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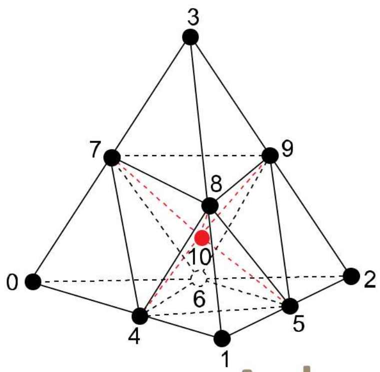

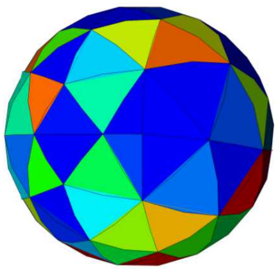

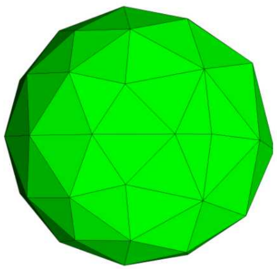

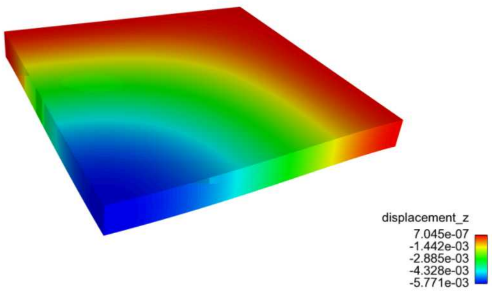

Advances in 10-node Composite Tetrahedral Elements for Solid Mechanics

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Sandia National Laboratories / CA

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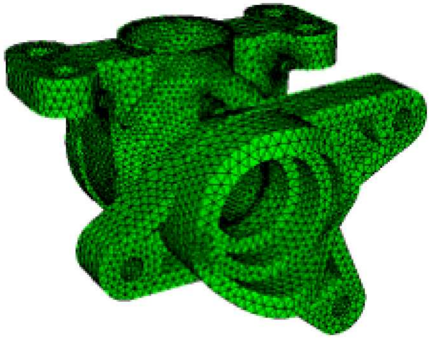
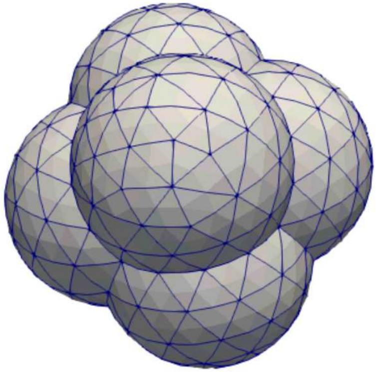
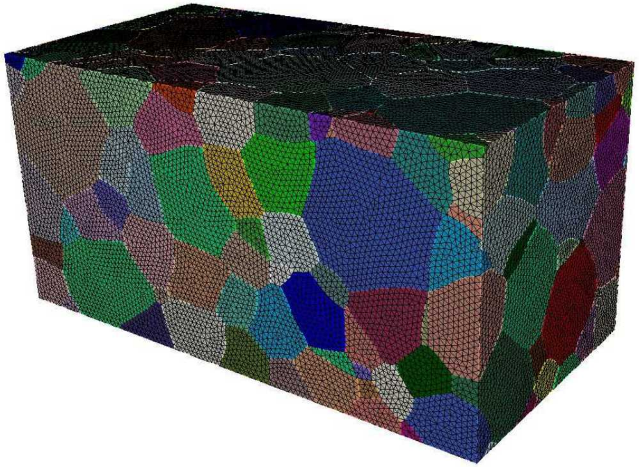
Motivation

- Automated hexahedral element meshing is elusive
- Manual hexahedral element meshing is time consuming ($$)
- Automated tetrahedral meshing is available, however standard linear tetrahedral elements perform poorly in solid mechanics applications
- Some higher order element, mixed formulation solutions exist, however production environment largely displacement based







Objectives

- Improve analysis throughput
- Provide an adequate alternative to hexahedral elements
  - Accuracy
  - Efficiency
  - Robustness
  - Feature integration (contact, coupled physics, user output, etc…)
- Explore tetrahedral formulations to relieve the meshing burden
  - Jay Foulk’s talk covers this and more …

What else do analysts need?

