SAND2019-4509PE

Hybrid System Modeling

PRESENTED BY

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Engineering Sciences Center

NM Tech Collaboration Meeting

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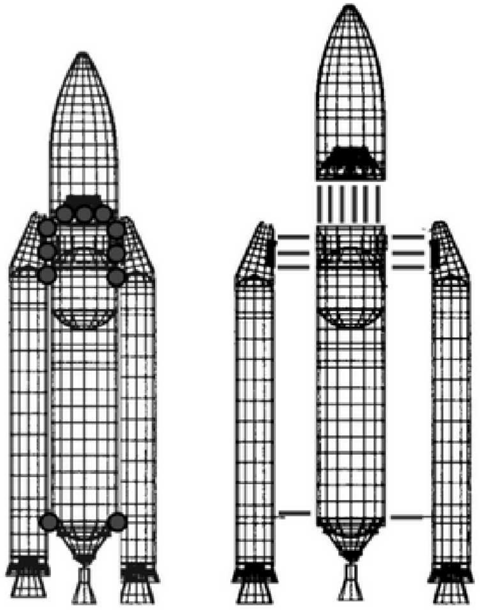
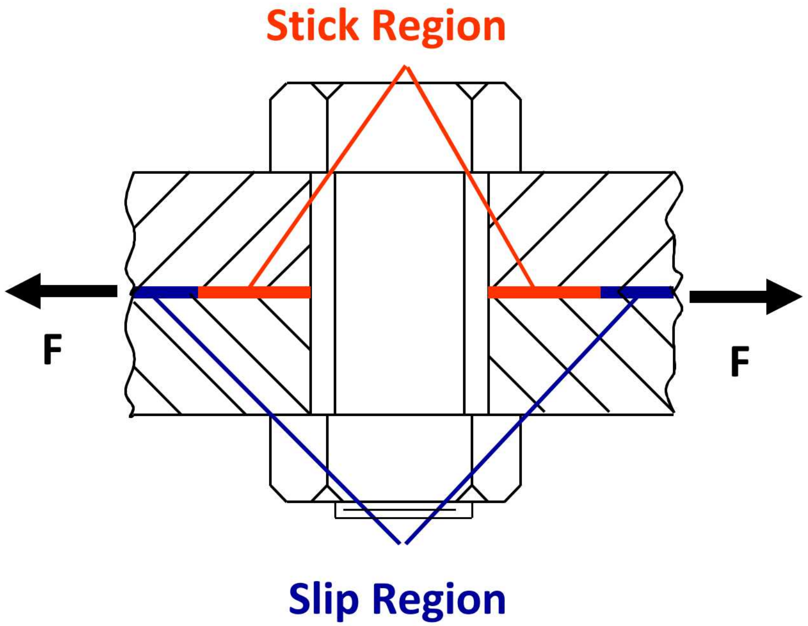
Outline

- Motivation
- Overview of Hybrid System Modeling (HSM)
- Examples of Linear HSM
- Nonlinear Dynamics
- Examples of Nonlinear HSM
- Future Work in System Modeling

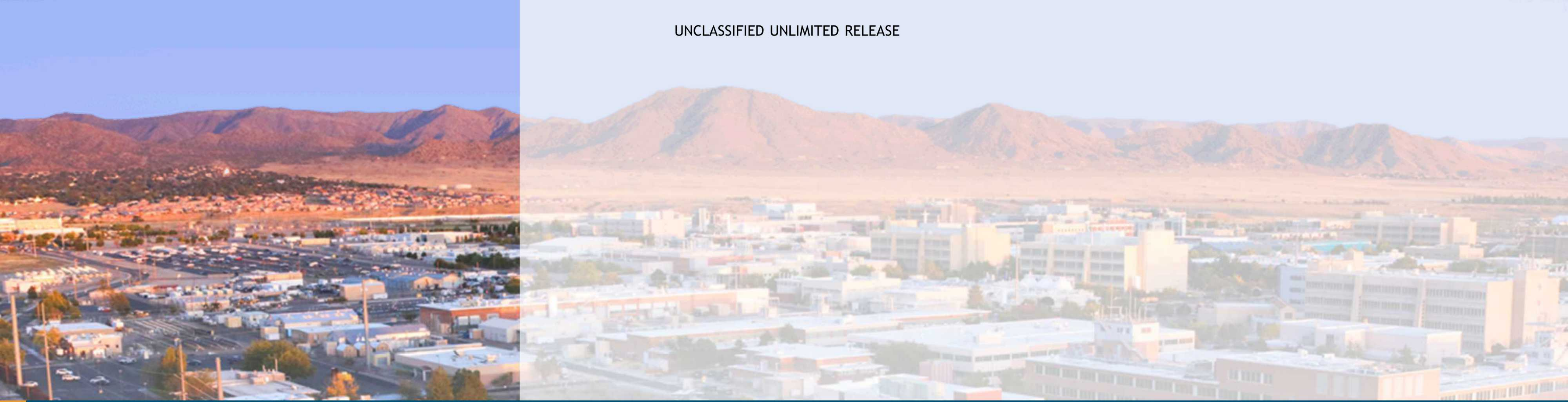
Introduction – Dan Roettgen

- **2007-2013:** Control Systems and Dynamics Engineer at General Electric – Aircraft Engines
  - Engineering Edison Development Program
  - Rotated through, design, analysis and test on the same engine program
- 2016: PhD in Engineering Mechanics from University of Wisconsin-Madison
  - "Experimental Dynamic Substructuring Using Nonlinear Modal Joint Models"
  - Funded through Sandia National Labs
- 2017: Joined Sandia National Laboratories as Post Doctoral Appointee
  - Studied experimental-hybrid substructuring, nonlinear joint dynamics and hardware-in-the-loop substructuring
- 2018: Converted to a full-time Senior Member of Technical Staff at Sandia National Labs
- Current research areas:
  - Experimental-Analytical Dynamic Substructuring: synthesis of experimental and numerical models
  - Nonlinear Experimental Dynamics: measurement techniques and identification processes





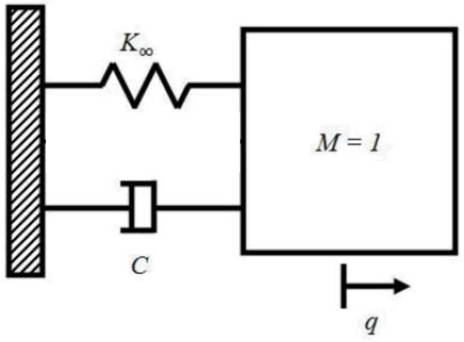
Keywords:
Structural dynamics; test-analysis correlation; nonlinear experimental dynamics; experimental-analytical substructuring

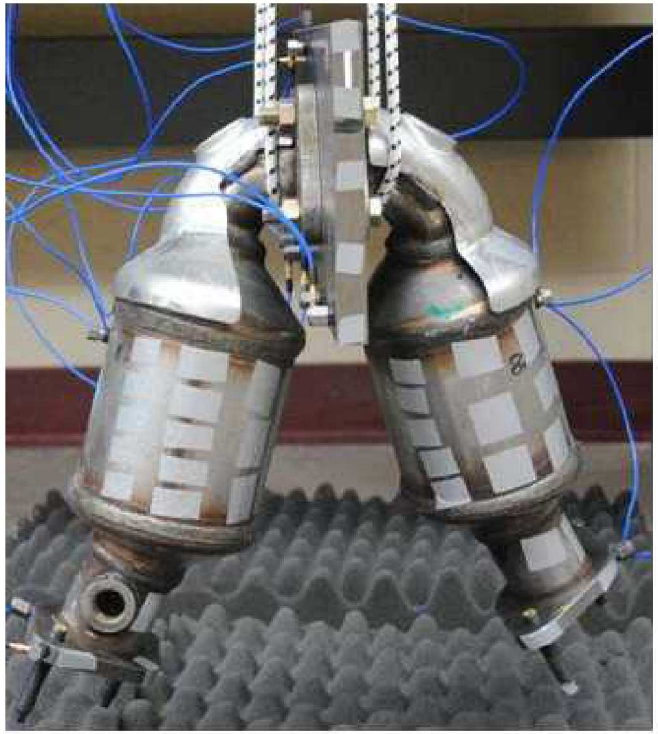


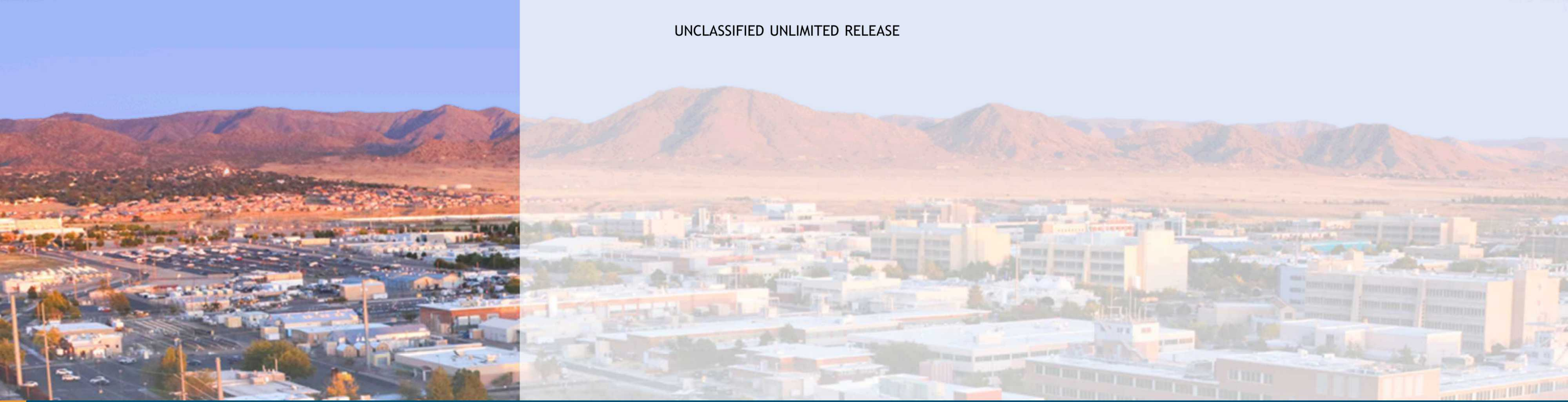
Motivation

Motivation

- **Simple physics of single degree of freedom models can often be used to describe the dynamics of a structure**
  - Linear modal approximations
  - Simplified models
- **Modeling complex systems can be daunting due to many complicated aspects:**
  - Unknown material properties
  - Incomplete design
  - Complicated geometry
  - Nonlinear dynamic response
- **Developing capabilities to model complex systems with simplified models is important to Sandia's missions**
  - This often involves an approach that combines both analytical and experimental models





