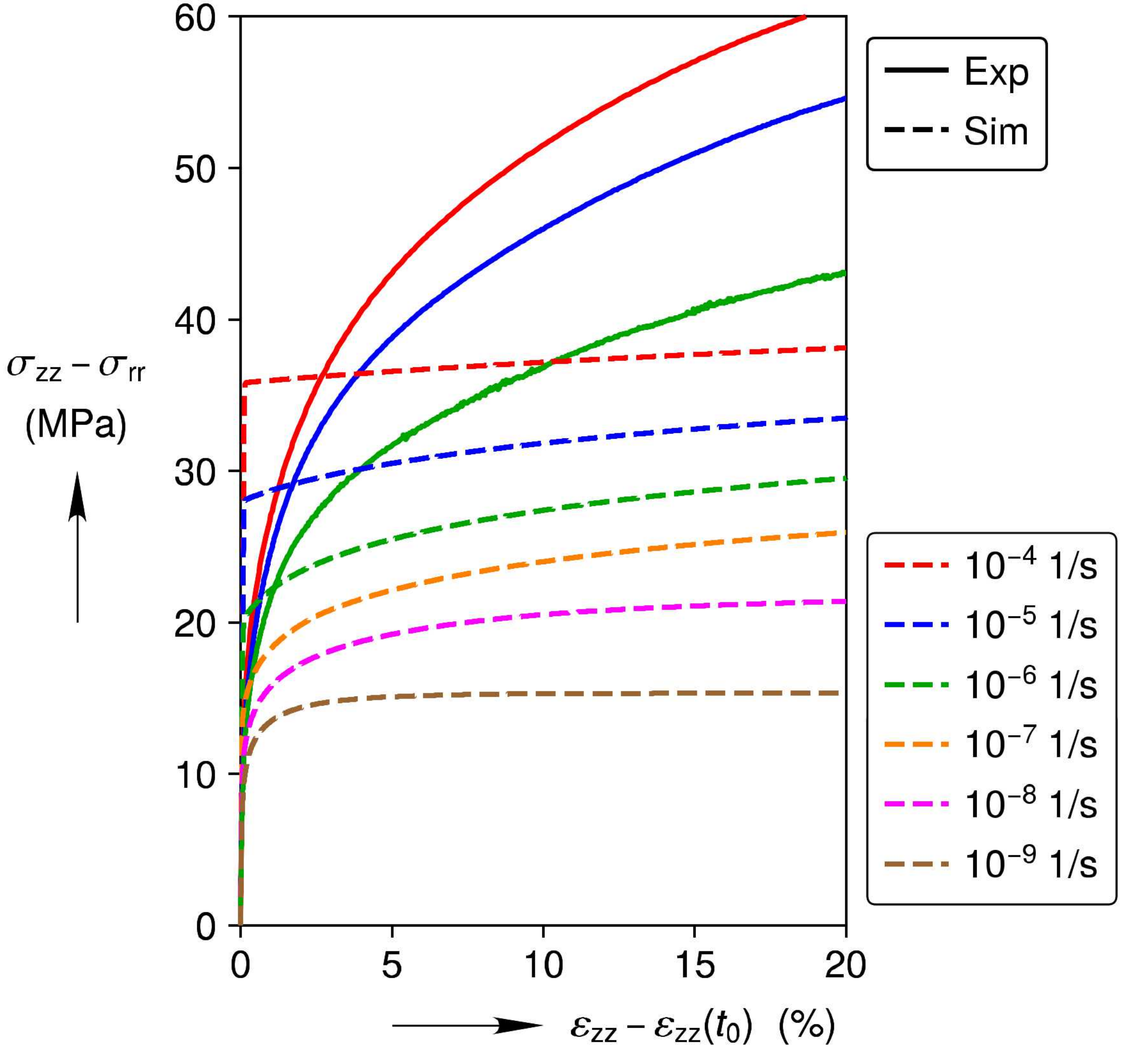
Mellegard, K., DeVries, K., and Callahan, G. 2007. Lode angle effects on the creep of salt. Proc. 6th Conference on the Mechanical Behavior of Salt. Pg. 9-15 (Modified)

Bauschinger Effect

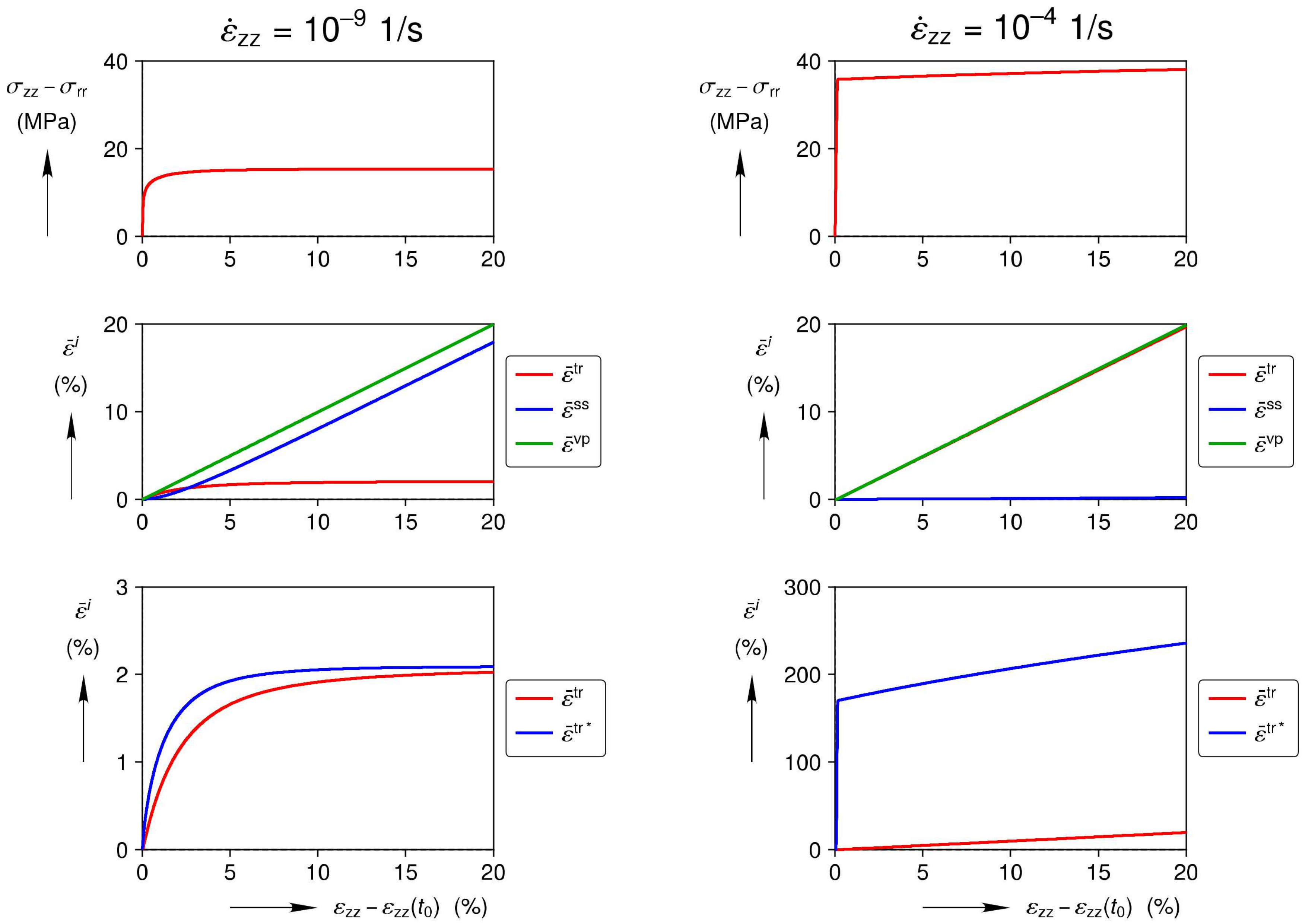


Yahya, O., Aubertin, M., and Julien, M. 2000. A unified representation of the plasticity, creep and relaxation behavior of rocksalt. International Journal of Rock Mechanics and Mining Sciences. Vol. 37. No. 5. Pg. 787-800 (Modified)

Munson-Dawson Predictions of CSR Tests



Closer Look at Munson-Dawson CSR Behavior



Model Formulation

Top Level Formulation

Strain Decomposition

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{\mathrm{el}} + \dot{\boldsymbol{\varepsilon}}^{\mathrm{vp}}$$

Isotropic, Linear, Hypoelasticity

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{\mathcal{C}} : \dot{\boldsymbol{\varepsilon}}^{\mathrm{el}}$$

$$\boldsymbol{\mathcal{C}} = (K - 2\,G/3)\,\boldsymbol{I} \otimes \boldsymbol{I} + 2\,G\,\boldsymbol{\mathcal{I}}$$

Associated Flow Rule

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{vp}} = \dot{\bar{\varepsilon}}^{\mathrm{vp}} \frac{\partial \bar{\sigma}^{\mathrm{vp}}}{\partial \boldsymbol{\sigma}}$$

Equivalent Viscoplastic Strain Rate

$$\dot{\bar{\varepsilon}}^{\mathrm{vp}} = H_{\mathrm{sr}} \left(\frac{\bar{\sigma}^{\mathrm{vp}}}{h} \right)^{H_{\mathrm{e}}}$$