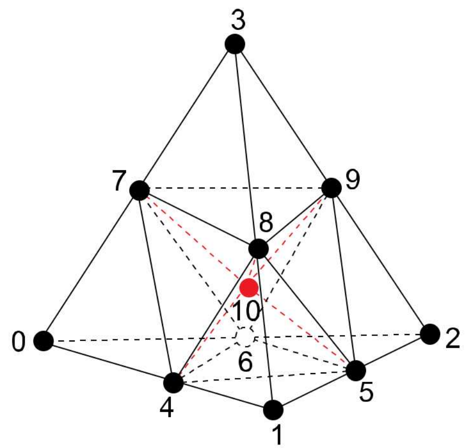
- Accuracy under nearly incompressible material behavior

- Conservative & sharp estimate of stable time step in explicit time integration

FORMULATION FOR NEARLY INCOMPRESSIBLE MATERIAL BEHAVIOR

Review of formulation

CT comprises 12 linear subtetrahedra



$$\boldsymbol{\varphi}(\boldsymbol{X}) := N_a(\boldsymbol{X})\boldsymbol{\varphi}_a \in U_h,$$
$$\bar{\boldsymbol{F}}(\boldsymbol{X}) := \lambda_\alpha(\boldsymbol{X})\bar{\boldsymbol{F}}_\alpha \in V_h,$$
$$\bar{\boldsymbol{P}}(\boldsymbol{X}) := \lambda_\alpha(\boldsymbol{X})\bar{\boldsymbol{P}}_\alpha \in V_h,$$

$$\boldsymbol{\varphi} \in U_h = \left\{ \boldsymbol{v} \in [H^1(B)]^3 \;\middle|\; \boldsymbol{v}|_{\text{subtets of } T} \in \mathcal{P}_1(\text{subtets of } T), T \in \mathcal{T}_h \right\}$$

$$\bar{\boldsymbol{F}}, \bar{\boldsymbol{P}} \in V_h = \left\{ \boldsymbol{V} \in [L^2(B)]^{3\times 3} \mid \boldsymbol{V}|_T \in \mathcal{P}_1(T), T \in \mathcal{T}_h \right\}$$

Thoutireddy, P., et al. "Tetrahedral composite finite elements." *International Journal for Numerical Methods in Engineering* 53.6 (2002): 1337-1351.

Ostien, J. T., Foulk III, J. W., Mota, A., Veilleux, M. G., "A 10-node composite tetrahedral finite element for solid mechanics." *International Journal for Numerical Methods in Engineering*107.13 (2016): 1145-1170.

Review of formulation

$$\Phi[\boldsymbol{\varphi}, \bar{\boldsymbol{F}}, \bar{\boldsymbol{P}}] := \int_B A(\bar{\boldsymbol{F}})\,\mathrm{d}V + \int_B \bar{\boldsymbol{P}} : (\boldsymbol{F} - \bar{\boldsymbol{F}})\,\mathrm{d}V$$
$$- \int_B R\boldsymbol{B} \cdot \boldsymbol{\varphi}\,\mathrm{d}V - \int_{\partial_T B} \boldsymbol{T} \cdot \boldsymbol{\varphi}\,\mathrm{d}S,$$

