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ABSTRACT

The bubble sliding after departing from nucleation site is typically observed in flow boiling systems and, the crucial impact on wall heat transfer has been evidenced through many experiments. As a result, the heat transfer modeling associated with sliding bubble has become one of the subjects of great attention in CFD boiling heat transfer community. The modeling efforts are primarily aimed at improving the existing Heat Flux Partitioning (HFP) model via the implementation of sliding bubble-induced heat transfer. The performance of HFP model depends inherently on the fidelity of sub-models used to predict the fundamental bubble parameters (e.g., bubble departure/lift-off diameter). In the same context, the accurate prediction of sliding bubble parameters (e.g., sliding bubble growth, sliding bubble velocity) is essential to achieving the successful heat transfer modeling associated with sliding bubble. Of many sliding bubble parameters, this paper deals with the sliding bubble velocity. Specifically, the force balance model was assessed in view of the sliding bubble velocity prediction. The parametric effect of key sub-models (e.g., drag force, bubble growth models) used in the force balance equation was investigated. The experimental data from Maity (2000) and Yoo et al. (2016) were used for demonstrating the model predictive ability. It was found that the force balance model proposed in this study was able to predict well the bubble sliding velocity based on the accurate prediction of bubble growth during sliding. The predictive performance was proven with the experimental data measured under various subcooled boiling conditions of both water and refrigerant (NOVEC-7000) flowing upward in vertical channels.

KEYWORDS
Sliding bubble, sliding bubble velocity, heat flux partitioning, subcooled boiling flow

1. INTRODUCTION

The significant heat transfer improvement due to sliding bubbles has been evidenced from many boiling experiments [1-3]. This has naturally caused a great deal of attention to the mechanism of sliding bubble-induced heat transfer and has led to associated modeling efforts for improved CFD boiling analysis. Due to the complexity of the phenomenon, however, the fundamental understanding of the heat transfer mechanism associated with sliding bubble is still largely lacking. In the literature, there are two types of experimental efforts to understand the sliding bubble behavior and the resulting wall heat transfer improvement: (1) one is the gas bubble sliding experiments which do not involve evaporation process [4] and the other is the steam (or vapor) bubble sliding experiments in which the bubbles grow along the sliding path due to evaporation [2]. Of these, the present study is concerned about the latter, i.e., vapor bubble sliding in flow boiling system. In particular, the force balance model that can be used to predict the sliding bubble parameters is the main interest of this paper.

The force balance model refers to the model framework to predict the fundamental bubble parameters such as bubble departure/lift-off diameter based on the analysis of forces acting on a bubble. The force balance model was originally proposed by Klausner et al. [5] and has subsequently been used with improvements by other researchers [6-8] as a method to 'mechanistically' predict the fundamental bubble parameters. The improvements applied to the original force balance model seem to be somewhat successful when viewed from the perspective of matching the limited available experimental data in the literature such as bubble departure diameter. However, the suitability or predictive performance of sub-models estimating the individual force components within the force balance equation has rarely been assessed (e.g., bubble growth model, drag force model). Also, the previous model improvements were often achieved with no physical evidence, using a few parameters adjusted to match the given set of experimental data. The empirical constant used in a bubble growth model or bubble-wall contact diameter are often subject to such adjustment. We should note, however, that such arbitrary tuning undermines the general mechanistic nature of force balance model and eventually make it have little difference with empirical fitting, and thus caution is required. In order to utilize the force balance model without deviating from its original purpose, the sub-models need be evaluated first to identify the true gaps and, if available, apply better ones. Specifically, when predicting the bubble lift-off diameter using force balance model, the trustworthy prediction cannot be expected without sub-models that correctly estimate the bubble parameters during the sliding process (e.g., bubble growth during sliding) prior to the occurrence of bubble lift-off. Adjusting unknown physics or unknown parameters (e.g., empirical constants) to match the experimental data of interest should be introduced in the last order.

So far, little research has been performed to investigate the suitability of sub-models used in the force balance model, especially in view of predicting the sliding bubble parameters. As mentioned above, the ability of force balance model to predict the sliding bubble parameters is prerequisite to the reliable prediction of bubble lift-off diameter, which is one of the most important fundamental bubble parameters required in the Heat Flux Partitioning (HFP) model framework.

