Modeling Ice Sheets with MALI

SAND2019-6031PE

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ETH, Zurich, Switzerland, June 12 2019

Center for Computing Research

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Funded by (ProSPect)

SciDAC Scientific Discovery through Advanced Computing

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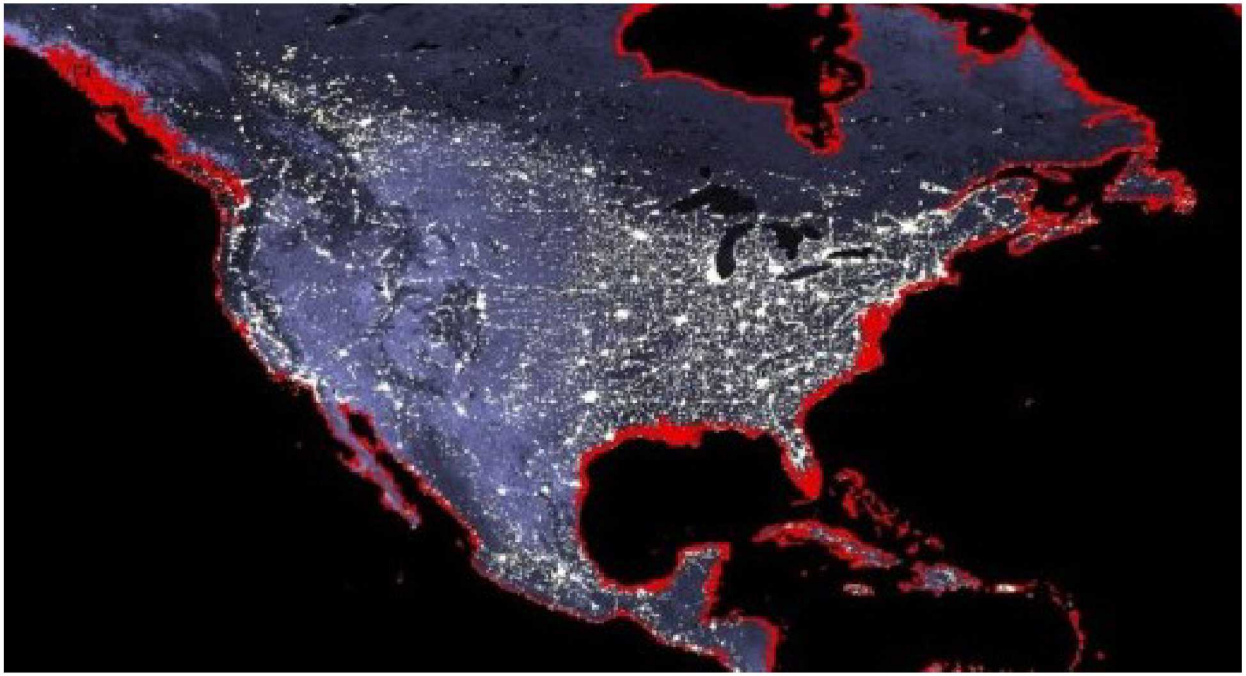
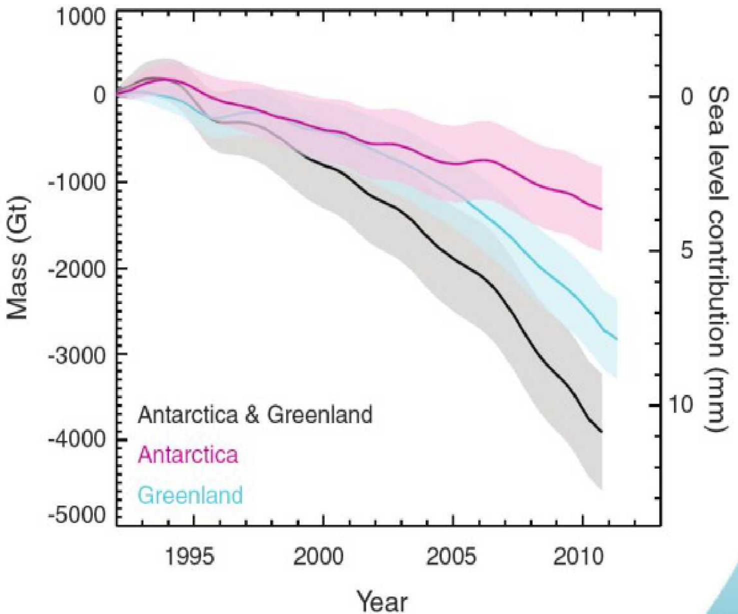
Brief introduction and motivation

- Greenland and Antarctica ice sheets store most of the fresh water on hearth.
- Modeling ice sheets (Greenland and Antarctica) dynamics is essential to provide estimates for sea level rise* and and fresh water circulation.
- Global mean sea-level is rising at the rate of 3.2 mm/yr and the rate is increasing.
- Latest studies suggest possible increase of 0.3 – 2.5m by 2100.



Map with 6 meters sea-level rise in red (NASA).



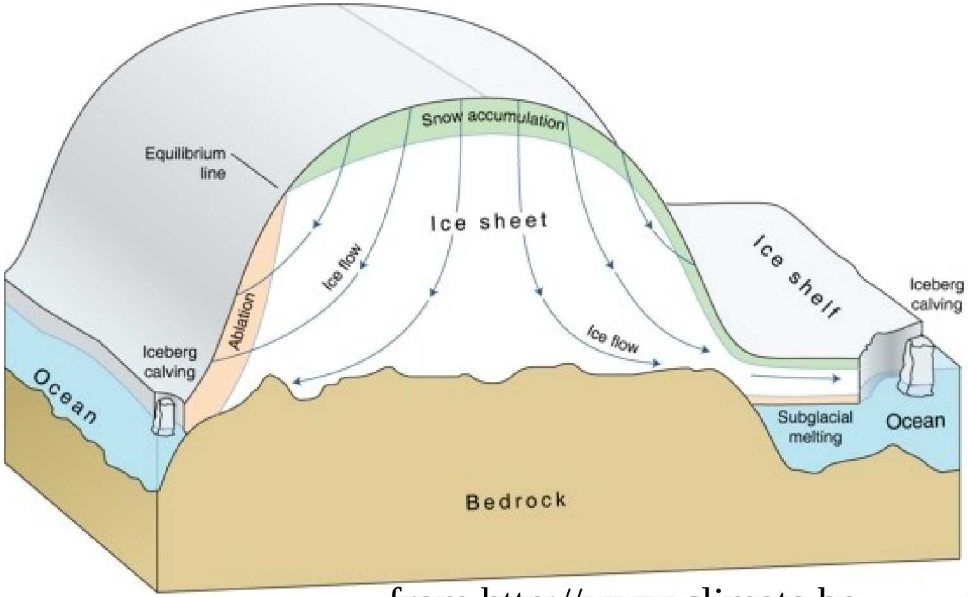
total mass loss of ice sheets in 1992-2011 (sheperd et al. 2012)

*DOE SciDAC project **ProSPect** (**Pro**babilistic **S**ea Level **P**roj**ect**ion from Ice Sheet and Earth System Models), Institutes: LANL, LBNL, SNL, ONL, NYU, Univ. of Michigan

Brief introduction and motivation

- Ice behaves like a very viscous shear-thinning fluid (similar to lava flow) driven by gravity. Source: snow packing/water freezing. Sink: ice melting / calving in ocean.
- Greenland and Antarctica have a shallow geometry (thickness up to 4 km, horizontal extensions of thousands of km).

Perito Moreno glacier



from http://www.climate.be

Outline:

- Ice sheet equations
- MALI model
- Model initialization
- Ensemble ice sheet modeling of ocean melt variability at Thwaites glacier
- Uncertainty Quantification: Inference and Forward propagation

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Ice Sheet Modeling

Ice momentum equations

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Boundary condition at ice-bedrock interface :

$$(\sigma \mathbf{n} + \beta \mathbf{u})_{||} = \mathbf{0} \quad \text{on} \quad \Gamma_\beta, \qquad \mathbf{u} \cdot \mathbf{n} = 0$$

Ice Sheet Modeling

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with:

$$\sigma = 2\mu \mathbf{D} - pI, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Nonlinear viscosity:

$$\mu = \frac{1}{2}\alpha(T)\,|\mathbf{D}(\mathbf{u})|^{\frac{1}{n}-1}, \quad n \geq 1, \quad (\text{tipically } n \simeq 3)$$

Viscosity is singular when ice is not deforming

Stiffening/Damage factor

$$\mu^*(x,y,z) = \phi(x,y)\,\mu(x,y,z)$$

ϕ : stiffening factor that accounts for modeling errors in rheology

Ice Sheet Modeling

Main components of an ice model:

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Boundary condition at ice-bedrock interface :

$$(\sigma \mathbf{n} + \beta \mathbf{u})_{\|} = \mathbf{0} \quad \text{on} \quad \Gamma_\beta, \qquad \mathbf{u} \cdot \mathbf{n} = 0$$

- **Model for the evolution of the boundaries**
  (thickness evolution equation)

$$\frac{\partial H}{\partial t} = H_{flux} - \nabla \cdot (H \bar{\mathbf{u}}), \qquad \bar{\mathbf{u}} = \frac{1}{H} \int_z \mathbf{u} \, dz$$

- **Temperature, Basal hydrology**
- **Coupling with other climate components (e.g. ocean, atmosphere)**

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Stokes approximations in different regimes

$$\mathrm{Stokes}(\mathbf{u}, p)$$

$$\begin{cases} -\nabla \cdot (2\mu \mathbf{D}(\mathbf{u}) - p\mathbf{I}) = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Drop terms using **scaling argument** based on the fact that ice sheets are shallow

$$\mathbf{D}(u,v) = \begin{bmatrix} u_x & \frac{1}{2}(u_y + v_x) & \frac{1}{2}(u_z + \cancel{w_x}) \\ \frac{1}{2}(u_y + v_x) & v_y & \frac{1}{2}(v_z + \cancel{w_y}) \\ \frac{1}{2}(u_z + \cancel{w_x}) & \frac{1}{2}(v_z + \cancel{w_y}) & -(u_x + v_y) \end{bmatrix} \qquad \mathrm{u} := \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\mu = \mu(|\mathbf{D}(u,v)|)$$

Quasi-hydrostatic approximation

3rd momentum equation

$$-\cancel{\partial_x(\mu\, u_z)} - \cancel{\partial_y(\mu\, v_z)} - \partial_z(2\mu\, w_z - p) = -\rho g,$$

continuity equation

$$w_z = -(u_x + v_y)$$

$$\Longrightarrow \; p = \rho g(s - z) - 2\mu(u_x + v_y)$$

$$\mathrm{FO}(u,v)$$

First Order* or Blatter-Pattyn model

$$-\nabla \cdot \left(2\mu \tilde{\mathbf{D}} - \rho g(s-z)\mathbf{I}\right) = \mathbf{0}$$

$$\text{with } \tilde{\mathbf{D}}(u,v) = \begin{bmatrix} 2u_x + v_y & \frac{1}{2}(u_y + v_x) & \frac{1}{2}u_z \\ \frac{1}{2}(u_y + v_x) & u_x + 2v_y & \frac{1}{2}v_z \end{bmatrix}$$

