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Tutorial on MIMO Vibration

PRESENTED BY

Ryan Schultz, 7 August 2019

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MIMO Tutorial – Outline (General)

- 2 Parts:
  - Part 1: Fundamentals of MIMO Random Vibration Testing
  - Part 2: Advanced Topics
- Objective:
  - Give an introduction to theory and techniques for MIMO random vibration testing
  - Develop understanding of terminology used in MIMO
  - Provide example problems to demonstrate various test configurations
  - Introduce advanced topics
- Audience:
  - Folks with basic understanding of the objectives of vibration testing, some background in vibration and linear system dynamics

MIMO Tutorial – Outline (Part 1)

- Part 1: Fundamentals of MIMO Random Vibration Testing

1. Introduction: What are we trying to do?
2. Random Vibration & Shock Environments – Some Examples
3. Linear Systems
   - Input-Output Relationships
   - Single Input & Multiple Input
   - Linear vs. Power Space Equations
4. Quantifying Responses
   - RMS, APSD, CPSD, Coherence, Phase, Linear Spectrum
5. Input Estimation
   - Transient (Linear Space)
   - Stationary (Power Space)
6. Controlling a MIMO Test
   - Description of the General Control Process
7. Where are we with MIMO?

MIMO Tutorial – Outline (Part 2)

- Part 2: Advanced Topics

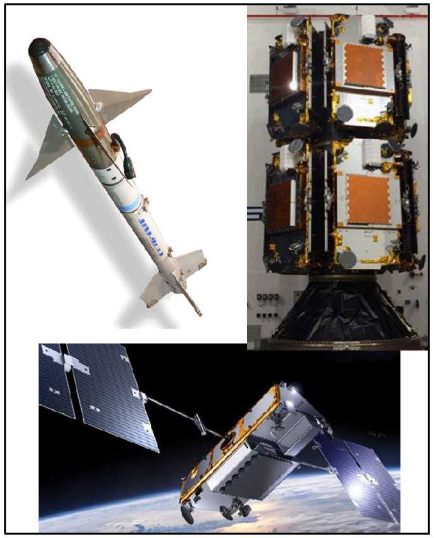
1. Review
   - MIMO Linear System Equations
   - Input Estimation & Control Process
2. Structure of CPSD & FRF Matrices
   - Expected Forms, Checks, and Numerical Corrections
   - Manipulating CPSD Matrices (extracting subsets, scaling, sign changes)
3. Signal Processing for MIMO Tests
   - Time Histories to CPSDs
   - Ensemble Averaging and Estimates
4. FRF Estimation Methods
5. Modes, Modal FRFs, and Modal-Space MIMO Equations
6. Fancy Input Estimation Techniques
   - Correlated vs. Uncorrelated Inputs
   - Regularized Inverse Solutions
   - Input Adjustments – Correcting Levels

INTRODUCTION

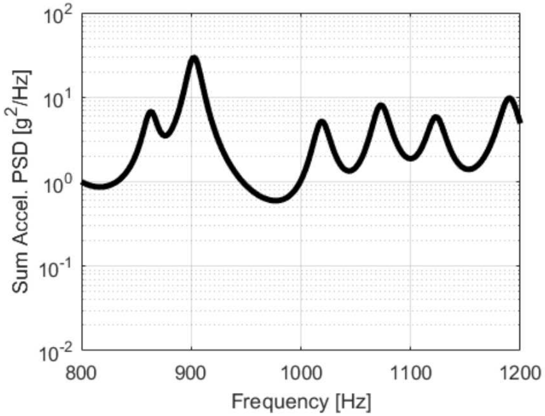
Let’s get some context

What are we trying to do?

Field Environments



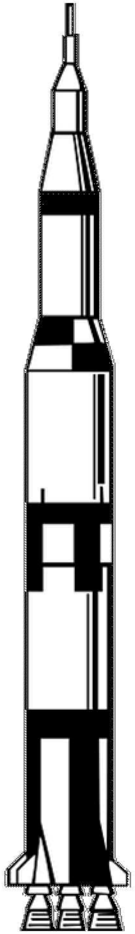
Excitation of Critical Systems & Components



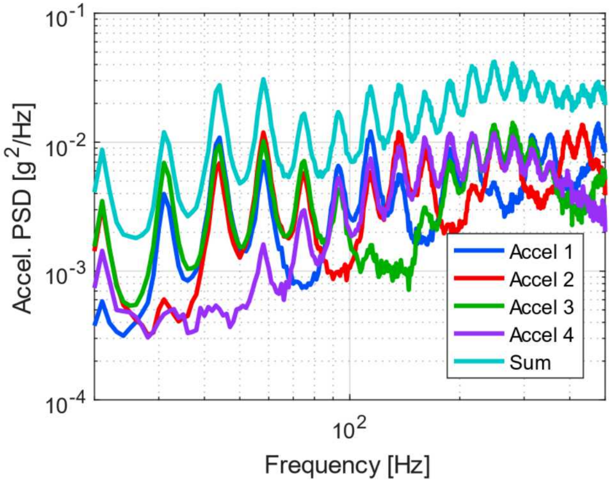
Response PSD

What are we trying to do?

System



- Can't just do a field test to analyze our systems
- Subject some device under test (DUT) to some inputs to achieve some desired response
  - Re-create vibration response from the field
  - Create vibration response to match a spec
- See if the DUT will fail/change due to these vibrations



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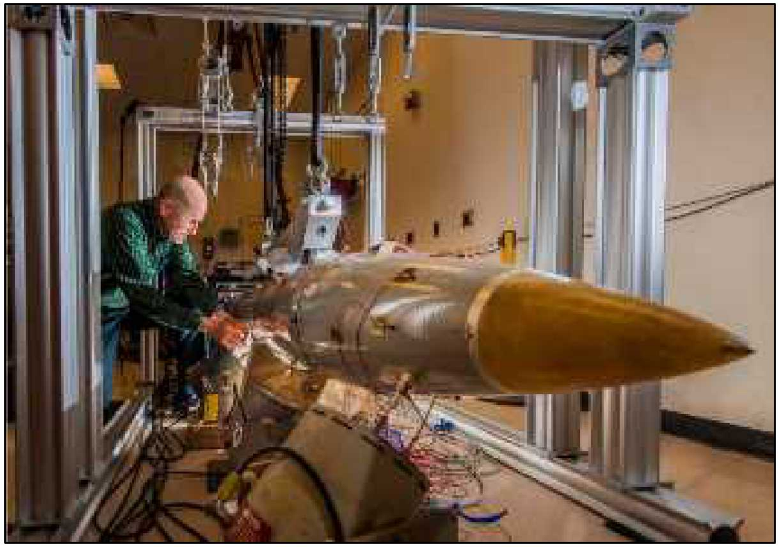
Types of MIMO Vibration Tests

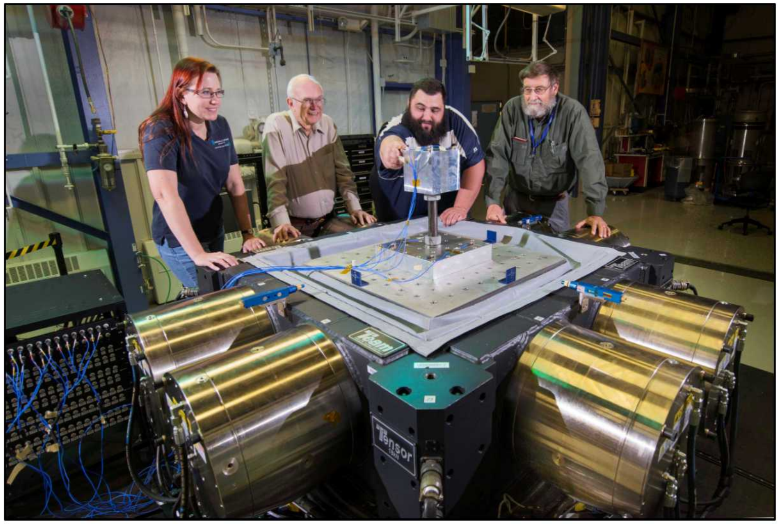
Single-Axis Shaker

SIMO

MIMO



Multi-Shaker



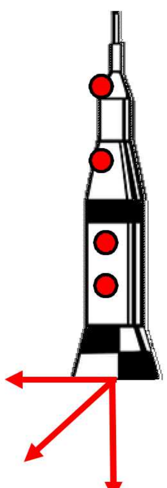
Multi-Axis, 6-DOF Shaker

6-DOF vs. Multi-Shaker

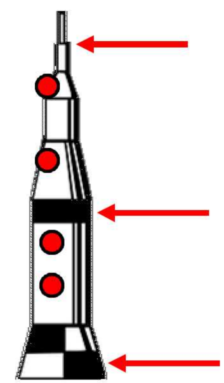
- Both are MIMO tests
- Difference in how inputs get to the system