This paper investigates the force balance model and its constituent sub-models from the perspective of predicting the sliding bubble parameters, especially sliding bubble velocity. A parametric study was performed for the various sub-models available in the literature and the results were compared with experimental data. The experimental data used in this study were taken under various subcooled boiling conditions of water and refrigerant flowing upward through vertical channels. We examined the suitability of the sub-models used in the force balance equation as well as the physical assumptions that have long been employed by previous researchers, and then a new base line force balance model has been proposed.

2. MODEL DESCRIPTION

In this study, the formulation of a force balance model to predict the bubble sliding velocity is based on the one proposed by Klausner et al [5]:

$$\sum F_x = F_b sin\varphi + F_{s,x} + F_{qs} + F_{AM.x} \tag{1}$$

$$\sum F_y = F_b cos\varphi + F_{s,y} + F_{AM.y} + F_l + F_h + F_{cp} \quad (2)$$

(where F_x and F_y denotes force components in x and y directions, respectively, F_b is buoyancy force, F_s is surface tension force, F_{qs} is quasi-steady drag force, F_{AM} is added mass force, F_l is lift force, F_h is hydrodynamic pressure force, and F_{cp} is contact pressure force.)

Figure 1 illustrates all the force components and various physical parameters needed to formulate the force balance equations, as shown in Eqs. (1) and (2), parallel (x-direction) and perpendicular (y-direction) to a heated wall, respectively. In order to predict the sliding bubble velocity parallel to a heated wall, which is the major concern of this study, the Eq. (1) and associated force components should be well-defined and therefore we focus on them in this paper.

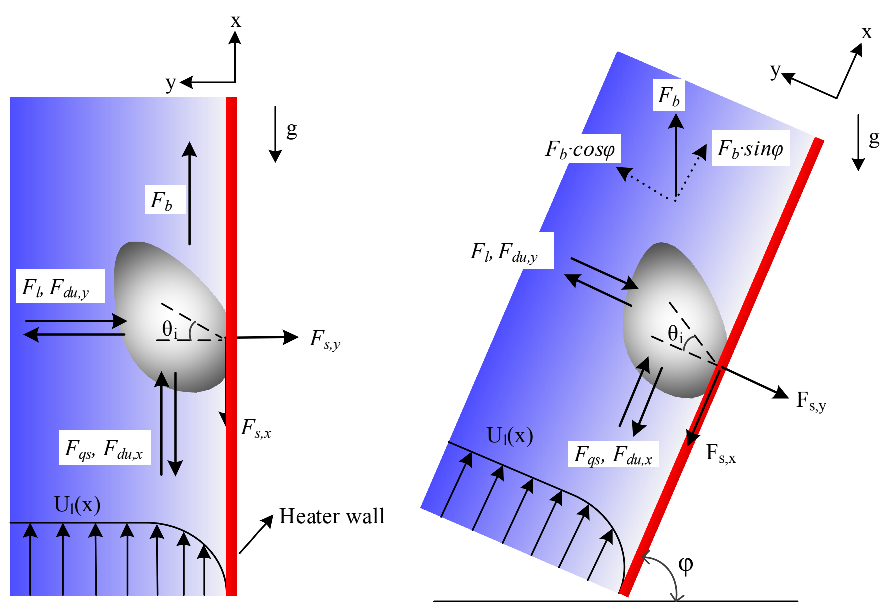


Figure 1. Force components acting on a sliding bubble along a vertical and inclined surface

In Table I, summarized are the details of each force component used in Eq. (1) and the modeling methods which have typically been employed by previous researchers. The physical parameters that must be determined (through modeling) to solve Eq. (1) are also presented in this table. Given the modeling methods of individual force components as well as of the essential physical parameters, the momentum conservation equation for a sliding bubble, i.e., Eq. (1) can be solved numerically as follows:

$$\sum F_x = m_b \frac{du_b}{dt} = m_b \frac{u_b^{n+1} - u_b^n}{\Delta t} = F_b sin\varphi + F_{s,x} + F_{qs} + F_{AM.x} \quad (3)$$

All parameters used in Eq. (3) are defined in the Nomenclature section. It should be noted that the modeling of individual force components used in Eq. (3) can be further simplified by introducing the assumptions such as 'one-dimensional' and 'steady-state' flow of working fluid (i.e., du_l/dt=0 in estimating the $F_{AM,x}$). To solve the Eq. (3), the initial velocity of sliding bubble was assumed to be zero (i.e., u_b=0), meaning that the bubble does not move while growing at a nucleation site before departure. Also, the initial value of sliding bubble diameter or departure bubble diameter (D_0) was given the same as the experimental observation. That is, the bubble growth at a nucleation site before departure (i.e., prior to sliding) was not modeled in this study. The wall superheat along the sliding path was given as boundary condition for each simulation case based on the corresponding experimental measurements [9, 10].