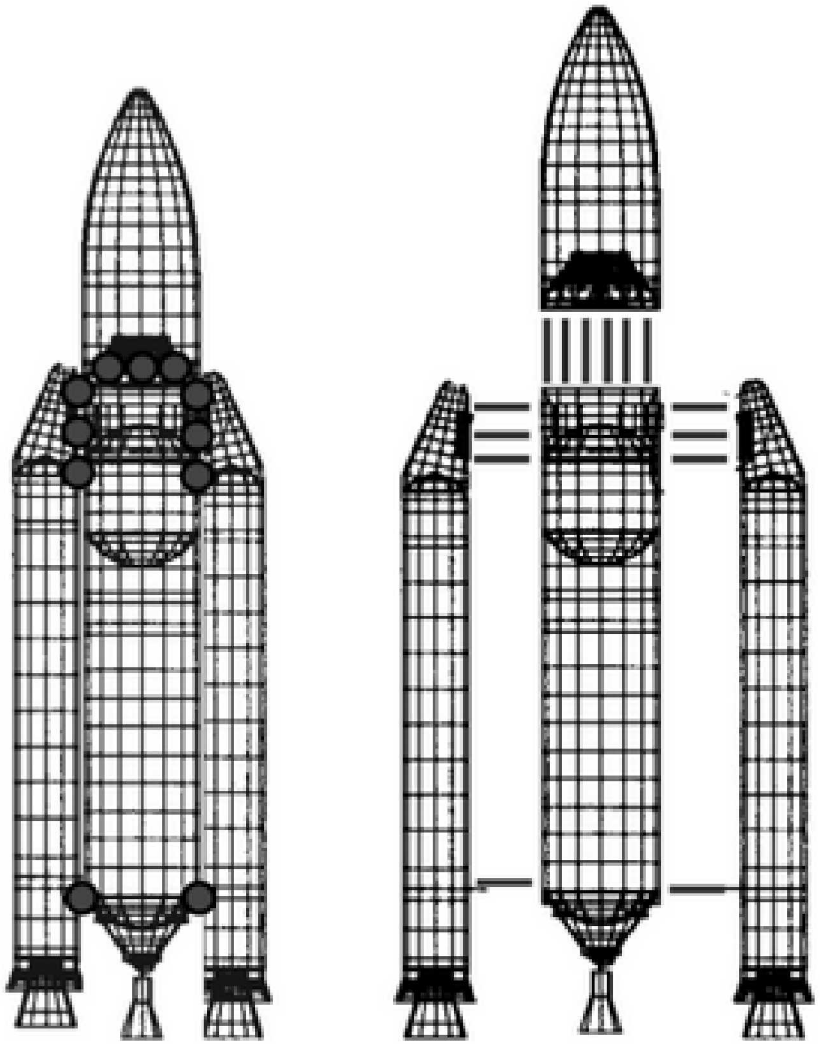


Overview of Hybrid System Modeling

Dynamic Substructuring

- **Hybrid System Modeling Background**
  - This, often termed as Dynamic Substructuring
  - Initially used as a model reduction basis to simplify computation finite element methods[1-3]
  - Follows the same concepts as massively parallel computing

- **Take a complicated problem and break it into simpler parts**
  - Often a prediction is comprised of analytical and experimental models to best estimate the response of a complicated assembly
  - Once complete this method allows for rapid design change evaluation that is critical to a fast paced design cycle

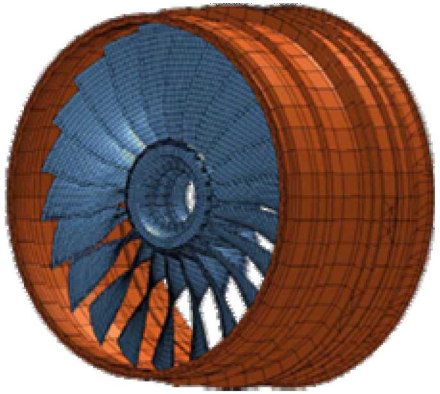


Overview of Hybrid System Models

- **Often we are tasked with analyzing systems where one or more components are difficult to model analytically**
  - Unknown material properties
  - Complicated geometry
  - Difficult to model
  - Produced and designed by an outside vendor

- **The dynamics of these parts can have profound impact on the system level performance**
- **Experimental models are often the optimal choice for characterizing the impact of non modeled hardware it's the system level response**

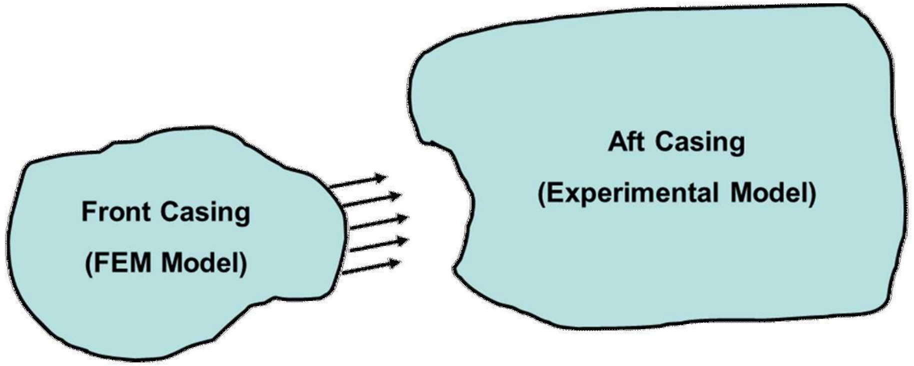
FEM Front Casing

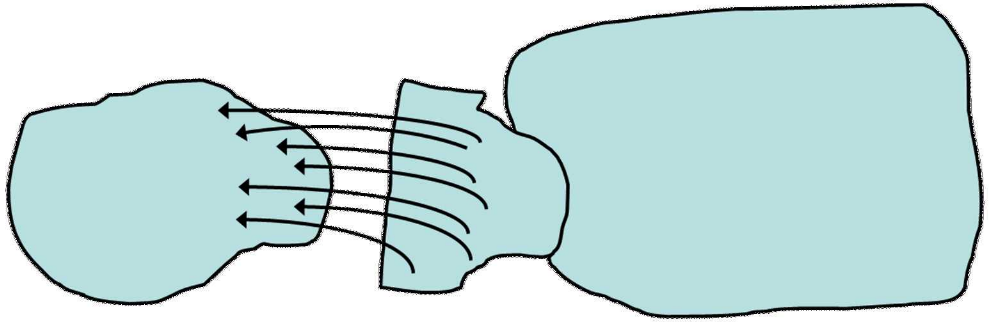


Experimental Model of Casing

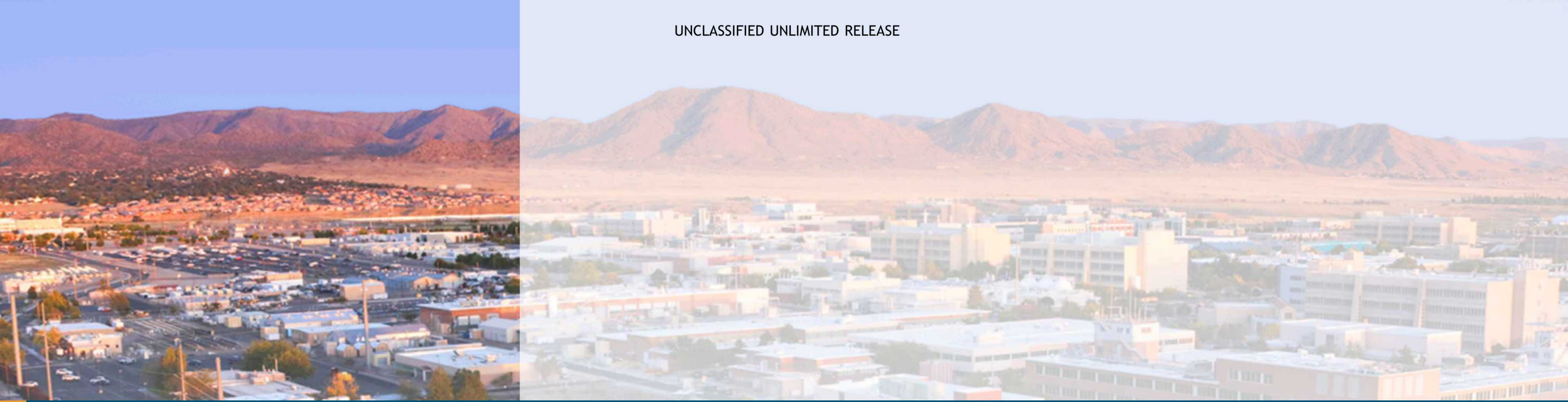
Overview of Experimental Modal Analysis (EMA)

- **Component Mode Synthesis**
- At Sandia the subcomponent models are often represented by a combination of SDOF modal models