Multi-Axis, 6-DOF Shaker

Multi-Shaker



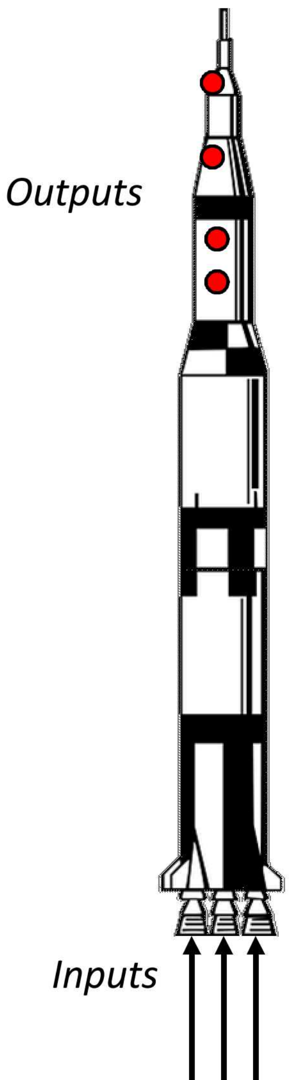
Multi-Direction Base Inputs



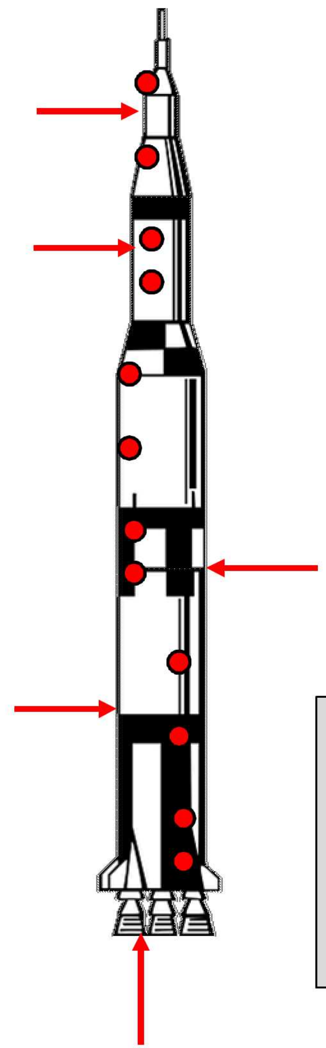
Multiple Point Force Inputs

Field vs. Lab

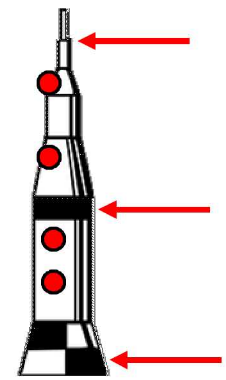
Field Environment Loads



Lab Shaker Loads (Full System Test)



Lab Shaker Loads (Component Test)



We usually can't perfectly match the loads or the system from the field in the lab

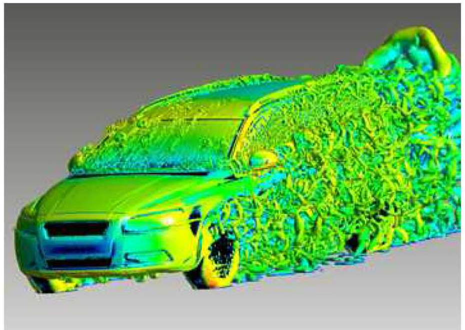
- **Inputs are different (pressure vs. shaker forces)**
- **System is different**
- **Can only test a sub-system or component**
- **Boundary conditions are different**

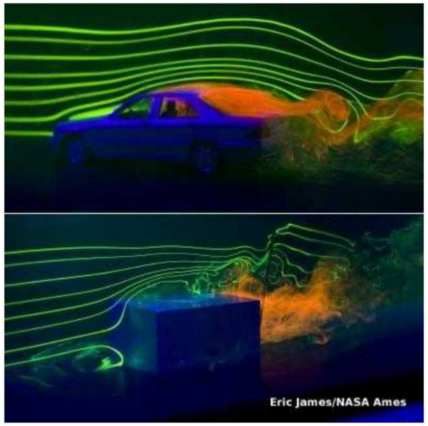
VIBRATION ENVIRONMENTS

What are the loads?

Random Vibration & Shock

- These are the typical vibration environments
- Random vibration:
  - Stationary – levels and spectral content do not change with time
- Shock:
  - Transient – single event, levels and spectral content change with time





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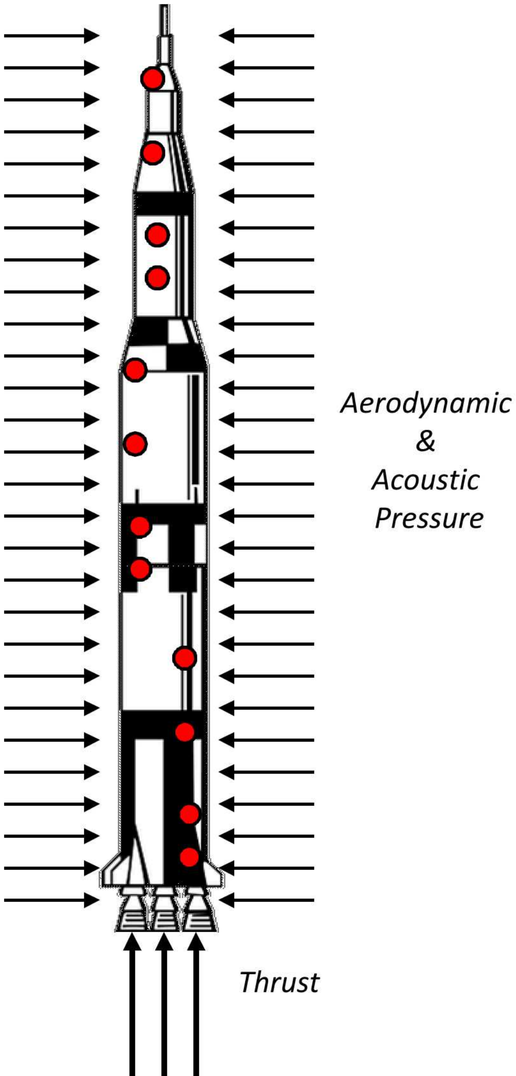
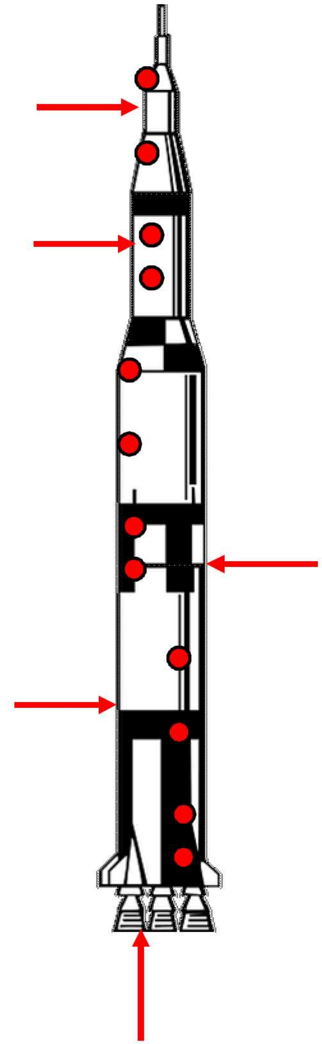
Random Vibration Loads

Field Environment Loads

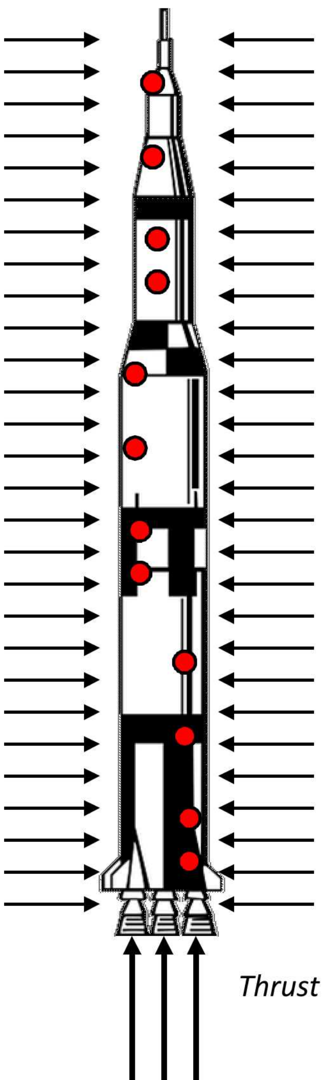


Lab Shaker Loads
(Full System Test)

