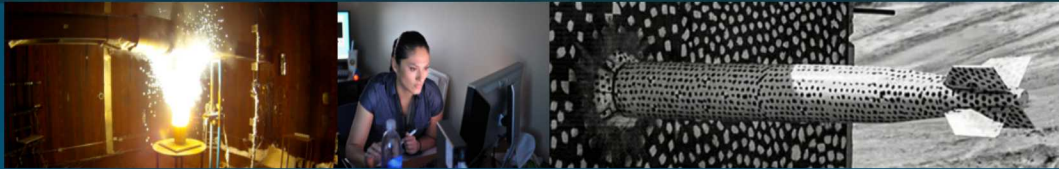




Sandia  
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Laboratories

SAND2019-9862PE

# Effective Pair-Potentials for Modelling High Energy-Density Matter



PRESENTED BY

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COLLABORATORS

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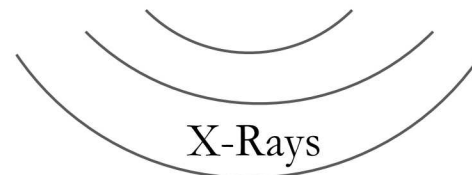
- Quantify interfacial mixing in ICF experiments
- Assess accuracy of transport models needed to design ICF experiments

## Experimental Set-up

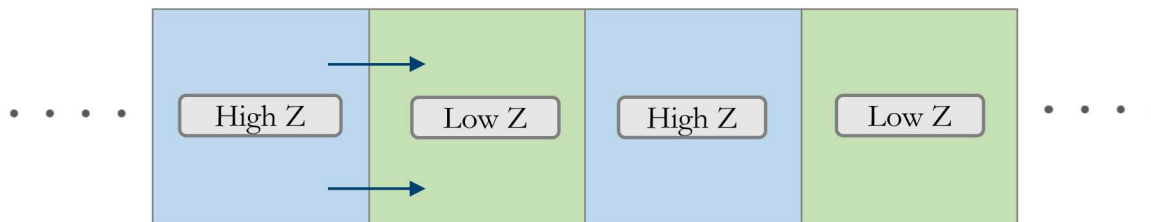


Focus of this work

- Specifics:
  - High Z: Vanadium
  - Low Z: CH Foam



X-Rays



Interfaces



## ICF experiments explore very disparate regimes

### Solids

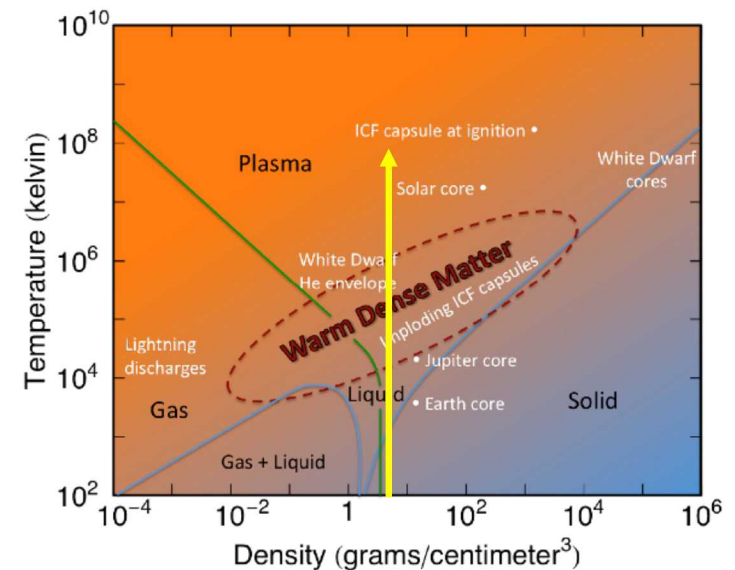
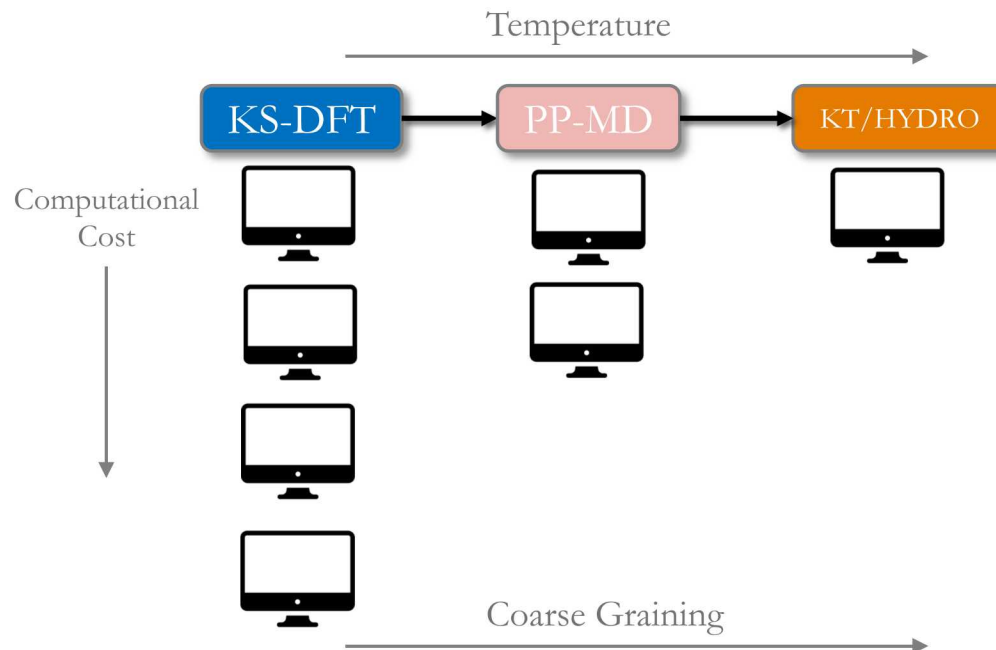
- Ions are very strongly coupled, electrons are extremely degenerate
- Density Functional Theory (DFT) – uses on-the-fly N-body potential