The overall calculation procedure to numerically solve the Eq. (3), including the time-step and physical parameters updates, are illustrated in Figure 2. Given the initial and boundary conditions, the physical parameters such as bubble size [$D(t)$] and local liquid velocity [$u_l(y)$] were computed first before estimating the individual force components. The bubble was assumed to begin to slide when the sum of the estimated values of force components parallel to a wall (x-direction) exceeded zero. While the bubble slid, the computed results of sliding bubble velocity as well as other important physical parameters such as sliding bubble growth was extracted and analyzed which will be discussed in Section 4.

Table I. Equations typically used for modeling force components (parallel to a wall)

Constituent model	Modeling equation	Remarks
Buoyancy force ($\overrightarrow{F_b}$)	$\overrightarrow{F_b} = (\rho_l - \rho_v)\frac{4\pi R^3}{3}\vec{g}$	
Quasi-steady drag force (F_{qs})	- Mei and Klausner (1992) [11]: $\frac{\vec{F}_{qs}}{6\pi\rho_l v_l U_{l,bulk} R} = \frac{2}{3} + \left[\left(\frac{12}{Re}\right)^{0.65} + 0.796^{0.65}\right]^{-1/0.65}$ - Drag force equation: $\vec{F}_{qs} = \frac{1}{2}C_D\rho_l(u_l - u_v)\lvert u_l - u_v\rvert\pi R^2$	- $Re = \frac{2U_{l,bulk}R}{v_l}$ - The definitions of drag coefficient (C_D) depend on authors (see Table III).
Added mass force ($\vec{F}_{AM}$) [12]	$\vec{F}_{AM.x} = \frac{1}{2}\cdot\frac{4}{3}\pi\rho_l R^3\left(\frac{d\overrightarrow{u_l}}{dt} - \frac{d\overrightarrow{u_b}}{dt}\right) + 2\pi\rho_l(\overrightarrow{u_l} - \overrightarrow{u_b})\dot{R}$	
Surface tension force ($\vec{F}_s$) [5]	$\vec{F}_{s,x} = -1.25 d_w\sigma\frac{\pi(\alpha-\beta)}{\pi^2-(\alpha-\beta)^2}(\sin\alpha - \sin\beta)$ $\vec{F}_{s,y} = -d_w\sigma\frac{\pi}{\alpha-\beta}(\cos\beta - \cos\alpha)$	α, β, and d_w should be determined.
Near-wall liquid velocity	- Hinze (1976) [13]: $u^+ = \frac{u_l(y)}{u_\tau} = \frac{1}{k}\cdot ln\left[1 + k\frac{yu_\tau}{v}\right] + c\left[1 - exp(-\frac{yu_\tau/v}{\chi}) - \frac{yu_\tau/v}{\chi}exp(-0.33\frac{yu_\tau}{v})\right]$ - Logarithmic equations [8, 14, 15]: $u^+ = \frac{u_l(y)}{u_\tau} = C_1\, ln\, y^+ + C_2$	- $k=0.4$, $\chi=11$, $c=7.4$, and u_τ should be determined. - C_1 and C_2 depend on the distance from a heated wall (i.e., y^+).
Bubble growth [16]	- Zuber (1961) [16]: $R(t) = \frac{2b}{\sqrt{\pi}}Ja\sqrt{\alpha_l t}$	The empirical constant b should be determined.

3. Experimental Data

The experimental databases from Maity (2000) [10] and Yoo et al. [3, 9] were used for the present study of force balance model. In both experiments, the sliding bubbles moving upward through a vertical square channel were visually observed under various subcooled flow boiling conditions. As a working fluid, the Maity and Yoo et al. employed deionized water and refrigerant (NOVEC-7000), respectively. Both experiments were performed at atmospheric system pressure and the Joule heating was applied from one surface of the square to induce the boiling. The nucleation of spurious bubbles on the heating surface, which could interfere with the visual observation, was avoided effectively from the both experiments by having a single artificial cavity on the smooth heater surface. The detailed experimental conditions used in this study to evaluate the force balance model performance are summarized in Table II. This table provides the bubble departure diameter or initial sliding bubble diameter (D_0) measured at each experimental condition because this was used as initial condition for the present force balance model.