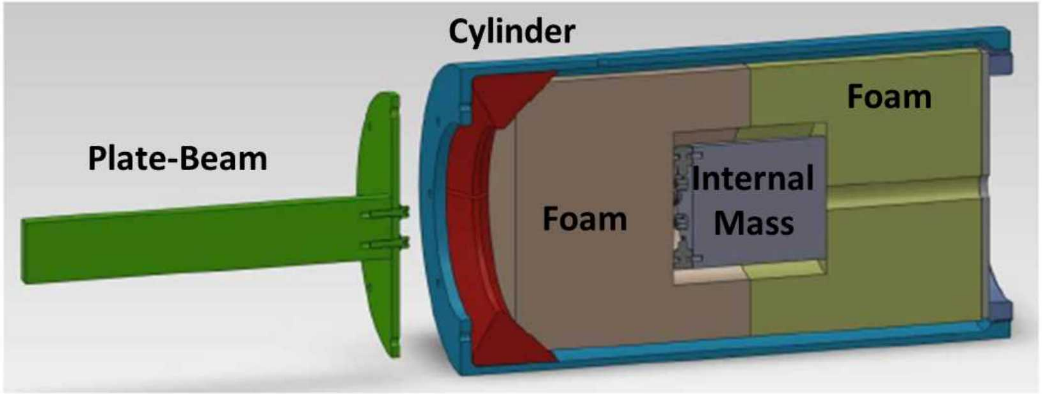


- It is important to get an accurate modal basis when completing modal substructuring
- In order to create quality subcomponent models the interface is often mass loaded to approximate the boundary conditions of the assembly
- This method is referred to as the Transmission Simulator Method

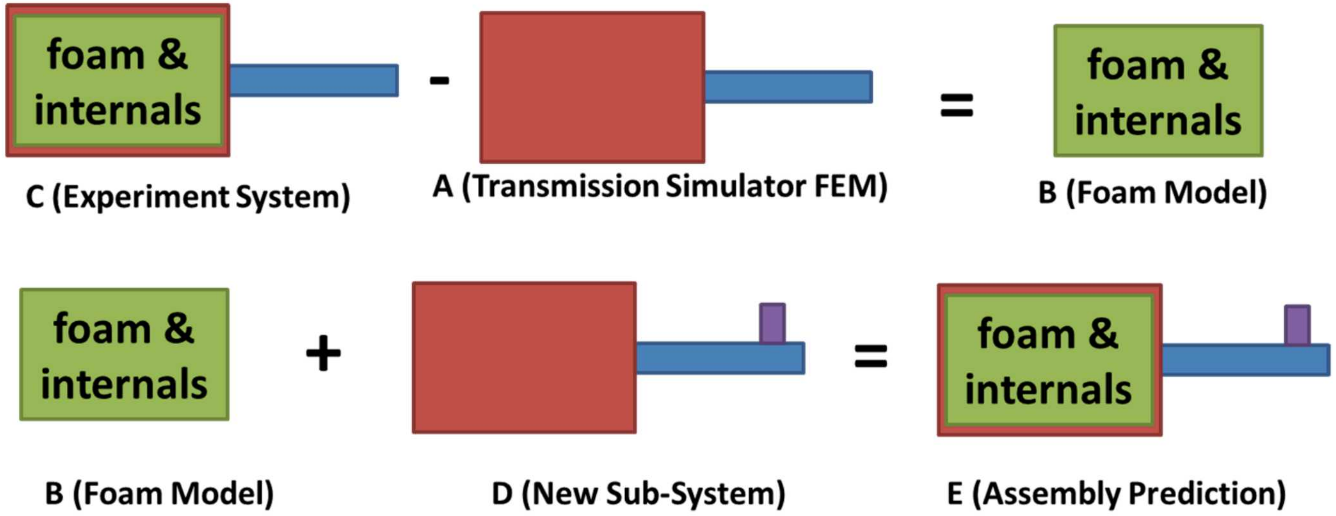


Examples of Linear HSM

Cylinder Plate Beam Example



- Many unknowns in the system including the joint, foam compression, and internal contact loads between the internal mass and foam
- There is a large continuous interface in this problem which should provide a substructuring challenge



Transmission Simulator Method Theory I of II

- To begin using the Transmission Simulator method, set up a system of equations with the modal equations of motions of each component in block diagonal form:

$$\begin{bmatrix} \boldsymbol{I}_C & 0 & 0 \\ 0 & \boldsymbol{I}_D & 0 \\ 0 & 0 & -\boldsymbol{I}_{TS} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{q}}_C \\ \ddot{\boldsymbol{q}}_D \\ \ddot{\boldsymbol{q}}_{TS} \end{Bmatrix} + \begin{bmatrix} \left[\ddots \omega_C^2 \ddots \right] & 0 & 0 \\ 0 & \left[\ddots \omega_D^2 \ddots \right] & 0 \\ 0 & 0 & -\left[\ddots \omega_{TS}^2 \ddots \right] \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_C \\ \boldsymbol{q}_D \\ \boldsymbol{q}_{TS} \end{Bmatrix} = \begin{Bmatrix} \Phi_C^T F_C \\ \Phi_D^T F_D \\ \Phi_{TS}^T F_{TS} \end{Bmatrix}$$

- Next, write the constraints of the system forcing connection degrees of freedom to be equal in each system, where B is a Boolean matrix:

$$\boldsymbol{B} \begin{Bmatrix} \boldsymbol{x}_C \\ \boldsymbol{x}_D \\ \boldsymbol{x}_{TS} \end{Bmatrix} = \boldsymbol{B} \begin{bmatrix} \boldsymbol{\Phi}_C & \boldsymbol{0} & -\boldsymbol{\Phi}_{TS} \\ \boldsymbol{0} & \boldsymbol{\Phi}_D & -\boldsymbol{\Phi}_{TS} \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_C \\ \boldsymbol{q}_D \\ \boldsymbol{q}_{TS} \end{Bmatrix} = \{\boldsymbol{0}\}$$

- Finally define a new synthesized set of coordinates, η, such that

$$\boldsymbol{q} = \boldsymbol{L}\boldsymbol{\eta}$$

Transmission Simulator Method Theory I of II

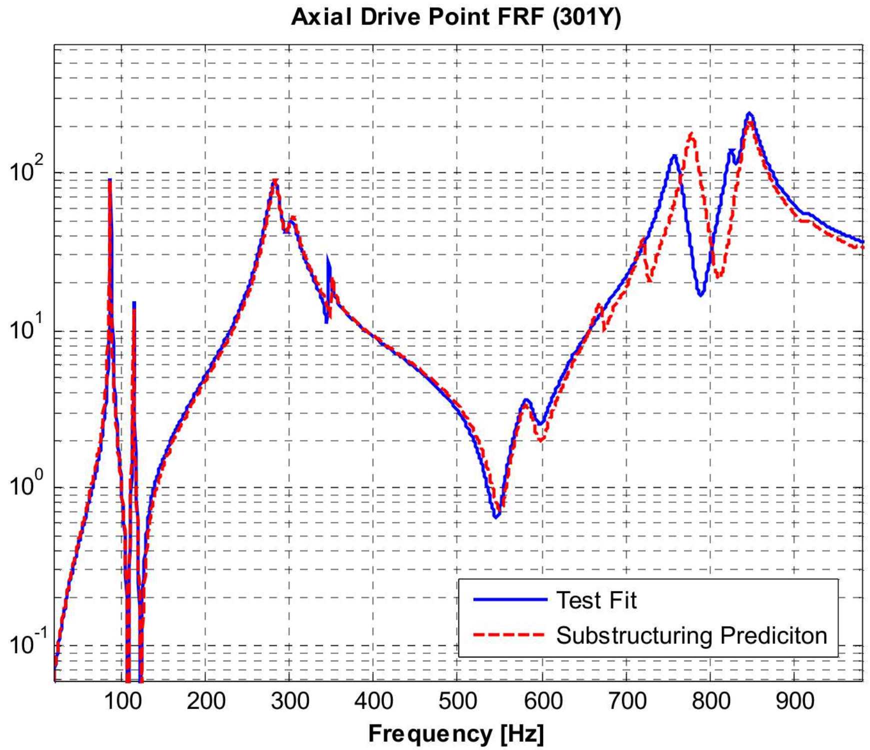
- In order for the constraint equation to be satisfied in the general sense we can solve for the transformation matrix L:

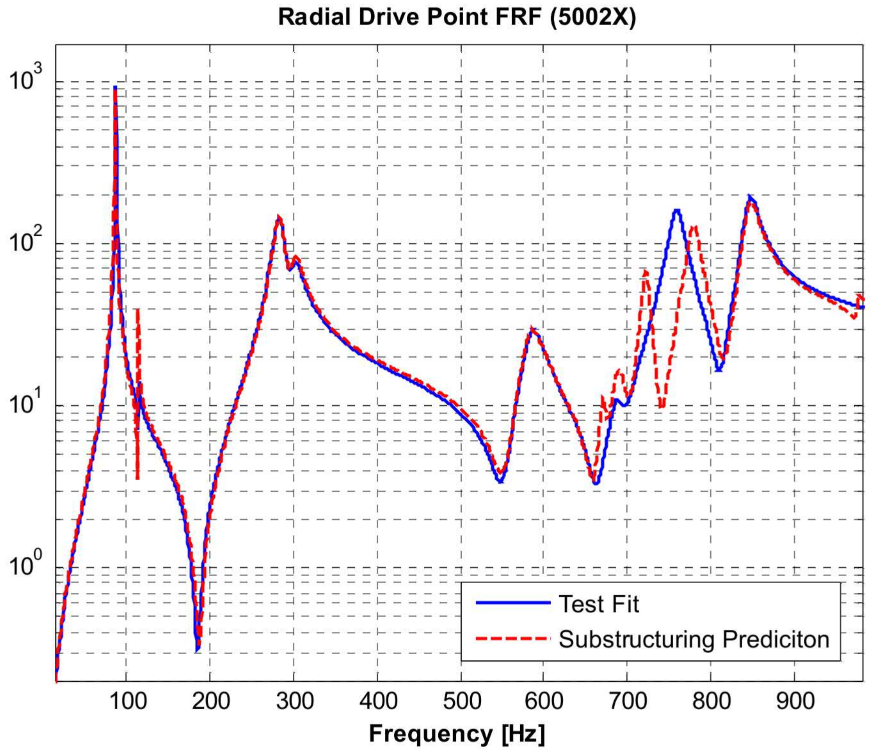
$$\boldsymbol{B}\begin{bmatrix}\boldsymbol{\Phi}_C & \boldsymbol{0} & -\boldsymbol{\Phi}_{TS}\\ \boldsymbol{0} & \boldsymbol{\Phi}_D & -\boldsymbol{\Phi}_{TS}\end{bmatrix}\boldsymbol{L}\begin{Bmatrix}\boldsymbol{\eta}_C\\ \boldsymbol{\eta}_D\\ \boldsymbol{\eta}_{TS}\end{Bmatrix}=\{\boldsymbol{0}\}\rightarrow \boldsymbol{L}=null\left(\boldsymbol{B}\begin{bmatrix}\boldsymbol{\Phi}_C & \boldsymbol{0} & -\boldsymbol{\Phi}_{TS}\\ \boldsymbol{0} & \boldsymbol{\Phi}_D & -\boldsymbol{\Phi}_{TS}\end{bmatrix}\right)$$