### Warm Dense Matter (WDM)

- Ions are strongly coupled, electrons are partially degenerate
- Pair-potential molecular dynamics (PP-MD) – pre-computed pair potentials

### Hot Plasma

- Ions weakly coupled, electrons no longer degenerate
- Solve a kinetic equation - McBGK



$$1 \text{ eV} \approx 1.16 \times 10^4 \text{ K}$$



- The **Yukawa (linear OF-DFT) model** is a general purpose widely used pair potential across plasmas physics. The following approximations are made:
  1. The bound electrons have no physical extent which shrinks bound cloud to a point. This gives us the effective ionization  $\langle Z \rangle$
  2. Considers a linearization of remaining free electrons density. This gives a spherical electron screening cloud with radius  $\lambda_{TF}$
- **Kohn Sham DFT (KS-DFT)** includes richer QM effects and is a higher fidelity method
  - Solving the Kohn Sham equations gives the electron density

You can use either of the above methods in a molecular dynamics code to evolve the ion species

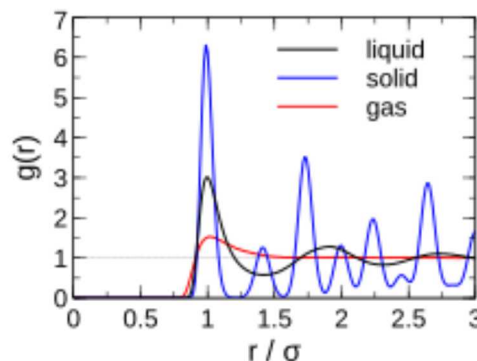
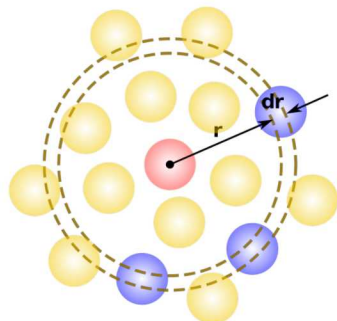
- Yukawa PP-MD uses **screened** Coulomb potential for ion-ion interactions
  - Sarkas – developed at MSU
- KS-DFT-MD uses **bare** Coulomb potential for ion-ion interactions
  - Vienna ab initio Simulation Package (VASP)

What are the macroscopic properties, like self-diffusion, that each model predicts?

## Validating Results: Correlation Functions



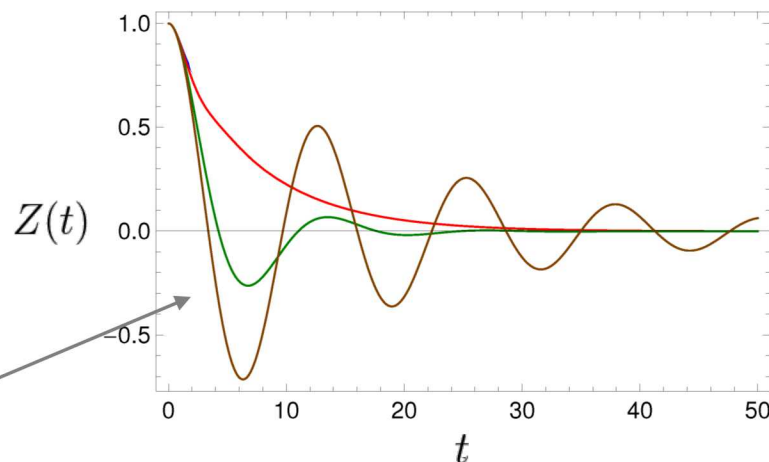
- **Radial Distribution Function (RDF):** Spatial correlation function
  - Helps identify the physical state of the system