Stationary points yield discrete formulation:

$$\delta_{\bar{\boldsymbol{P}}}\Phi = 0 \rightarrow \bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{X}),\ \text{discrete gradient operator}$$

$$\delta_{\bar{\boldsymbol{F}}}\Phi = 0 \rightarrow \bar{\boldsymbol{P}},\ \text{stress projection onto } V_h$$

$$\delta_{\boldsymbol{\varphi}}\Phi = 0 \rightarrow \boldsymbol{R}_a(\boldsymbol{\varphi}) = 0, \text{weak form of equilibrium}$$

$$\boldsymbol{R}_a(\boldsymbol{\varphi}) := \int_\Omega \left.\frac{\partial A(\boldsymbol{F})}{\partial \boldsymbol{F}}\right|_{\boldsymbol{F}=\bar{\boldsymbol{F}}} \cdot \bar{\boldsymbol{\mathcal{B}}}_a\,\mathrm{d}V - \int_\Omega R\boldsymbol{B}N_a\,\mathrm{d}V - \int_{\partial_T\Omega} \boldsymbol{T}N_a\,\mathrm{d}S$$

Goals for nearly incompressible formulation

- Project pressures into a lower order space to avoid locking
- Maintain symmetry of stiffness matrix – conjugate gradient is the workhorse linear solver in our production code
- Preserve displacement FEM formulation to fit existing code architecture

Five-field formulation for nearly incompressible behavior

$$\Phi^\star[\boldsymbol{\varphi}, \bar{\boldsymbol{F}}, \bar{\boldsymbol{P}}, \bar{J}^\star, \bar{p}^\star] := \int_B A\left(\left(\frac{\bar{J}^\star}{\bar{J}}\right)^{1/3} \bar{\boldsymbol{F}}\right) \mathrm{d}V + \int_B \bar{\boldsymbol{P}} : (\boldsymbol{F} - \bar{\boldsymbol{F}})\, \mathrm{d}V + \int_B \bar{p}^\star(\bar{J} - \bar{J}^\star)\, \mathrm{d}V$$
$$- \int_B R\boldsymbol{B} \cdot \boldsymbol{\varphi}\, \mathrm{d}V - \int_{\partial_T B} \boldsymbol{T} \cdot \boldsymbol{\varphi}\, \mathrm{d}S,$$

Piecewise constant approximation - $\bar{J}^\star, \bar{p}^\star \in W_h = \left\{ c \in L^2(B) \mid c|_T \in \mathcal{P}_0(T), T \in \mathcal{T}_h \right\}$

Stationarity wrt pressure leads to Jacobian projection:

$$\delta_{\bar{p}^\star} \Phi^\star = 0 \to \bar{J}^\star = \frac{1}{V_\Omega} \int_\Omega \bar{J}\, \mathrm{d}V,$$

This defines modified gradient operator:

$$\bar{\boldsymbol{\mathcal{B}}}_a^\star = \left(\frac{\bar{J}^\star}{\bar{J}}\right)^{1/3} \bar{\boldsymbol{\mathcal{B}}}_a \qquad \bar{\boldsymbol{F}}^\star := \left(\frac{\bar{J}^\star}{\bar{J}}\right)^{1/3} \bar{\boldsymbol{F}} = \sum_a \bar{\boldsymbol{\mathcal{B}}}^\star \boldsymbol{x}_a$$

Five-field formulation for nearly incompressible behavior

Let $\bar{\boldsymbol{P}}^{\star} := \left. \dfrac{\partial W(\boldsymbol{F})}{\partial \boldsymbol{F}} \right|_{\boldsymbol{F}=\bar{\boldsymbol{F}}^{\star}}$.

Stationarity wrt Jacobian defines projection of pressure:

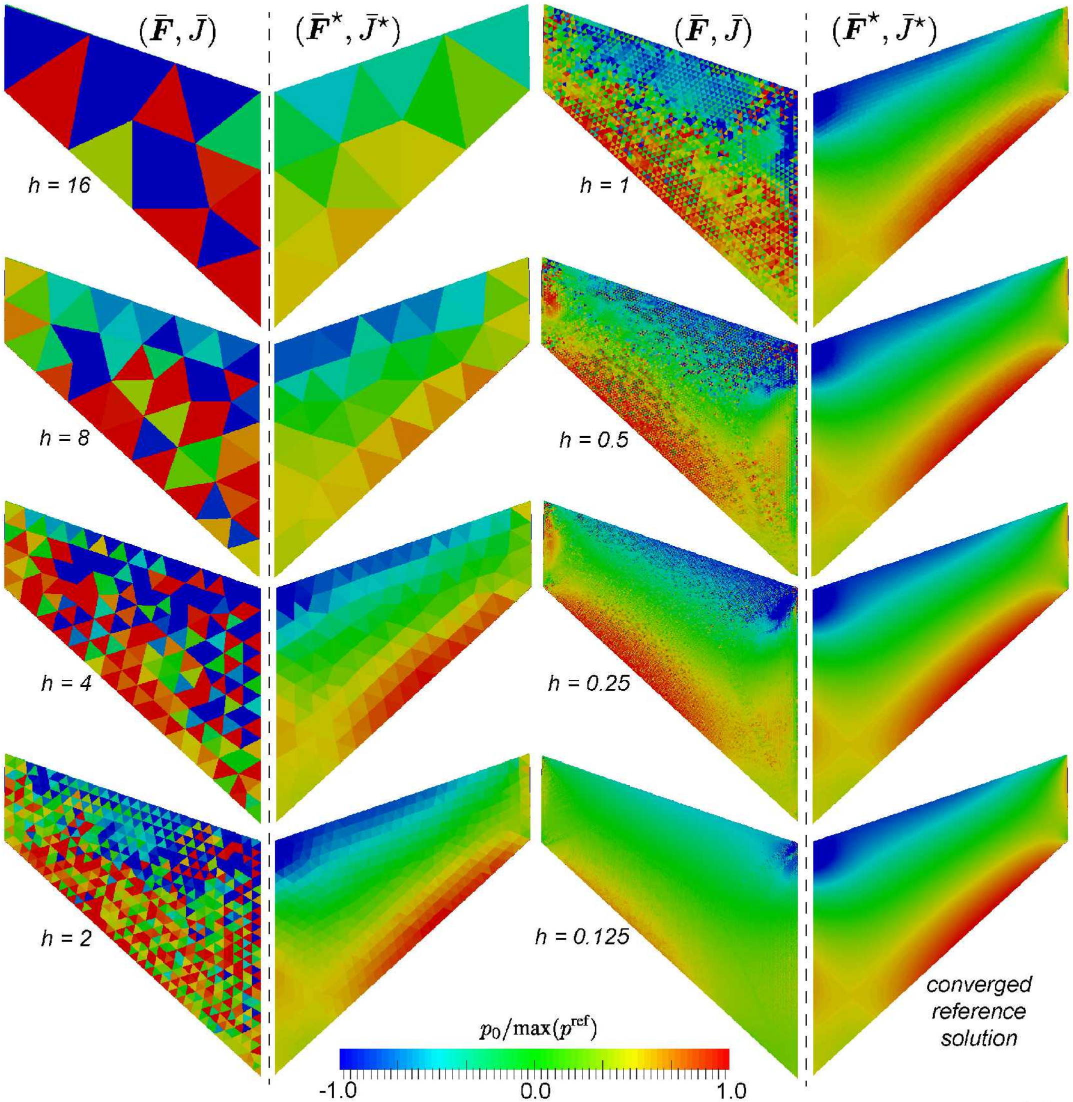
$$\delta_{\bar{J}^{\star}} \Phi^{\star} = 0 \rightarrow \bar{p}^{\star} = \frac{1}{V_{\Omega}} \int_{\Omega} \frac{\operatorname{tr}(\bar{J}^{\star-1} \bar{\boldsymbol{P}}^{\star} \bar{\boldsymbol{F}}^{\star T})}{3} \, \mathrm{d}V$$