Equivalent Viscoplastic Stress

$$\bar{\sigma}^{\mathrm{vp}} = \sqrt{\tfrac{3}{2}\,(\boldsymbol{\sigma}^{\mathrm{d}} - \boldsymbol{b}) : (\boldsymbol{\sigma}^{\mathrm{d}} - \boldsymbol{b})}$$

(rate independent hardening ignored)

Yahya, O., Aubertin, M., Julien, M. 2000. A unified representation of the plasticity, creep and relaxation behavior of rocksalt. International Journal of Rock Mechanics and Mining Sciences. Vol 37. No 5. Pg. 787-800

Equivalent Viscoplastic Stress

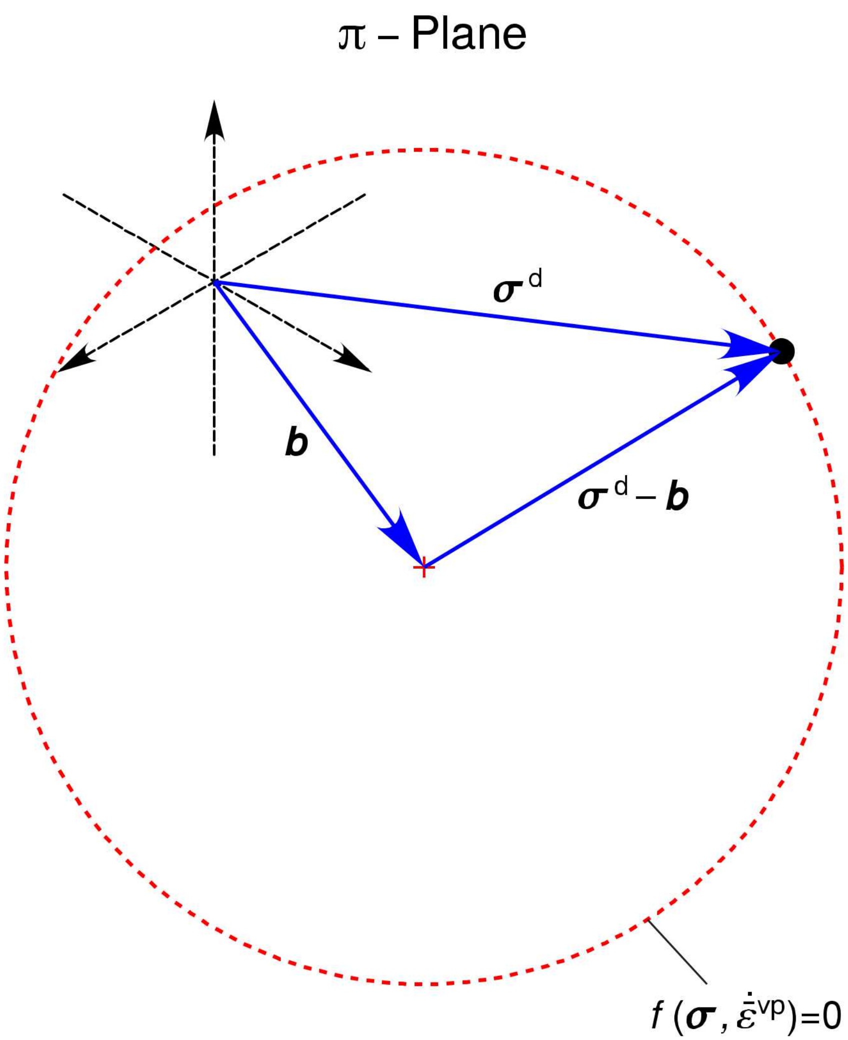
$$\bar{\sigma}^{\mathrm{vp}} = \sqrt{\tfrac{3}{2}\,(\boldsymbol{\sigma}^{\mathrm{d}} - \boldsymbol{b}) : (\boldsymbol{\sigma}^{\mathrm{d}} - \boldsymbol{b})}$$

if $\boldsymbol{\sigma}^{\mathrm{d}}$ and $\boldsymbol{b}$ aligned

$$\bar{\sigma}^{\mathrm{vp}} = \bar{\sigma} - \bar{b}$$

$$\bar{\sigma} = \sqrt{\tfrac{3}{2}\,\boldsymbol{\sigma}^{\mathrm{d}} : \boldsymbol{\sigma}^{\mathrm{d}}}$$

$$\bar{b} = \sqrt{\tfrac{3}{2}\,\boldsymbol{b} : \boldsymbol{b}}$$



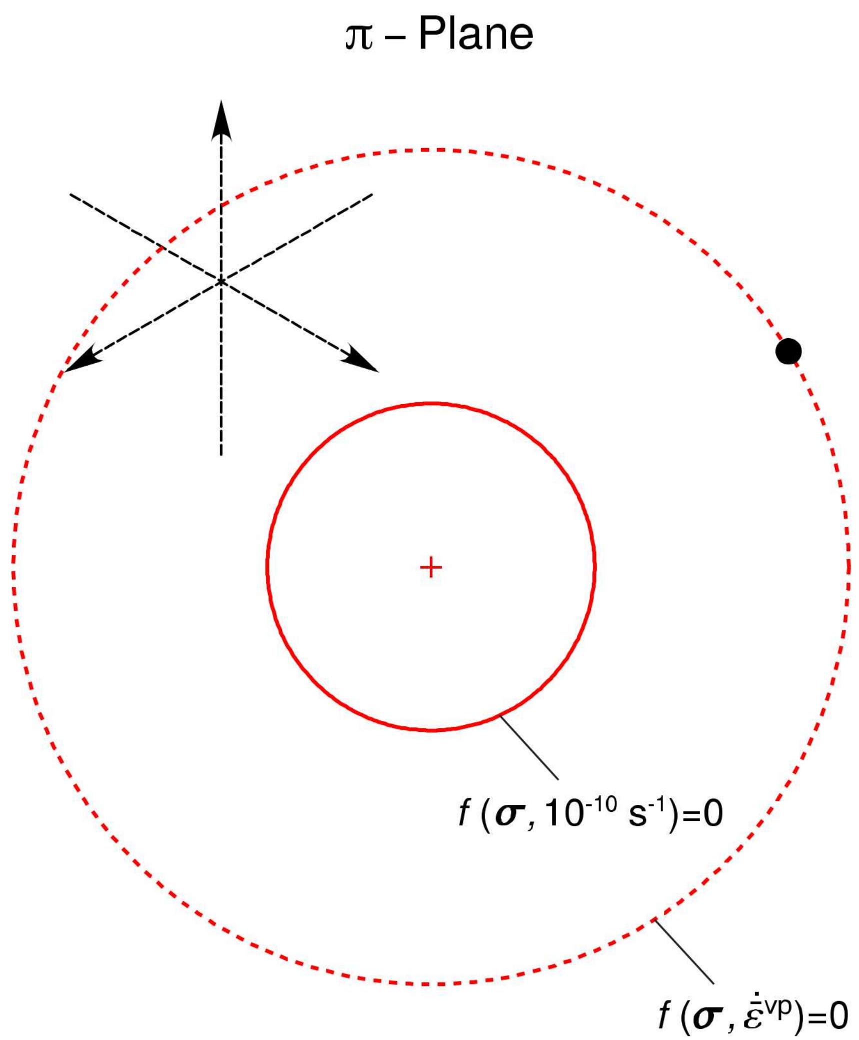
Rate-Dependent Yield Function

$$\dot{\varepsilon}^{\mathrm{vp}} = H_{\mathrm{sr}} \left(\frac{\bar{\sigma}^{\mathrm{vp}}}{h} \right)^{H_{\mathrm{e}}}$$

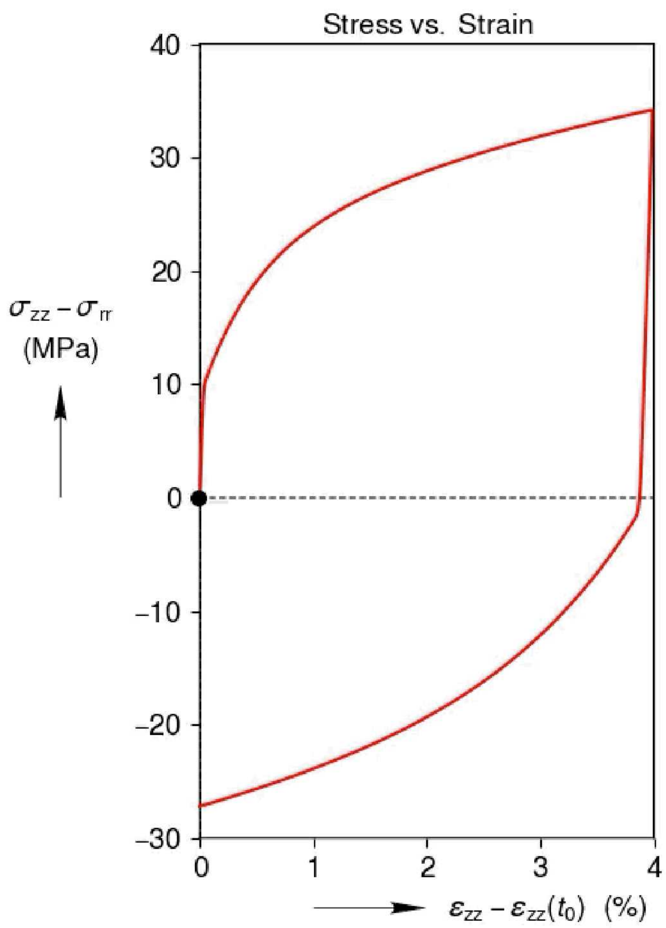
$$f(\boldsymbol{\sigma}, \dot{\varepsilon}^{\mathrm{vp}}) = \bar{\sigma}^{\mathrm{vp}} - h \left(\frac{\dot{\varepsilon}^{\mathrm{vp}}}{H_{\mathrm{sr}}} \right)^{1/H_{\mathrm{e}}} = 0$$