- **Velocity-Autocorrelation Function (VACF):** Time correlation function
  - Give us information on individual particle motion

Normalized VACF:

$$Z(t) = \frac{\langle \mathbf{v}(t) \cdot \mathbf{v}(0) \rangle}{\langle \mathbf{v}(0) \cdot \mathbf{v}(0) \rangle}$$



“Caging” or “backscattering” of particles





- **Macroscopic transport coefficients:** Self-diffusion
  - How far a particle moves over time as it moves from regions of high concentration to low concentration

Two examples of how to compute the self-diffusion coefficient

- From the VACF: 
$$D = \frac{1}{3} \int_0^\infty dt \langle \mathbf{v}(t) \cdot \mathbf{v}(0) \rangle$$
- From Einstein's Relation: 
$$D = \lim_{t \rightarrow \infty} \frac{|\mathbf{r}_i(t) - \mathbf{r}_i(0)|^2}{6t}$$

Can compare these expressions with those obtained from the **Stanton and Murillo** analytic theory

## Molecular dynamics evaluation of self-diffusion in Yukawa systems

Cite as: Physics of Plasmas **7**, 4506 (2000); <https://doi.org/10.1063/1.1316084>  
Submitted: 30 May 2000 . Accepted: 15 August 2000 . Published Online: 19 October 2000