- The original system of equations can now be synthesized using the transformation L, enforcing the constraints onto the uncoupled system:

$$\boldsymbol{L}^T\begin{bmatrix}\boldsymbol{I}_C & 0 & 0\\ 0 & \boldsymbol{I}_D & 0\\ 0 & 0 & -\boldsymbol{I}_{TS}\end{bmatrix}\boldsymbol{L}\begin{Bmatrix}\ddot{\boldsymbol{\eta}}_C\\ \ddot{\boldsymbol{\eta}}_D\\ \ddot{\boldsymbol{\eta}}_{TS}\end{Bmatrix}+\boldsymbol{L}^T\begin{bmatrix}\left[\ddots\omega_C^2\ddots\right] & 0 & 0\\ 0 & \left[\ddots\omega_D^2\ddots\right] & 0\\ 0 & 0 & -\left[\ddots\omega_{TS}^2\ddots\right]\end{bmatrix}\boldsymbol{L}\begin{Bmatrix}\boldsymbol{\eta}_C\\ \boldsymbol{\eta}_D\\ \boldsymbol{\eta}_{TS}\end{Bmatrix}=\boldsymbol{L}^T\begin{Bmatrix}\boldsymbol{\Phi}_C^T F_C\\ \boldsymbol{\Phi}_D^T F_D\\ \boldsymbol{\Phi}_{TS}^T F_{TS}\end{Bmatrix}$$

Linear Predictions



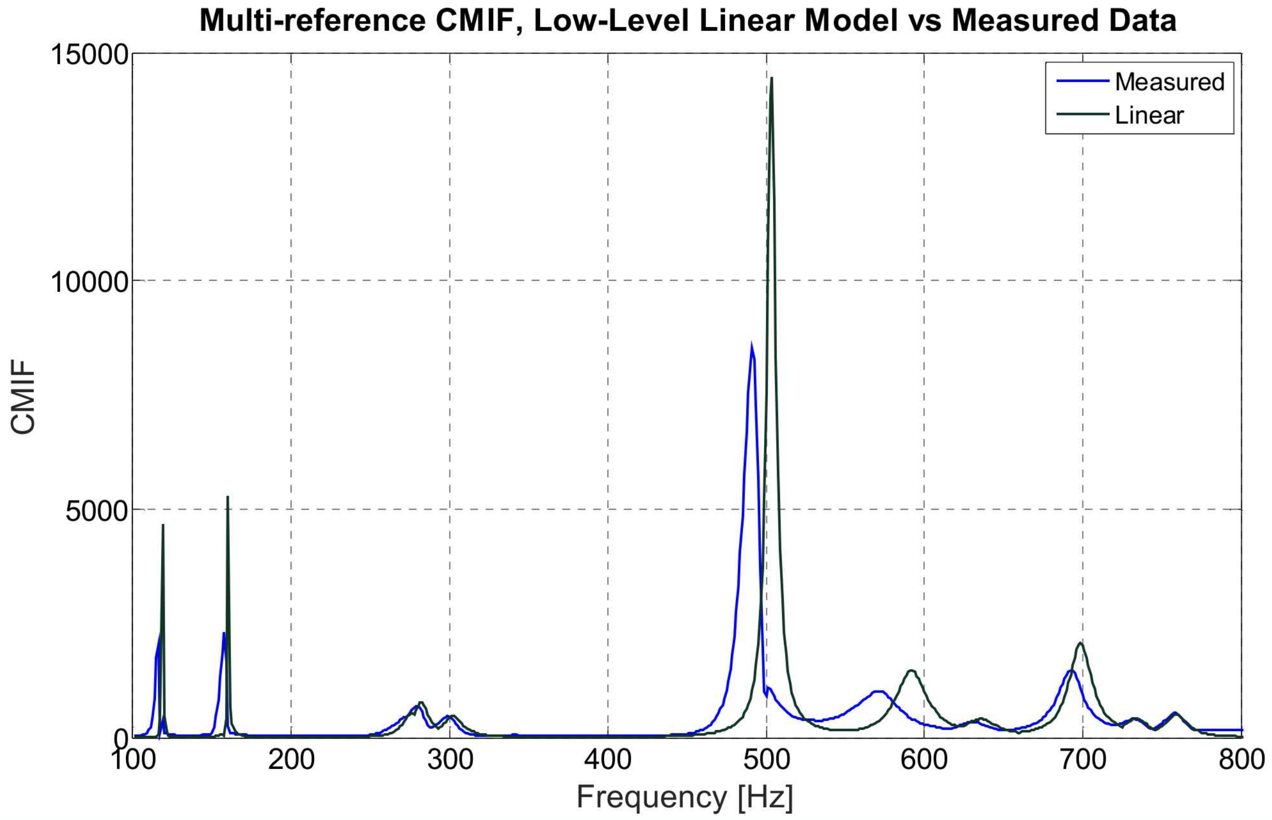


- Substructuring predictions led to good drive point FRF comparisons for both axial and radial excitation points

What about nonlinearity?

Nonlinear Predictions

- Linear predictions for this system looked excellent
- This current method does not account for any nonlinearities due to joints
- What happens when the system is loaded higher force levels?



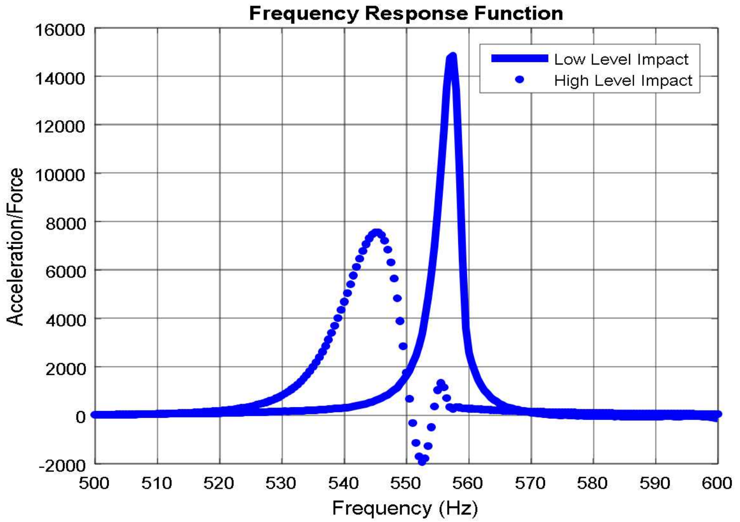
Can these nonlinearities be accounted for in the HSM process?

Nonlinear Dynamics

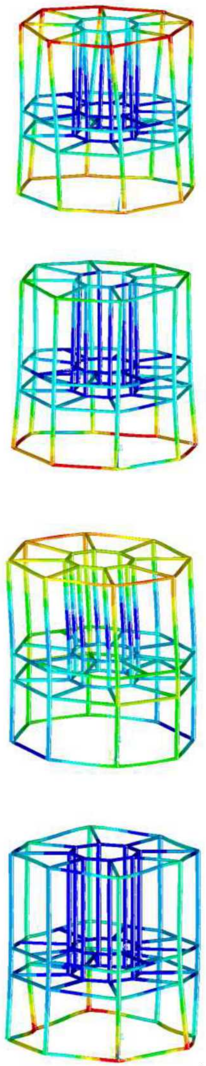
Nonlinear Modal Model Motivation

- Often we treat a structures dynamic response as linear, which means the response scales with forcing amplitude.

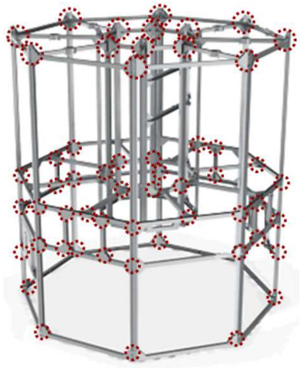
- Many industries rely on bolted joints to connect subcomponents. The frictional interfaces at these joints cause an otherwise linear system to have a nonlinear response, observed as a change in damping and stiffness with response amplitude



- Many constitutive elements have been formed to characterize these responses when the nonlinearity is caused by joints (Iwan, Palmov, Smallwood, etc.)



- If we have many joints, it becomes cumbersome to identify the parameters of each joint separately!