**Dukowicz, Price and Lipscomb, 2010. J. Glaciol*

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Stokes approximations in different regimes

$$\mathrm{FO}(u, v)$$

Ice regime:
grounded ice with frozen bed

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & \frac{1}{2}u_z \\ 0 & 0 & \frac{1}{2}v_z \\ 0 & 0 & w_z \end{bmatrix}$$

$$p = \rho g(s - z)$$

$$\mathrm{SIA}(u, v)$$

Shallow Ice Approximation

Ice regime:
shelves or fast sliding grounded ice

$$\mathbf{D} = \begin{bmatrix} u_x & \frac{1}{2}(u_y + v_x) & 0 \\ \frac{1}{2}(u_y + v_x) & v_y & 0 \\ 0 & 0 & w_z \end{bmatrix}$$

$$p = \rho g(s - z) - 2\mu(u_x + v_y)$$

$$\mathrm{SSA}(u, v)$$

Shallow Shelf Approximation

$$\text{Hybrid models}, \simeq SIA + SSA$$

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MPAS-Albany Landice model (MALI)

Algorithm and Software

ALGORITHM	SOFTWARE TOOLS
Finite Volume on Voronoi Meshes	MPAS
Linear Finite Elements on test/hexas	Albany
Quasi-Newton optimization (L-BFGS)	ROL
Nonlinear solver (Newton method)	NOX
Krylov linear solvers/Prec	AztecOO/ML, Belos/MueLu
Automatic differentiation	Sacado

MPAS: *Model for Prediction Across Scales, fortran finite volume library:*

- *works on Voronoi Tessellations*
- *conservative Lagrangian schemes for advecting tracers*
- *evolution of ice thickness*

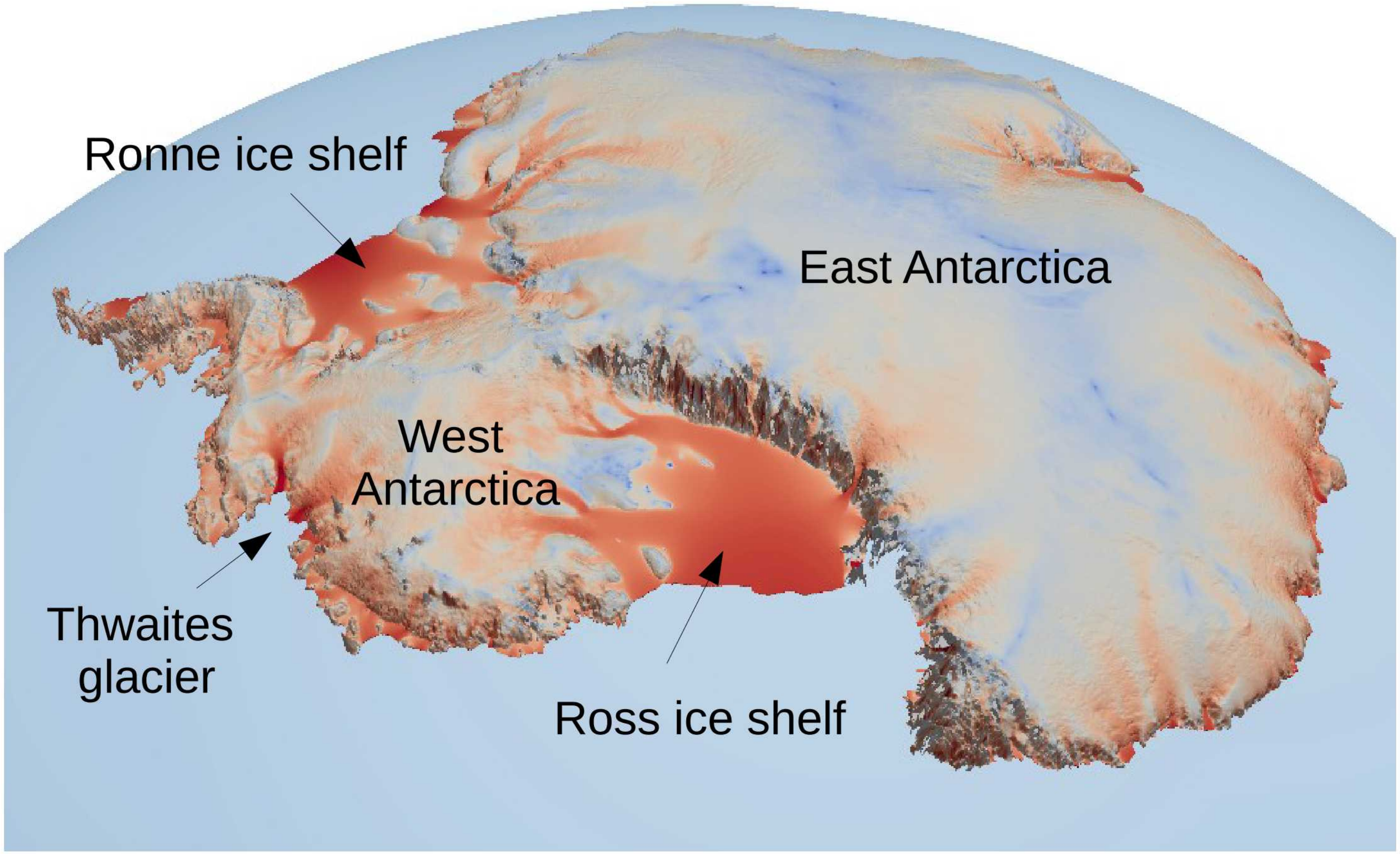
Albany: C++ finite element library built on Trilinos to enable multiple capabilities:

- Jacobian/adjoints assembled using automatic differentiation (Sacado).
- nonlinear and parameter continuation solvers (NOX/LOCA)
- large scale PDE constrained optimization (Piro/ROL)
- linear solver and preconditioners (Belos/AztecOO, ML/MeuLu/Ifpack)

Hoffman, et al. GMD, 2018
Tuminaro, Perego, Tezaur, Salinger, Price, SISC, 2016.
Tezaur, Perego, Salinger, Tuminaro, Price, Hoffman, GMD, 2015
Perego, Price, Stadler, JGR, 2014

MPAS-Albany Landice model (MALI)

Antarctic Ice Sheet velocity



Colored by ice sheet velocity surface velocity
(blue = slow, red = fast)

Model initialization

(using PDE-constrained optimization)

GOAL

Find ice sheet initial state that

- matches observations (e.g. surface velocity, temperature, etc.)
- is in compliance with flow model and climate forcing

estimating unknown (basal friction) or poorly known parameters (bed topography)

Bibliography

- *Arthern, Gudmundsson*, J. Glaciology, 2010
- *Price, Payne, Howat and Smith*, PNAS, 2011
- *Petra, Zhu, Stadler, Hughes, Ghattas*, J. Glaciology, 2012
- *Pollard DeConto*, TCD, 2012
- W. J. J.Van Pelt et al., The Cryosphere, 2013
- *Morlighem et al.* Geophysical Research Letters, 2013
- *Goldberg and Heimbach*, The Cryosphere, 2013
- *Michel et al.*, Computers & Geosciences, 2014
- *Perego, Price, Stadler,* Journal of Geophysical Research, 2014
- *Goldberg et al.*, The Cryosphere Discussions, 2015

Deterministic Inversion

PDE-constrained optimization problem: cost functional

Problem: find initial conditions such that the ice matches available observations.

Optimization problem:

find β and H that minimize the functional* $\mathcal{J}$

$$\mathcal{J}(\beta,\phi) = \int_\Omega \frac{1}{\sigma_u^2}|\mathbf{u}-\mathbf{u}^{obs}|^2\,ds \qquad \text{surface velocity mismatch}$$

$$+\int_\Omega \frac{1}{\sigma_\phi^2}|\phi-1|^2\,ds \qquad \text{stiffening factor mismatch}$$

$$+\mathcal{R}(\beta,\phi) \qquad \text{regularization terms.}$$

subject to ice sheet model equations (FO or Stokes)

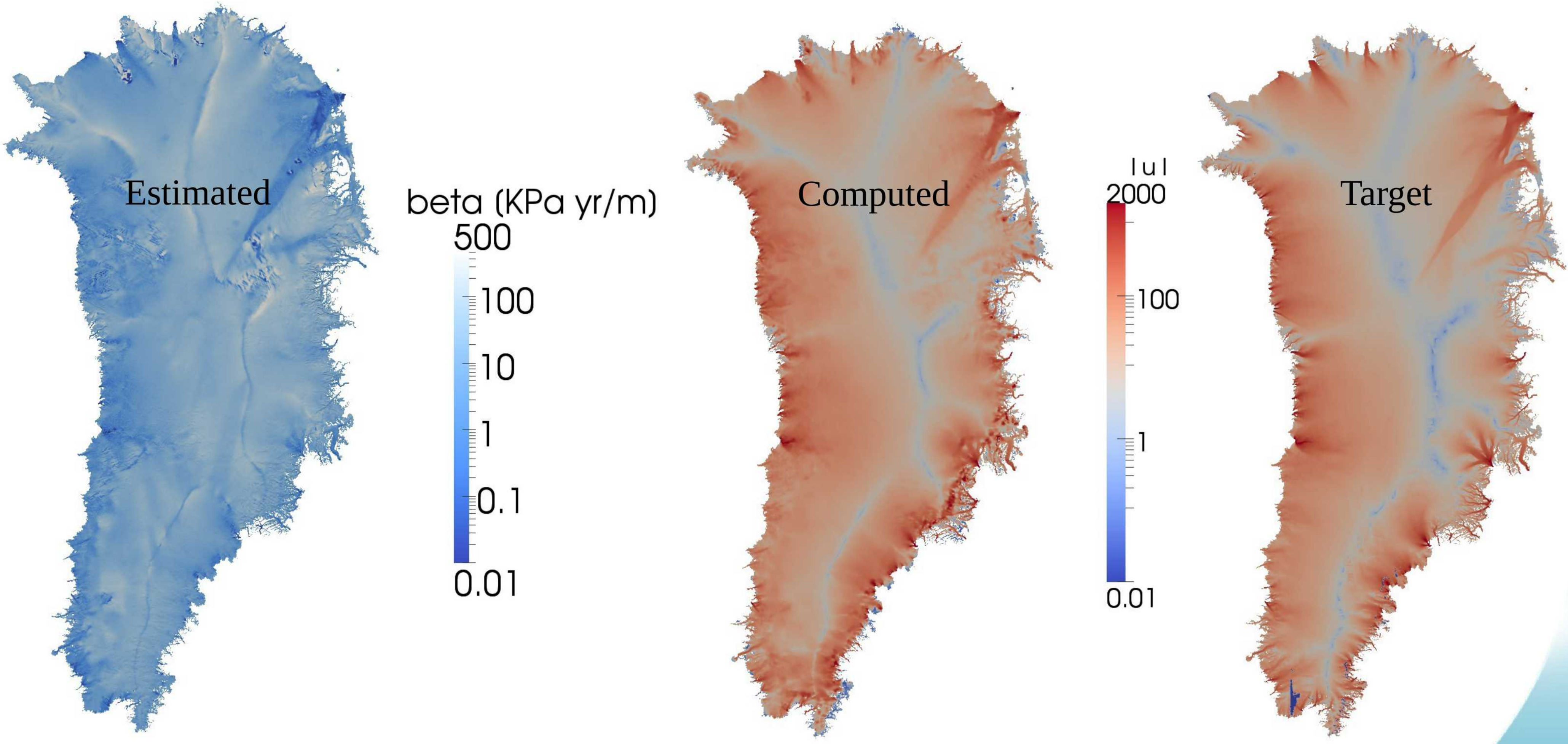
$\mathbf{u}$: computed depth averaged velocity
ϕ: stiffening factor
β: basal sliding friction coefficient
$\mathcal{R}(\beta,\phi)$ regularization term