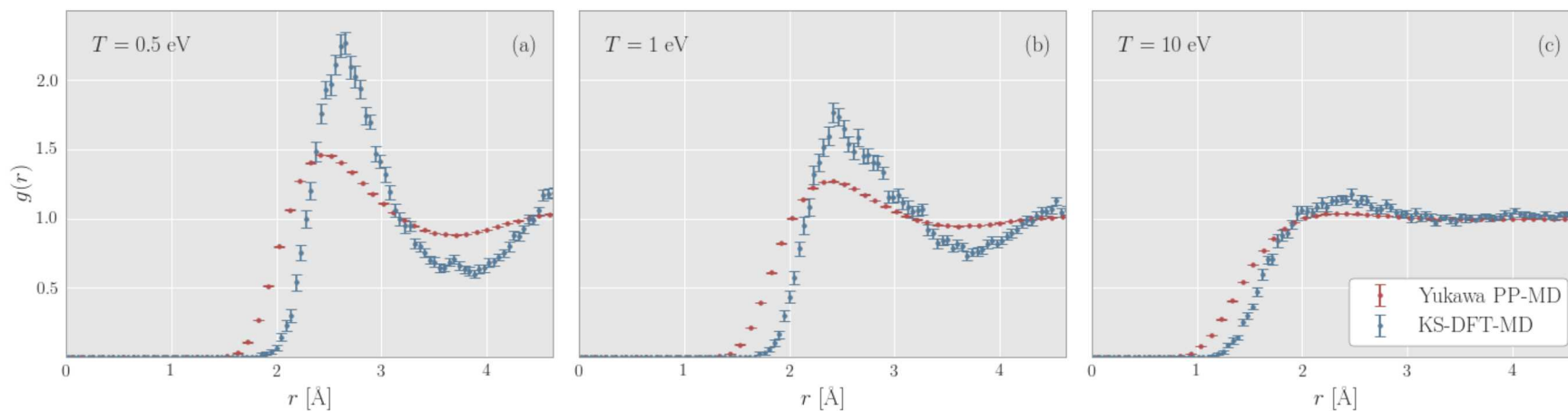
H. Ohta, and S. Hamaguchi



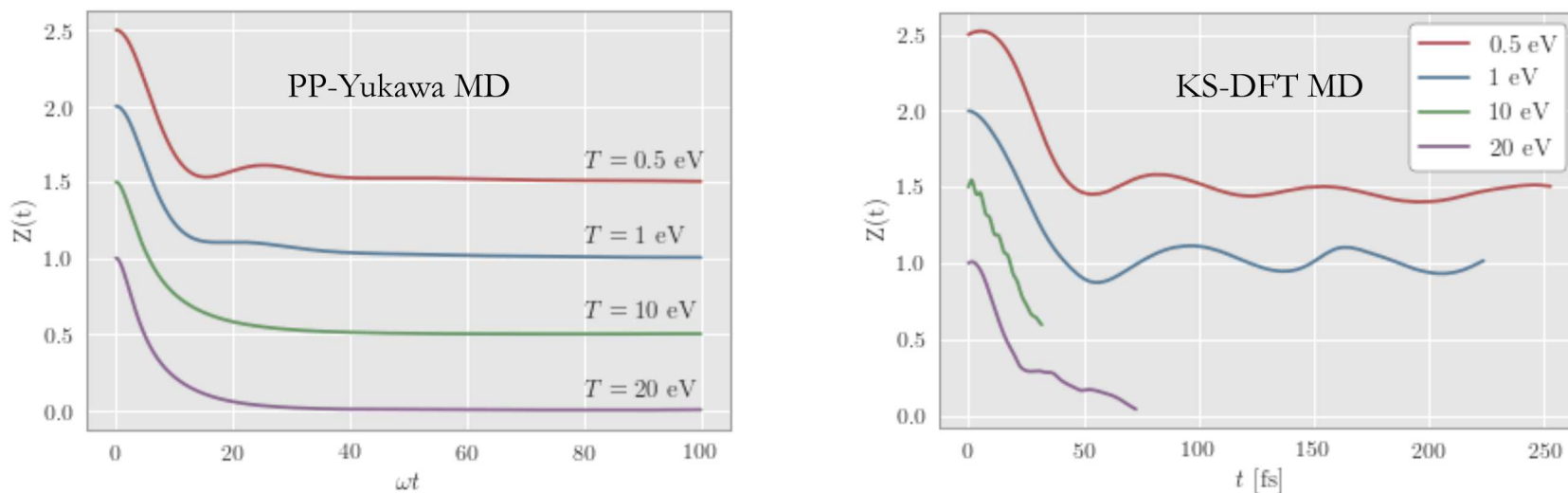
Value of  $D$  compared from two expressions above – only slight difference in values



## Comparison of RDFs:



## Comparison of VACFs:



## Why Do We Need a Pair-Potential?



- **Goal:** Can we avoid using KS-DFT to determine the electronic structure by generating the “best” pair potential for a pair potential MD simulation?

**Best** = pair-potential that incorporates as much information from KS-DFT as possible

- Why do we care?

Temp [ev]	PP-MD CPU Hrs	KS-DFT-MD CPU Hrs
5	5.7e-1	1.20042e+4
10	4.6e-1	1.08590e+4
20	4.8e-1	5.80032e+3

CPUs: 1

Particles: 500

Timesteps: 30K

CPUs: 36

Particles: 54

Timesteps: <=12K

- Simulations that can take multiple weeks across many CPUs can be reduced to minutes on a single CPU.
- This information can be used to inform transport coefficients in a hydro model.



# Generating Effective Pair Potentials from Force Matching



- **Main idea of FM:** Take KS-DFT-MD data (e.g. positions, forces, energies, stresses, etc.) and minimize a cost function to fit a pair-potential.
- To perform the force matching. We make use of the open-source software **potfit**



## On fitting a gold embedded atom method potential using the force matching method

J. Chem. Phys. **123**, 204719 (2005); <https://doi.org/10.1063/1.2124667>

Gregory Grochola<sup>1</sup>, Salvy P. Russo, and Ian K. Snook

- The target function to be minimized can be defined as

$$Z(\xi) = \sum_{k=1}^{3N} u_k (F_k(\xi) - F_k^0)^2 + \sum_{l=1}^M w_E (E_l(\xi) - E_l^0)^2 + \sum_{l=1}^{6M} w_S (S_l(\xi) - S_l^0)^2$$

Force Components

Energy Components

Stress Components

- The optimization routines in **potfit** include:

- Simulated annealing
- Differential evolution

High-level optimization

- Least-squares

Low-level optimization

- **Differential Evolution (Genetic Algorithm)**

1. **Mutation:** change the current state of the set of starting potentials
2. **Crossover:** mix terms from other potentials in set to generate new set of potentials
3. **Selection:** determine if new forms are favorable or not

# Analytic Potential Forms for Force Matching



- We use the VASP data to fit the analytic forms of the following potentials

1. Vanilla **Yukawa**

$$u^Y(r) = \frac{Z^2 e^2}{r} e^{-r/\lambda_{TF}}$$

parameters to fit: 2

2. Yukawa with a **short-range repulsion** term (SRR Yukawa)

$$u^{Y+SRR}(r) = \frac{a}{r^4} + \frac{Z^2 e^2}{r} e^{-r/\lambda_{TF}}$$

parameters to fit: 3

3. The **exact gradient-corrected** potential (EGS)

$$u_{\text{mon}}^{EGS}(r) = \frac{Z^2 e^2}{2r} [(1 + \alpha) e^{-r/\lambda_-} + (1 - \alpha) e^{-r/\lambda_+}]$$