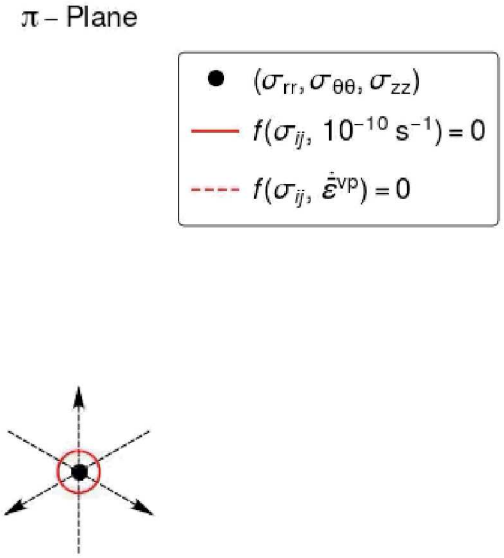
if $\boldsymbol{\sigma}^{\mathrm{d}}$ and $\boldsymbol{b}$ aligned

$$f(\boldsymbol{\sigma}, \dot{\varepsilon}^{\mathrm{vp}}) = \bar{\sigma} - \bar{b} - h \left(\frac{\dot{\varepsilon}^{\mathrm{vp}}}{H_{\mathrm{sr}}} \right)^{1/H_{\mathrm{e}}} = 0$$

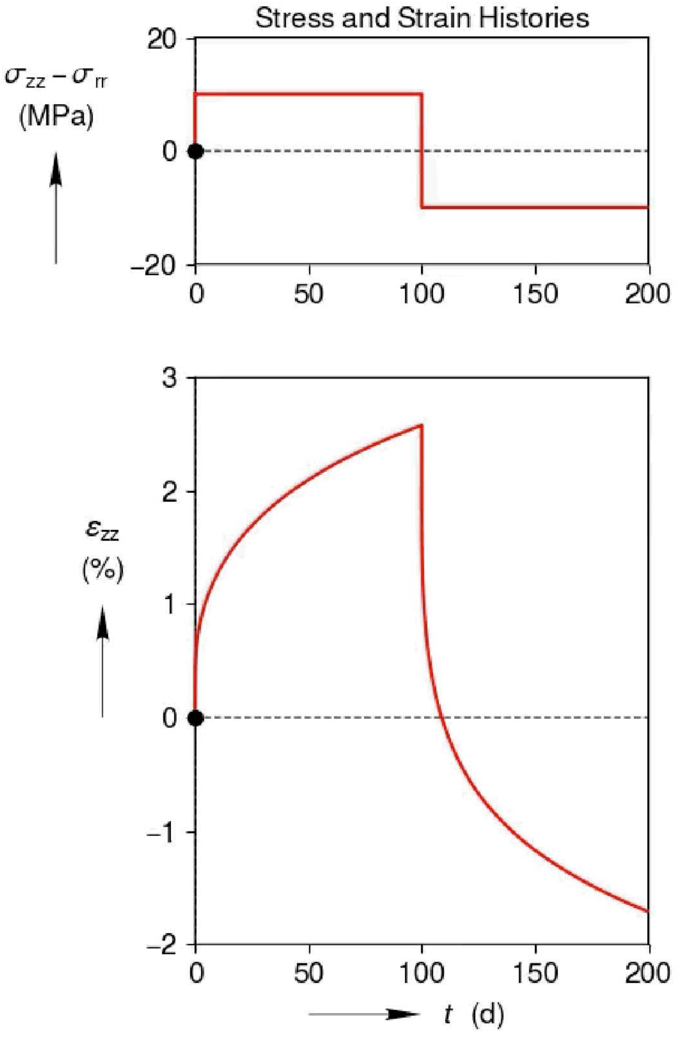


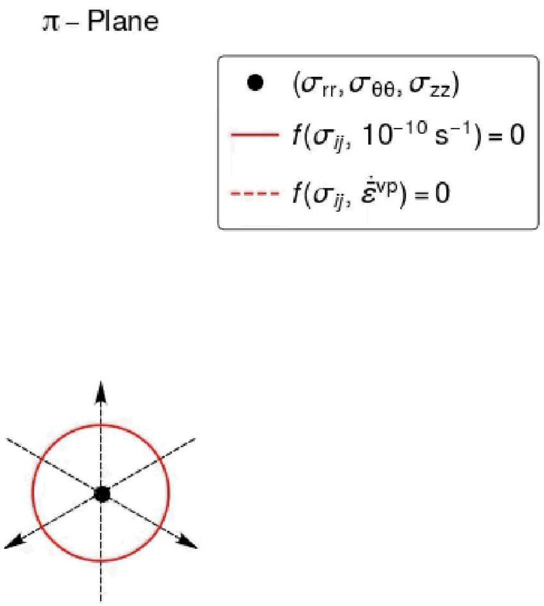
Bauschinger Effect



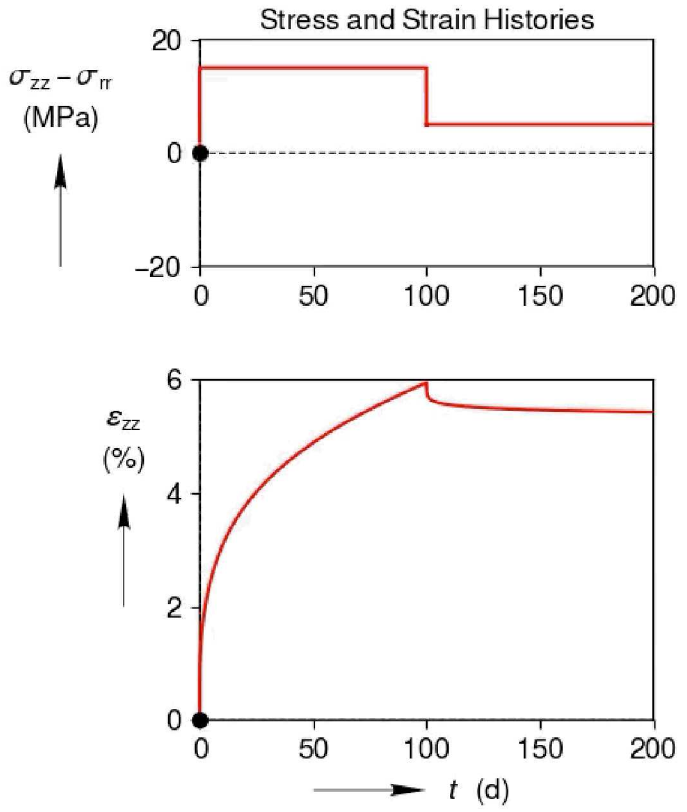


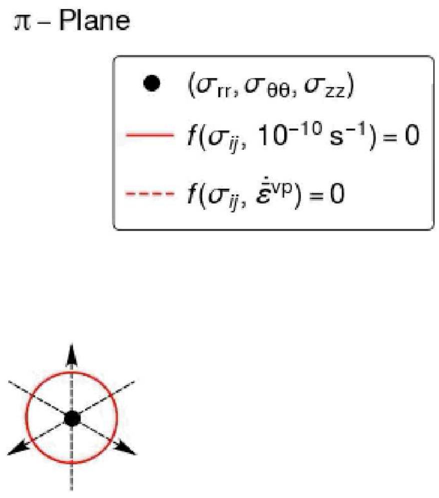
Triaxial Compression to Triaxial Extension Creep