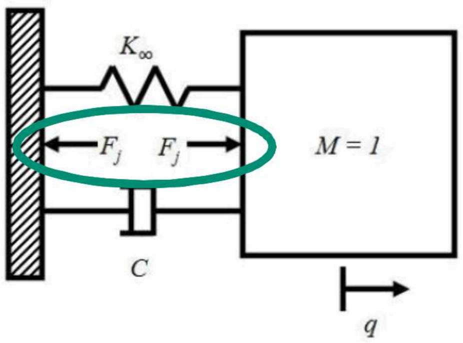


$$\begin{Bmatrix} F_S^1, K_T^1, \chi^1, \beta^1 \\ F_S^2, K_T^2, \chi^2, \beta^2 \\ F_S^3, K_T^3, \chi^3, \beta^3 \\ \vdots \end{Bmatrix}$$

- Experimental evidence has shown that many jointed structures can be tested and represented with uncoupled weakly nonlinear modes.[2]

- This comes with two main assumptions
  - Energy transfer between modes remains negligible
  - The mode shapes of the nonlinear system are preserved at all amplitudes

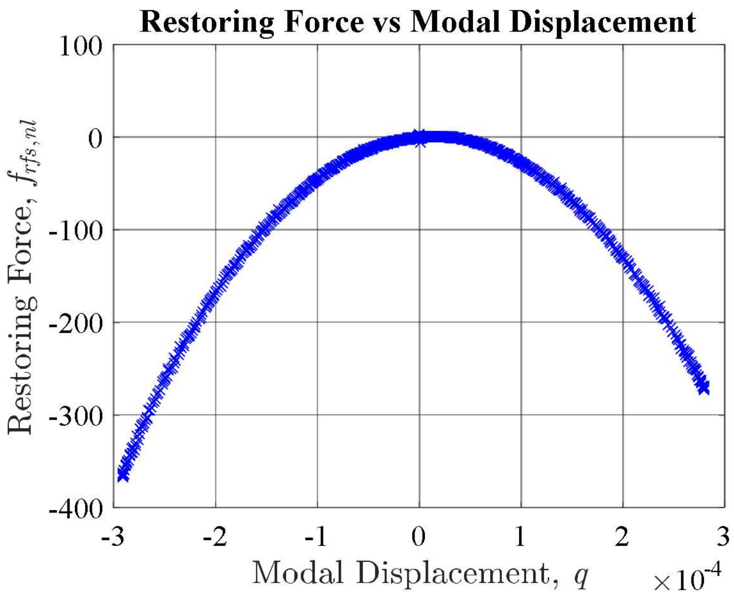
Motivation for Maximizing Modal Amplitude



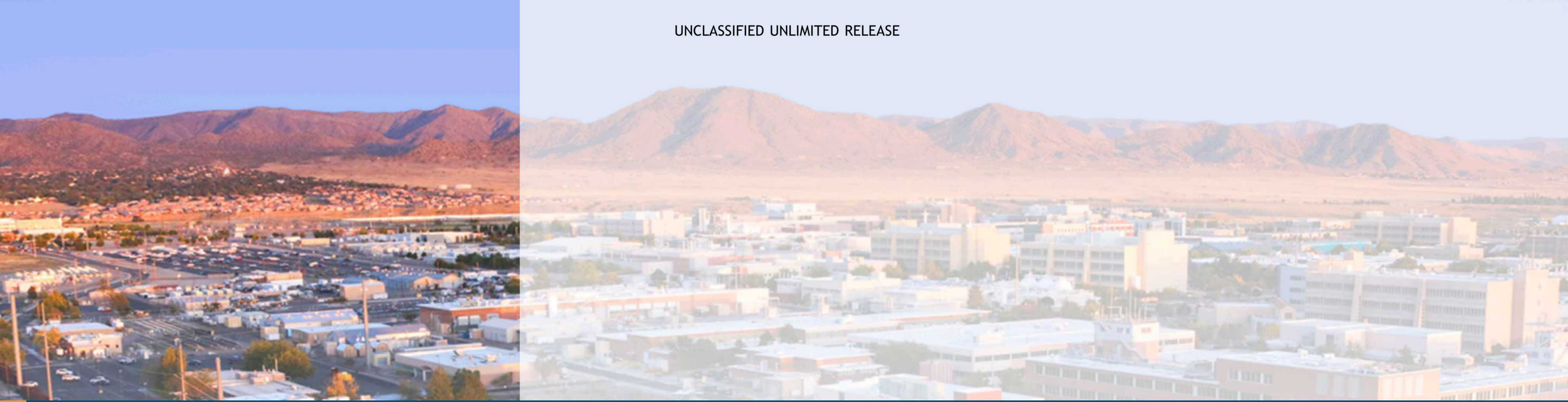
- Current Sandia research has been focused on nonlinear modal modeling
- This theory augments a traditional modal model with the addition of a nonlinear forcing element to capture nonlinear response on a mode-by-mode basis

$$\ddot{q} + c_0\dot{q} + k_0 q + c_1\dot{q}|\dot{q}| + c_2\dot{q}^3 + k_1 q|q| + k_2 q^3 = \phi^T F$$

- Nonlinear modal models are often characterized by the amplitude dependent natural frequency and damping observed in the system
- These measurements are only accurate within the amplitude range tested
- Extrapolation with nonlinear models is inadvisable, so higher responses must be achieved in testing

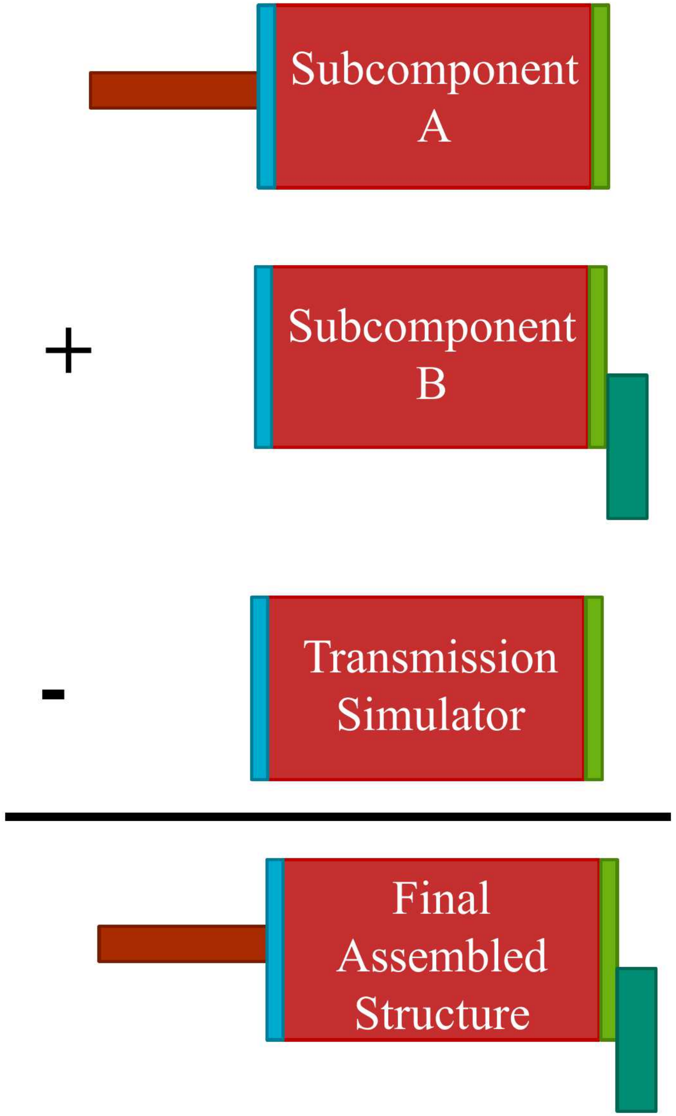


In order to reach operational vibration levels the modal amplitude excited needs to be increased



Demonstrating Nonlinear HSM

Nonlinear HSM Example



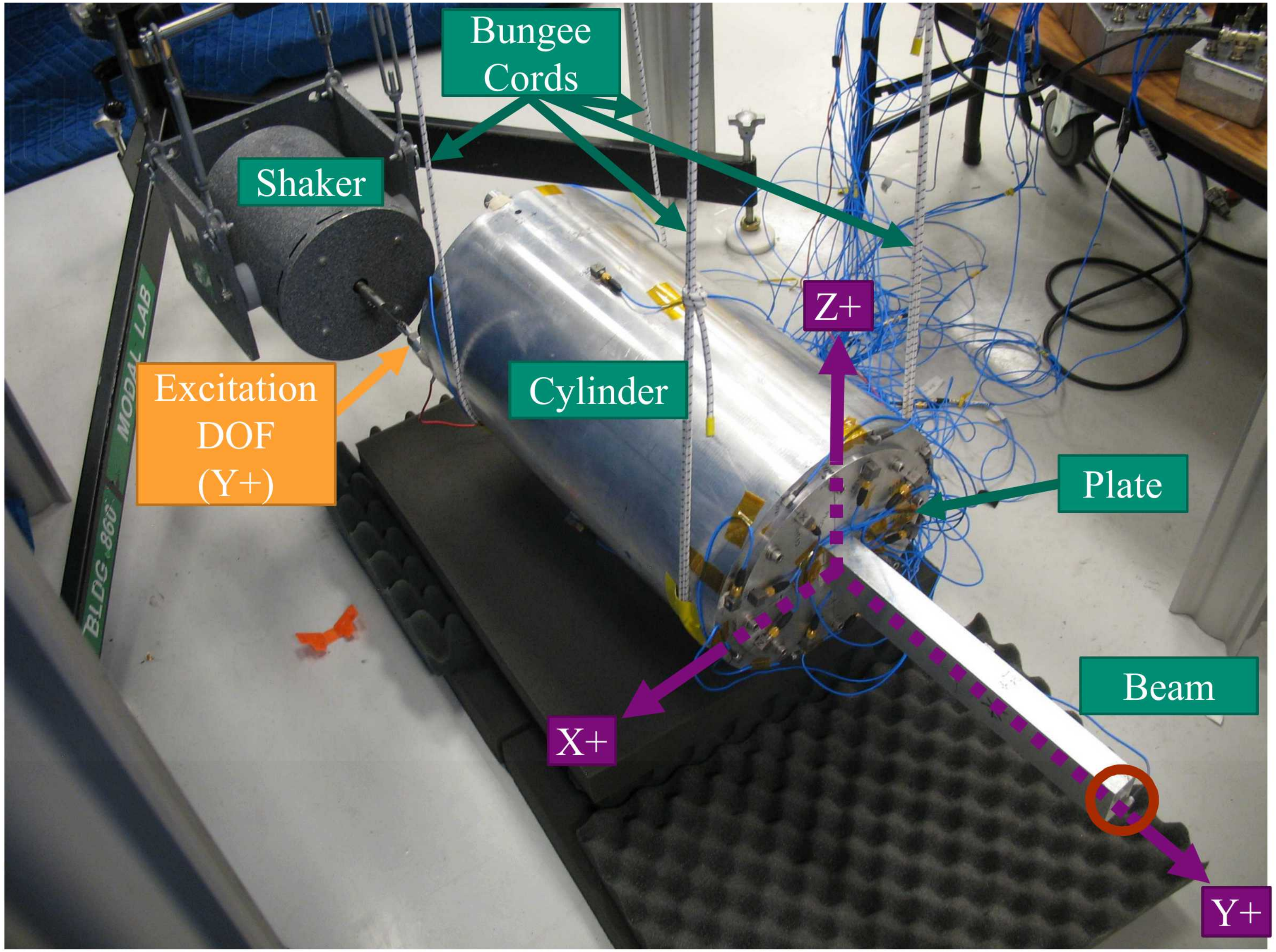
Subcomponent A
- Consists of the Beam, Plate, Cylinder, and Aft Plug
- Low-Level Testing and Linear Modal Identification
- High-Level Testing and Non-Linear Modal Identification