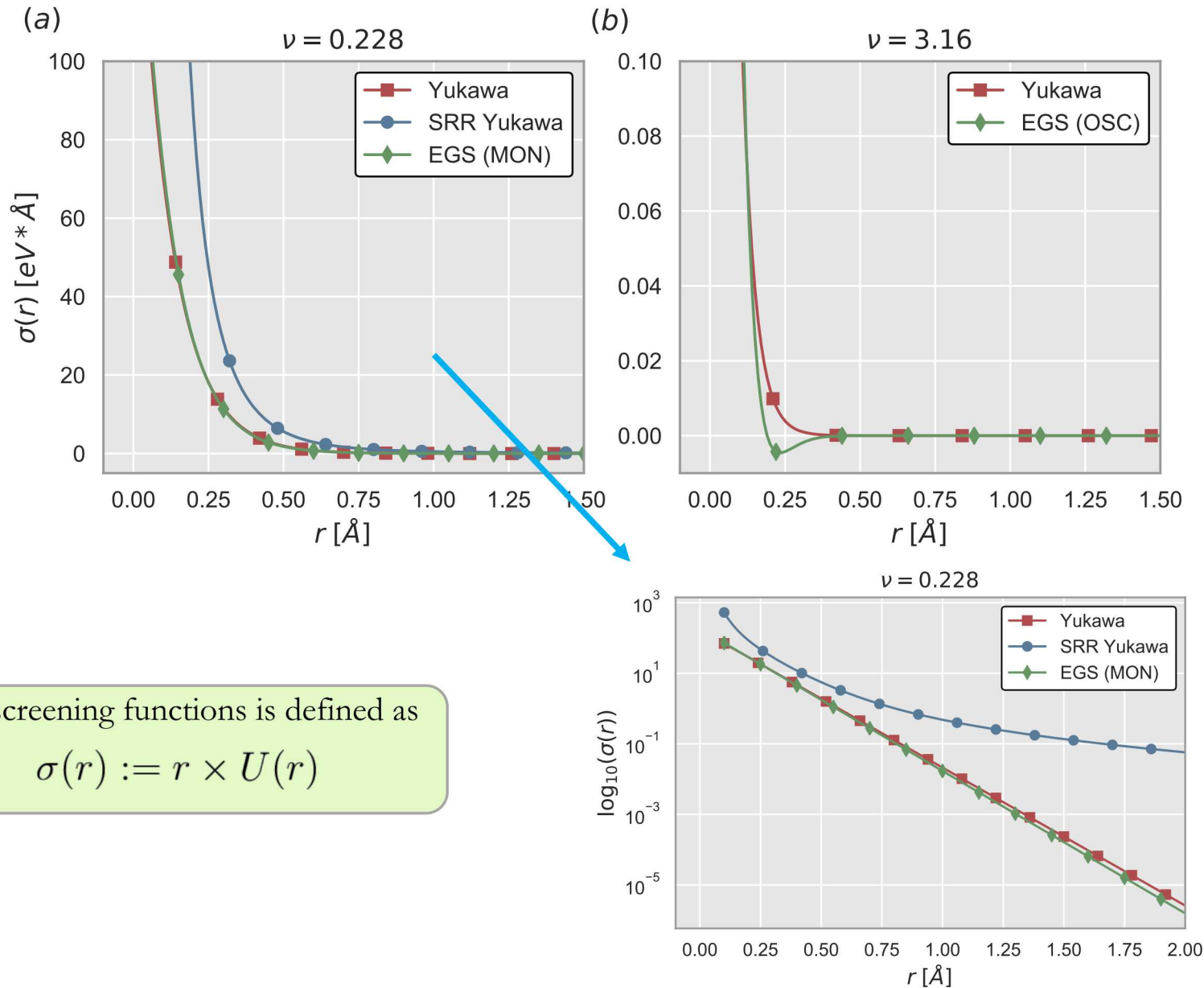
parameters to fit: 4

$$u_{\text{osc}}^{EGS}(r) = \frac{Z^2 e^2}{2r} [\cos(r/\gamma_-) + \alpha' \cos(r/\gamma_-)] e^{-r/\gamma_+}$$