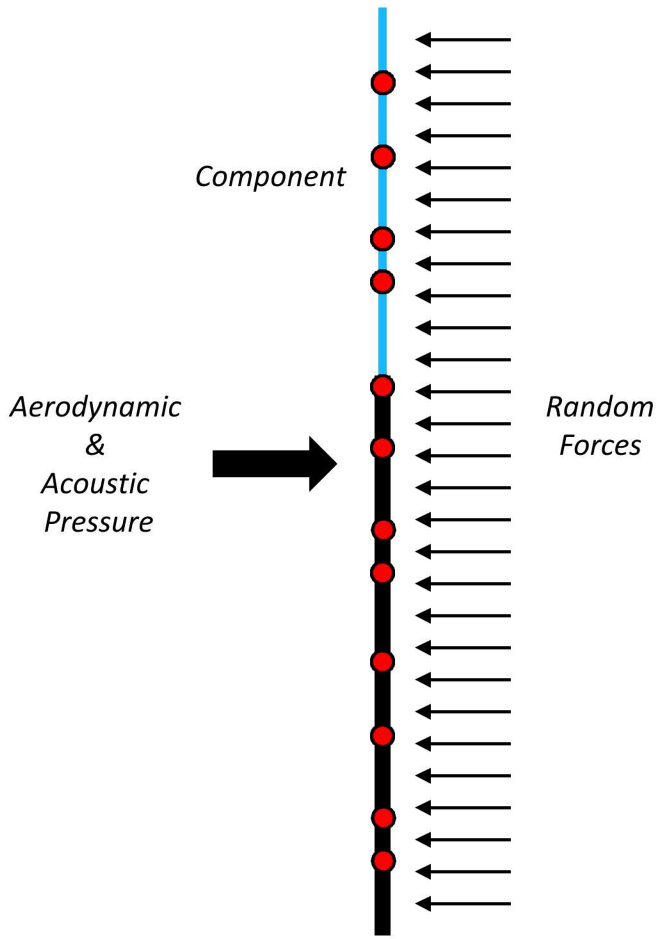


Random Vibration Example

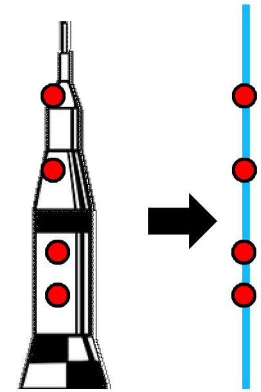
Example System



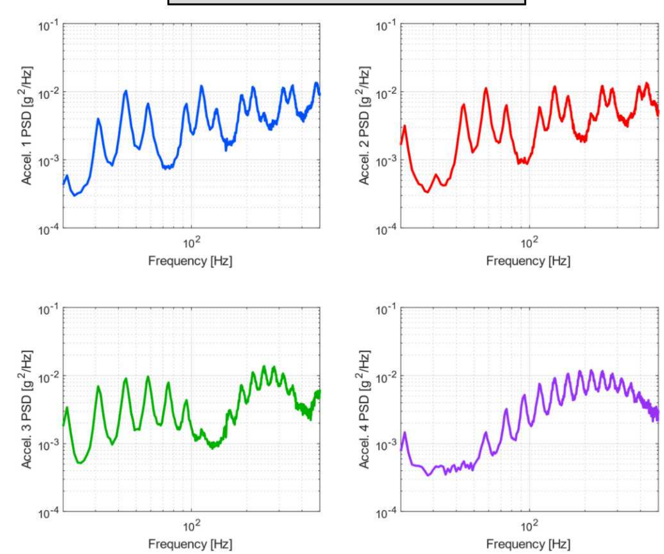
Simple Beam Model



Grab Responses from the Component



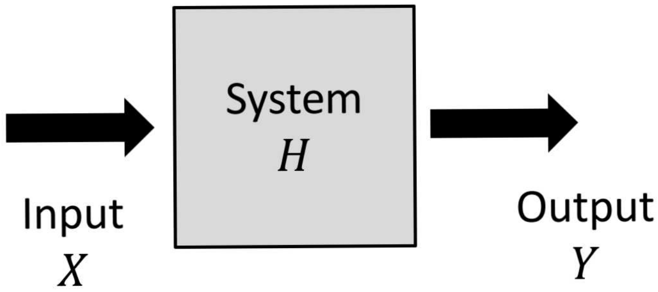
Response PSDs



LINEAR SYSTEMS

What other type of system is there anyway?

Linear Systems

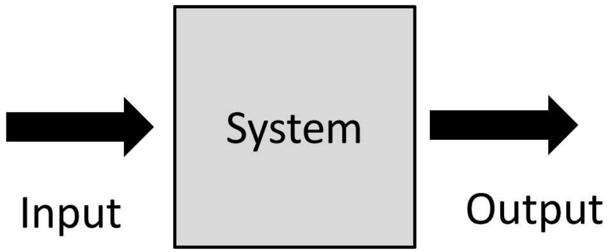


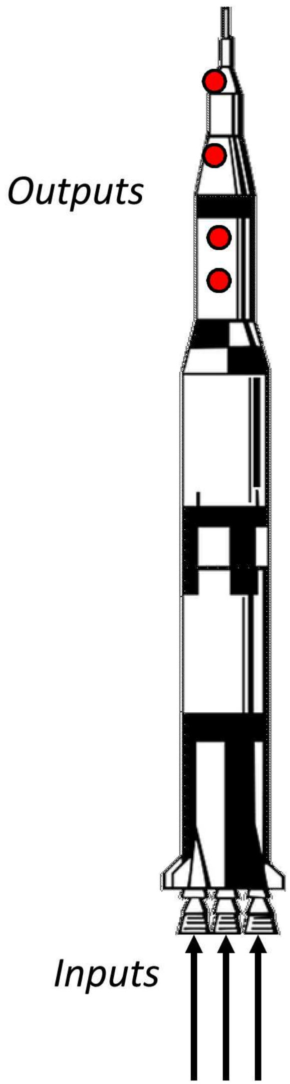
Linear Systems

- *Disclaimer: Systems are generally nonlinear, but we pretend they are linear because it makes things easier*
- Linear: Output scales linearly with input
- Basic Input-Output Relationship:

$$X_y = H_{yx} X_x$$

- Input: X_x
- System: H_{yx}
- Output: X_y
- System dynamics captured in FRF matrix, H_{yx}



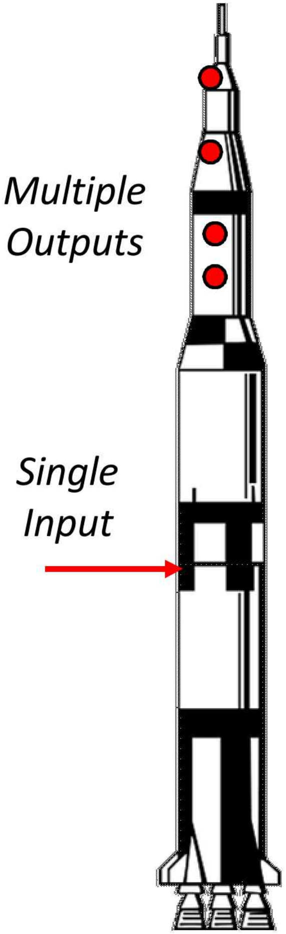


Linear Systems – Single Input

- One input, one output (SISO)
- One input, multiple outputs (SIMO)

$$X_y = H_{yx} X_x$$

- Sizes:
  - $X_x, 1x1$
  - $H_{yx}, Mx1$
  - $X_y, Mx1$

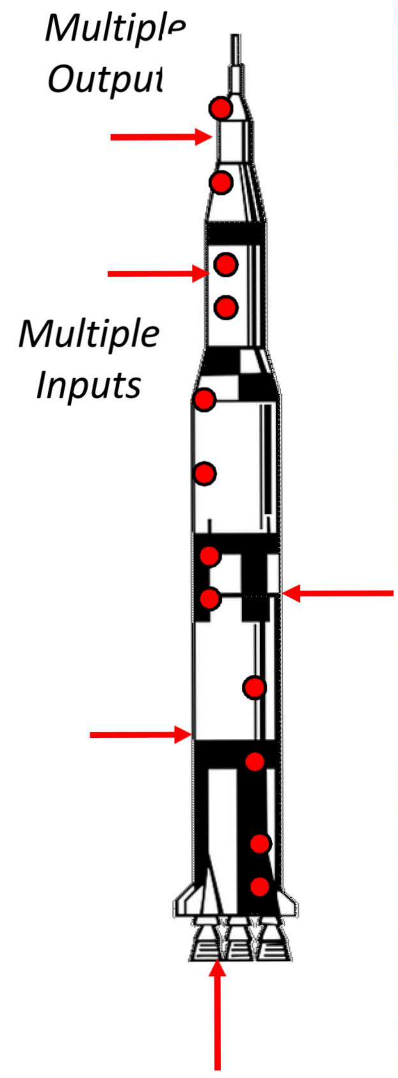


Linear Systems – Multiple Input

- Multiple input, multiple output (MIMO)

$$X_y = H_{yx} X_x$$

- Sizes:
  - $X_x, Nx1$
  - H_{yx}, MxN
  - $X_y, Mx1$



Linear Systems – Linear & Power Space Forms of the Equations

- $Y = HX$ is the MIMO linear system equation in linear space, meaning the units of the inputs or outputs are in [Volts-s] or [lbf-s] or [g-s]
  - Y, X are the linear spectrum of the outputs and inputs
- Typically, the power-space form of the equation is used, allowing the inputs and outputs to be in terms of PSDs
  - Why? Utilize ensemble averaging, which reduces noise in the PSD estimates

Linear Space

$$X_y = H_{yx} X_x$$

Power Space (Linear Squared)

$$S_{yy} = H_{yx} S_{xx} H_{yx}^H$$

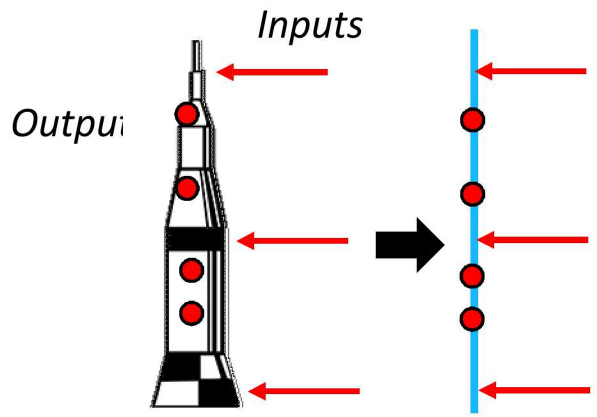
X_y, X_x are vectors of linear spectra

S_{yy}, S_{xx} are CPSD matrices

H_{yx} is the same in both equations

Note: A^H is Hermitian (conjugate transpose) of matrix A

FRF Matrices – What's in there?