Subcomponent B
- Consists of the Plate, Cylinder, Aft Plug, and Tail
- Low-Level Testing and Linear Modal Identification
- High-Level Testing and Non-Linear Modal Identification

Transmission Simulator
- Consists of the Plate, Cylinder, and Aft Plug
- Represented by a Finite Element Model
- Considered Linear for Substructuring Predictions
- Used to Select Measurement Locations

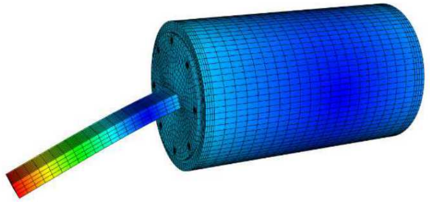
Prediction of Final Assembled Structure
- Represented by a Substructuring Prediction
- Compared to Low-Level and High-Level Truth Testing

Test Set-Up and Linear Modal Analysis Results

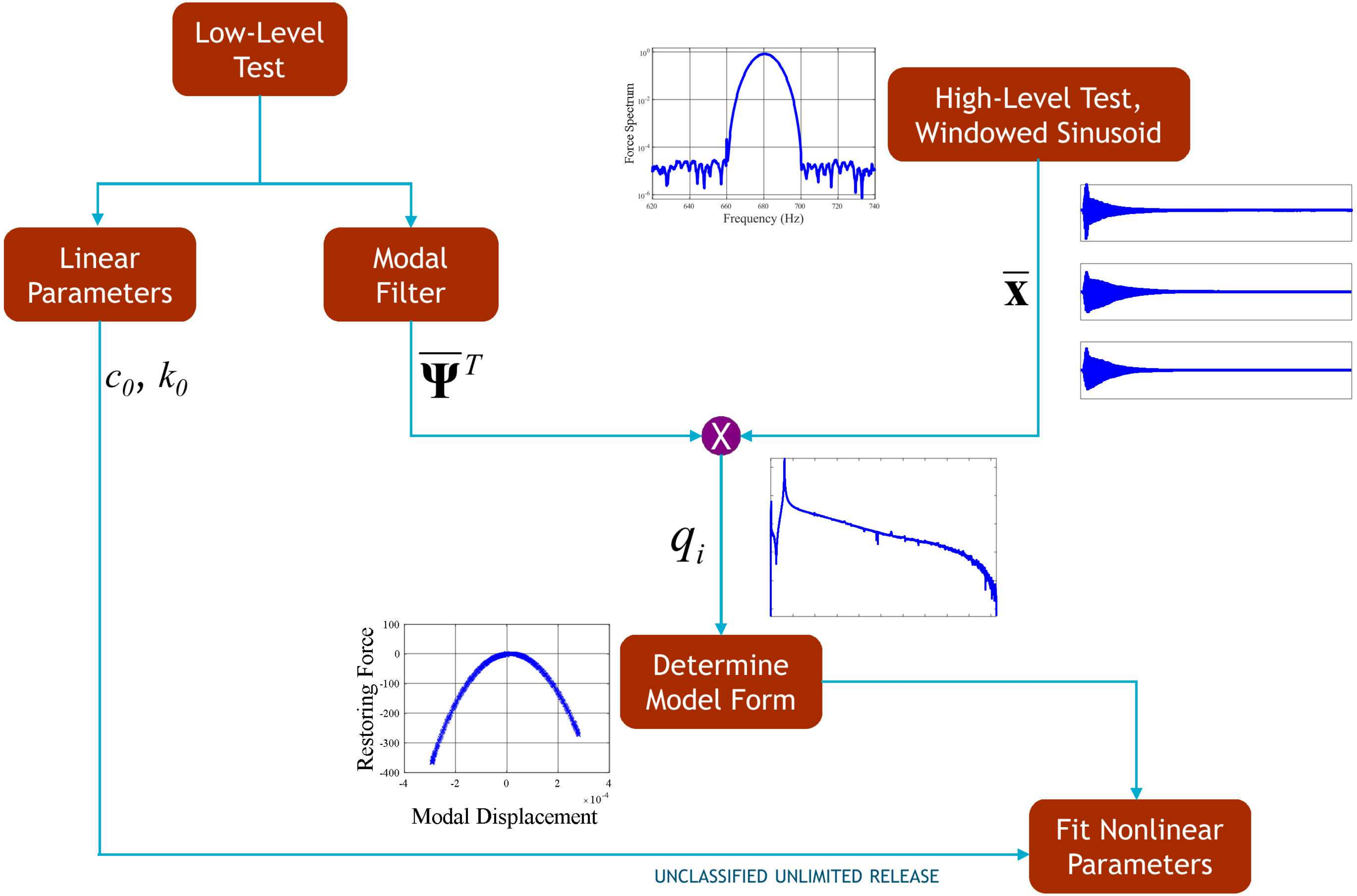


Mode*	Frequency (Hz)	Damping (%)	Description
7	130.0	0.397 %	1st bend of Beam in X-direction
8	172.6	0.322 %	1st bend of Beam in Z-direction
9	385.8	0.069 %	Ovaling of Cylinder
10	391.9	0.083 %	Ovaling of Cylinder
11	551.6	0.278 %	Axial mode
12	945.4	0.413 %	Ovaling of Cylinder
13	948.3	0.513 %	Ovaling of Cylinder
14	1025.7	0.076 %	2nd bend of Beam in X-direction

***Rigid body modes not listed**

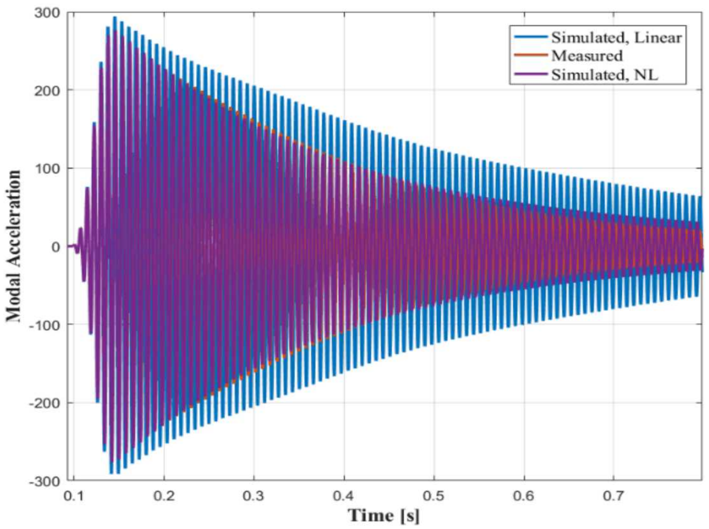


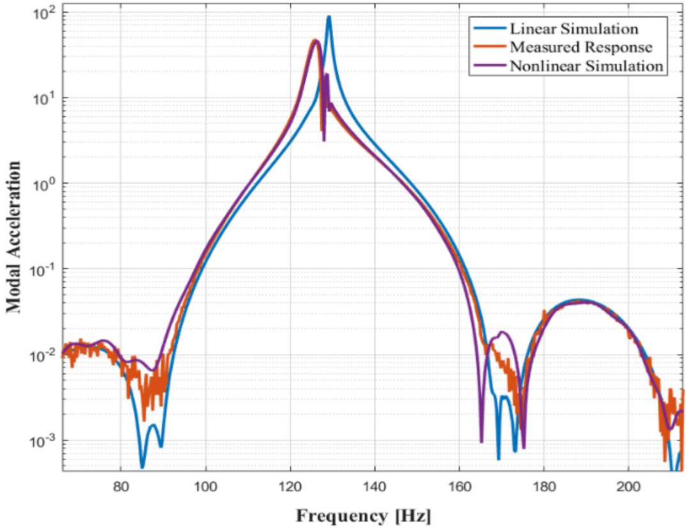
Nonlinear Modal Model Identification Process



Nonlinear Modal Model Identification Process

- Nonlinear modes were identified for each structure
- Each mode was fit using the restoring force surface method as described earlier and simulated to ensure a quality match to measured data





- Nonlinearities were fit using a restoring force surface technique

$$\begin{bmatrix} \dot{q}|\dot{q}| & \dot{q}^3 & q|q| & q^3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ k_1 \\ k_2 \end{bmatrix} = \phi^T F - \ddot{q} - c_0\dot{q} - k_0 q$$