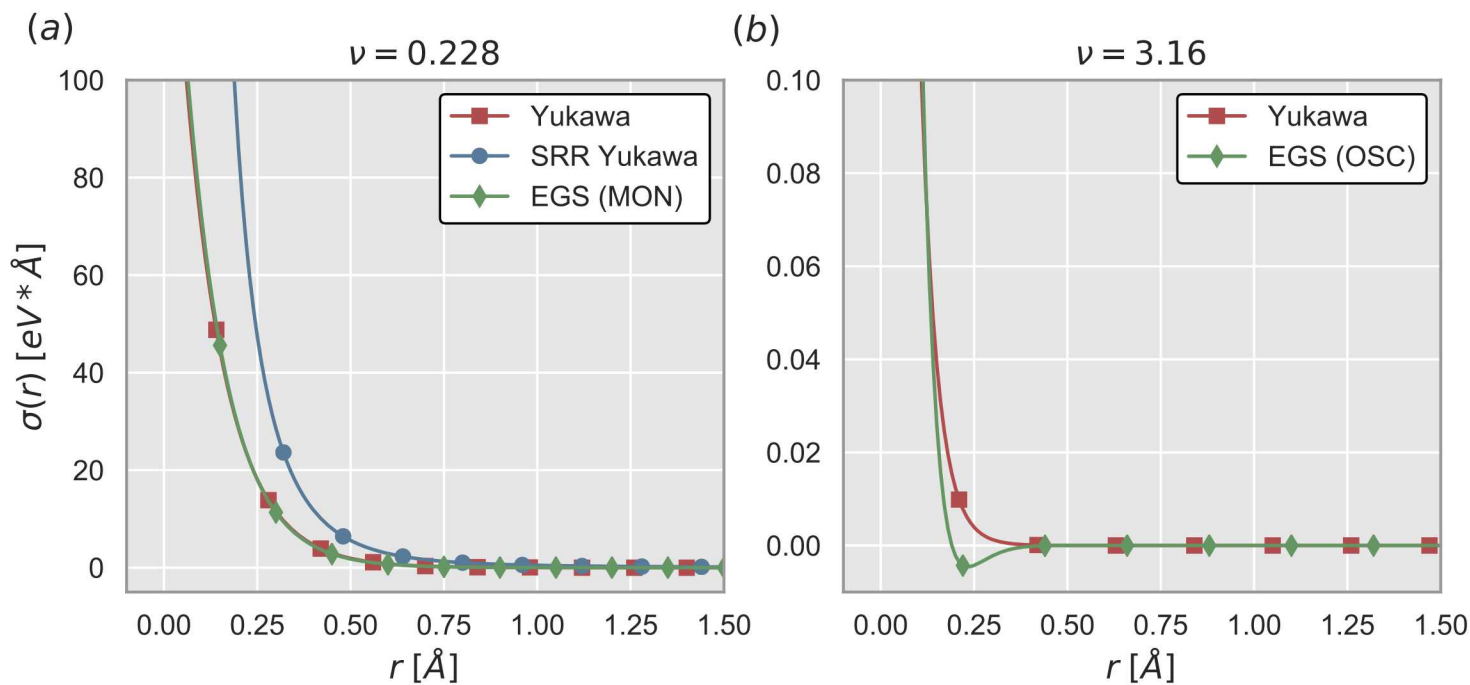
$$\nu = \frac{3\sqrt{8\beta}}{\pi} \lambda \mathcal{I}'_{-1/2}(\bar{\eta})$$

**gradient correction strength parameter:**  
When  $\nu > 1$ , the oscillatory form of EGS is used

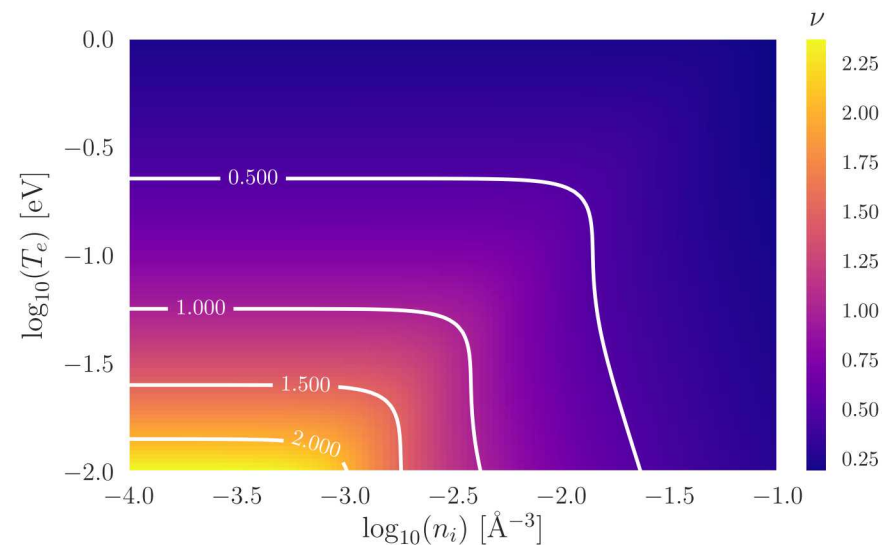
# Comparison of Screening Functions



# Comparison of Screening Functions



The heatmap to the right shows the regions when the oscillatory behavior from the EGS potential is expected ( $\nu > 1$ )