*_Perego, Price, Stadler,_ Journal of Geophysical Research, 2014

Greenland Inversion

velocity mismatch only, tuning basal friction

Inversion with 1.6M parameters



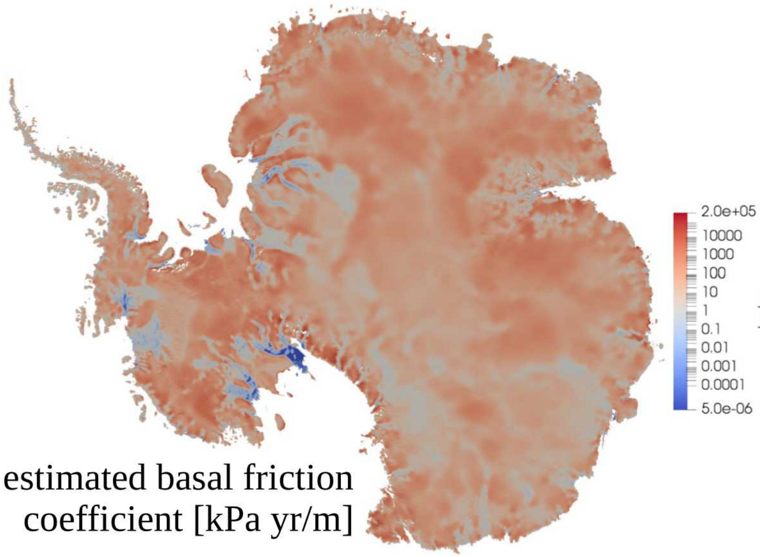
Basal friction coefficient (kPa yr/m)

surface velocity magnitude (m/yr)

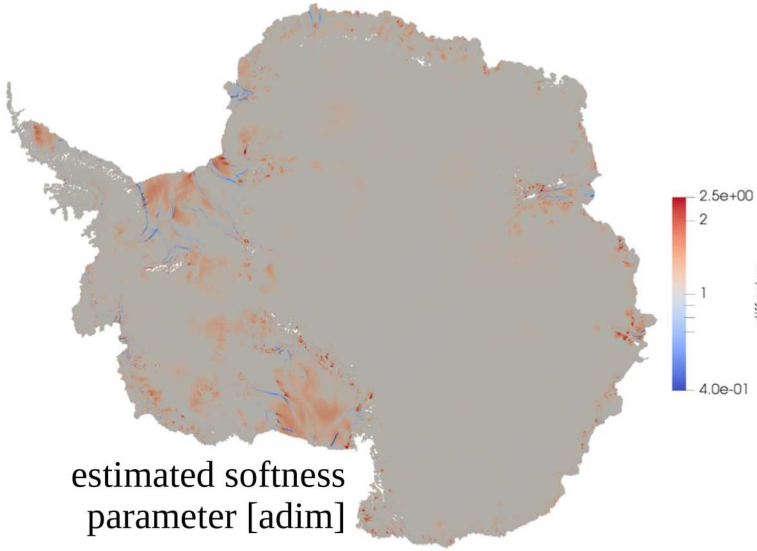
Geometry (Morlighem et al., Nature Geo., 2014)

Antarctica Inversion

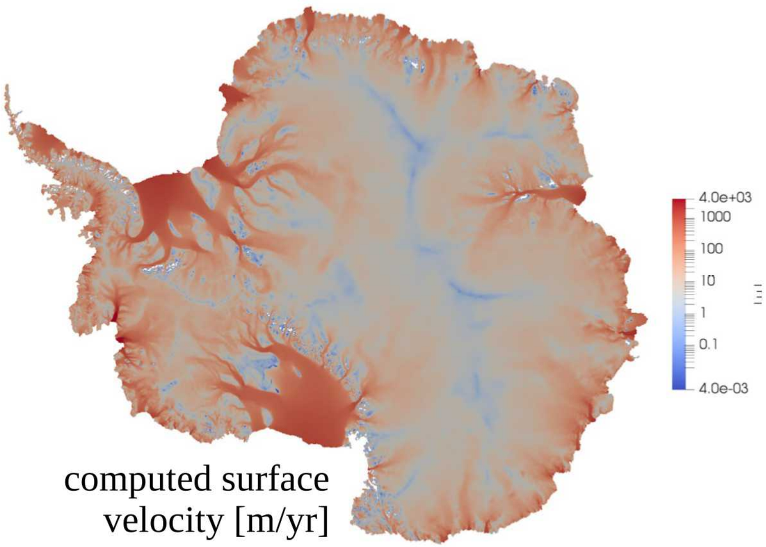
velocity and stiffening mismatches, tuning basal friction and stiffening



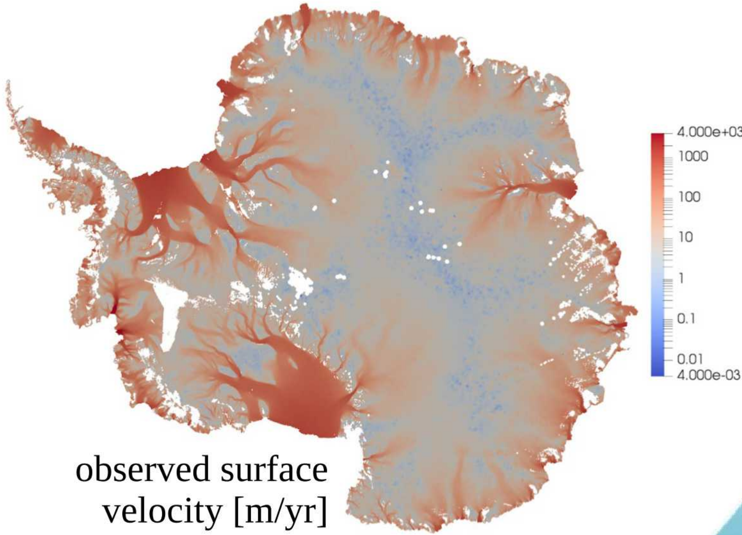
estimated basal friction coefficient [kPa yr/m]



estimated softness parameter [adim]



computed surface velocity [m/yr]

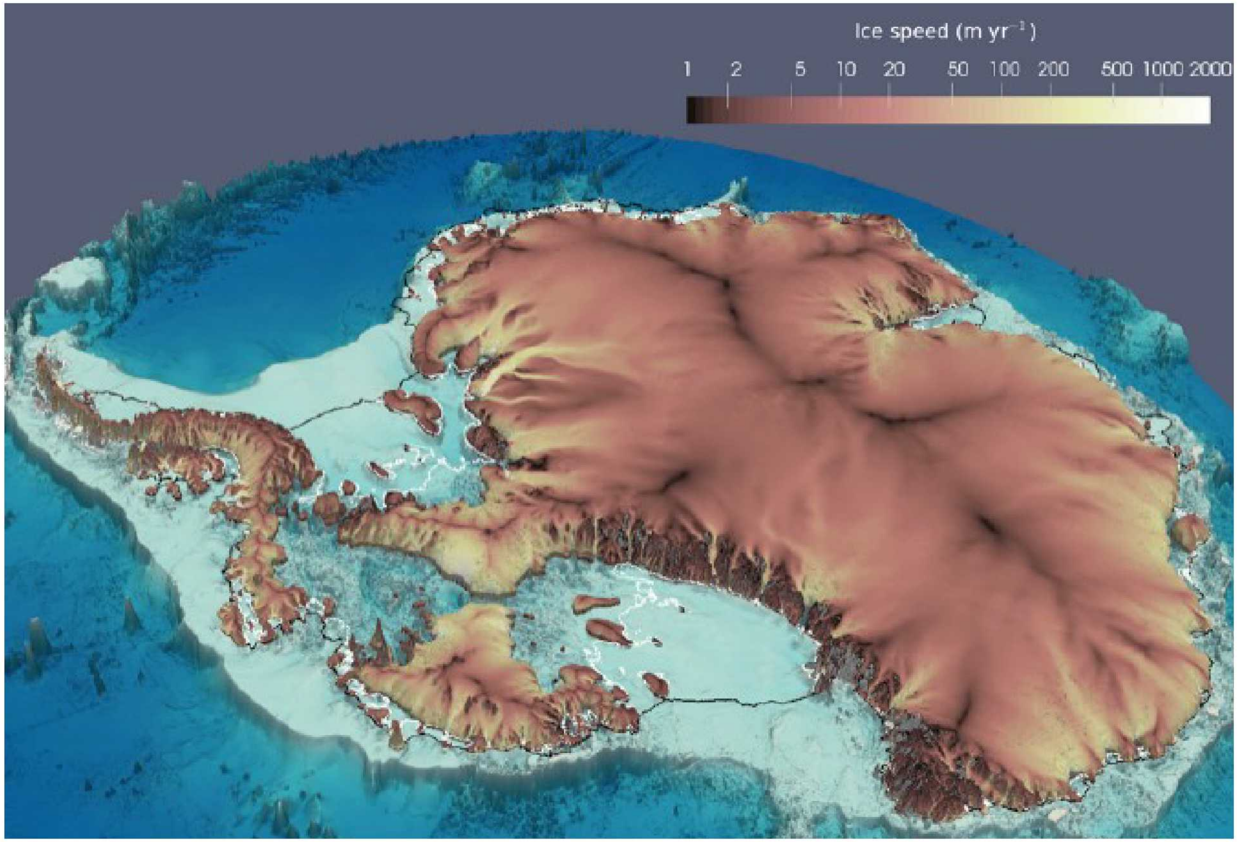


observed surface velocity [m/yr]