Stationarity wrt deformation defines weak equilibrium:

$$\boldsymbol{R}_a(\boldsymbol{\varphi}) = \int_{\Omega} \left(\bar{\boldsymbol{P}}^{\star} - \frac{1}{3} \operatorname{tr}(\bar{\boldsymbol{P}}^{\star} \bar{\boldsymbol{F}}^{\star T}) \bar{\boldsymbol{F}}^{\star -T} + \bar{J} \bar{p}^{\star} \bar{\boldsymbol{F}}^{\star -T} \right) \cdot \bar{\boldsymbol{\mathcal{B}}}_a^{\star} \, \mathrm{d}V - \boldsymbol{F}_a^{\mathrm{ext}} = \boldsymbol{0}$$

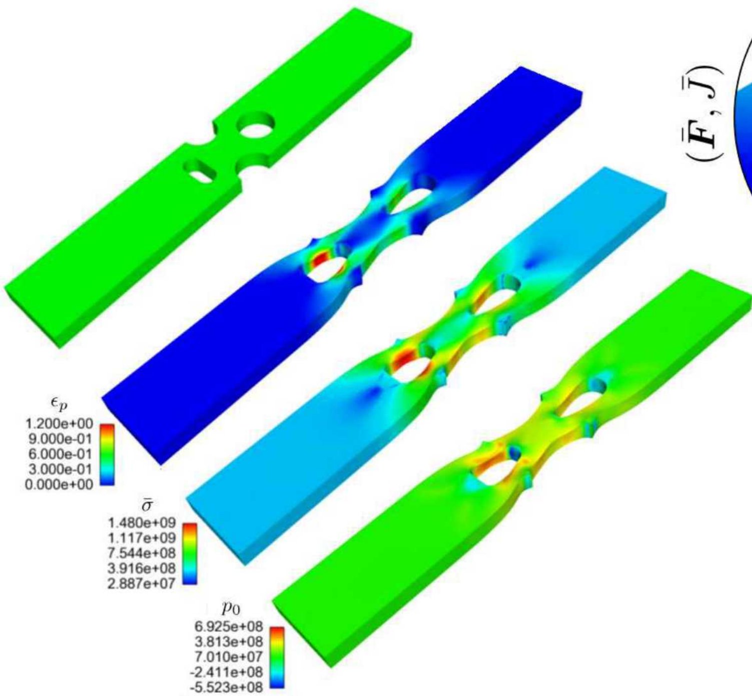
Test: Cook's Membrane

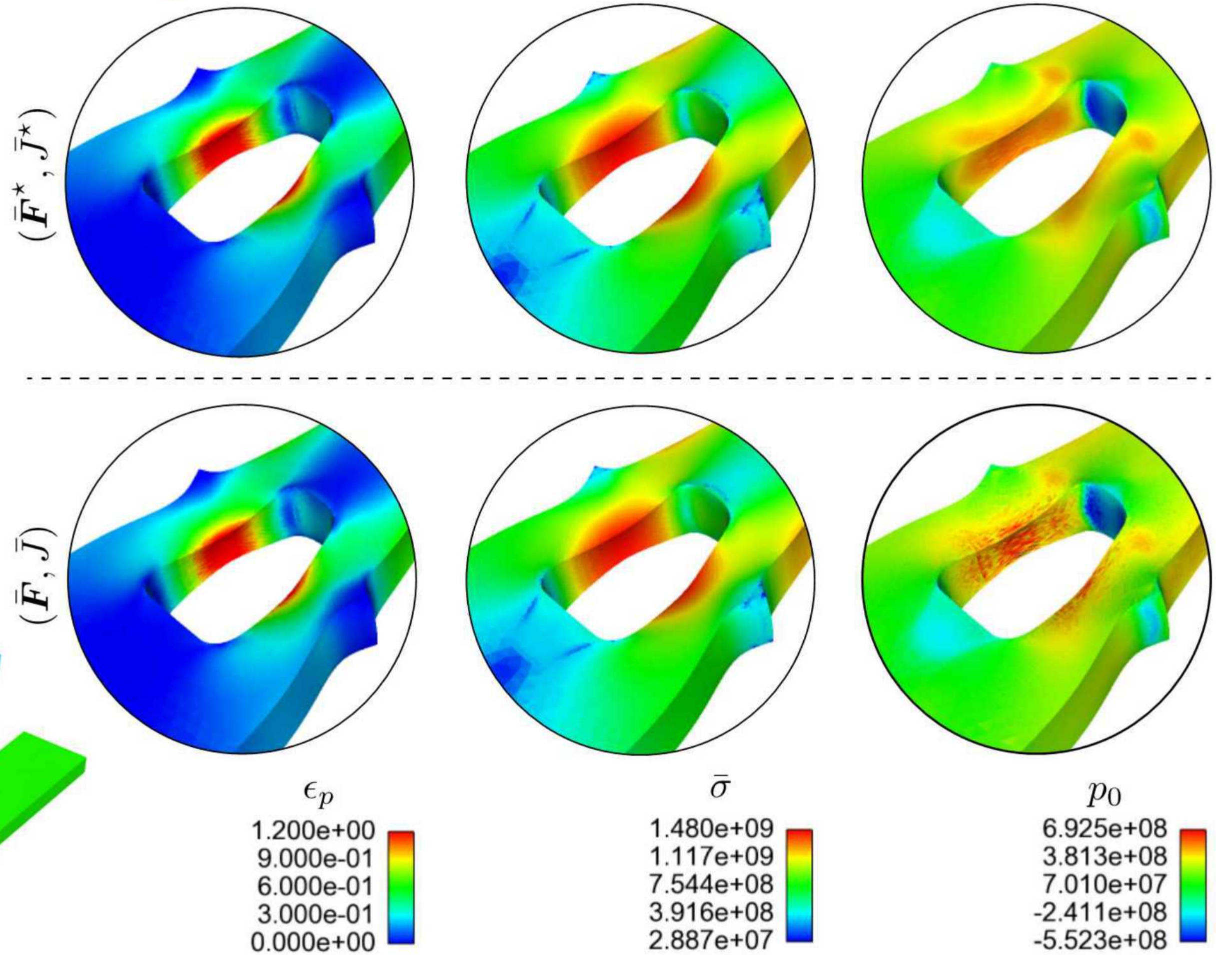
- Volume averaging the volumetric response alleviates locking