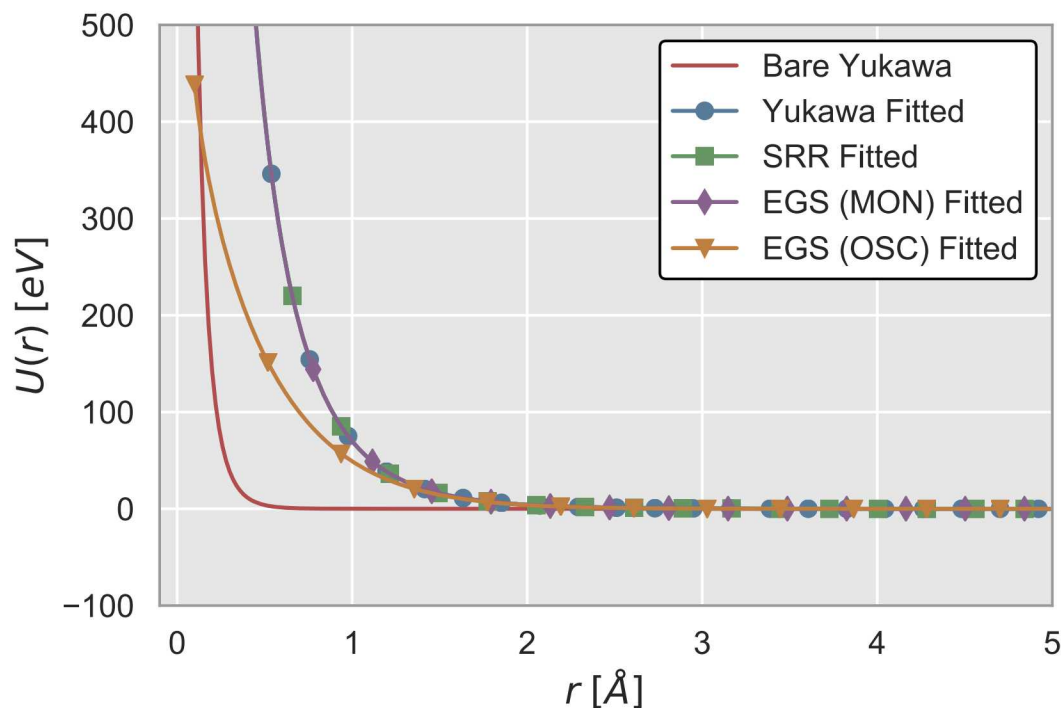


# Force Matched Potentials: Fitted Analytic Forms



- Optimization parameters:
  - Algorithm: Genetic Algorithm
  - Number of configurations from VASP:  $\sim 400$

Results for 0.5 eV VASP data



The **VASP** configurations are used as input to **potfit** and the objective function is minimized for the analytic forms on the previous slides

Can obtain effective ionization and screening lengths from these fitted potentials