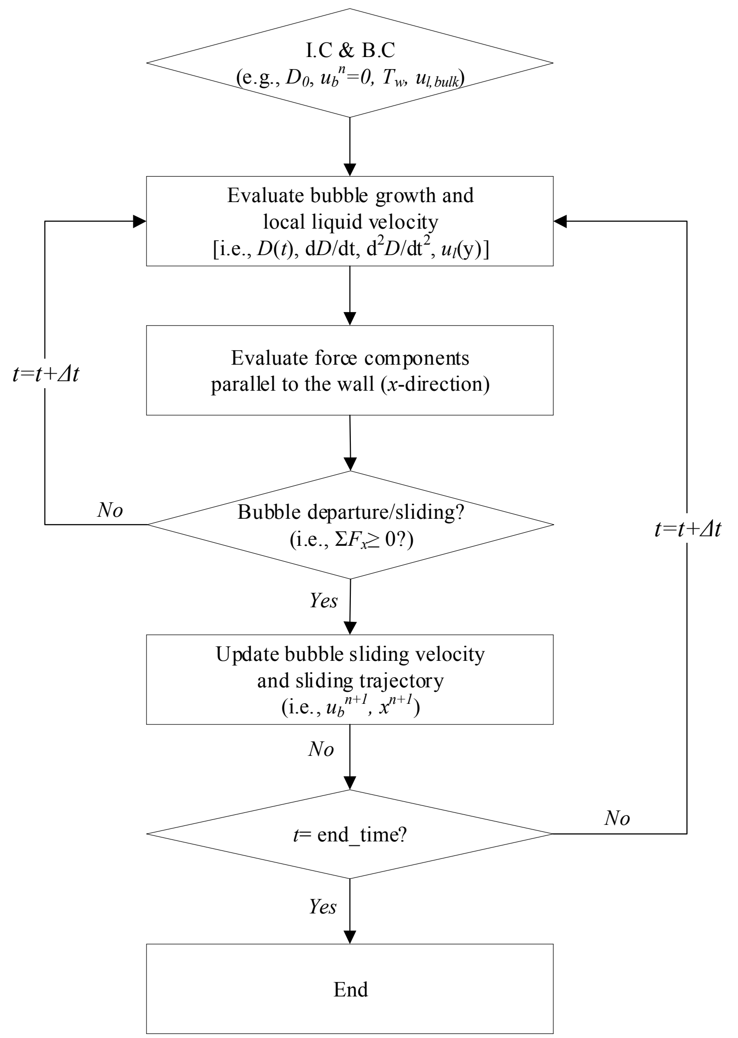


Figure 2. Calculation procedure for predicting the sliding bubble velocity

Table II. Experimental conditions used for the present study

Authors	**Test ID**	**Working fluid**	A_{flow} **[m²]**	$U_{l,bulk}$ **[m/s]**	ΔT_{sub} **[K]**	ΔT_{sup} **[K]**	D_0 **[mm]**
Maity (2000)	M1	Water	4.0E-4	0.25	0.3	5.9	1.2
	M3			0.077	0.6	5.0	1.4
Yoo et al. (2016)	Y5	NOVEC-7000	1.0E-4	0.30	13.5	3.6	0.078
	Y6			0.30	13.5	15.3	0.21
	Y7			0.30	13.5	18.5	0.23

* *All variables used in the table are defined in the Nomenclature section at the end of this paper.*

4. RESULTS AND DISCUSSION

4.1. Model Sensitivity

– Sensitivity study on quasi-steady drag force models (F_{qs})

Since Klausner et al. [5] first proposed a force balance model, the drag force model of Mei-Klausner [11], of which equation form is presented in Table I, has been used most widely in other subsequent studies [6-8, 11, 17, 18]. This model was originally proposed, based on a numerical analysis, to estimate the steady drag force acting on a spherical bubble moving in an unbounded uniform flow with no wall effect. Later, Mei et al. [19] extended the Mei-Klausner's work and proposed a similar correlation that was formulated to work at both low and high Reynolds number flows as follows:

$$\frac{\vec{F}_{qs}}{6\pi\rho_l \nu_l (u_l - u_b) R} = \frac{2}{3} + \left[\left(\frac{12}{Re_b}\right) + 0.75\left(\frac{3.315}{Re_b^{0.5}}\right) \right]^{-1} \tag{3}$$

Eq. (3) was employed by Thorncroft-Klausner [12] in their force balance model research.