Creep with Stress Drop





Isotropic Hardening

Evolution Equation

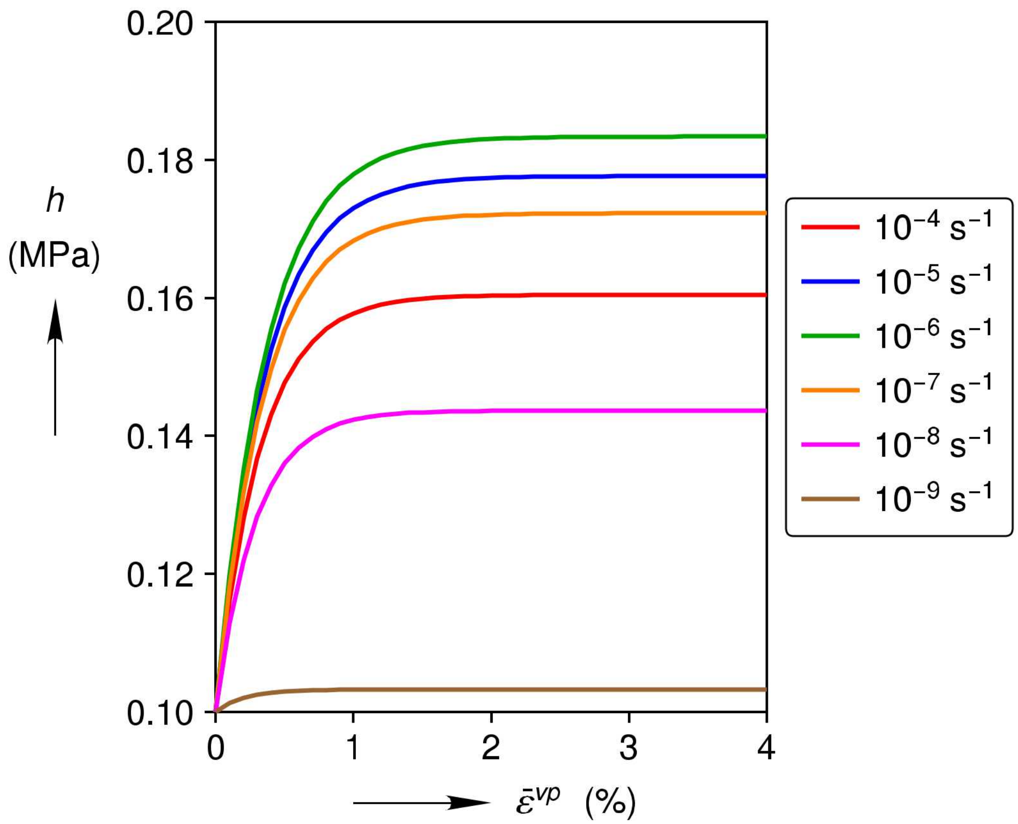
$$\dot{h} = H_r \left(\dot{\bar{\varepsilon}}^{vp} - \frac{h}{h^s} \dot{\bar{\varepsilon}}^{vp} \right)$$

Hardening Saturation
(precisely defined later)

$$h^s = h^s(\dot{\bar{\varepsilon}}^{vp})$$

Analytical Solution
(constant strain rate)

$$h = h^s \left[1 - \left(1 - \frac{h_0}{h^s} \right) \exp\left(- \frac{H_r\, \bar{\varepsilon}^{vp}}{h^s} \right) \right]$$



(rate independent hardening and thermal recovery ignored)

Kinematic Hardening

Evolution Equation

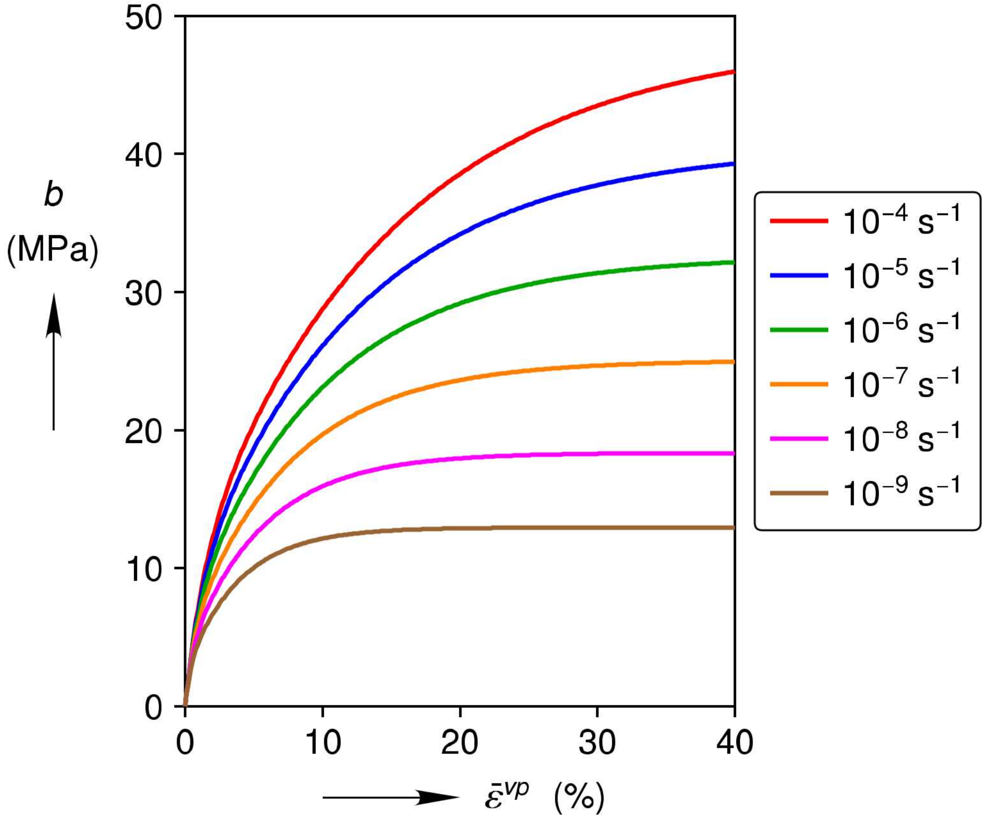
$$\dot{\boldsymbol{b}} = B_\mathrm{r}\left(\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^\mathrm{vp} - \frac{\boldsymbol{b}}{\bar{b}^\mathrm{s}}\,\dot{\bar{\varepsilon}}^\mathrm{vp}\right)$$

Hardening Saturation
(precisely defined later)

$$h^\mathrm{s} = h^\mathrm{s}(\dot{\bar{\varepsilon}}^\mathrm{vp})$$

Analytical Solution
(constant strain rate)

$$\bar{b} = \bar{b}^\mathrm{s}\left[1 - \exp\left(-\frac{B_\mathrm{r}\,\bar{\varepsilon}^\mathrm{vp}}{\bar{b}^\mathrm{s}}\right)\right]$$



(thermal recovery and secondary backstress ignored)

Secondary Creep

Hardening Saturations

$$h^{s} = \frac{\langle \bar{\sigma}^{s} - \bar{b}^{s} \rangle}{\left(\dfrac{\dot{\bar{\varepsilon}}^{vp}}{H_{sr}} \right)^{1/H_{e}}} \qquad \bar{b}^{s} = B_{c} \left(\frac{\bar{\sigma}^{s}}{A_{c}} \right)^{B_{e}} \qquad \bar{\sigma}^{s} = A_{c} \operatorname{arcsinh} \left[\left(\frac{\dot{\bar{\varepsilon}}^{vp}}{A_{sr}} \right)^{1/A_{e}} \right]$$

Equivalent viscoplastic strain rate at saturation

$$\dot{\bar{\varepsilon}}^{vp} = H_{sr} \left(\frac{\bar{\sigma}^{vp}}{h^{s}} \right)^{H_{e}}$$

σ^{d} and $\boldsymbol{b}$ aligned at saturation

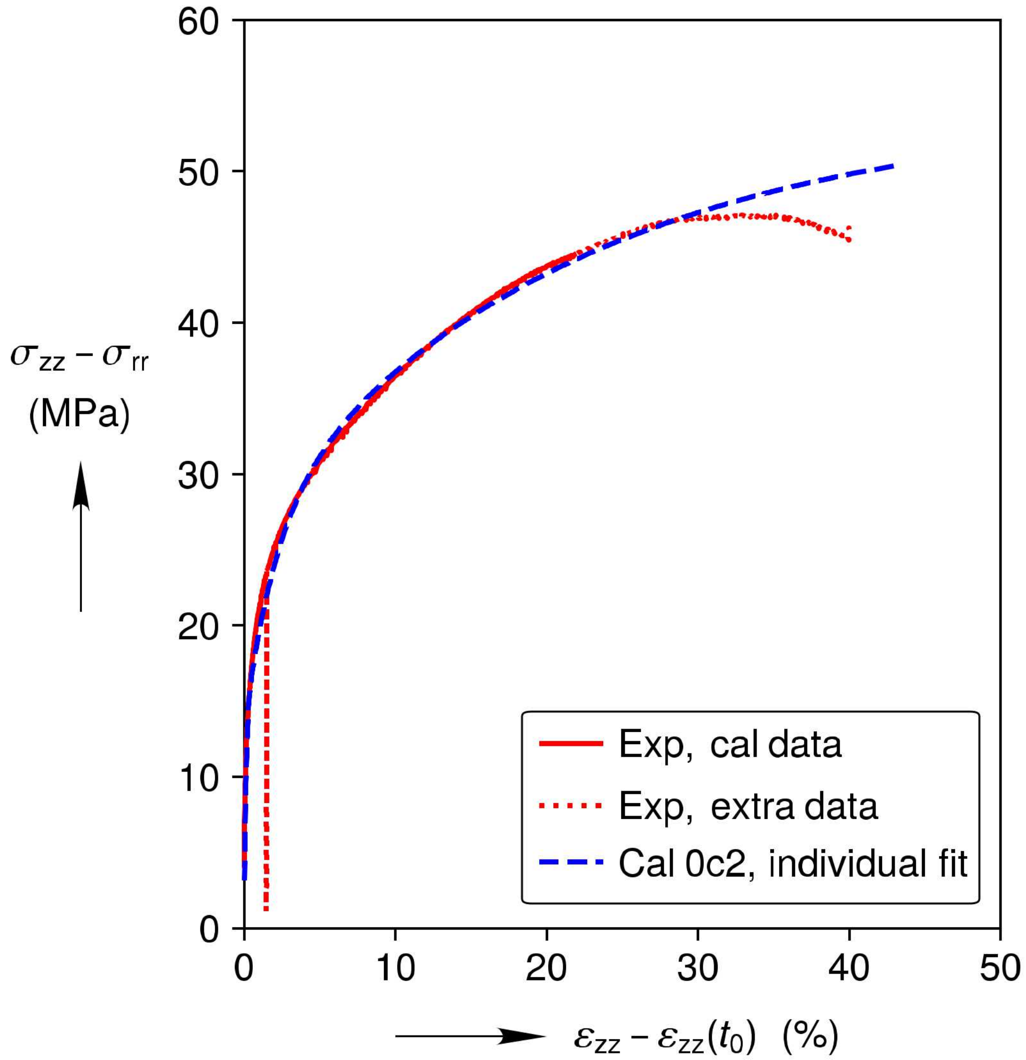
$$\bar{\sigma}^{vp} = \bar{\sigma} - \bar{b}^{s}$$