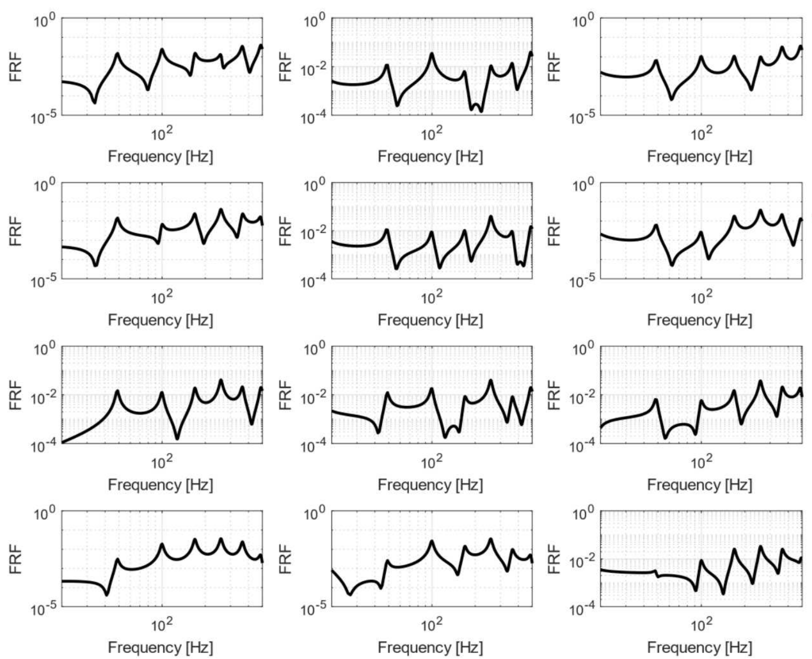


- Relates the outputs to the inputs
- FRF is outputs x inputs x frequencies
  - Rows = outputs
  - Cols = inputs
  - Pages = frequencies

$$H_{yx} = S_{yx}S_{xx}^{+}$$

4 Outputs, 3 Inputs

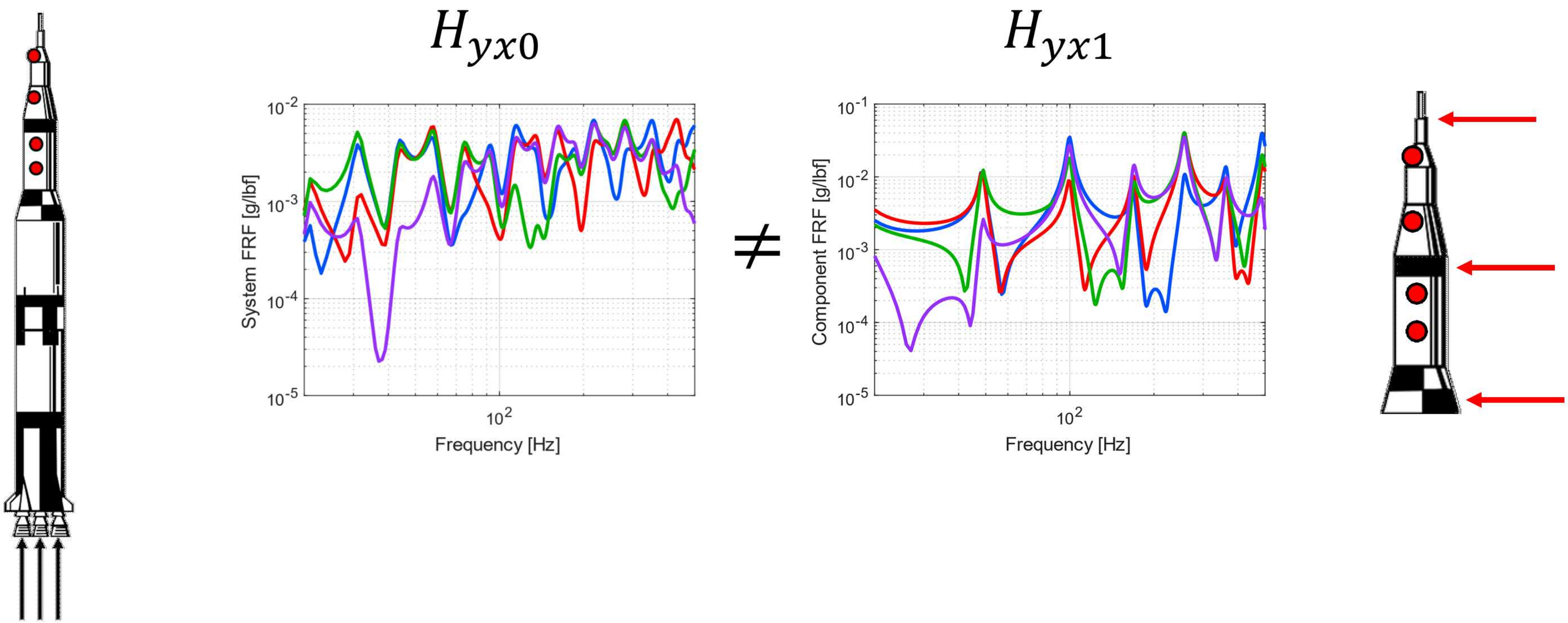
4 Output, 3 Input FRF Matrix



FRF between Output 2 and Input 3

Field vs. Lab Systems

- Often, Field system will be different than the Lab system
  - Change in boundary conditions
  - Change in design, assembly, variability, etc.
- FRFs will be different, affecting the ability to control & achieve accurate response

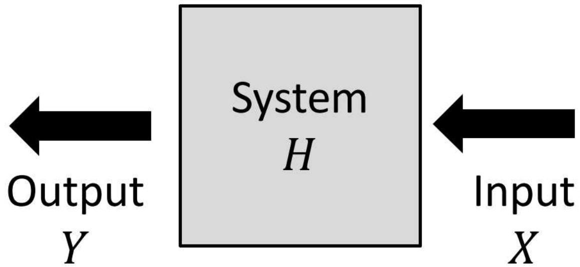


OUTPUT METRICS

It's complicated…

Gauge Response Represented by CPSD Matrices

$$S_{yy} = H_{yx} S_{xx} H_{yx}^{H}$$



CPSD Matrices – What's in there?

$$S_{xx}, S_{yy}$$

- Complex valued terms
- Size: MxMxF or NxNxF
  - Square, 3-dimensional
- Hermitian, Positive-Definite
  - Meaning…
  - $S_{xx} = S_{xx}^H$
  - Diagonals are real-valued
  - Off-Diagonals are conjugate-symmetric

$$S_{xx} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12}^* & S_{22} & S_{23} \\ S_{13}^* & S_{23}^* & S_{33} \end{bmatrix}$$

CPSD Matrices – What's in there?

- How is the CPSD matrix made?
  - Outer product of a vector of linear spectra with itself

$$[S_{xx}] = df\{X_x\}\{X_x\}^H$$

- Relationship between the APSDs, Coherence & Phase:
  - S_{ii}, S_{jj}: APSDs for the i-th and j-th signal (input or output signal)
  - γ_{ij}^2 : Coherence between the i-th and j-th signal
  - ϕ_{ij} : Phase between the i-th and j-th signal

$$S_{ij} = \sqrt{\gamma_{ij}^2 S_{ii} S_{jj}}\, e^{j\phi_{ij}}$$

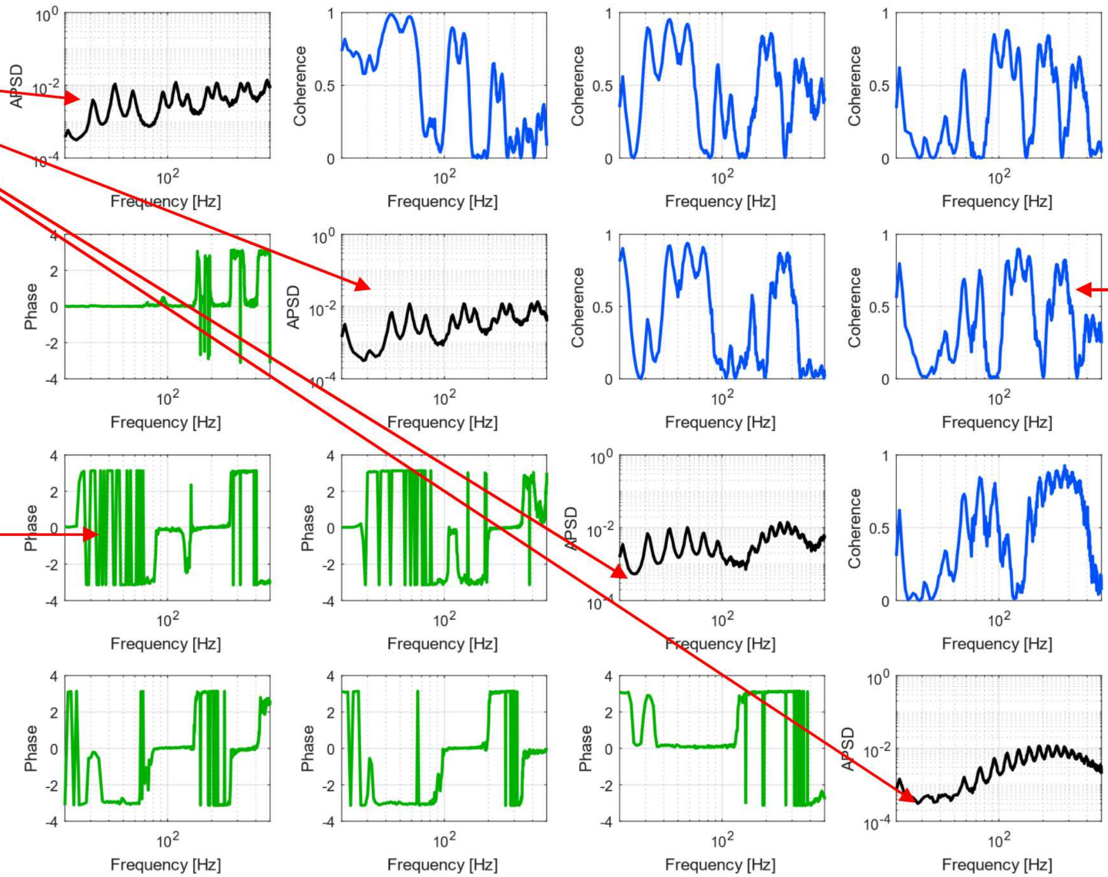
CPSD Matrices – What's in there?