On the other hand, a few of other authors such as Yeoh and Tu [14] and Xu et al. [15] estimated the quasi-steady drag force (F_{qs}) by employing the drag force equation which models the interfacial drag force between a bubble and surrounding liquid as follows:

$$\vec{F}_{qs} = \frac{1}{2} C_D \rho_l (u_l - u_v)|u_l - u_v| \pi R^2 \tag{4}$$

Eq. (4) requires a proper value of drag coefficient, C_D, in order to estimate the F_{qs}., and a variety of models/correlations to estimate the drag coefficient exist in the literature. The Table III presents several drag coefficient (C_D) models we tested in the context of force balance model. Although the Mei-Klausner drag force model was used predominantly by previous researchers in predicting the fundamental bubble parameters (e.g., bubble departure/lift-off diameter) using force balance model, little has been discussed for its predictive performance or suitability compared against other models available in the literature.

This study investigated the effect of drag force model, which is applied to the force balance equation [Eq. (1)], from the standpoint of predicting the sliding bubble velocity. The Figure 3 shows the predicted results of sliding bubble velocity depending on the drag force model selection. The predictions of force balance model were compared with experimental data of Maity [10] (M1, left side) and Yoo et al. [9] (Y7, right side). It is noted that in Figure 3 the models set-up other than drag force model followed the base line model set-up we proposed in this paper, which will be detailed in Section 4.2.

Figure 3 shows that all the drag force models tested showed little difference in the prediction of sliding bubble velocity at the initial phase of bubble sliding for both experiments, but the difference became significantly larger as the bubble continued to slide. In particular, the Mei-Klausner model [11], the most widely employed by previous researchers, ended up significantly overestimating the sliding bubble velocities measured from both Maity [10] and Yoo et al [9]. On the other hand, the drag force models of Delnoij et al [20], Ishii-Zuber [21], and Snyder et al [22] showed good agreement consistently with the sliding bubble velocities measured from the two experiments.

Table III. Drag coefficient models tested in this study

Authors	C_D	Remarks
Delnoij et al. [20]	$\begin{cases} C_D = 240 \ \ for \ \ Re_b \le 0.1 \\ C_D = (24/Re_b)(1+0.15Re_b^{0.687}) \ \ for \ \ 0.1 \le Re_b \le 1000 \\ C_D = 0.44 \ \ for \ \ Re_b \ge 1000 \end{cases}$	$Re_b = 2R(t)\lvert\Delta U\rvert/\upsilon_l$
Ishii and Zuber [21]	$\begin{cases} C_D = (24/Re_b)(1+0.1Re_b^{0.75}) \ \ for \ \ Re_b \le 500 \\ C_D = \frac{4}{3}\left\{\frac{g(\rho_l-\rho_v)R^2}{\sigma}\right\}^{0.5} \ \ for \ \ 500 \le Re_b \le 2\times10^5 \end{cases}$	
Lain et al.	$\begin{cases} C_D = 16/Re_b \ \ for \ \ Re_b \le 1.5 \\ C_D = \frac{14.9}{Re_b^{0.78}} \ \ for \ \ 1.5 \le Re_b < 80 \\ C_D = \frac{48}{Re_b}\left(1-\frac{2.21}{\sqrt{Re_b}}\right) + 1.86\times10^{-15}Re_b^{4.756} \ \ for \ \ 80 \le Re_b < 1500 \end{cases}$	
Snyder et al.	$\begin{cases} C_D = 24/Re_b \ \ for \ \ Re_b < 0.1 \\ C_D = \frac{24}{Re_b}\left\{1+\frac{3.6}{Re_b^{0.313}}\left(\frac{Re_b-1}{19}\right)^2\right\} \ \ for \ \ 0.1 \le Re_b \le 20 \\ C_D = \frac{24}{Re_b}\{1+0.15Re_b^{0.687}\} \ \ for \ \ Re_b > 20 \end{cases}$	
Mei and Klausner [11]	See Table I	
Mei et al. [19]	See Eq. (3)	

**All variables used in the above equations are defined in the Nomenclature section.*

It should be noted that no drag force models tested in the present work consider the presence of a wall which is required to strictly describe the drag force acting on a sliding bubble. However, the comparison shown in Figure 3 demonstrates how the drag force model selection affects the prediction of sliding bubble velocity and strongly suggests which model can be a better option in current situation where there is lack of rigorously applicable model.