Put all together to obtain

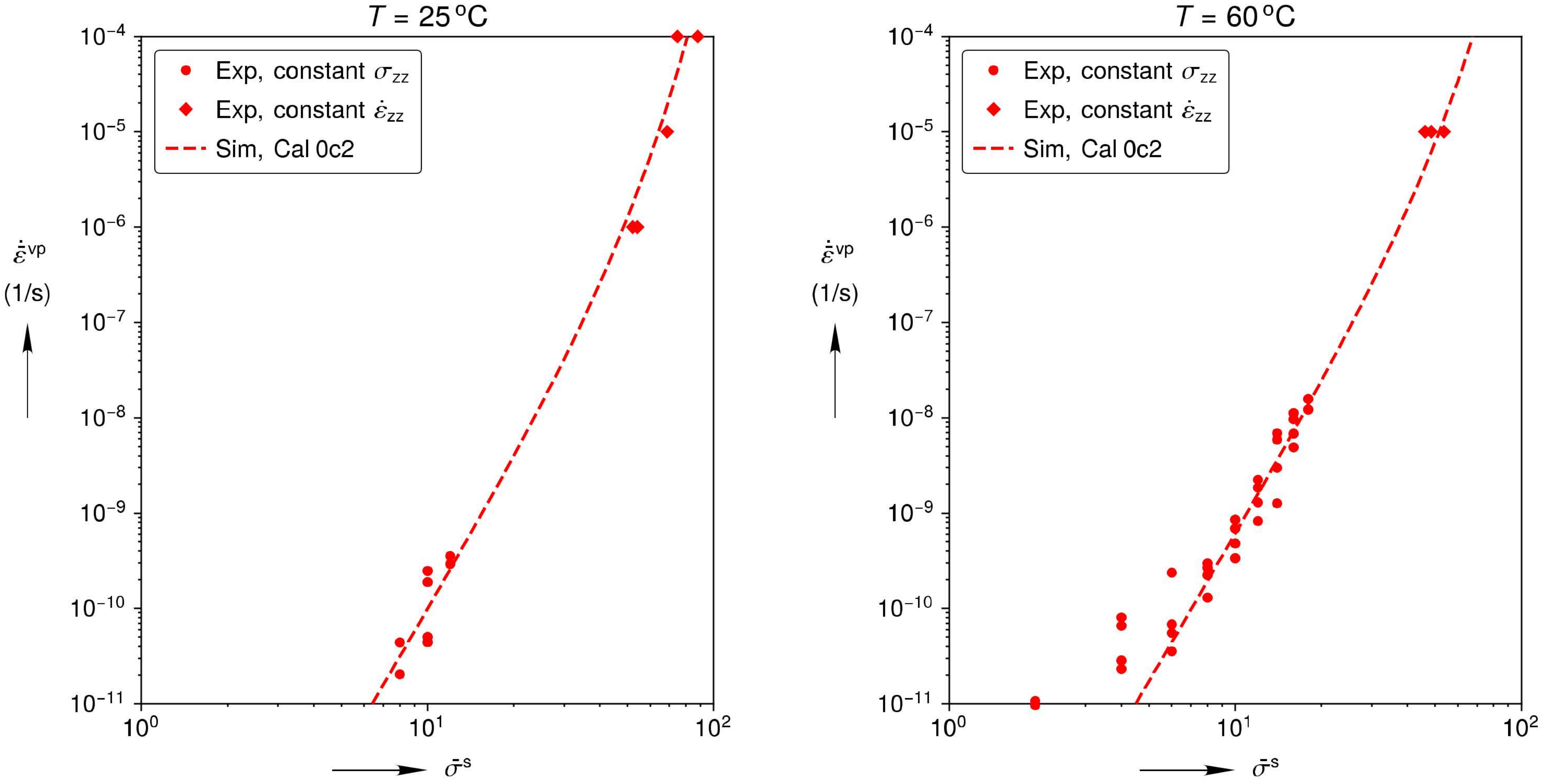
$$\dot{\bar{\varepsilon}}^{vp} = A_{sr} \left[\sinh \left(\frac{\bar{\sigma}^{s}}{A_{c}} \right) \right]^{A_{e}}$$

First Calibration Attempt

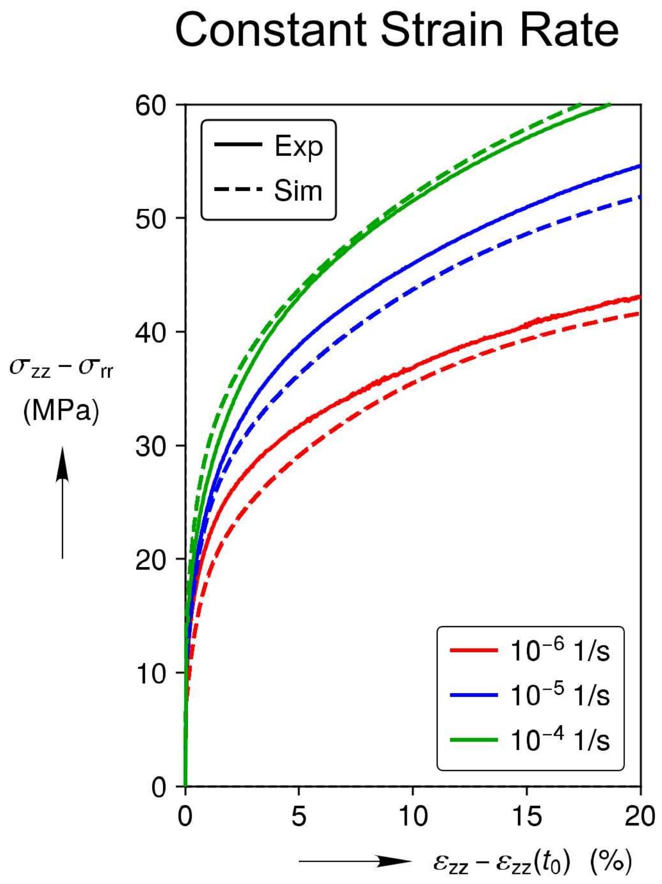
Calibrate Model Against Each Constant Strain Rate Test