Application – Large Deformation

Large deformation and necking of stainless steel 304L





Pressure solution is “noisy” without volume averaging

Affect on rank sufficiency

- Deficiency or degree 3 introduced on an isolated element
- Assemblies of elements possess full rank in all configurations tested
- No global hourglassing modes observed
- Local mode may be excited in certain situations

ESTIMATION OF STABLE TIME STEP

Critical Time Step

- Characteristic element length is computed from smallest of subtet inradii

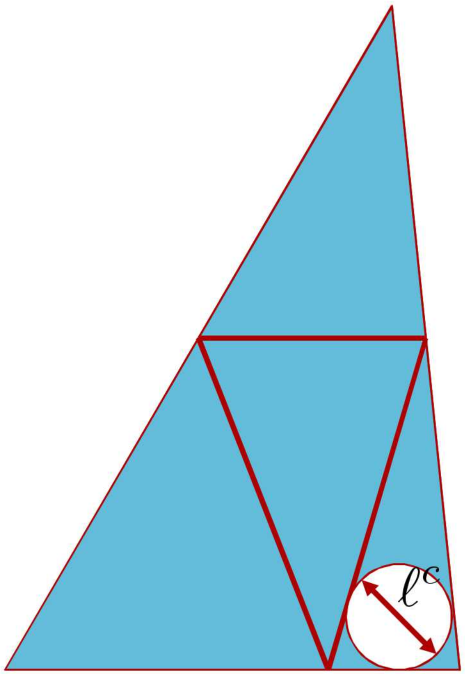
$$M = \lambda + 2\mu \quad \text{longitudinal modulus}$$