– *Sensitivity study on bubble growth models*

In order to accurately predict the sliding bubble velocity using force balance model, it is very important to have a reliable bubble growth prediction model. This is because the force components during the sliding process can only be estimated correctly based on the accurate bubble growth prediction. In this paper, two bubble growth models, i.e., Zuber [16] and Yoo et al. [23] were evaluated within the force balance model framework and the model effects on the prediction of sliding bubble parameters were observed.

The Zuber model was originally developed for pool boiling but has been employed widely for the prediction of bubble growth in flow boiling as well. The typical way to extend the Zuber model application to flow boiling is to adjust the empirical constant (b) (see the specific equation presented in Table I). The typical range of this empirical constant applied to the flow boiling prediction is about b=0.8–1.73 [6-8, 12, 14, 15, 17]. On the other hand, Yoo et al. model [23] was developed specifically for the growing bubbles in the sliding process, which was validated with experimental data measured in subcooled boiling conditions of water and refrigerants flowing upward though a vertical channel. A certain level of empiricism was also employed in Yoo et al. model, but the validation was performed with various sets of subcooled flow boiling data without arbitrary adjustment of empirical constant.

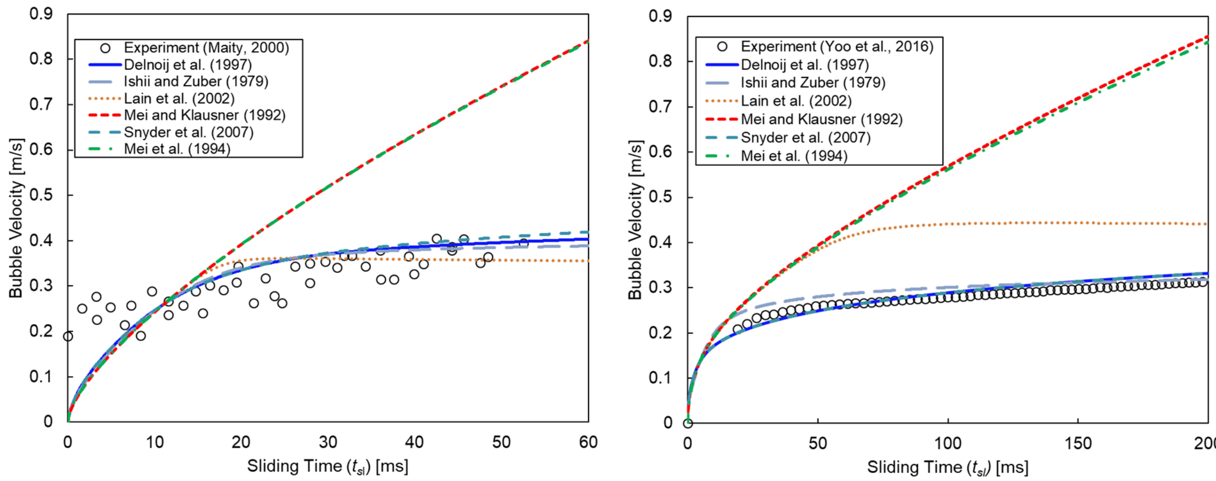


Figure 3. Effect of drag force models on the prediction of sliding bubble velocity [Experimental data: Maity (M1, left ride), Yoo (Y7, right ride)]