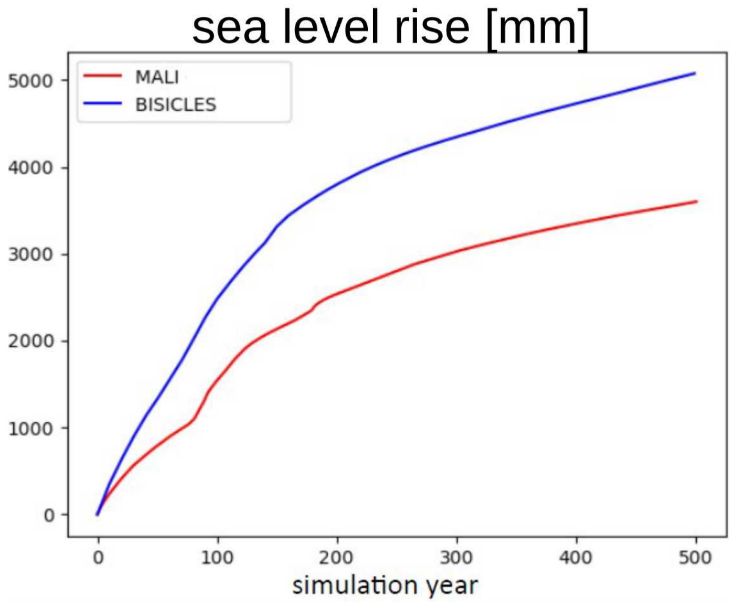
simulation details

#parameters: 2.5M	#cores: 8640
#unknowns: 30M	#nodes:180
machine: Edison (NERSC)	#hours:18

Ice sheet response under extreme (unrealistic) forcing



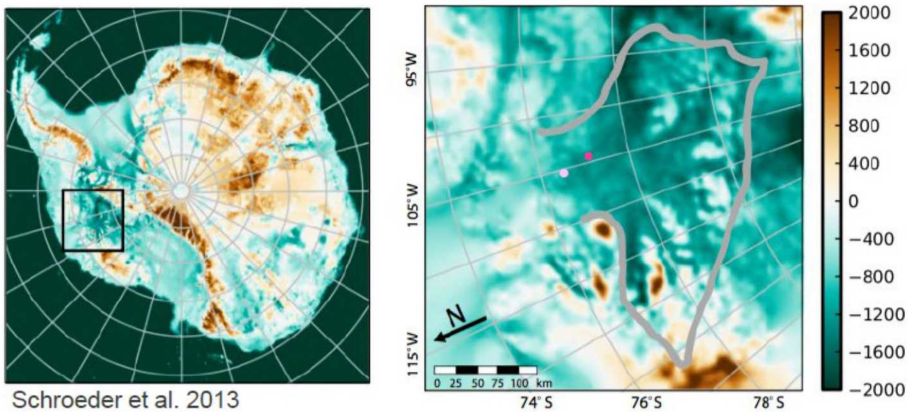
Simulation by Tong Zhang and Matt Hoffman



ABUMIP targets the response of the ice sheet model to instantaneous removal of all ice shelves, to understand the sensitivity of ice sheet to extreme climate forcing

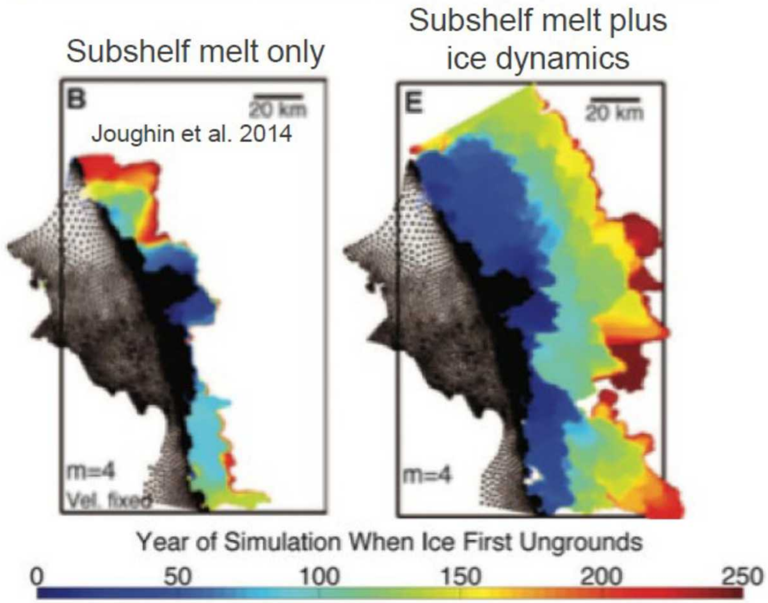
Ensemble ice sheet modeling of ocean melt variability at Thwaites glacier
(slides and most of work courtesy of Matt Hoffman)

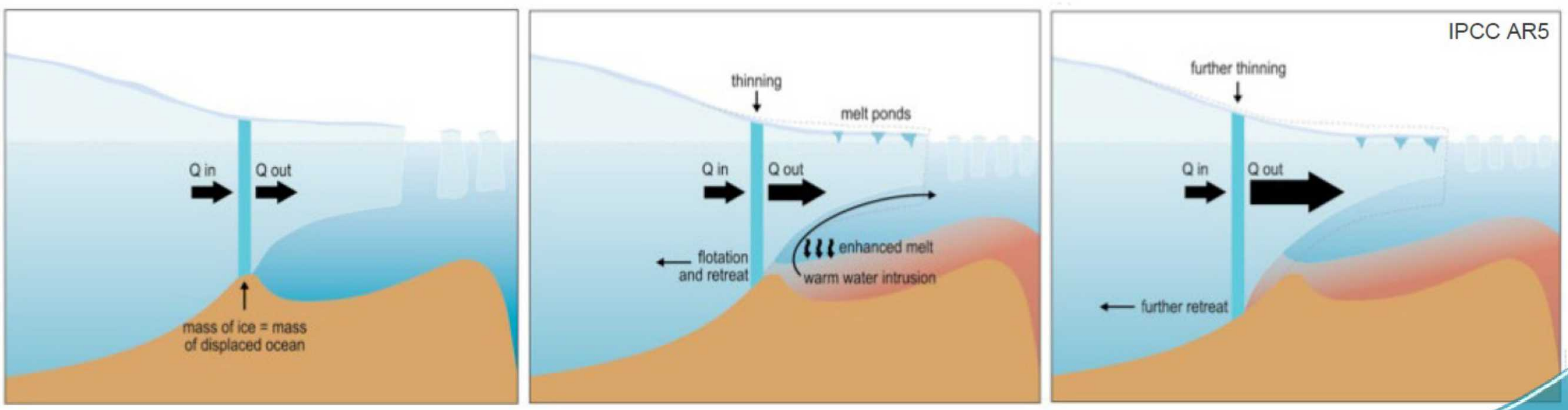
Marine ice sheet with overdeepened basin



Schroeder et al. 2013

Marine Ice Sheet Instability predicted





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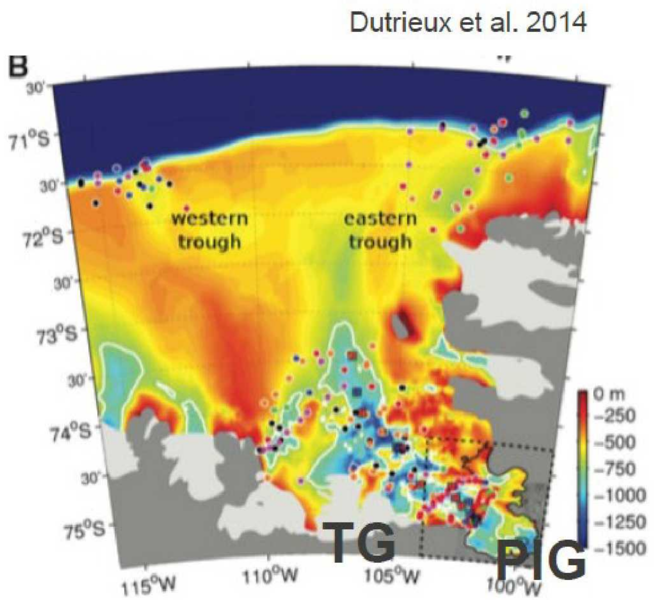
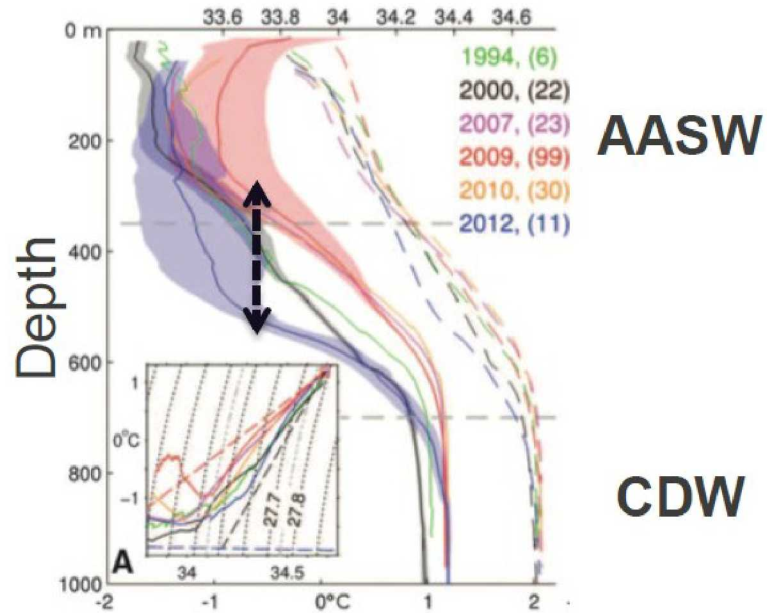
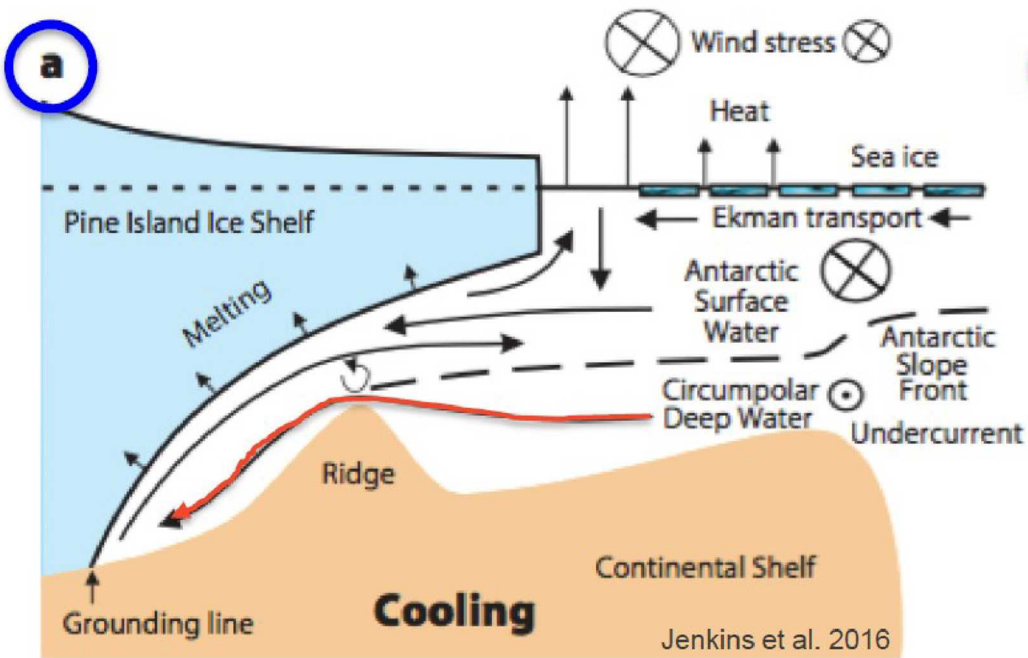
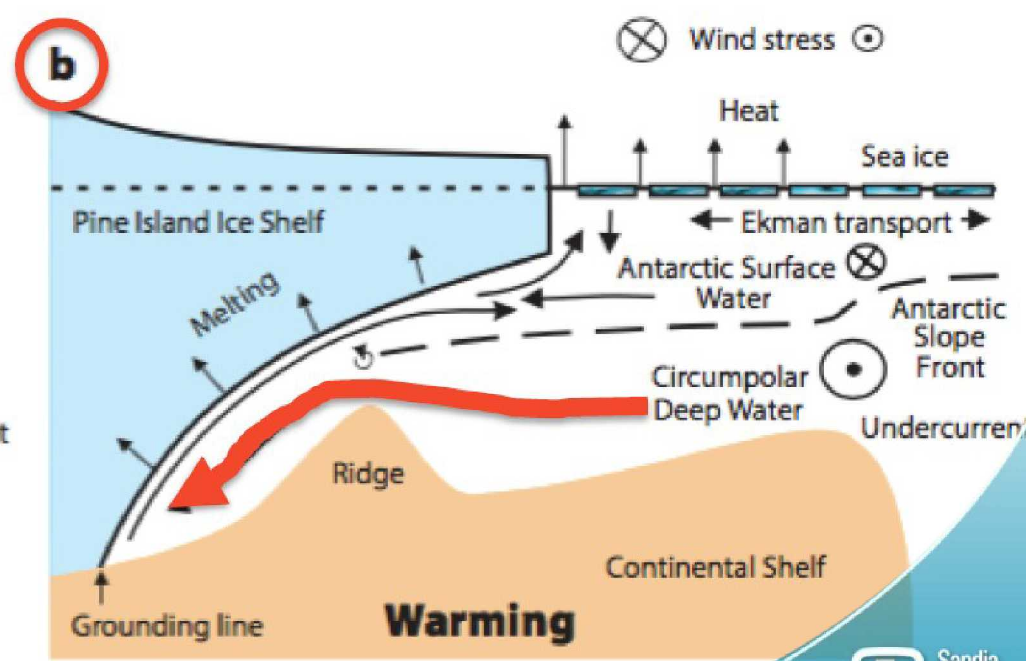
Ocean Water Masses Controlling Ice Melting