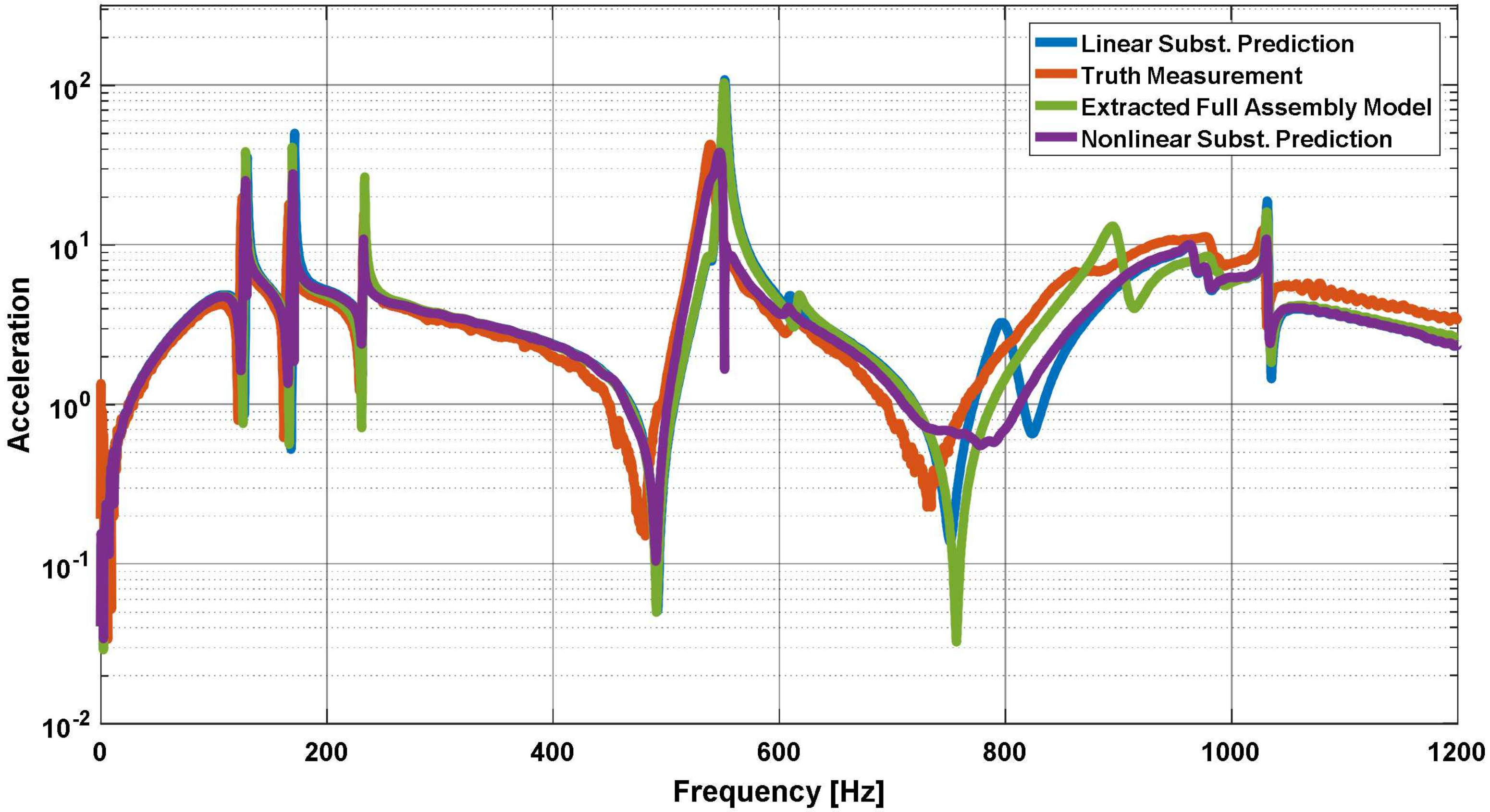
Known Unknown Known

$$\begin{bmatrix} c_1 \\ c_2 \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \dot{q}|\dot{q}| & \dot{q}^3 & q|q| & q^3 \end{bmatrix}^+ \left[\phi^T F - \ddot{q} - c_0\dot{q} - k_0 q\right]$$

These polynomial terms will represent the nonlinearity found in each subcomponent mode

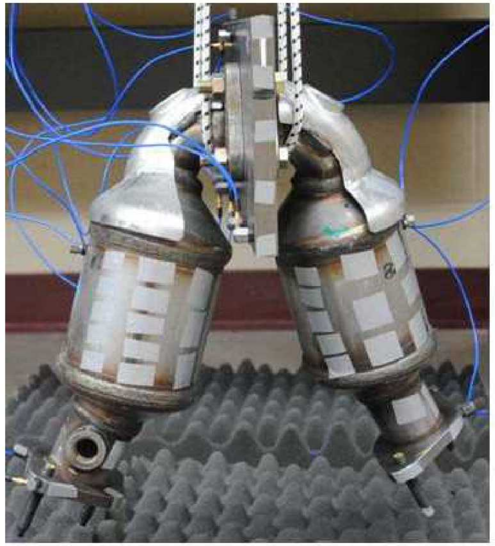
Nonlinear HSM Prediction

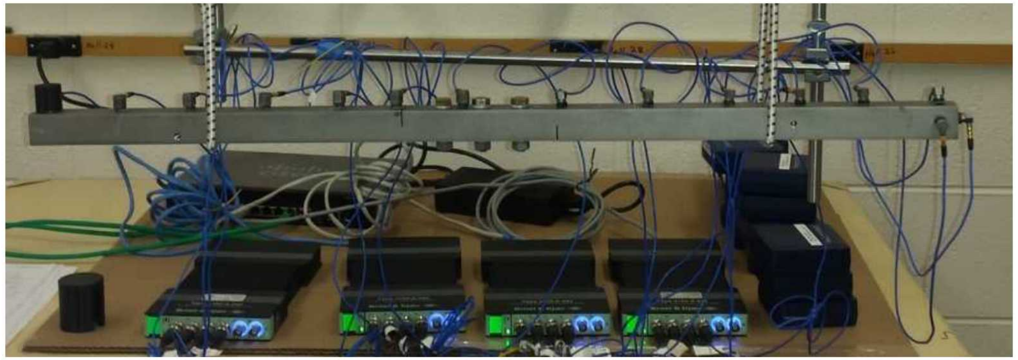
- Using numerical integration, the response of the assembled structure due to a high-level chirp input was calculated
- This included the nonlinear subcomponent modal model forces which were transformed into the space of the assembled structure using the linear synthetization matrix, L

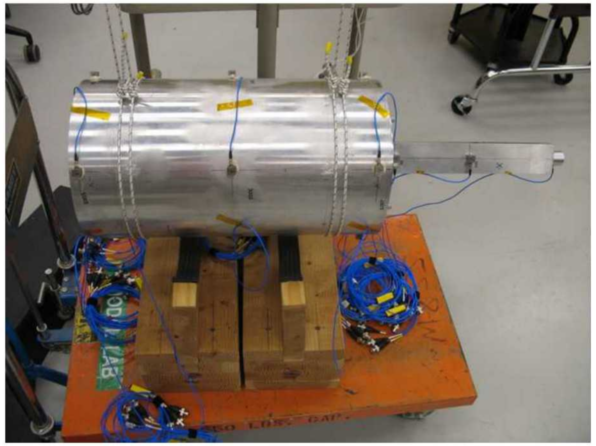


Hybrid System Modeling Closing Thoughts

Nonlinear System Modeling





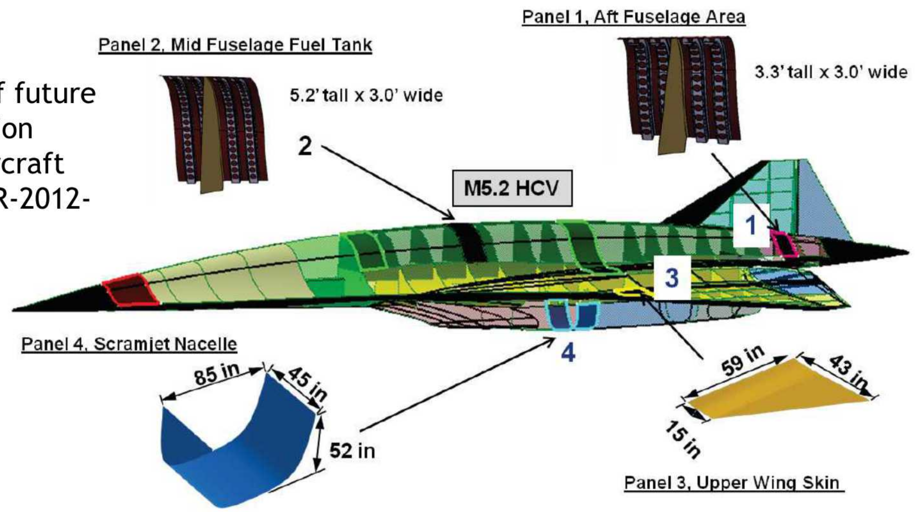


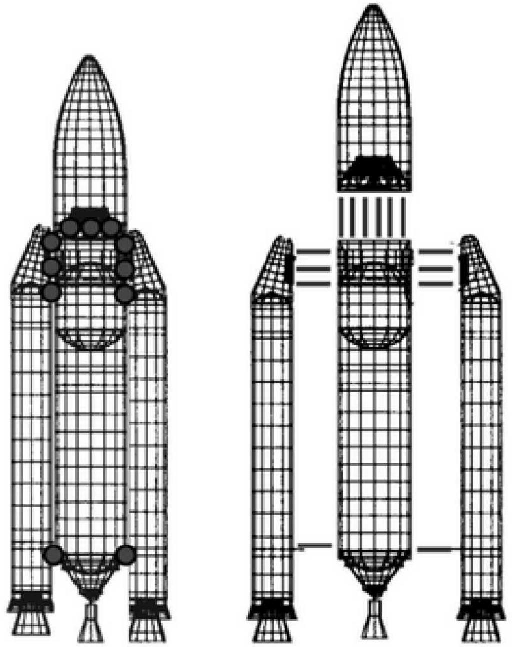
- Hybrid System Modeling is a form of dynamic substructuring that is used to combine experimental and analytical models to best predict assembled dynamic response
- There are many ways to connect subsystem models, the technique used most at Sandia is the Transmission Simulator Method
- Currently Sandia is developing techniques to apply these methods to structures that exhibit nonlinear response and has tested this process on a few structures