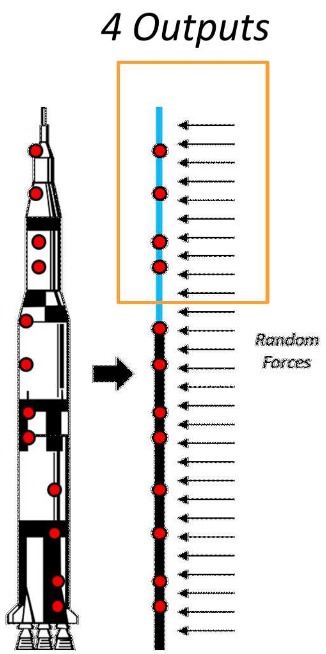
4x4 Syy Output CPSD Matrix

APSDs on the diagonal

Coherence between Signal 2 and Signal 4

Phase between Signal 1 and Signal 3

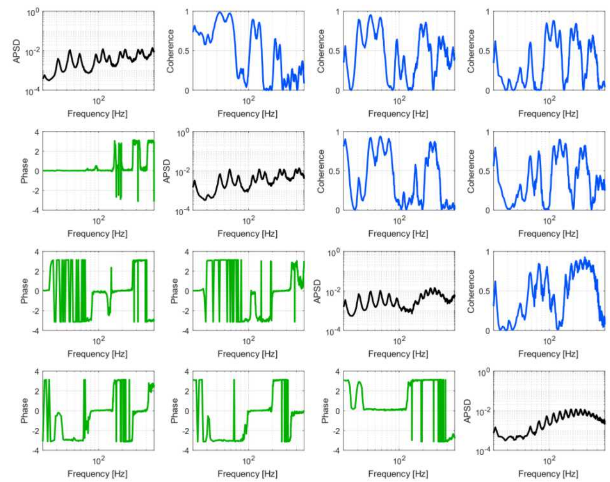




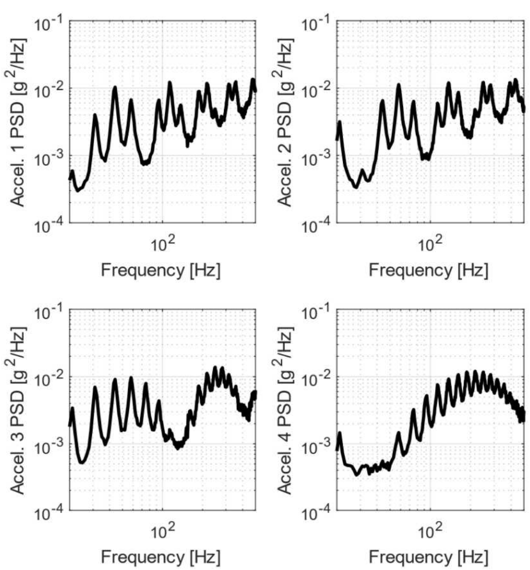
Cross-terms (off-diagonals) represent a combination of APSDs, Coherence & Phase

$$S_{ij} = \sqrt{\gamma_{ij}^2 S_{ii} S_{jj}}\; e^{j\phi_{ij}}$$

Too Much Data in the CPSD Matrix…What To Do?

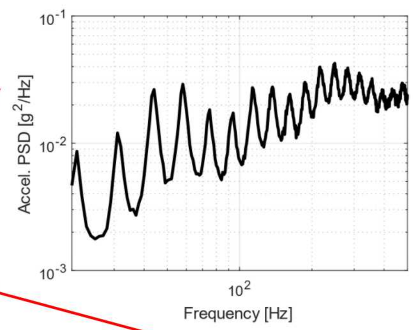


APSDs from the CPSD:



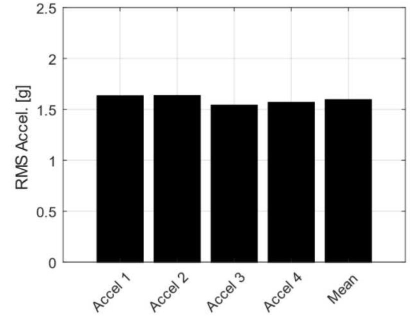
No Cross Terms

Sum of APSDs



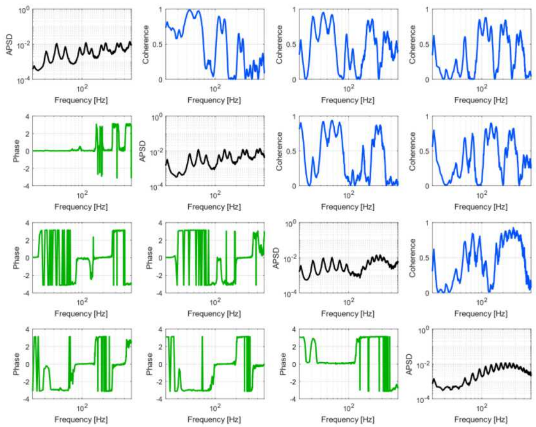
Condense Down Spatial Dimension

RMS of APSDs



Condense Down Frequency Dimension

CPSD Matrices – What's in there?



$$S_{ij} = \sqrt{\gamma_{ij}^2 S_{ii} S_{jj}}\, e^{j\phi_{ij}}$$

APSDs from the CPSD:

$$G_{ii} = S_{ii}$$

NOTE: G_{ii} must be real and positive

Coherence from the CPSD:

$$\gamma_{ij}^2 = \frac{\left|S_{ij}\right|^2}{S_{ii} S_{jj}}$$

NOTE: γ_{ij}^2 must be real between 0 and 1

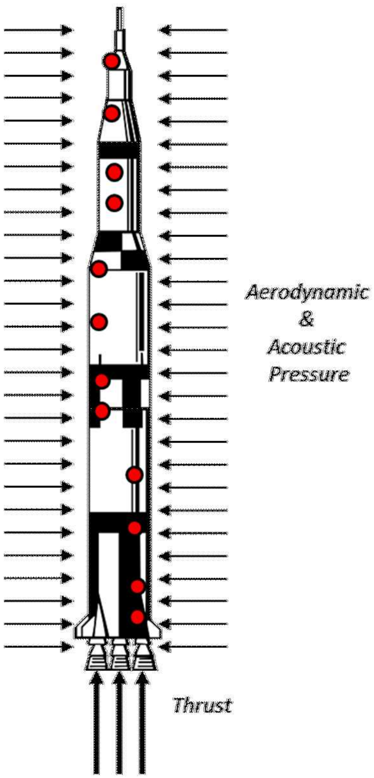
Phase from the CPSD:

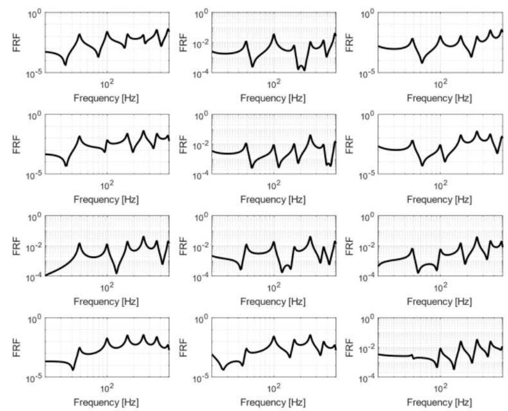
$$\phi_{ij} = \tan^{-1}\left(\frac{Im\left(S_{ij}\right)}{Re\left(S_{ij}\right)}\right)$$

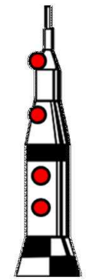
Our System So Far

Full System & Field Loads	FRF Matrix of Full System & Component	Measured Field Response of Component
S_{xx}	H_{yx}	$S_{yy} = H_{yx} S_{xx} H_{yx}^H$

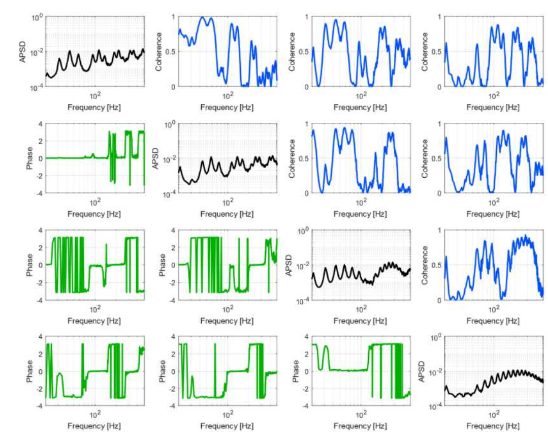
Field Environment Loads







$$S_{xx} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12}^* & S_{22} & S_{23} \\ S_{13}^* & S_{23}^* & S_{33} \end{bmatrix}$$



INPUT ESTIMATION

Gotta tell the shakers what to do

Determine Lab Inputs to Match Some Response

$$S_{yy} = H_{yx} S_{xx} H_{yx}^{H}$$

Desired Known Unknown

Input Estimation

- Determine inputs to best match some desired responses
- Stationary Problems:

$$S_{xx} = H_{yx}^{+} S_{yy} H_{yx}^{+H}$$

- Transient Problems:

$$X_x = H_{yx}^{+} X_y$$

- Notes:
  - A^+ is the pseudo-inverse of a matrix A, thus A can be rectangular
  - Shown here are the direct methods for estimating inputs (using a direct inverse of the FRF matrix). These can also be solved using iterative approaches, other optimization techniques

Input Estimation – Some Details

- There are really some different FRF & CPSD matrices floating around here
- Terminology:
  - Subscript 0 = Field system, inputs & responses
  - Subscript 1 = Lab system, inputs & responses
- Objective of Lab test and Input Estimation is to make inputs, S_{xx1}, which make a lab system, H_1, respond as S_{yy1} which best match the measured field response, S_{yy0}

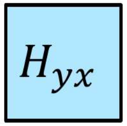
$$S_{yy0} = H_{yx0} S_{xx0} H_{yx0}^{H}$$ Field Test

$$S_{xx1} = H_{yx1}^{+} S_{yy1} H_{yx1}^{+H}$$ Input Estimation

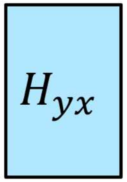
$$S_{yy1} = H_{yx1} S_{xx1} H_{yx1}^{H}$$ Lab Test