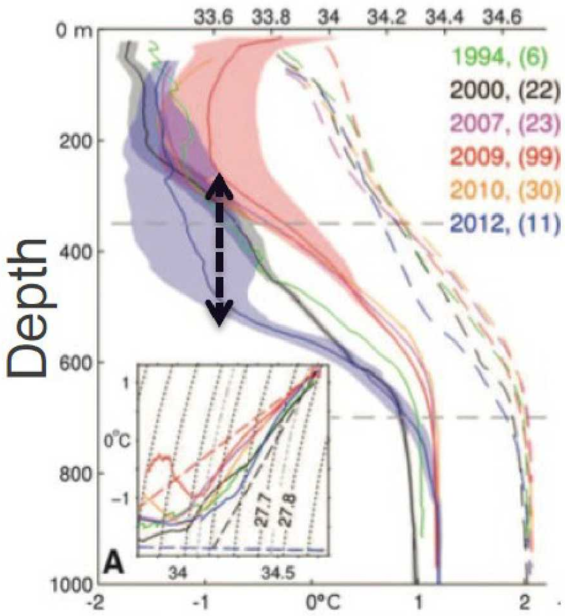


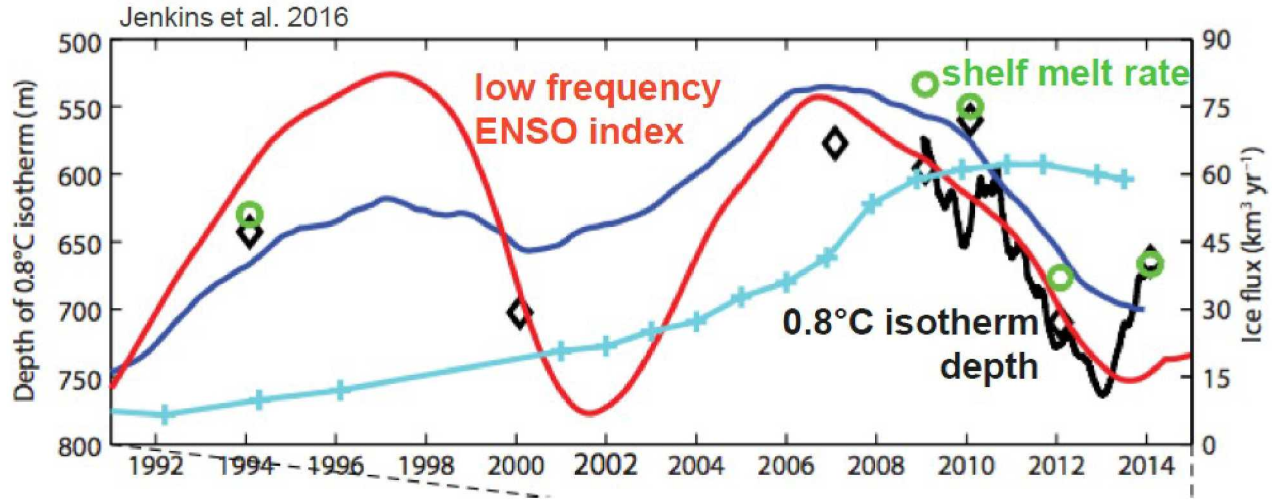






Climate Variability affecting Antarctic subshelf melting





- El Nino/Southern Oscillation (2-7 yr)
- Southern Annular Mode (20-30 yr)
- Pacific Decadal Oscillation (15-25 yr, 50-70 yr)
- Atlantic Multidecadal Oscillation (50-80 yr)

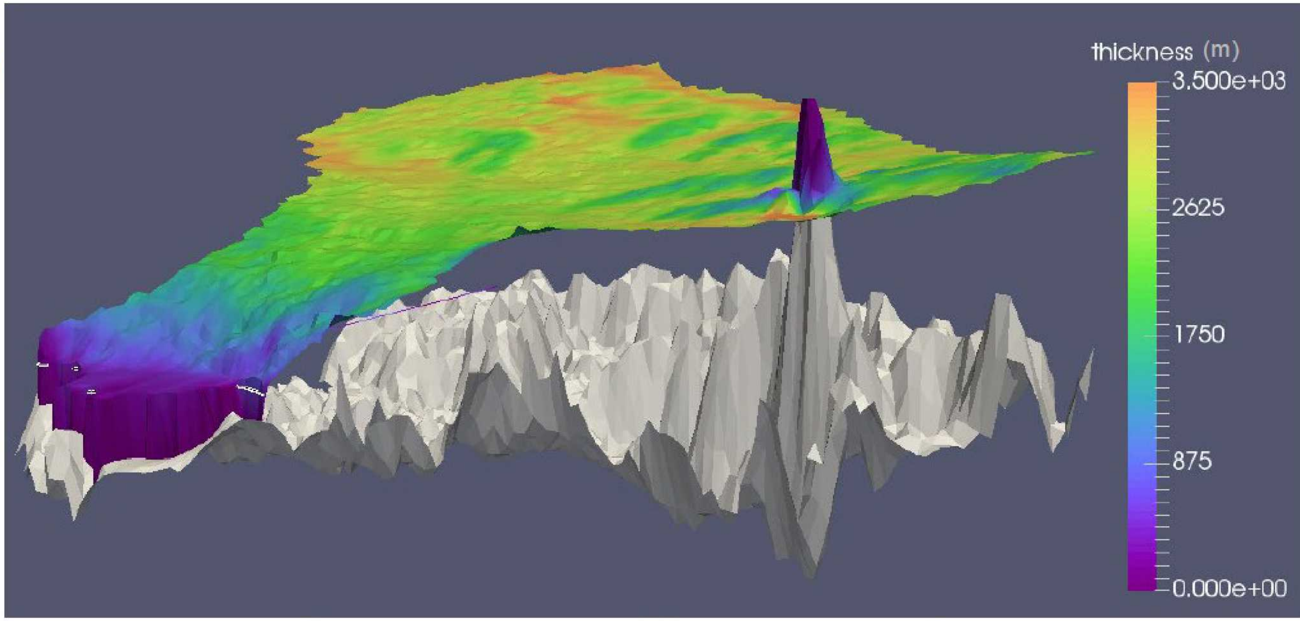
How might climate variability affect marine ice sheet stability?

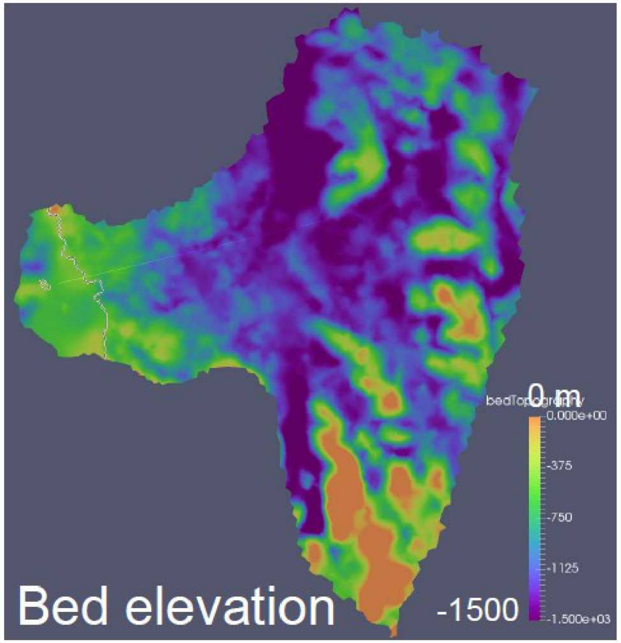
Model Setup

- MPAS-Albany Land Ice (MALI)
- 3d First-order momentum balance approx. (Blatter/Pattyn)
- Variable resolution regional grid (1-8 km)