$$c = \sqrt{\frac{M}{\rho}} \quad \text{P-wave speed}$$

$$\ell^c = 2\ r_{in} \quad \text{min } \textbf{subtet} \text{ inradius}$$

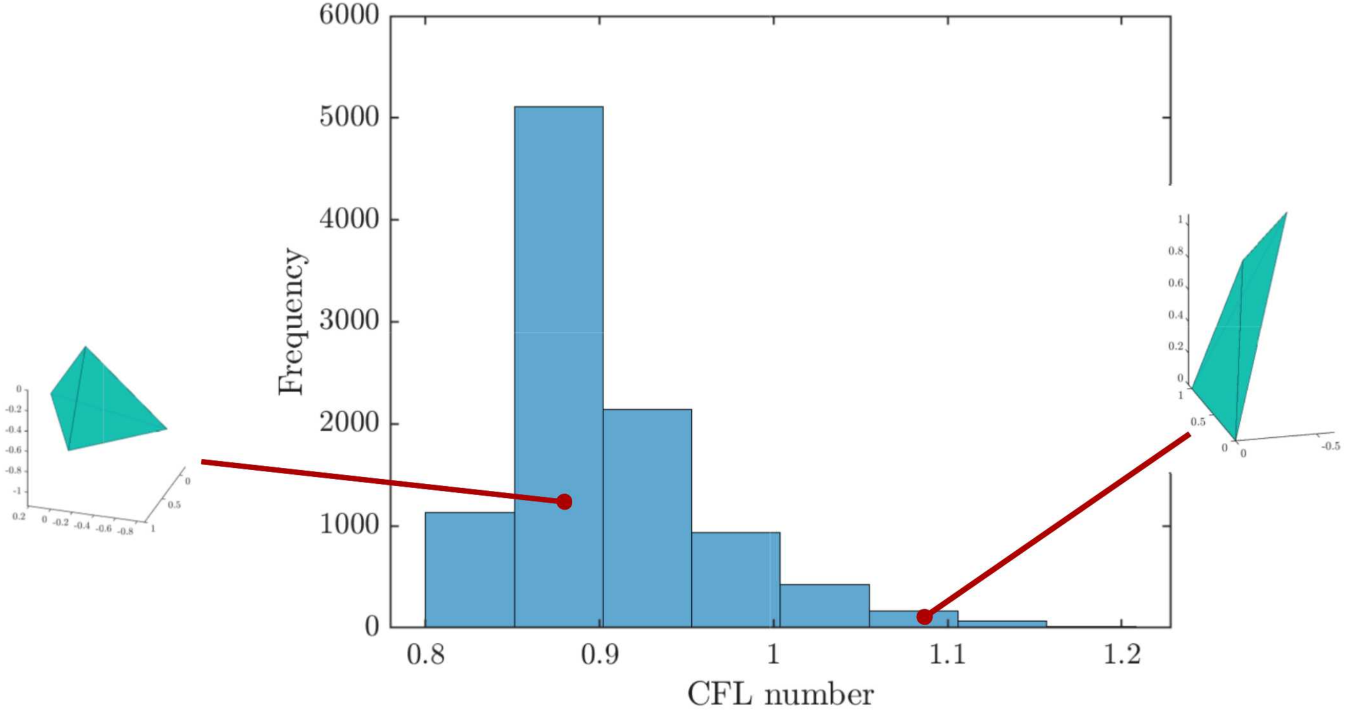
$$\Delta t_{crit} \approx \frac{C\ell^c}{c}$$

$$C \approx 2 \quad \text{(empirical)}$$



Timestep sharpness & conservatism

- Numerical experiments on 10,000 randomly generated tets
- Compare estimated timestep to exact critical timestep (from spectral analysis)



- Global spectrum typically more compressed, adding extra conservatism

Conclusions and Future Work

- Nearly incompressible formulation
  - New variational formulation for pressure & Jacobian projection
  - Smooth pressure fields obtained in tests
  - Local to elements, no new global fields added
  - Numerical inf-sup testing needed
- Critical timestep estimation
  - Simple estimate based on easily computed geometric data
  - Numerical testing suggests estimate is conservative for reasonable meshes
  - Work continuing to identify simple higher-order shape metric to sharpen estimates
  - … particularly for elements with kinked edges