Input Estimation – Over-Determined, Under-Determined

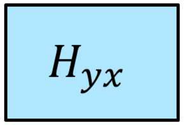
- Square: Same number of inputs as responses, H_{yx} is MxM=NxN
  - Can achieve perfect control (match the responses), however, the forces can be high



- Over-Determined: Rectangular, H_{yx} is MxN and M>N
  - More outputs than inputs
  - Most typical in MIMO testing
  - No solutions, so find a least-squares solution using pseudo-inverse



- Under-Determined: Rectangular, H_{yx} is MxN and N>M
  - More inputs than outputs
  - Infinite solutions, so find a minimum norm solution or solution with most nonzero components (results vary by the solution algorithm!)



$$S_{xx} = H_{yx}^{+} S_{yy} H_{yx}^{+H}$$

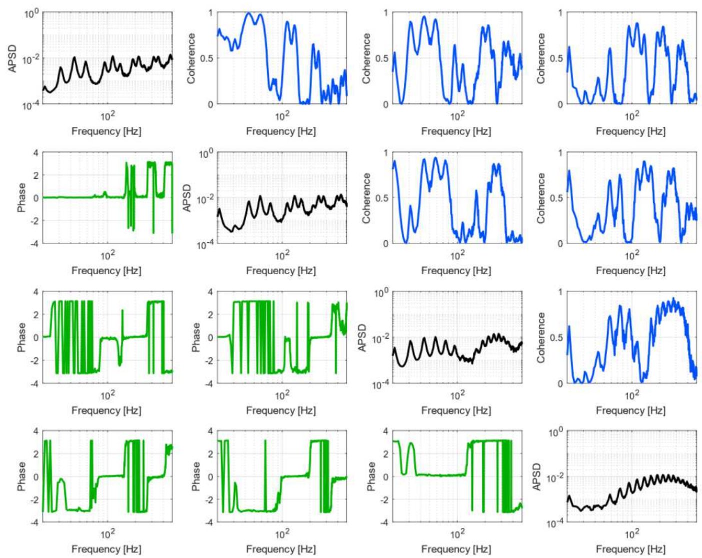
Note: This Equation Tries to Match ALL the Terms in S_{yy0}

Inputs which best match the target response CPSD in terms of the:
- **Levels (APSDs)**
- **Coherence**
- **Phase**

$$S_{yy0} = H_{yx0} S_{xx0} H_{yx0}^{H}$$ Field Test

$$S_{xx1} = H_{yx1}^{+} S_{yy1} H_{yx1}^{+H}$$ Input Estimation

$$S_{yy1} = H_{yx1} S_{xx1} H_{yx1}^{H}$$ Lab Test



Do you care about the cross-terms?

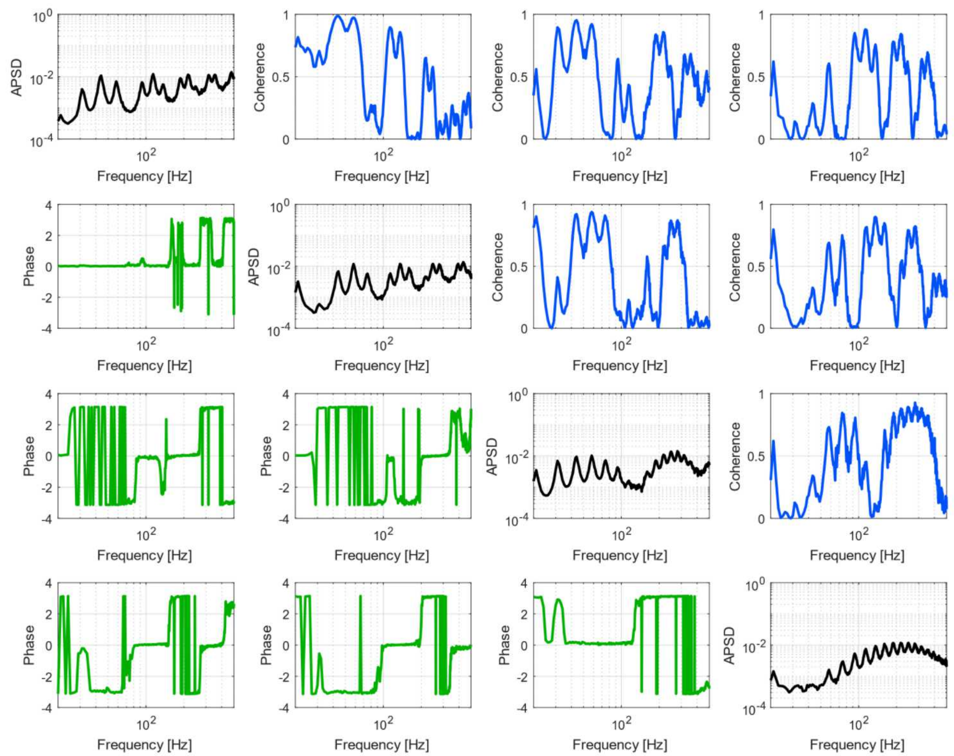
Are you confident in the measurement of the cross-terms?

How do you write a spec for this?

Specs & Target Responses

Specs: Current Practice is to Use Field Time History Data to Form CPSDs

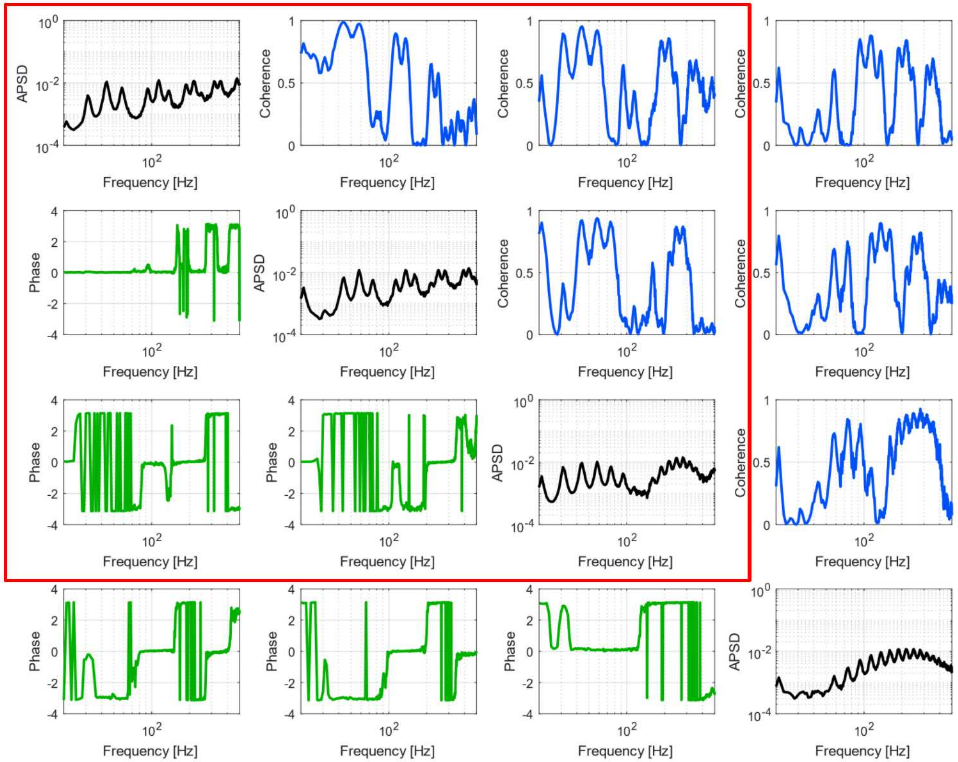
- **Need the full CPSD matrix**
- **Measurements must be time-aligned for useful phase**
- **Future work should focus on spec development**



Specs & Target Responses

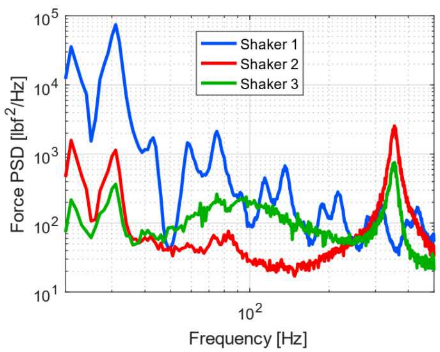
Typical Practice is to Use a Subset of the Field Measurements as "Target" Responses

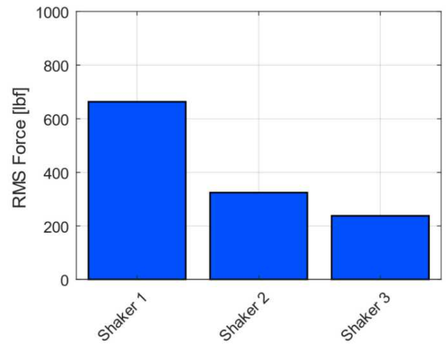
- **Remaining measurements can be used as reference**