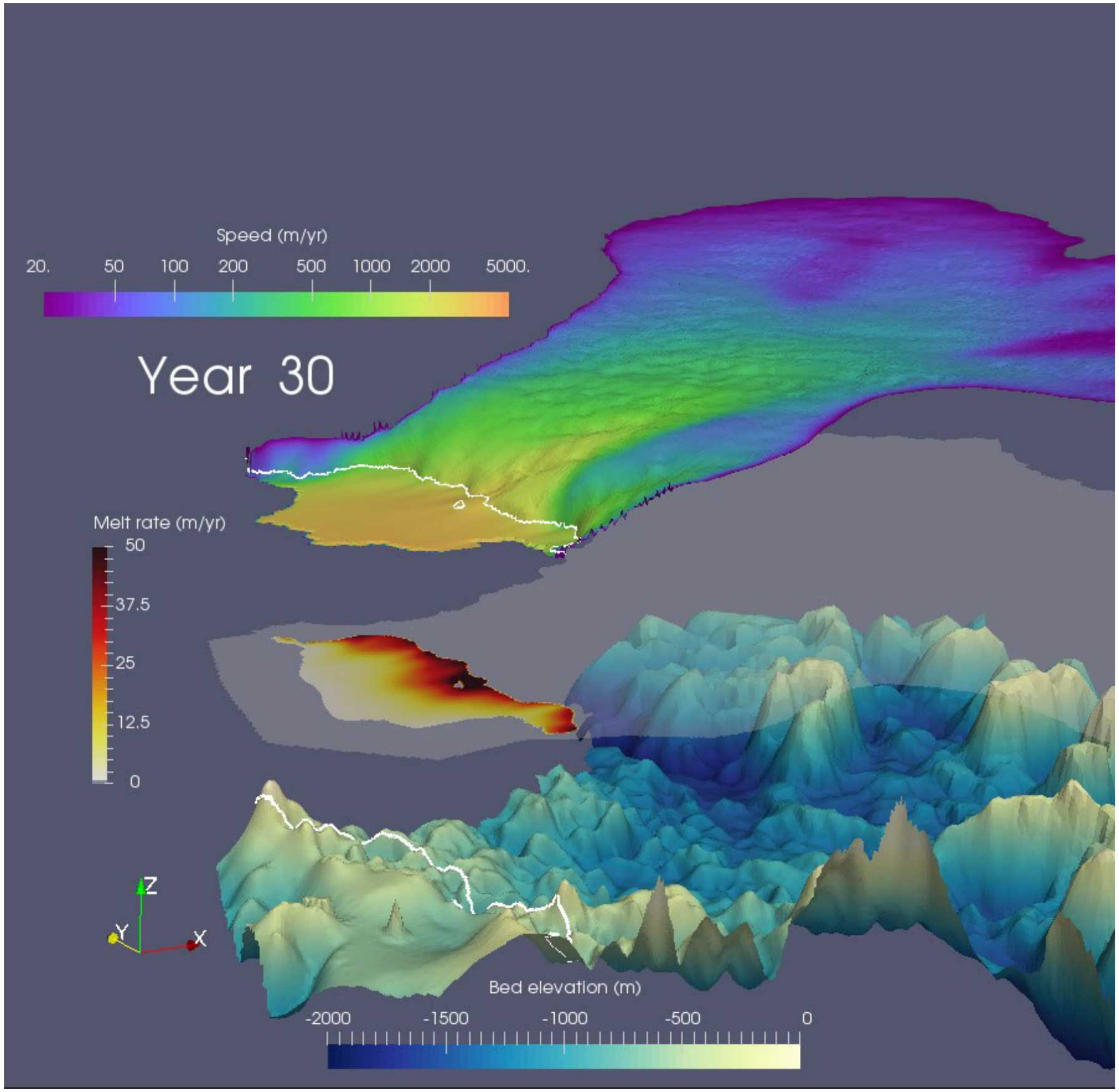
- Thickness, bed elevation from BEDMAP2
- Linear basal friction law
- Basal friction parameter optimized from InSAR surface velocity
- Fixed temperature field (pers. comm. Frank Pattyn)
- Calving front fixed in time
- SMB from RACMO2

- *Validated by observed grounding line flux transient*

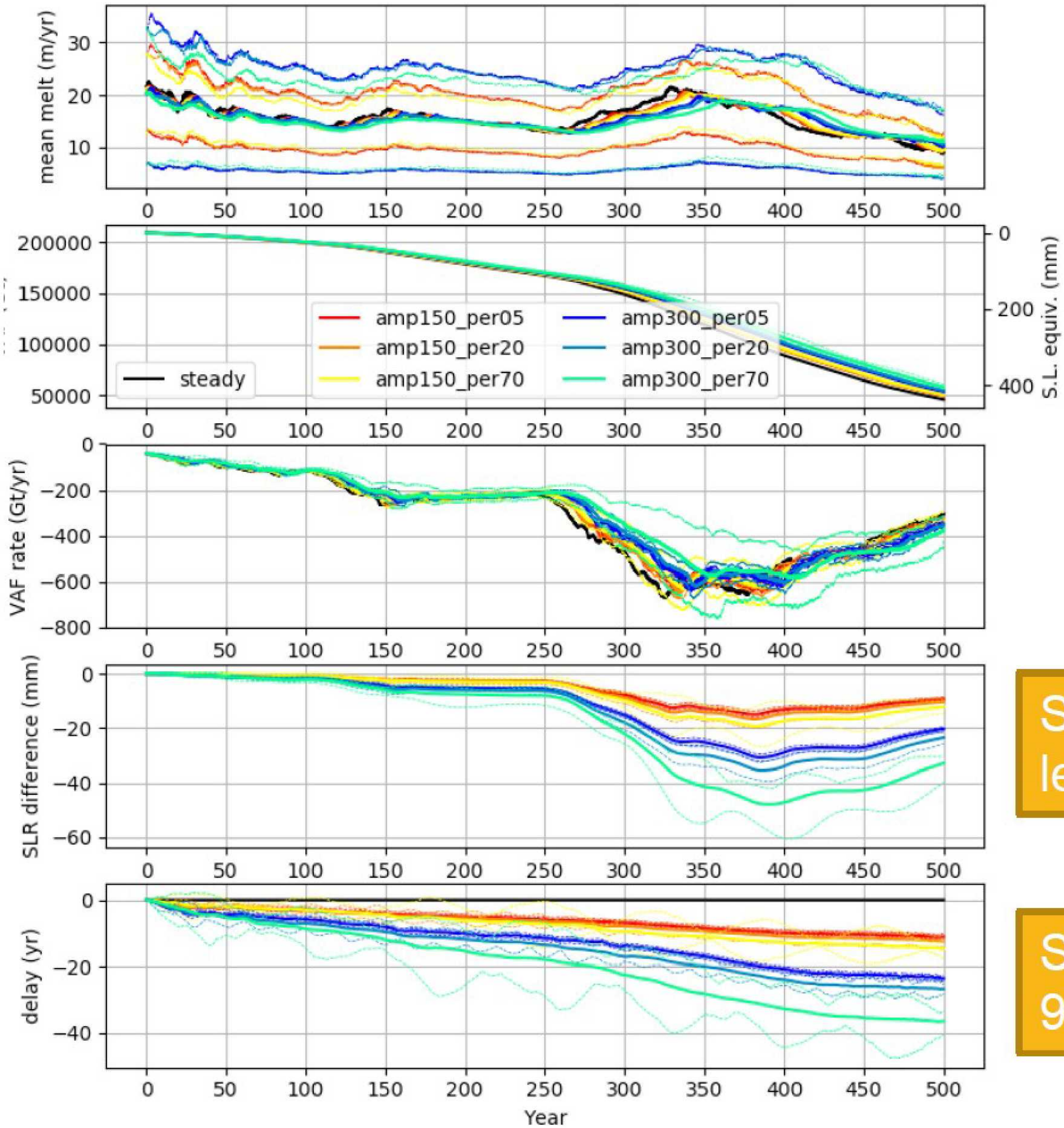




Results: single run (amplitude=300m, period =20yr)