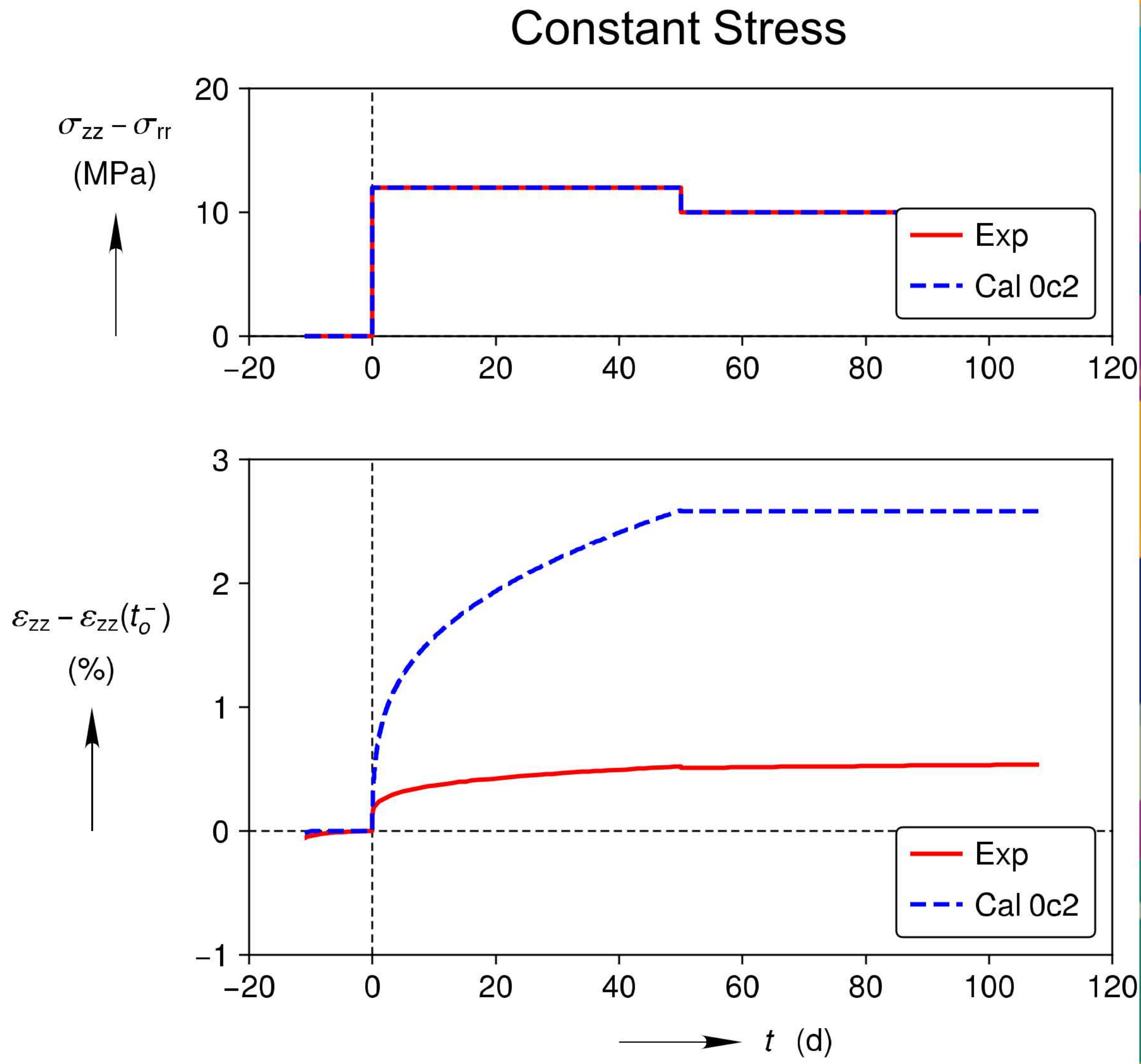


Equivalent Strain Rate vs. Saturation Stress



Comparisons Against Test Data





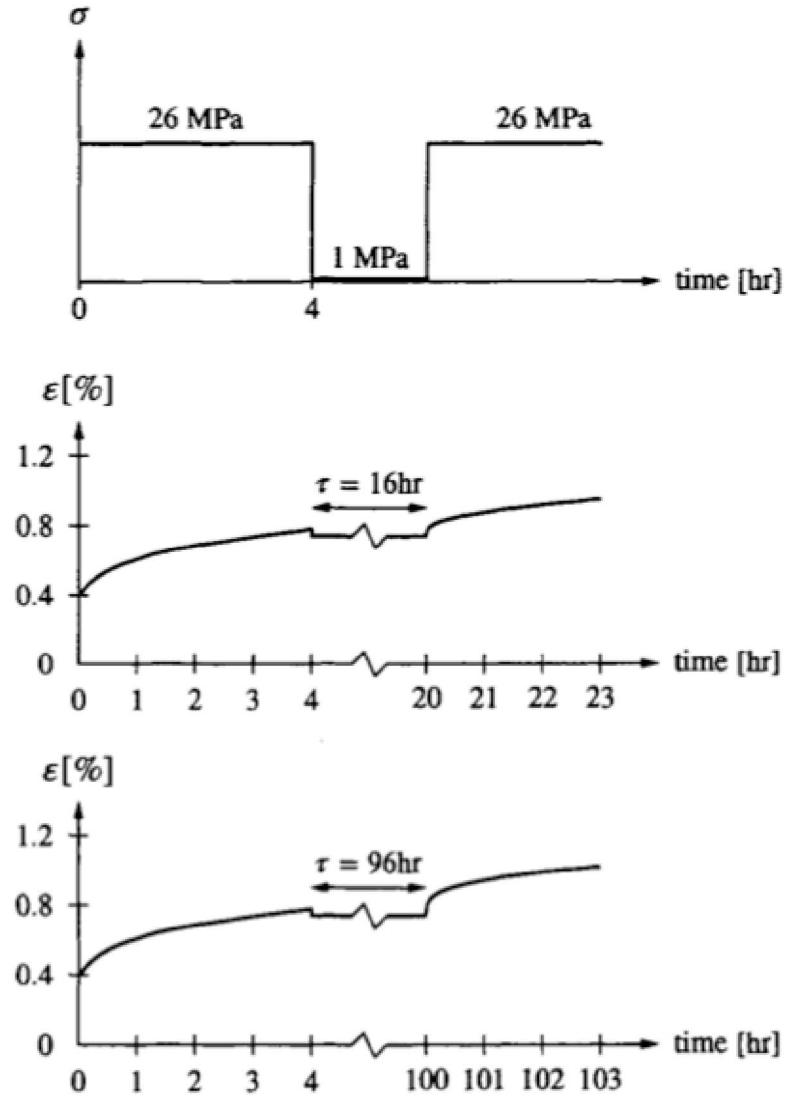
Summary

1. Rational thermodynamic framework helps keep models organized.
2. Kinematic hardening allows one to capture the Bauschinger effect and transient creep phases during non-proportional loading.
3. Kinematic hardening may be important during:
   a. Initial room closure
   b. Compaction of engineered barriers
   c. Rubble pile compaction
   d. Cyclic loading of liquid and gas storage caverns
4. Model calibration is challenging
   a. Must distinguish between isotropic and kinematic hardening
   b. Difficult to match both constant strain rate and constant stress tests

Extra Slides

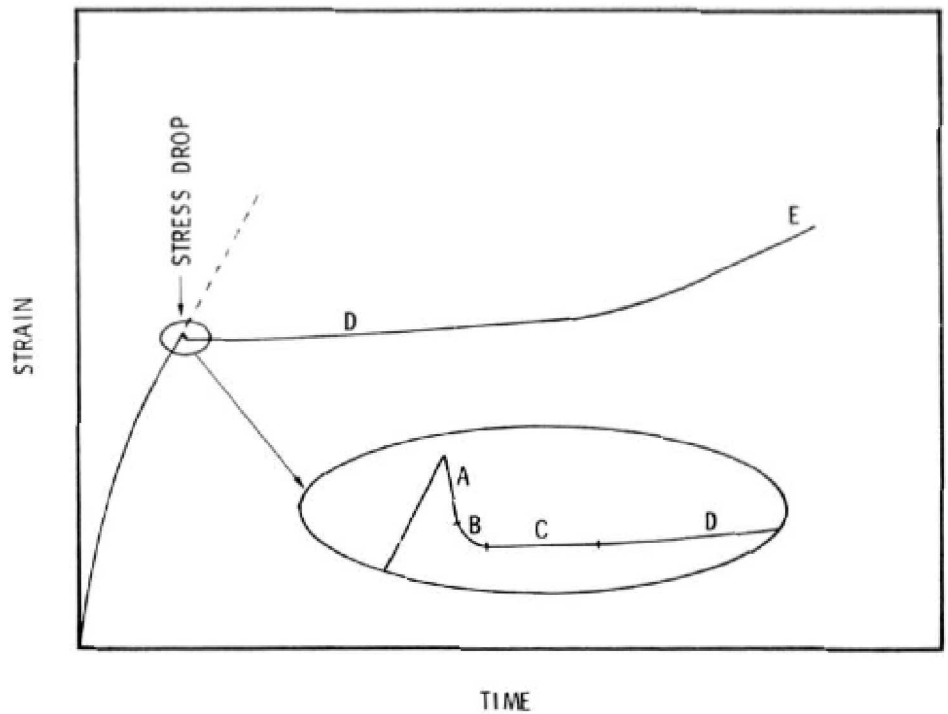
Examples of Thermal (Static) Recovery

Aluminum 1100 at 300°F



Wang, T. T. and Onat, E. T. (1968). Nonlinear mechanical behavior of 1100 aluminum at 300F. *Acta Mechanica*, vol. 5, pg. 54-70

Salt Behavior (Schematic)



Munson, D.E. and Dawson, P.R. (1982). Transient creep model for salt during stress loading and unloading. Sandia National Laboratories. SAND82-0962

Thermal (Static) Recovery

1. Add a term to hardening evolution equations

$$\dot{h} = H_\mathrm{r} \left(\dot{\bar{\varepsilon}}^\mathrm{vp} - \frac{h}{h^\mathrm{s}} \, \dot{\bar{\varepsilon}}^\mathrm{vp} \right) - H_\mathrm{tr} \, h^{H_\mathrm{te}}$$