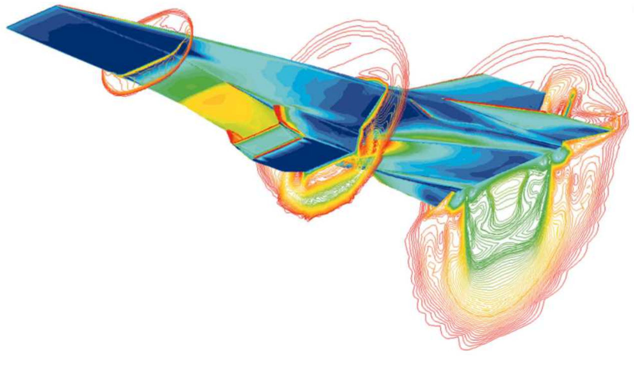
Future of HSM

- Intelligent design of structures that experience critical loads rely on the development of these tools
- This exciting work is the way we are preparing fore the future in the experimental structural dynamics group at Sandia
- As we strive to understand critical load levels, high-level excitation techniques and analysis collaboration are key areas of future focus

Exploratory design of future reusable, long duration cruise high-speed aircraft from AFRL-RQ-WP-TR-2012-0280





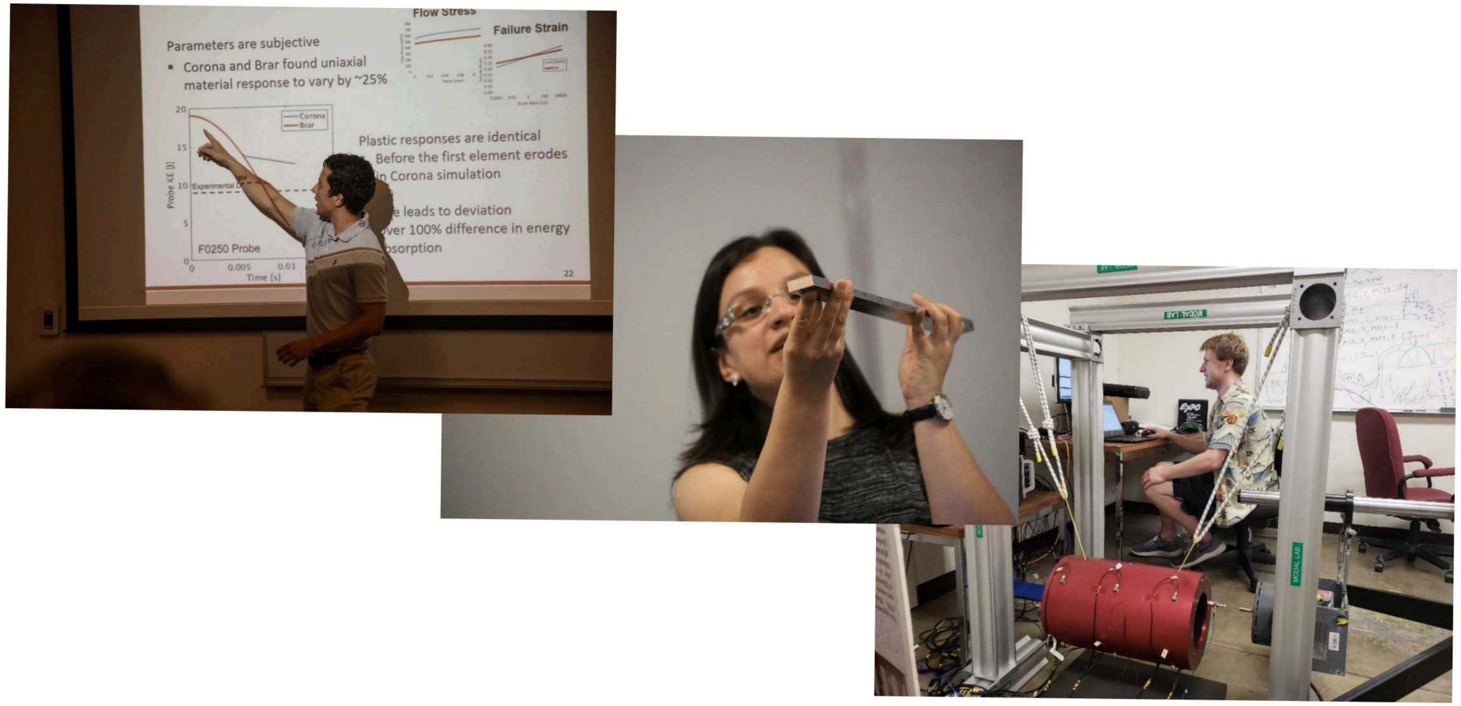


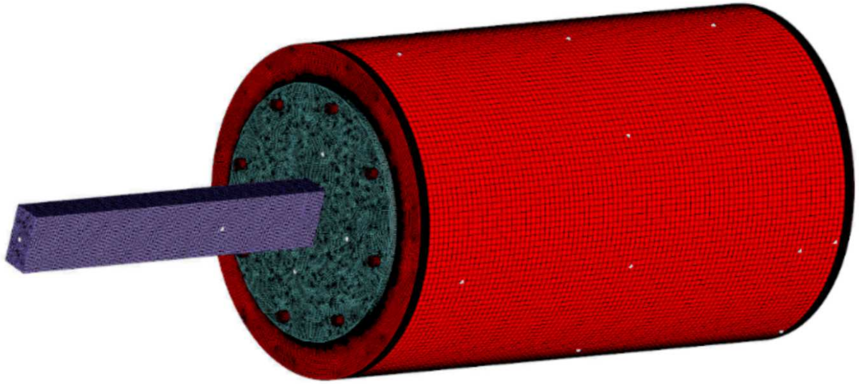
NASA X-43A experimental unmanned hypersonic vehicle (left) CFD simulation of Mach 7 flight (right) Mach 7 wind tunnel test full-scale X-43A. Images courtesy of www.dfrc.nasa.gov/Gallery/Photo/X-43A/Large/

Sandia Outreach

Nonlinear Mechanics and Dynamics Institute

- Collaborative research institute addressing topics in both computational and experimental mechanics and dynamics
  - 7-week summer program: June 17th through August 1st
  - 7 research projects for 2019
  - 3-4 staff and faculty mentors, 3 students per team
- Hosted by Sandia National Laboratories and the University of New Mexico

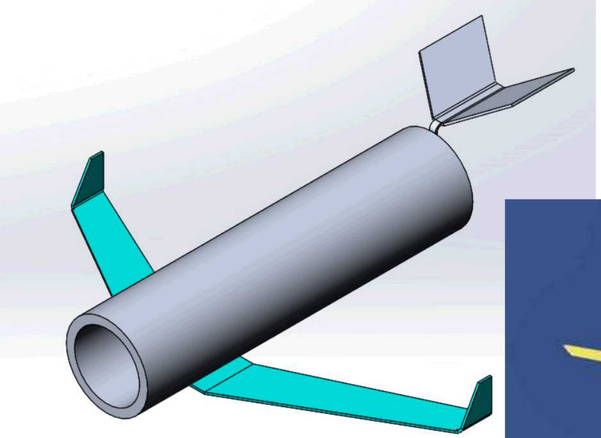


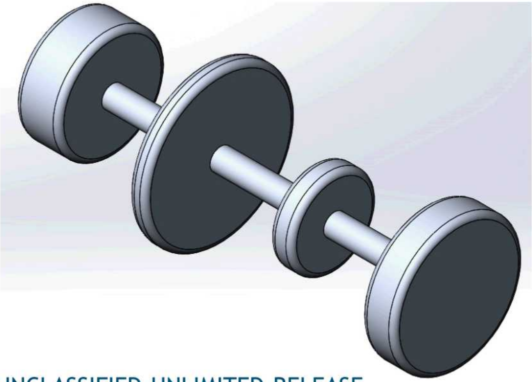
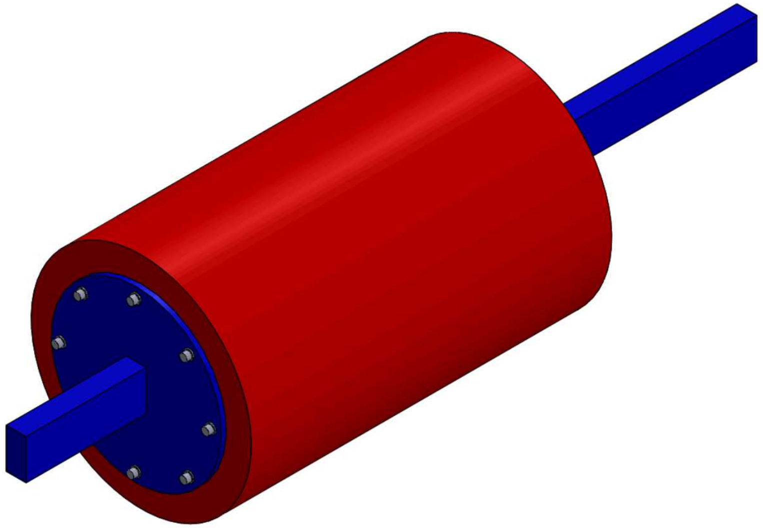


Society of Experimental Mechanics – Substructuring Blackbox Challenge

- Sandia is working with the Dynamic Substructuring group within SEM to propose a substructuring blackbox challenge in 2020
- Universities would be provided a FEM model and Experimental system to connect using different HSM techniques
- At the conclusion of the challenge teams can discuss their approach and see how other groups approached the same problem with the goal of fostering collaboration in the field of HSM.

Potential Structure Designs



Relevant Publications

- Roettgen, D. R. and Allen, M. S., "Nonlinear characterization of a bolted, industrial structure using a modal framework," Mechanical Systems and Signal Processing, Vol. 84, Part B, Pages 152-170. dx.doi.org/10.1016/j.ymssp.2015.11.010
- Allen, M. S., Roettgen, D. R., Kammer, D. C. and Mayes, R. L. " Modal Substructuring using modal Iwan models Part I: Simulated Experiments," Mechanical Systems and Signal Processing, (In Submission)
- Allen, M. S., Roettgen, D. R., Kammer, D. C., and Mayes, R. L. " Modal Substructuring using modal Iwan models Part II: Experimental Demonstration," Mechanical Systems and Signal Processing, (In Submission)
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Questions?

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