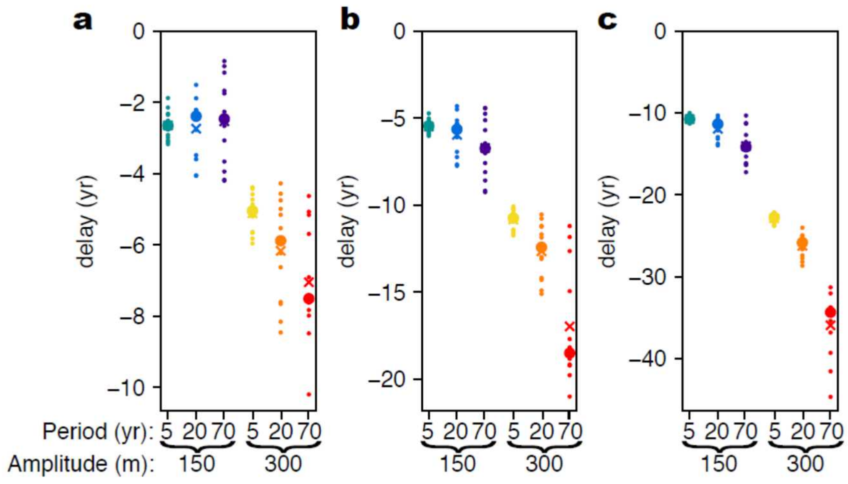


Results: all ensembles



Mass loss delay enhanced by

- **Larger amplitude**
- **Longer period**

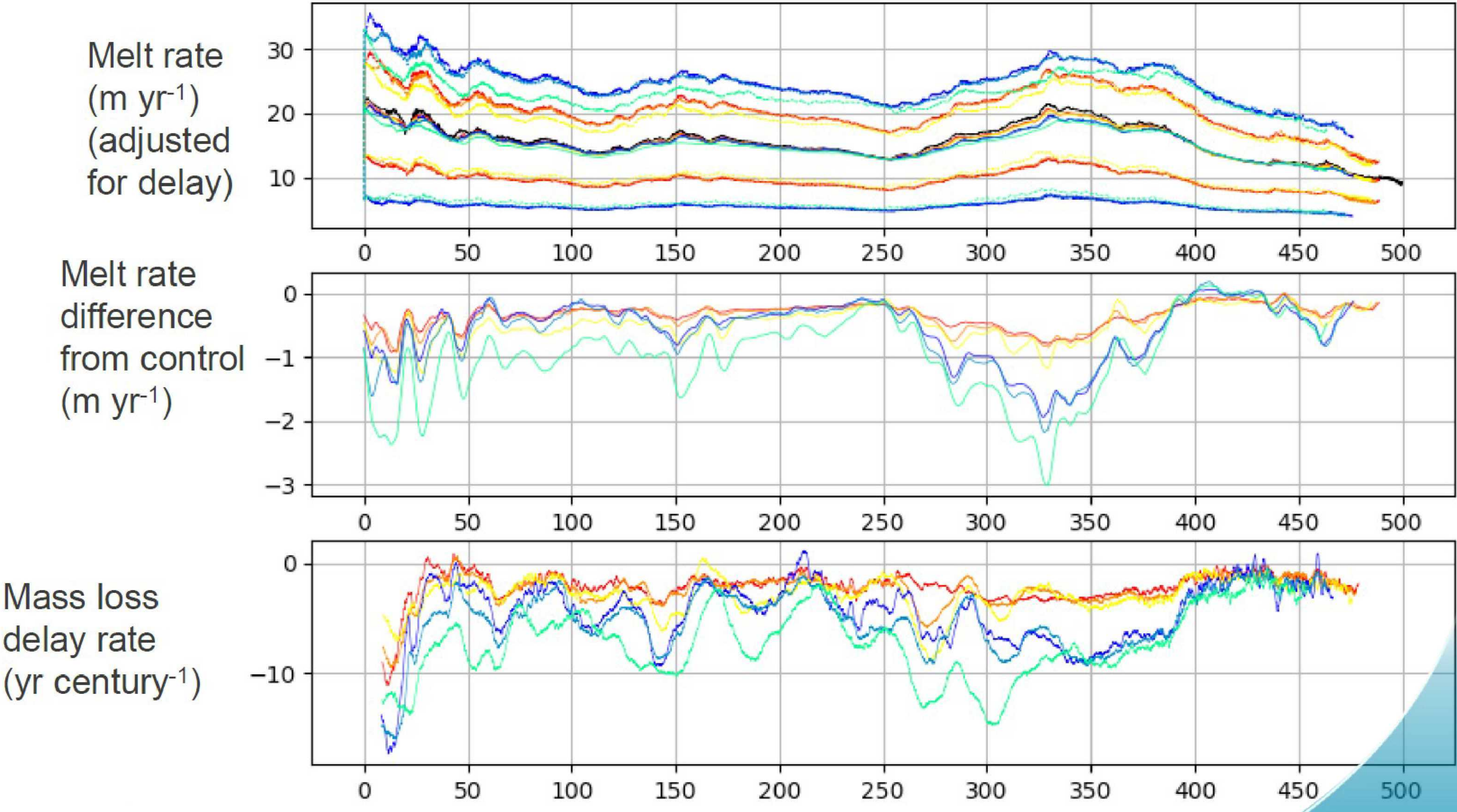


SLR is 10-40 mm less after 500 years

SLR is delayed 9-43 years

Mechanism for delay in mass loss

1. Asymmetric melt forcing – primary mechanism (~75% of delay)

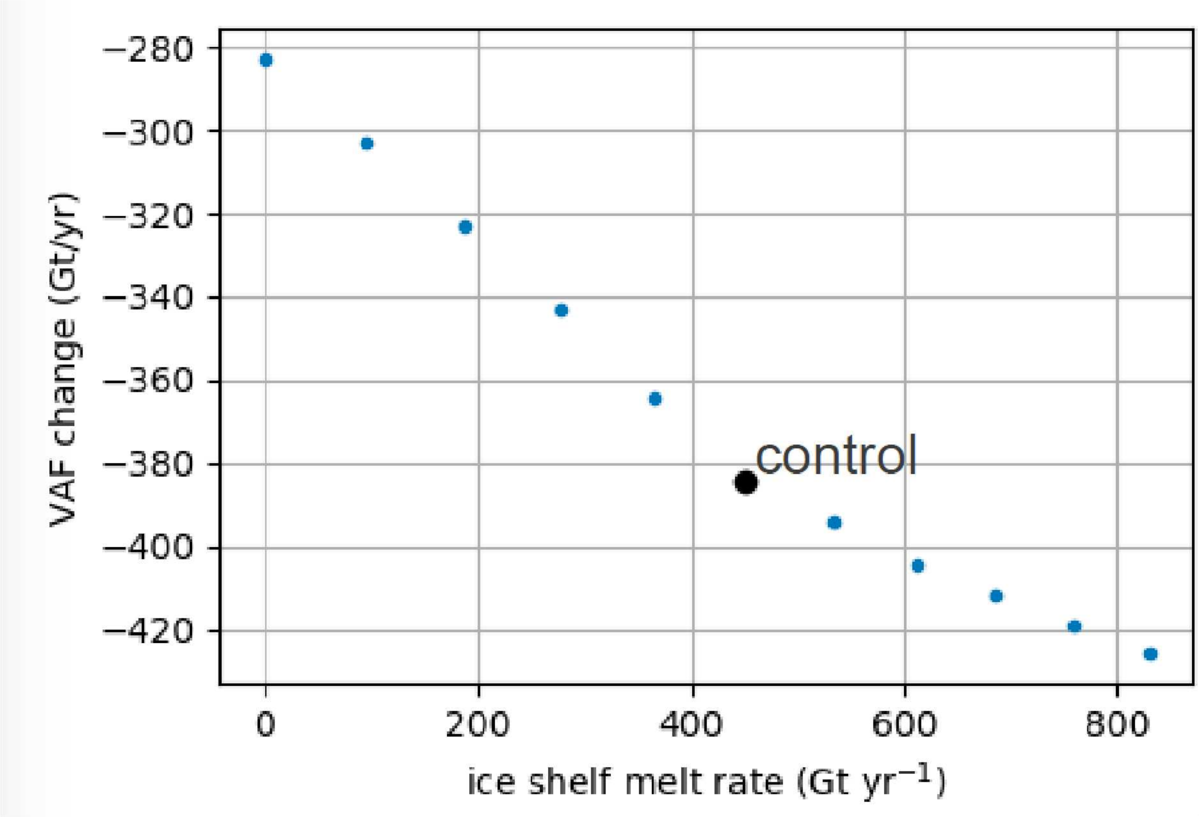


Mechanism for delay in mass loss

2. Nonlinear ice dynamic response to ice shelf melting

– secondary mechanism (~25% of delay)

- Decreasing melt → large decrease in mass loss
- Increasing melt → small increase in mass loss



Conclusion

Subshelf melt variability affects grounding line evolution and sea level contribution

- Variable runs always retreat *less* than steady runs
- Effects small (~3%) for realistic (?) modes of variability
- ~10% *less* SLR for plausible large amplitude, long period variability after hundreds of years
- Decadal rates of change can differ by up to 50%
  - implications for interpreting mass balance observations, e.g., GRACE

- Caveats: parameterized melt, simplistic variability, uncertain bed topography

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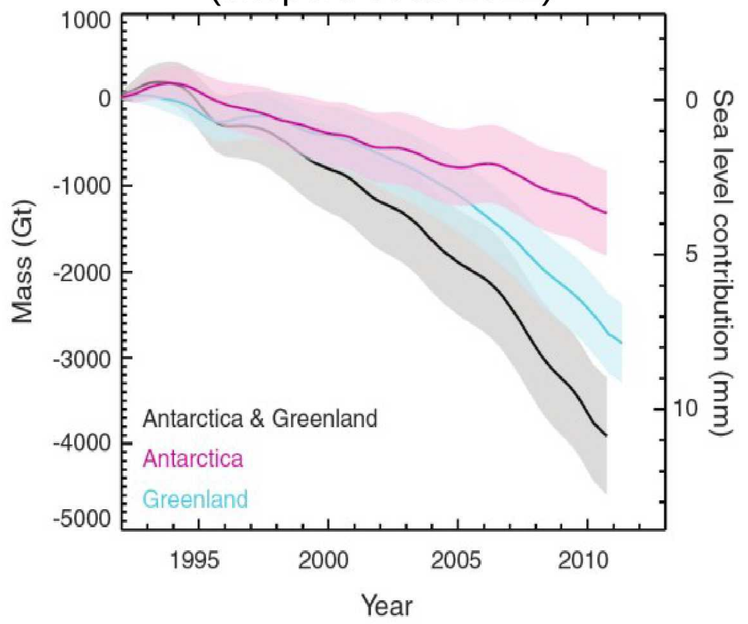
Uncertainty Quantification

Ultimate goal:
quantify the Sea Level Rise and related uncertainties

(sheperd et al. 2012)



Current Work flow:

- Perform *adjoint-based deterministic inversion* to estimate initial ice sheet state (i.e. characterize the present state of ice sheet to be used for performing prediction runs).
- *Bayesian inference*: Gaussian posterior low-rank approximation; use deterministic inversion to characterize the parameter distribution (i.e, use the inverted field as mean field of the parameter distribution and approximate its covariance using sensitivities/Hessian).
- *Forward Propagation*

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Bayesian Inference

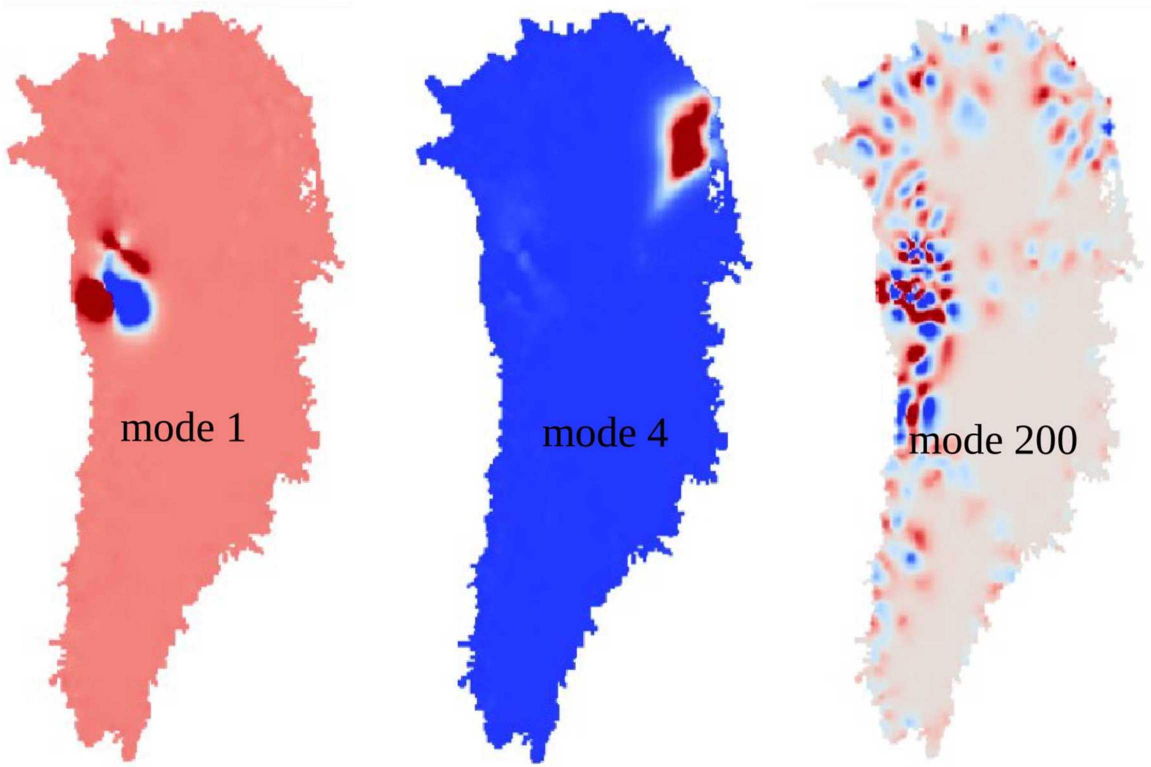
Gaussian approximation*

It is possible to approximated the distribution of the basal friction informed by the velocity data with a Gaussian distribution* using the Hessian of the objective functional of the initialization problem.

$$\Sigma_{\mathrm{post}} = \left(H + \Sigma_{\mathrm{prior}}^{-1}\right)^{-1} = \left(\Sigma_{\mathrm{prior}} H + I\right)^{-1} \Sigma_{\mathrm{prior}}$$

Compute approximation of posterior using low-rank approximation:

$\Sigma_{\mathrm{prior}} H \approx W_r \Lambda_r V_r^T$ (low rank approximation - randomized SVD)