or

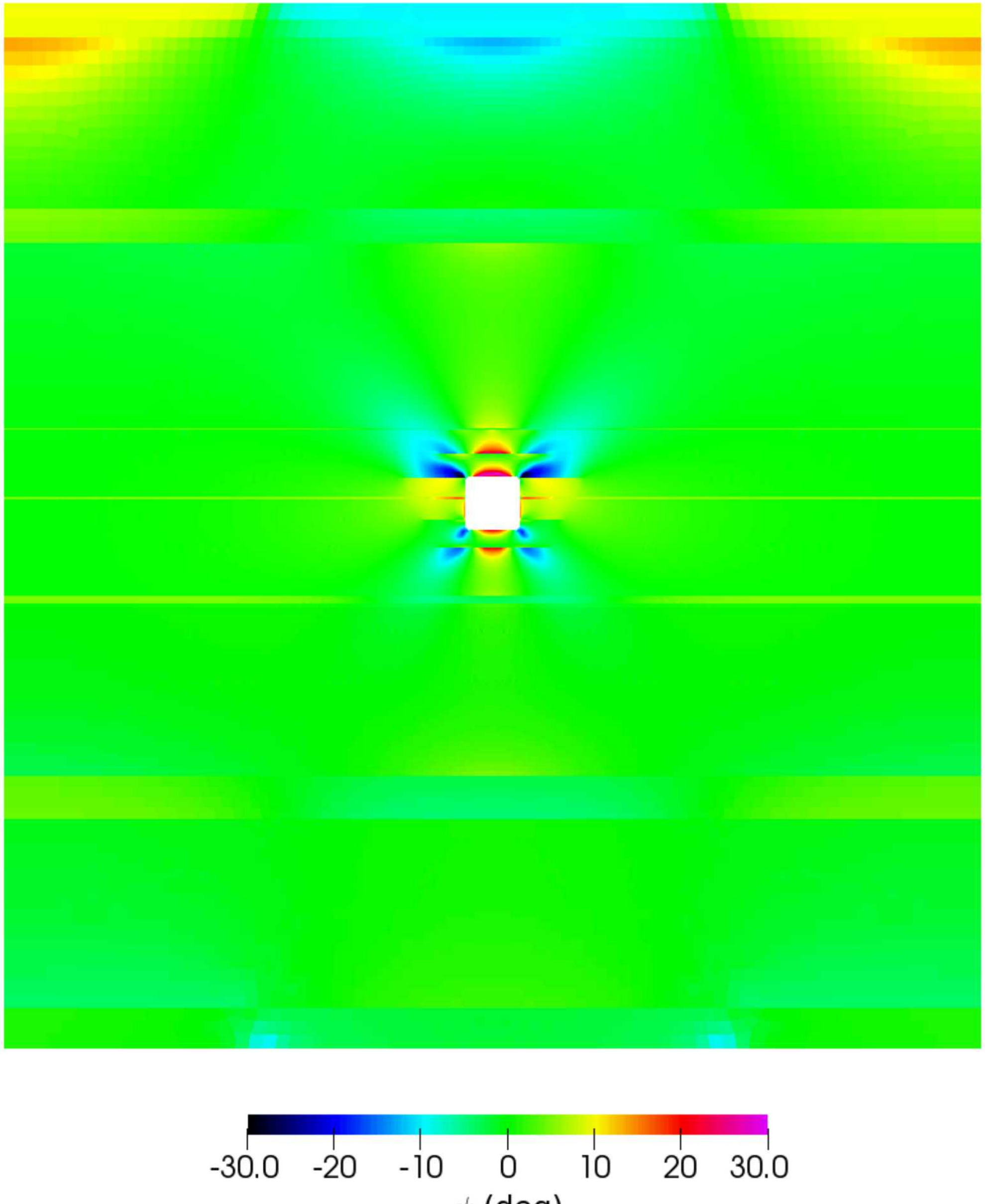
$$\dot{h} = H_\mathrm{r} \left(\dot{\bar{\varepsilon}}^\mathrm{vp} - \frac{h}{h^\mathrm{s}} \, \dot{\bar{\varepsilon}}^\mathrm{vp} \right) - H_\mathrm{tr} \left\langle h - h^\mathrm{s} \right\rangle^{H_\mathrm{te}}$$

2. Internal state variable conjugate to h is no longer $\bar{\varepsilon}^\mathrm{vp}$

3. Responsible for higher secondary creep rate above 80°C ?

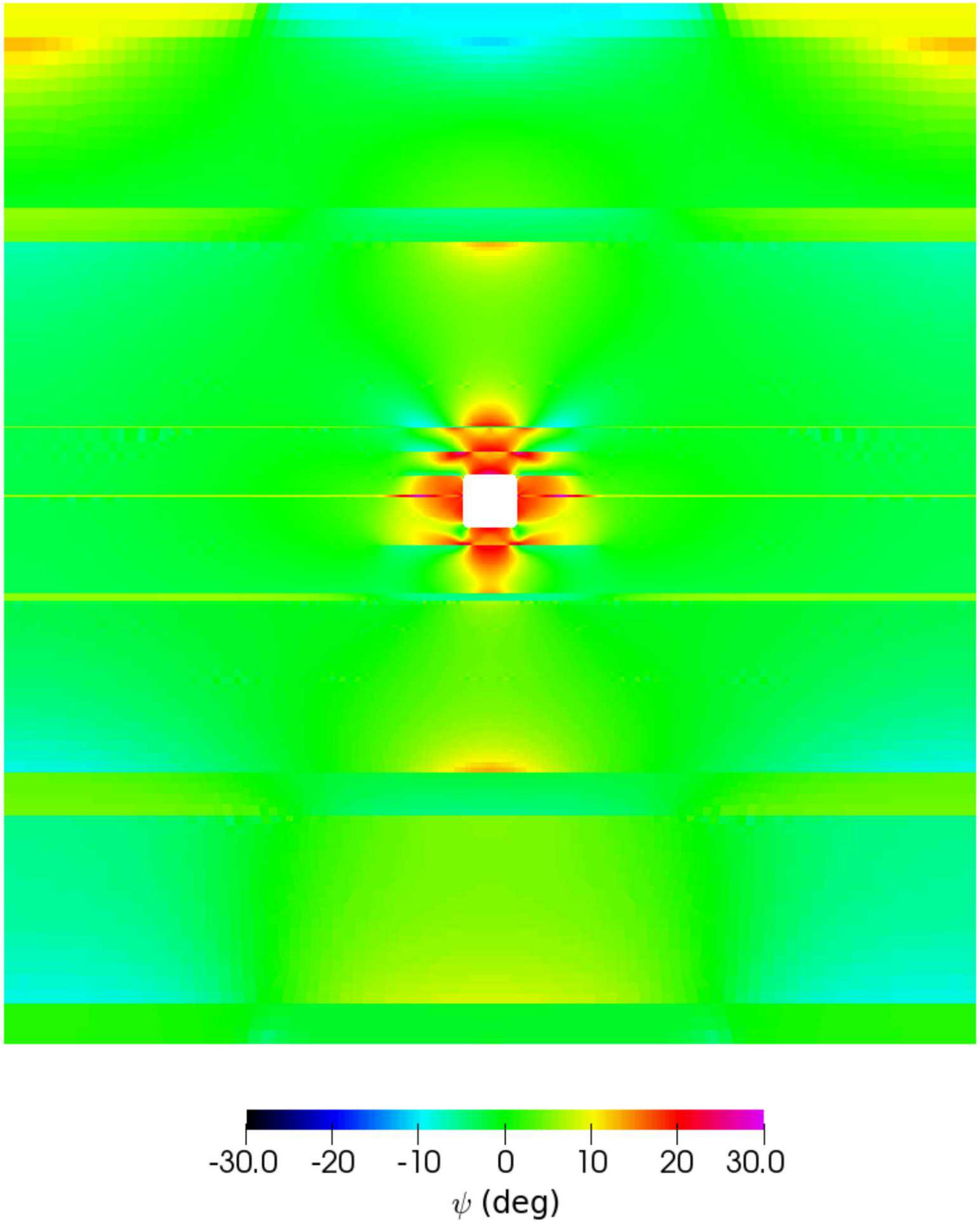
Lode Angle Around Room D (MD Model)

t = 0 yr



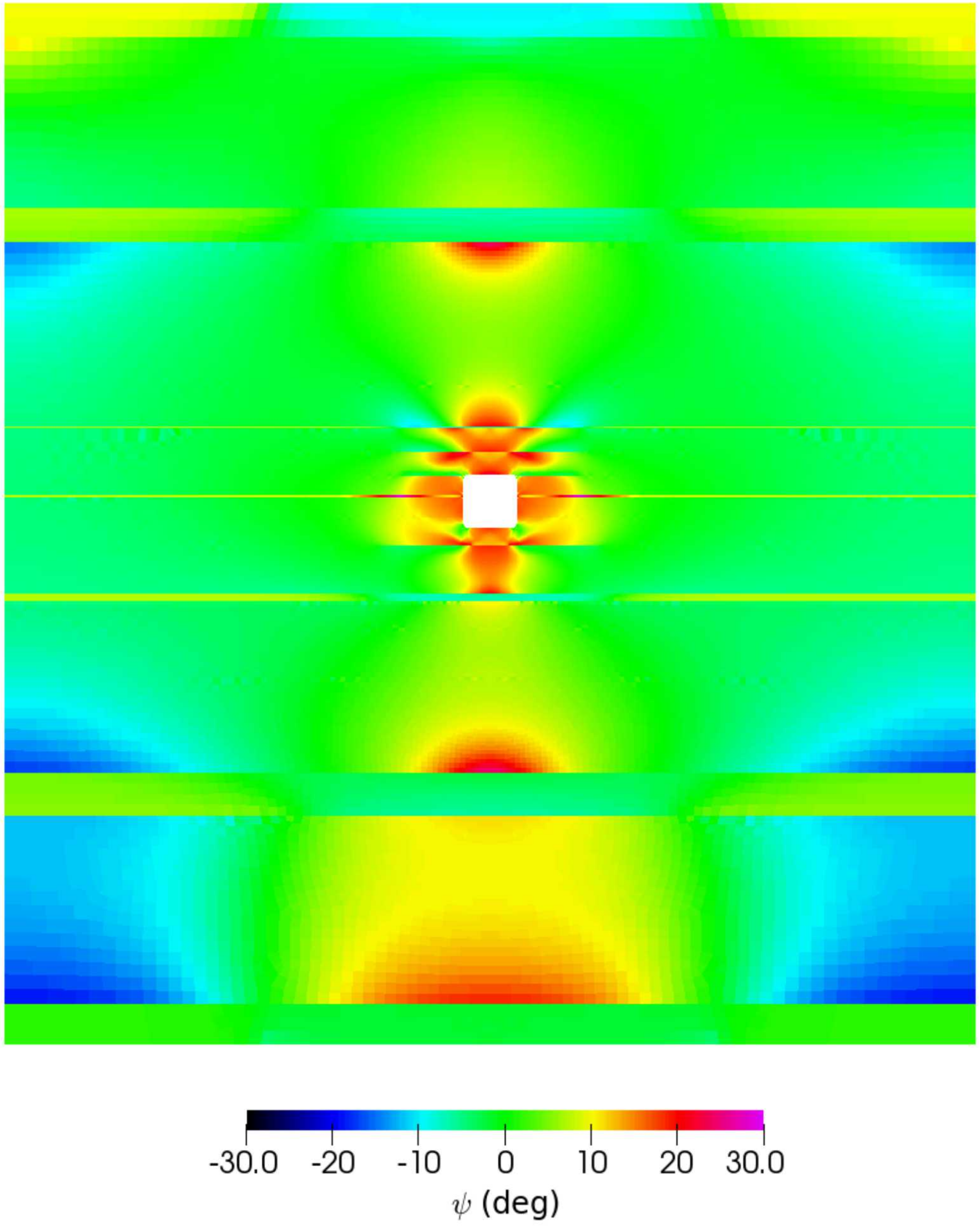
Lode Angle Around Room D (MD Model)