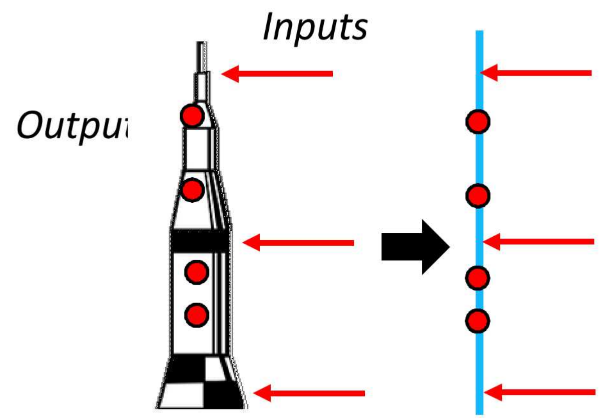


Lab Test Results

Estimated Inputs

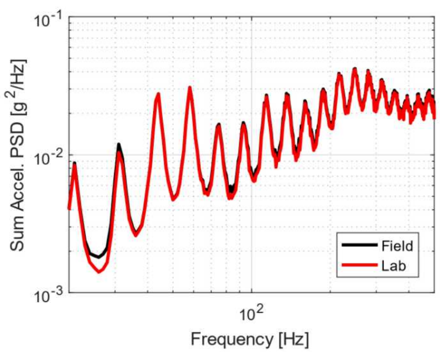


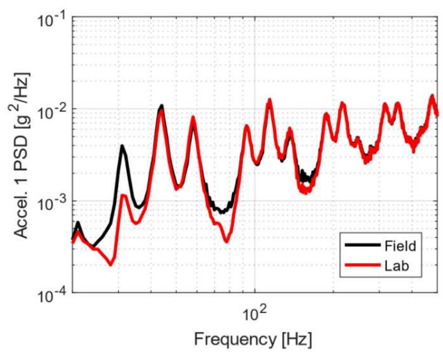


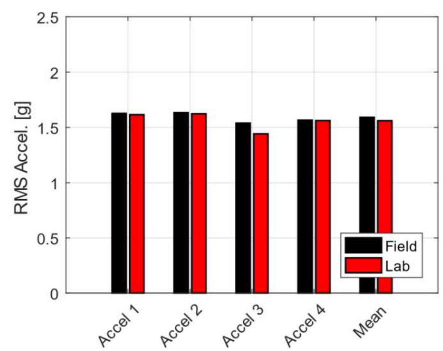


4 Outputs, 3 Inputs

Lab Response

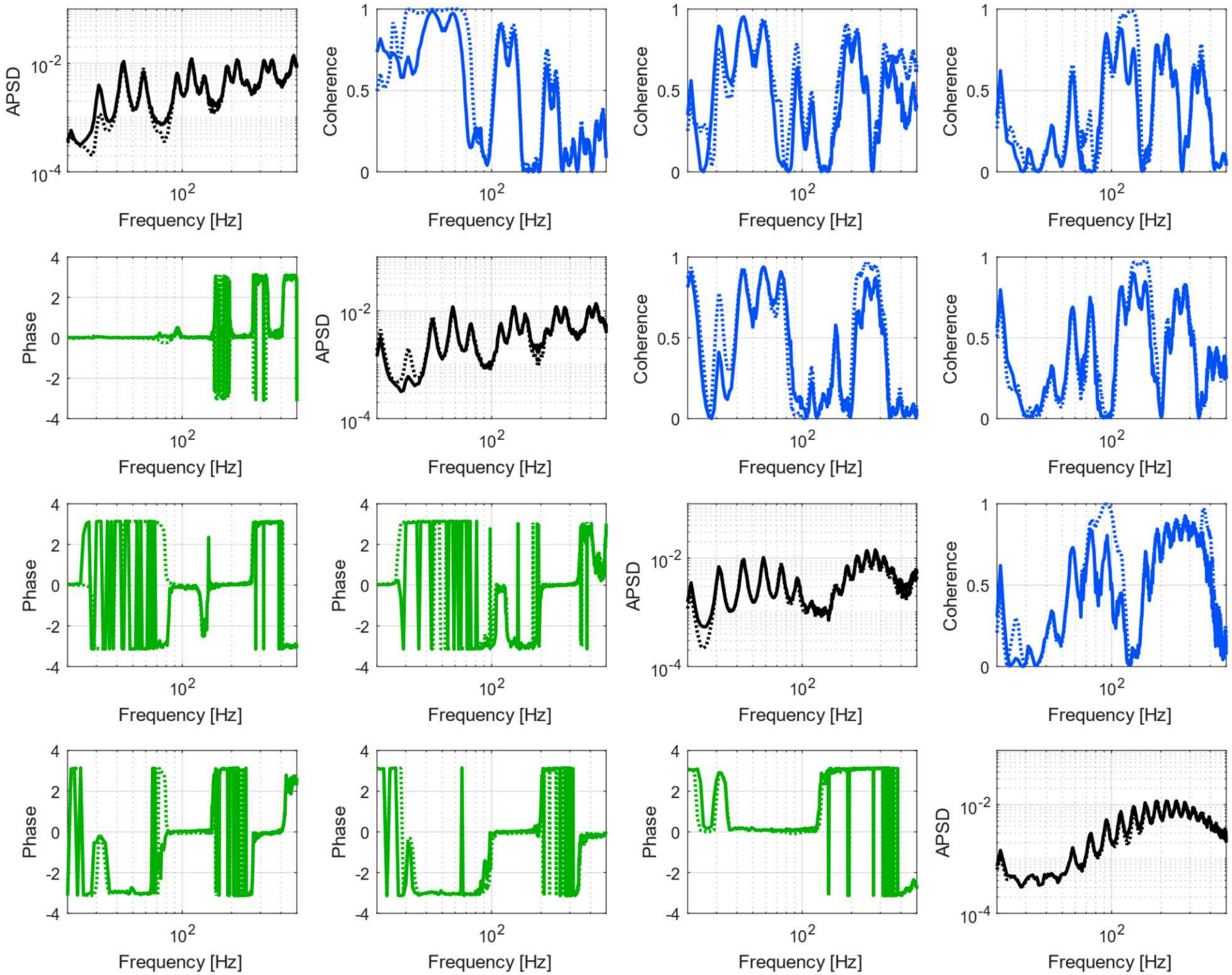


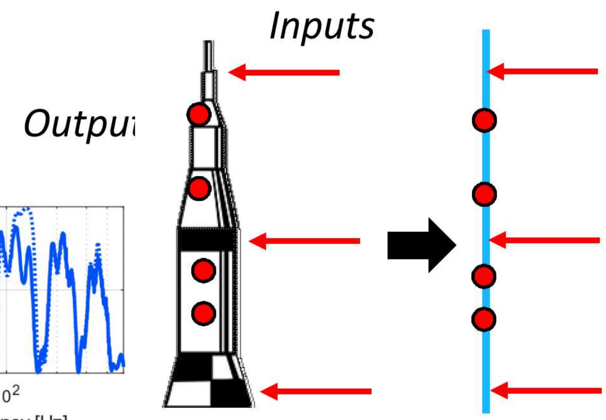




Lab Test Results

Entire CPSD Matrix





4 Outputs, 3 Inputs

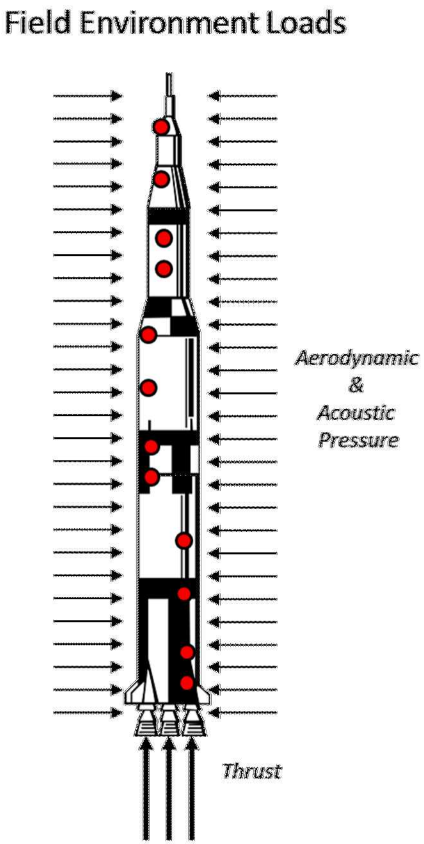
A MIMO VIBRATION TEST

What goes on under the hood

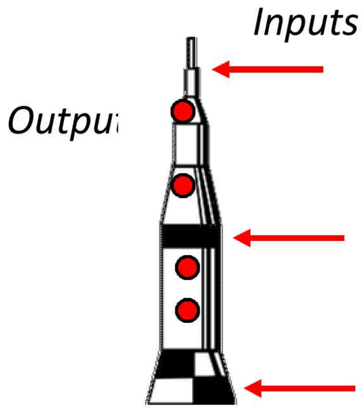
Controlling a MIMO Test

- Step through the process…what is going on in a MIMO test?

$$S_{yy0}$$



$$H_{yx1}$$



$$S_{xx1} = H_{yx1}^{+} S_{yy0} H_{yx1}^{+H}$$

$$S_{yy1} = H_{yx1} S_{xx1} H_{yx1}^{H}$$

Controlling a MIMO Test

- Step through the process…what is going on in a MIMO test?

1. Measure the Field response, form S_{yy1}
2. Setup the Lab test (mount shakers, put gauges at Field locations etc.)
3. Measure the Lab FRF matrix, H_{yx1}, using uncorrelated inputs
4. Estimate the Lab inputs, S_{xx1}
5. Convert the inputs from CPSDs to time histories, send to the shakers
6. Measure the Lab response, S_{yy1}
7. *optional*
   1. Adjust the input levels based on predicted or actual response error (closed loop)
   2. Adjust the FRF matrix based on the levels/spectrum of the test inputs

BOILED DOWN

If you take away just one thing today…

Key Equations

$$X_y = H_{yx} X_x$$

Outputs System Inputs

$$S_{yy} = H_{yx} S_{xx} H_{yx}^H$$

Outputs System Inputs

X_y	Output Linear Spectrum Mx1, [g-s]
H_{yx}	System FRF Matrix MxN, [g/lbf]
X_x	Input Linear Spectrum Nx1, [g/lbf]
S_{yy}	Output CPSD MxM [g2/Hz]
S_{xx}	Input CPSD NxN [lbf2/Hz]