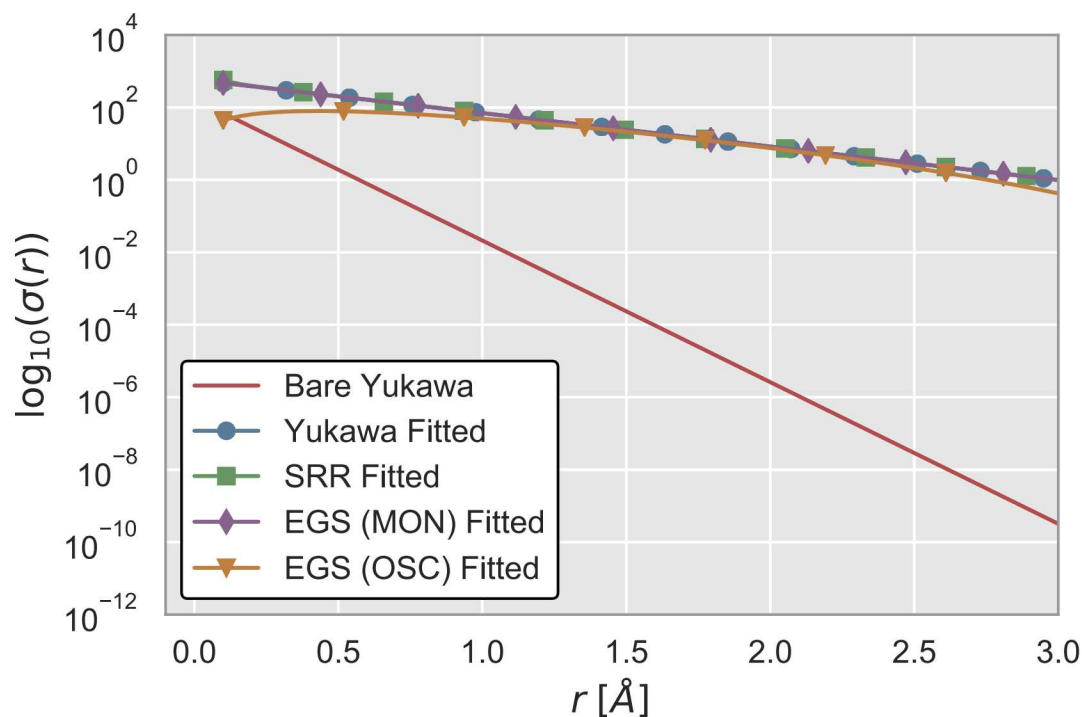


# Screening Functions for Force Matched Analytic Potentials



- Optimization parameters:
  - Algorithm: Genetic Algorithm
  - Number of configurations from VASP:  $\sim 400$

Results for 0.5 eV VASP data



Even at 3 Angstroms,  
forces non-negligible  
relative to **bare Yukawa**  
potential

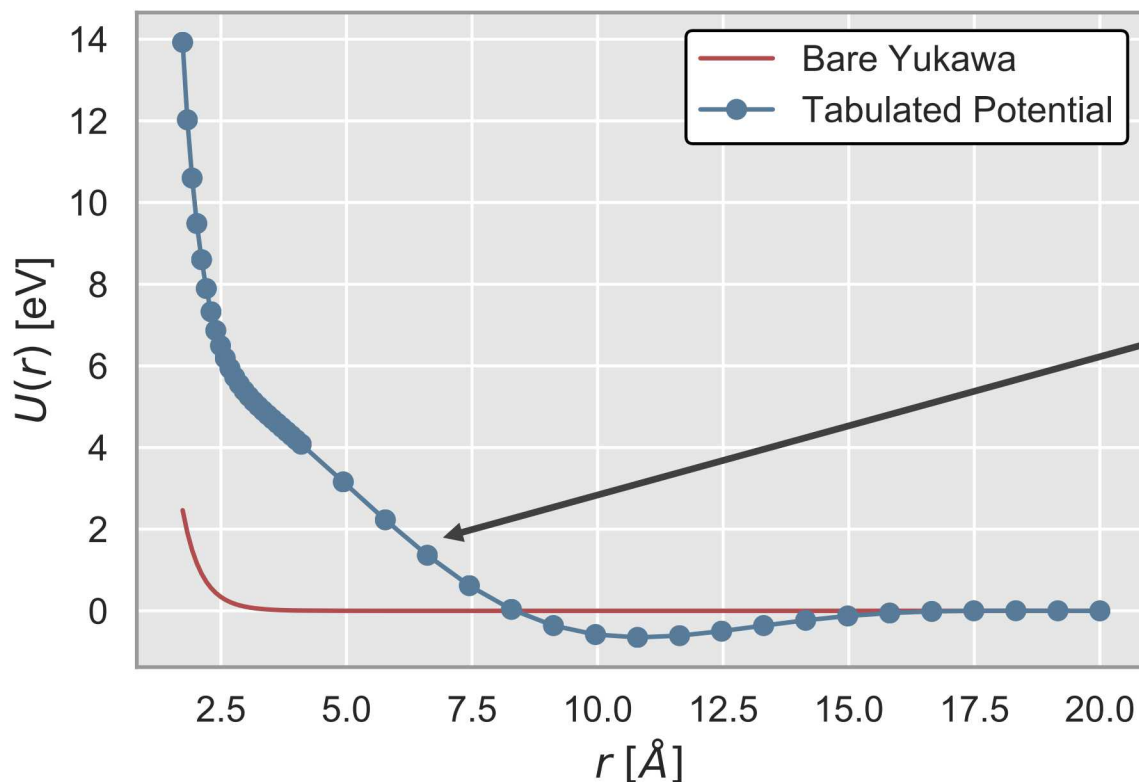
## Force Matched Potentials: Tabulated Form\*



- Tabulated potentials: A series of splines fit to VASP data

**Pros:** No restriction on the functional form of the potential

**Cons:** Number of nodes/knots results in a very high number of parameters to optimize



Each point on the blue curve is a parameter to optimize (45 parameter optimization problem!)

\*Preliminary results



- Compare RDFs, VACFs, and self-diffusion coefficients for fitted potentials
- Finalize tabulated potential work
  - Lots of options to play around with here, potfit metric for accuracy is ambiguous
- Obtain data for more temperature and densities to determine transferability to other materials (e.g. Al, Be, Fe)