t = 1 d



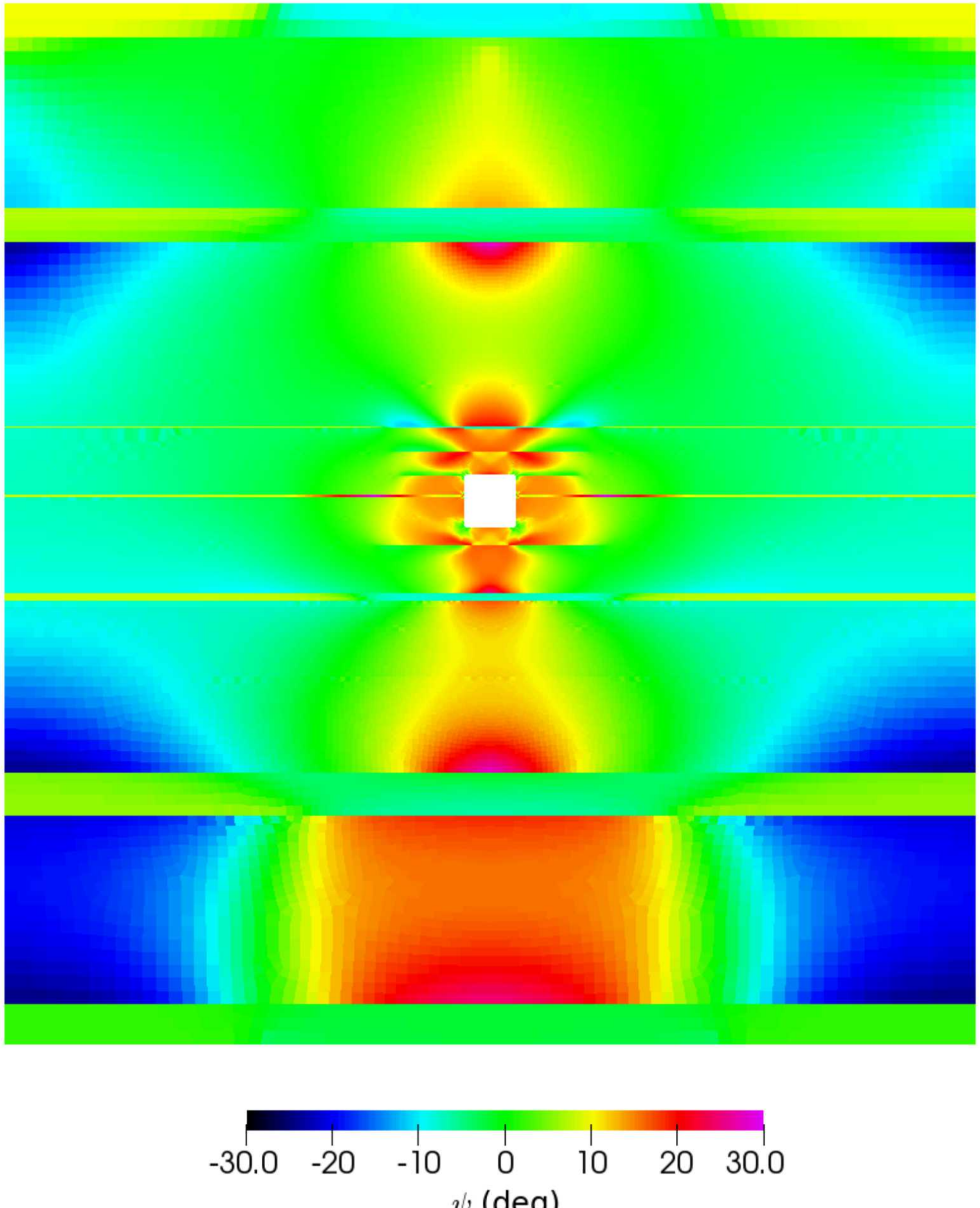
Lode Angle Around Room D (MD Model)

t = 10 d



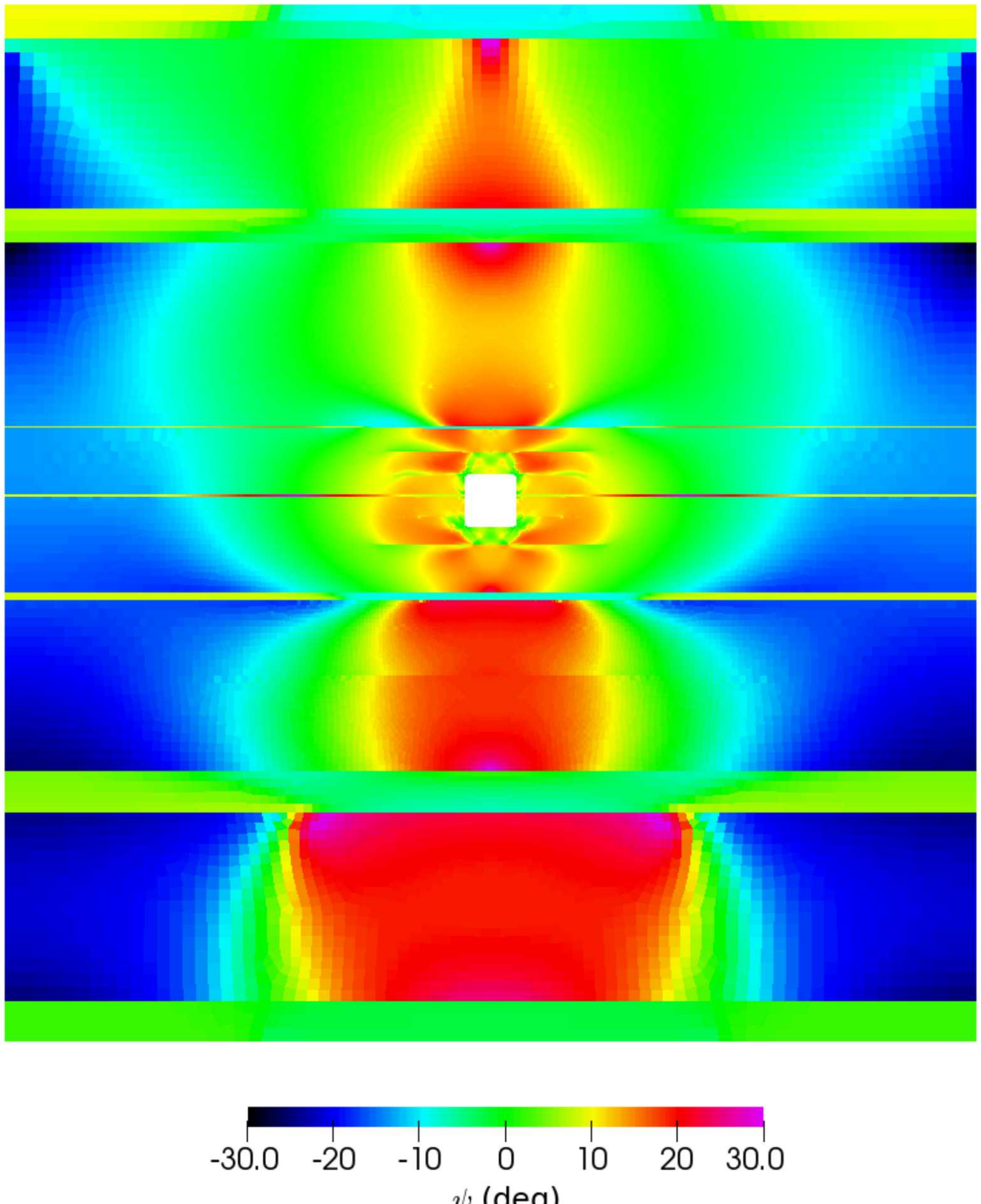
Lode Angle Around Room D (MD Model)

t = 100 d



Lode Angle Around Room D (MD Model)

t = 1000 d



Lode Angle Around Room D (MD Model)

t = 2850 d