Eigenvectors

*T. Isaac, N. Petra, G. Stadler, O. Ghattas, JCP, 2015

Bayesian Inference

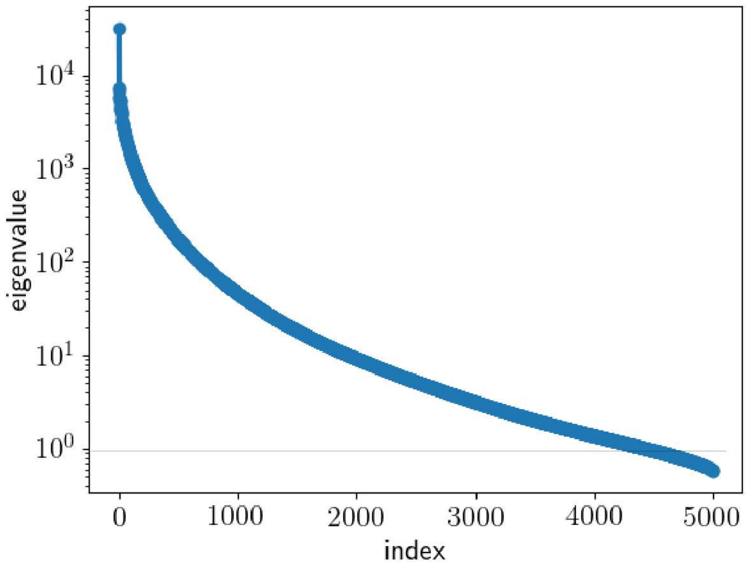
Gaussian approximation*

It is possible to approximated the distribution of the basal friction informed by the velocity data with a Gaussian distribution* using the Hessian of the objective functional of the initialization problem.

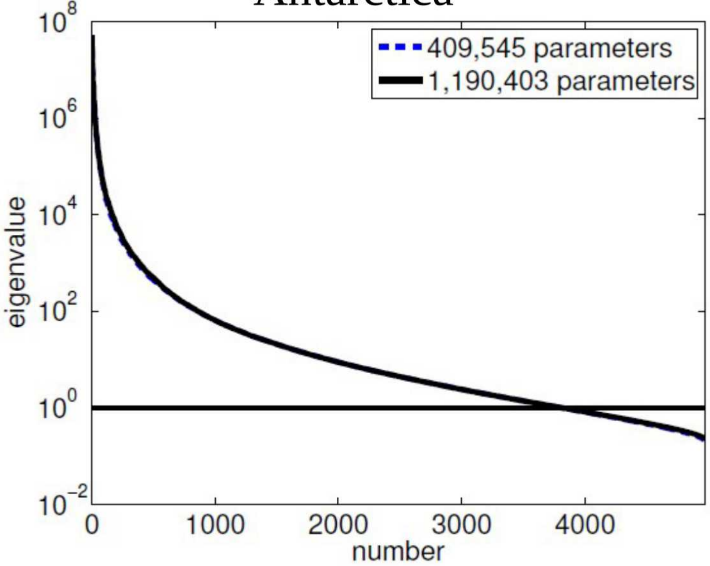
$$\Sigma_{\text{prior}} H \approx W_r \Lambda_r V_r^T \quad \text{(low rank approximation - randomized SVD)}$$

$$\Sigma_{\text{post}} = \Sigma_{\text{prior}} - W_r \Lambda_r W_r^T + \mathcal{O}\left(\sum_{i=r+1}^{N} \frac{\lambda_i}{1+\lambda_i} \right)$$ (Sherman-Morrison-Woodbury formula)

Greenland

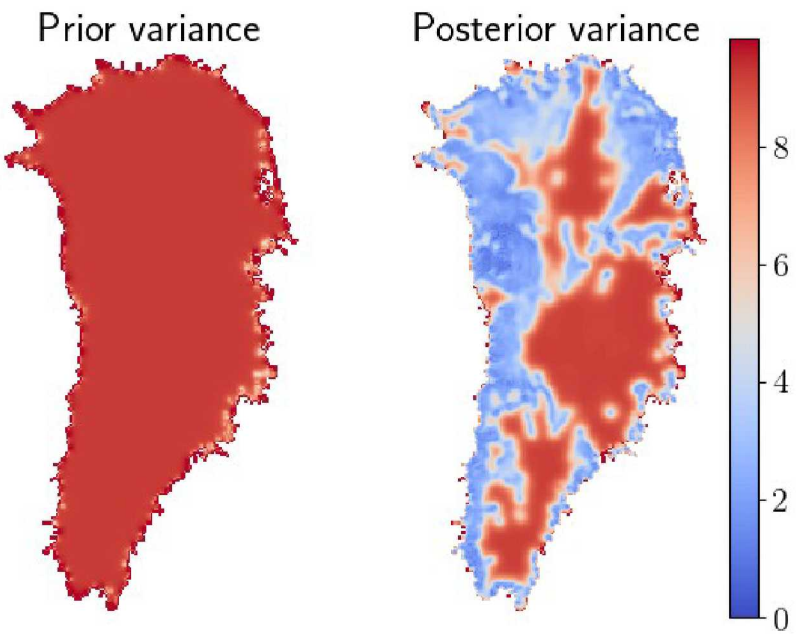


Antarctica*

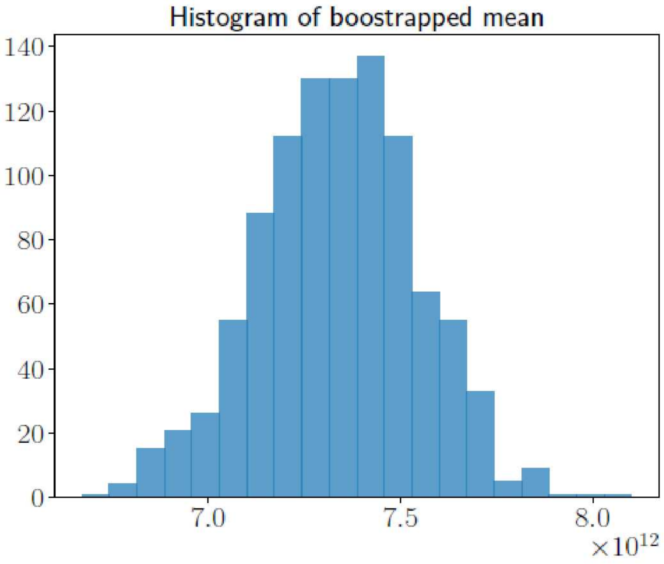


*T. Isaac, N. Petra, G. Stadler, O. Ghattas, JCP, 2015

Bayesian Inference and Forward Propagation



variance of the prior and posterior distribution of the basal friction



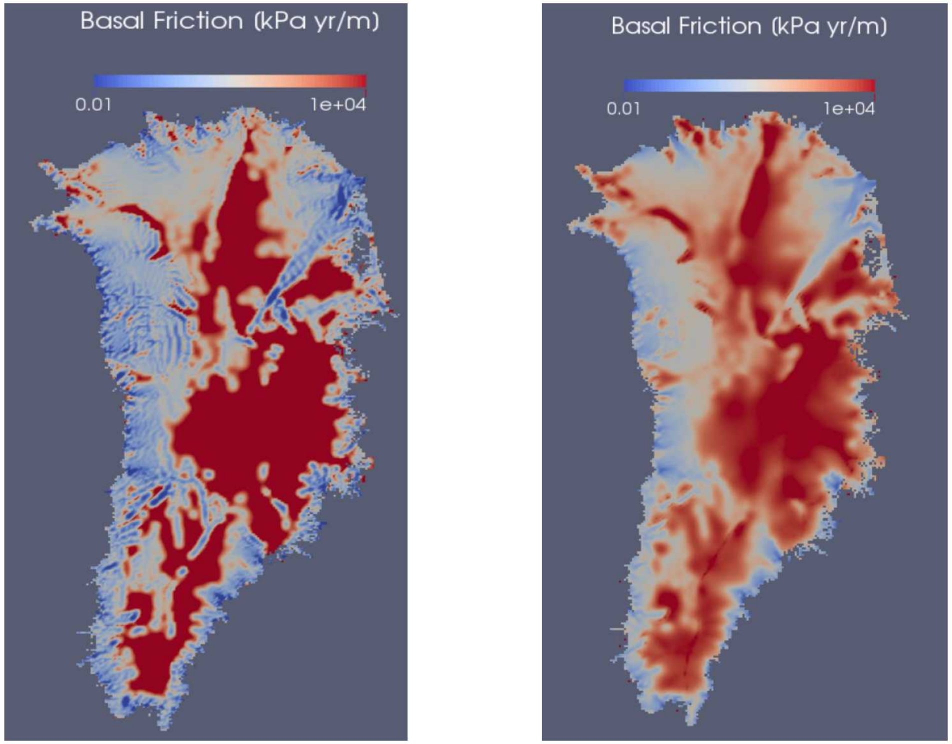
sea level change after 100 years due to Greenland from 100 fwd simulations

Challenge: dimension of parameter space is too high for forward propagation. We want to mitigate this with a multifidelity approach, using lower fidelity models

Bayesian Inference and Forward Propagation

dimension reduction by adding physics

Subglacial hydrology models rely on an handful of parameters that, to first approximation can be considered uniform.

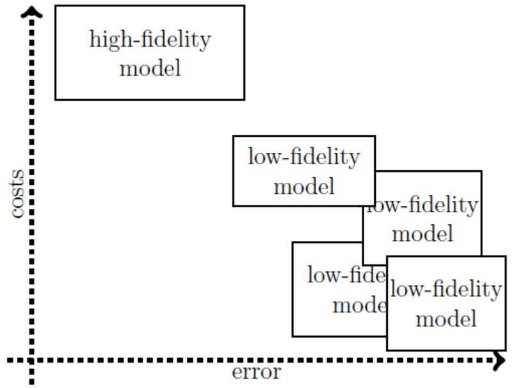


Two-step estimation of basal friction parameters
Estimate spatial dependent basal friction by minimizing mismatch between observed surface velocity and FO surface velocity (usual basal friction estimation)
Calibrate the basal hydrology model by matching that (target) basal

Left: target basal friction [kPa yr/m], from FO calibration
Right: basal friction computed w/ calibrated hydrology model

Considerations on Bayesian Calibration and Uncertainty Propagation

- **Bayesian Inference:**
  High dimensional parameter space. Even if we accept the Gaussian approximation for the posterior, forward propagation is still unfeasible. Performing the Bayesian calibration to recover the true distribution for the parameters is also unfeasible.



- **Strategies for forward propagation:**
  -adopt a multifidelity strategy.
  - build emulator (polynomial chaos, Neural Networks, ...) of the forward model and sample emulator (issue: lots of model runs needed to build emulator)
  - use cheap physical models (e.g. SIA, SSA) or low resolution solves to reduce the cost of building the emulator.
  - use sensitivitives and active subspace methods.
  - use techniques such as the compressed sensing to adaptively select significant modes and the basis for the parameter space.
  - Improve fidelity of the model (e.g. physicalbased model for sliding considering subglacial hydrology) to reduce the parameter space.

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Thank you!