*****shown for a single frequency line. In general, the 3rd dimension is frequency, making Hyx MxNxF, Sxx NxNxF, etc.*

The Process, in Equations

$$S_{yy0} = H_{yx0} S_{xx0} H_{yx0}^{H}$$

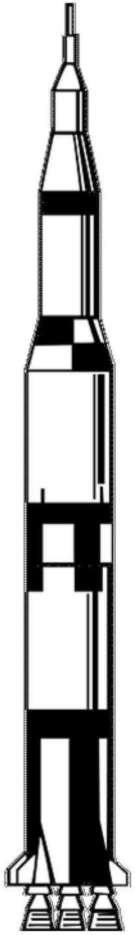
$$S_{yyFRF} = H_{yx1} S_{xxFRF} H_{yx1}^{H}$$

$$S_{xx1} = H_{yx1}^{+} S_{yy1} H_{yx1}^{+H}$$

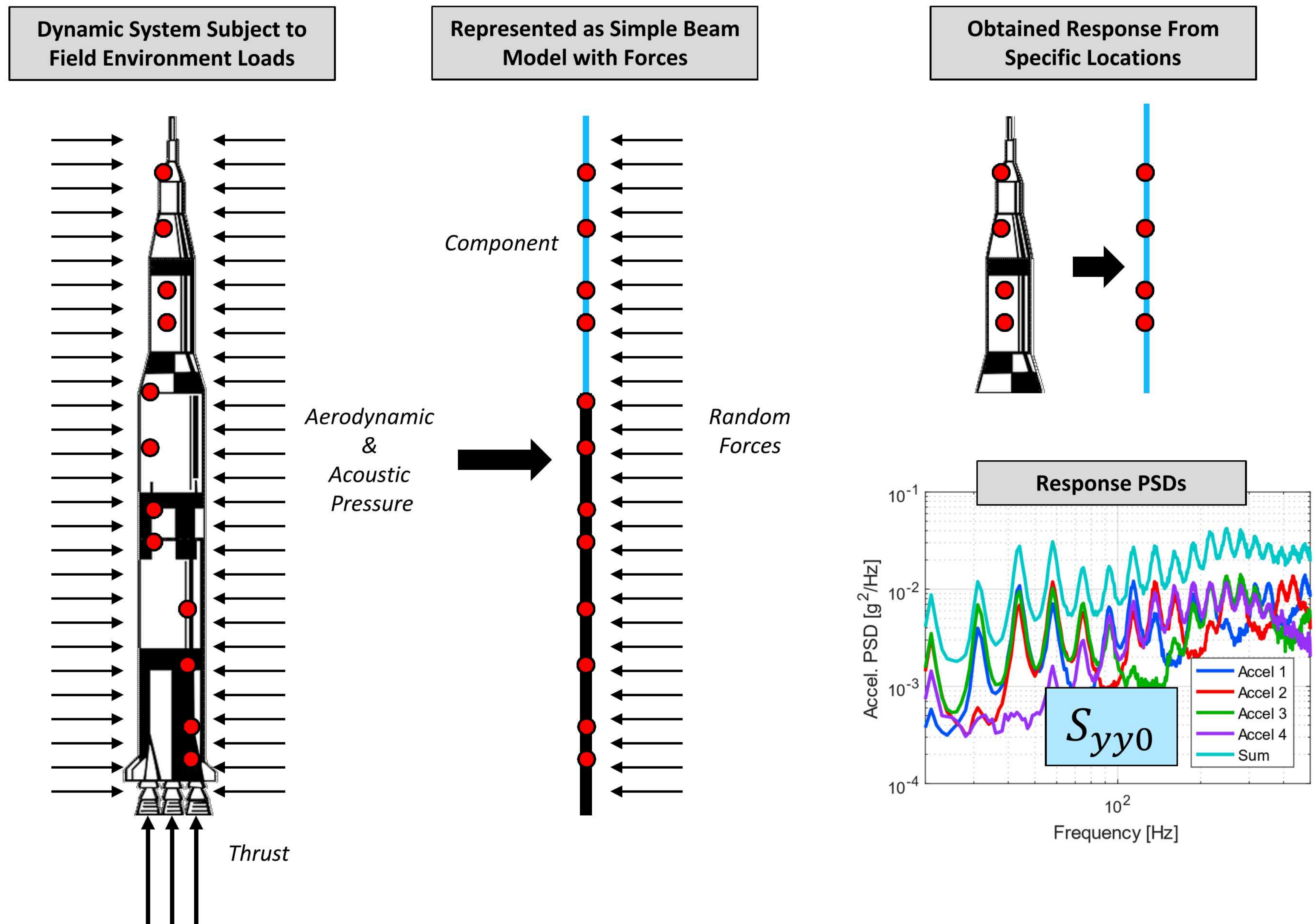
$$S_{yy1} = H_{yx1} S_{xx1} H_{yx1}^{H}$$

What Did We Do?

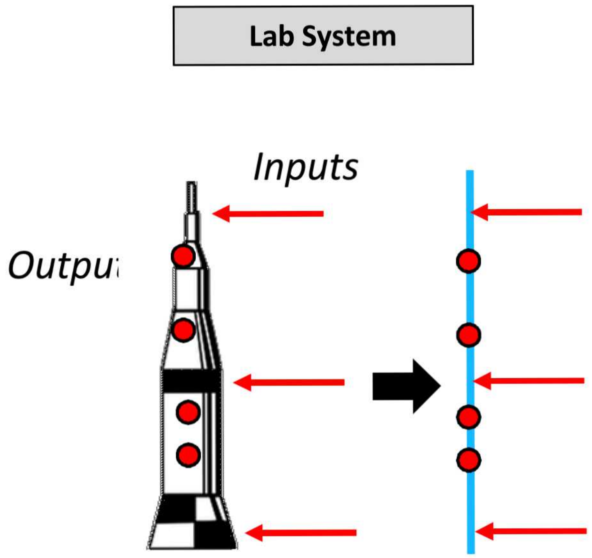
Dynamic System



What Did We Do?



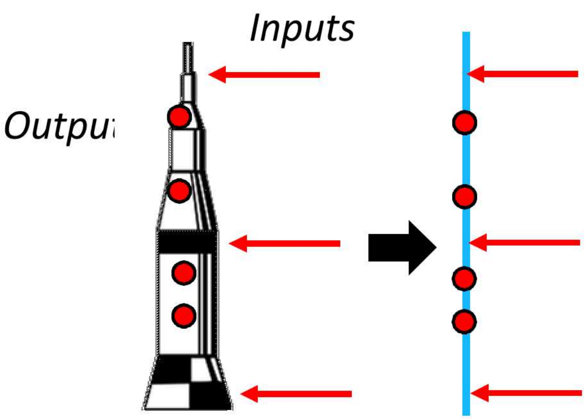
What Did We Do?



4 Outputs, 3 Inputs

What Did We Do?

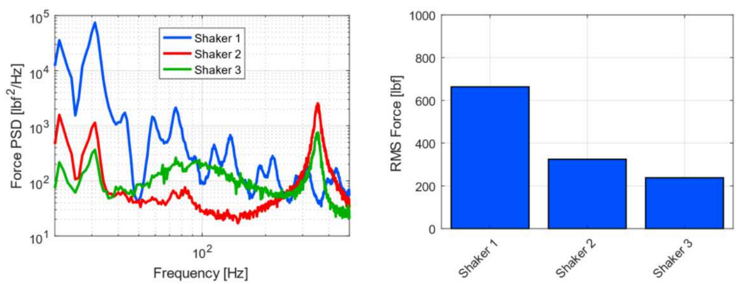
Lab System



4 Outputs, 3 Inputs

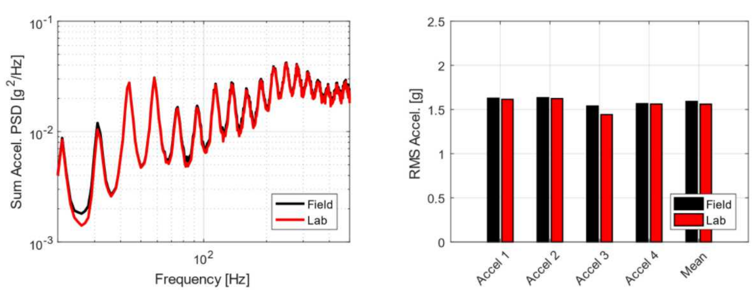
Solve Input Estimation Problem

$$S_{xx1} = H_{yx1}^{+} S_{yy0} H_{yx1}^{+H}$$



Obtain Lab Response from Lab Inputs

$$S_{yy1} = H_{yx1} S_{xx1} H_{yx1}^{H}$$



WHERE ARE WE WITH MIMO?

We're not done yet…

Where Are We? What's Next?

- MIMO vibration testing provides many benefits:
  - *Higher accuracy – match the Field response more closely*
  - *Improved controllability – can tailor a test to specific objectives*

- However, it introduces several new challenges:
  - *What do you do if you can't get* S_{yy0} *from the Field measurements? Specification development for MIMO is an open area of research*
  - *How can you implement limits across many gauges?*
  - *Where do you locate shakers and how do you control them to get a desired result?*
  - *How can you solve the control (input estimation) problem to achieve specific results?*
  - *How can you design fixtures to allow the Lab system to respond like the Field system?*
  - *How do you control different types of inputs simultaneously? Base shake + multiple shakers, etc.*

There are many opportunities for improvement, and opportunities to establish best practices for this next generation test technique

NEXT TIME

More to come

MIMO Tutorial – Outline (Part 2)

- Part 2: Advanced Topics

1. Review
   - MIMO Linear System Equations
   - Input Estimation & Control Process
2. Structure of CPSD & FRF Matrices
   - Expected Forms, Checks, and Numerical Corrections
   - Manipulating CPSD Matrices (extracting subsets, scaling, sign changes)
3. Signal Processing for MIMO Tests
   - Time Histories to CPSDs
   - Ensemble Averaging and Estimates
4. FRF Estimation Methods
5. Modes, Modal FRFs, and Modal-Space MIMO Equations
6. Fancy Input Estimation Techniques
   - Correlated vs. Uncorrelated Inputs
   - Regularized Inverse Solutions
   - Input Adjustments – Correcting Levels