In Figure 4, the effect of different bubble growth models on the prediction of sliding bubble velocity and sliding bubble growth is shown on the top and bottom rows, respectively. Also, the model predictions are compared with subcooled flow boiling data measured based on both water (Maity [10], left column) and refrigerant (Yoo et al. [9], right column). The top left of Figure 4 shows that the predicted sliding bubble velocity based on the Zuber model with empirical constants of b=0.8 or 1.73 tends to overestimate the experimental data. This is related to the significant over-prediction of bubble growth during sliding by Zuber model as shown on the bottom left of Figure 4. In contrast, when applying the Yoo et al. model, we can see in Figure 4 (left column) that both the sliding bubble velocity and sliding bubble growth are well-predicted (there was no tuning of any empirical constant). The similar comparison is shown on the right column of Figure 4, in which the experimental data of Yoo et al. [9] based on refrigerant (NOVEC-7000) is employed. It also shows that compared to Zuber model significantly overestimating the experimental data, Yoo et al. model leads to better agreement with experimental data on both sliding bubble parameters. Interestingly, Figure 4 on the right column also shows that the Zuber model can result in better agreement with experimental data on both sliding bubble parameters by employing b=0.1 which is much smaller value than those employed by previous researchers. However, it is still uncertain how this empirical constant should be adjusted depending on flow boiling conditions and working fluid, etc.

– Sensitivity study on near-wall liquid velocity modeling

The modeling of near-wall liquid velocity in two-phase flow is challenging and no generally applicable model exist yet in the literature. Thus, the single-phase turbulent wall function was used instead in this study to approximate the near-wall liquid velocity around a sliding bubble. Specifically, we computed the liquid velocity at the height of bubble center at each time step which was then used to estimate the associated force components such as drag force. In order to apply this method to two-phase flow, a proper assumption or modeling for the friction velocity (u_τ) is required. This study compared the three different methods used in the literature [5, 12, 17] for the modeling of two-phase friction velocity: (i) $u_\tau =$

$0.04U_{l,bulk}$, (ii) $u_\tau = 0.05U_{l,bulk}$, and (iii) $u_\tau \equiv \sqrt{\tau_w/\rho_l}$. In using (iii), the wall-shear stress (τ_w) was determined using the following equation:

$$\tau_w = 0.5C_f\rho_l U_{l,bulk}^2 \tag{5}$$

The fraction factor C_f used in Eq. (5) was then estimated using the Petukhov's formula [24]:

$$C_f/2 = [2.236\ln(Re_l) - 4.639]^{-2} \tag{6}$$

where Re_l is the bulk liquid Reynolds number defined based on the hydraulic diameter of a flow channel ($= D_h U_{l,bulk}/\nu_l$).

The predicted sliding bubble velocity depending on the friction velocity assumptions [(i)-(iii)] is shown in Figure 5. It shows that higher friction velocity assumption leads to higher sliding bubble velocity. This result is closely related to the variation of drag force acting on the sliding bubble. In both experimental cases of M1 and Y7 shown in Figure 5, the sliding bubble velocity was estimated higher than the local liquid velocity along the most sliding path ($u_l<u_b$). This means that the bubble motion along the sliding path was resisted by the surrounding liquid [i.e., the sign of drag force was negative according to the definition described in Eq. (4)]. Therefore, the higher local liquid velocity, caused by the higher friction velocity assumption, resulted in less magnitude of negative drag force, which in turn led to higher sliding bubble velocity. It was found from this study that the $u_\tau=0.05U_{l,bulk}$ showed the best agreement with the experimental data from both Maity and Yoo et al.

– Sensitivity study on the modeling of bubble contact diameter

One of the biggest uncertainties in the prediction of sliding bubble velocity using force balance model is associated with the physical parameters used to describe the surface tension force (F_s), i.e., (i) contact angle and (ii) bubble contact diameter (or bubble base diameter). As for the bubble contact diameter (d_w), the Yun et al (2012) and Sugrue et al (2016) assumed that it was 1/15 and 1/40 times smaller than the bubble diameter (D_b), respectively. In contrast, the experimental observation of Maity (2000) presented much larger values than those used in the previous models, i.e., $d_w=D_b/4$- $D_b/2$.

Figure 6 shows how the modeling of contact diameter affects the prediction of sliding bubble velocity. It shows that in the water-based Maity experiment (left side of Figure 6), the surface tension force tended to make the sliding bubble move faster as the contact diameter became smaller or neglected. The effect of different contact diameter modeling, however, was insignificant in the refrigerant-based Yoo et al experiment (right side of Figure 6). This is due to the fact that the relative contribution of surface tension force itself became smaller in the refrigerant because the surface tension of refrigerant (i.e., NOVEC-7000) was about 6 times smaller than that of water.

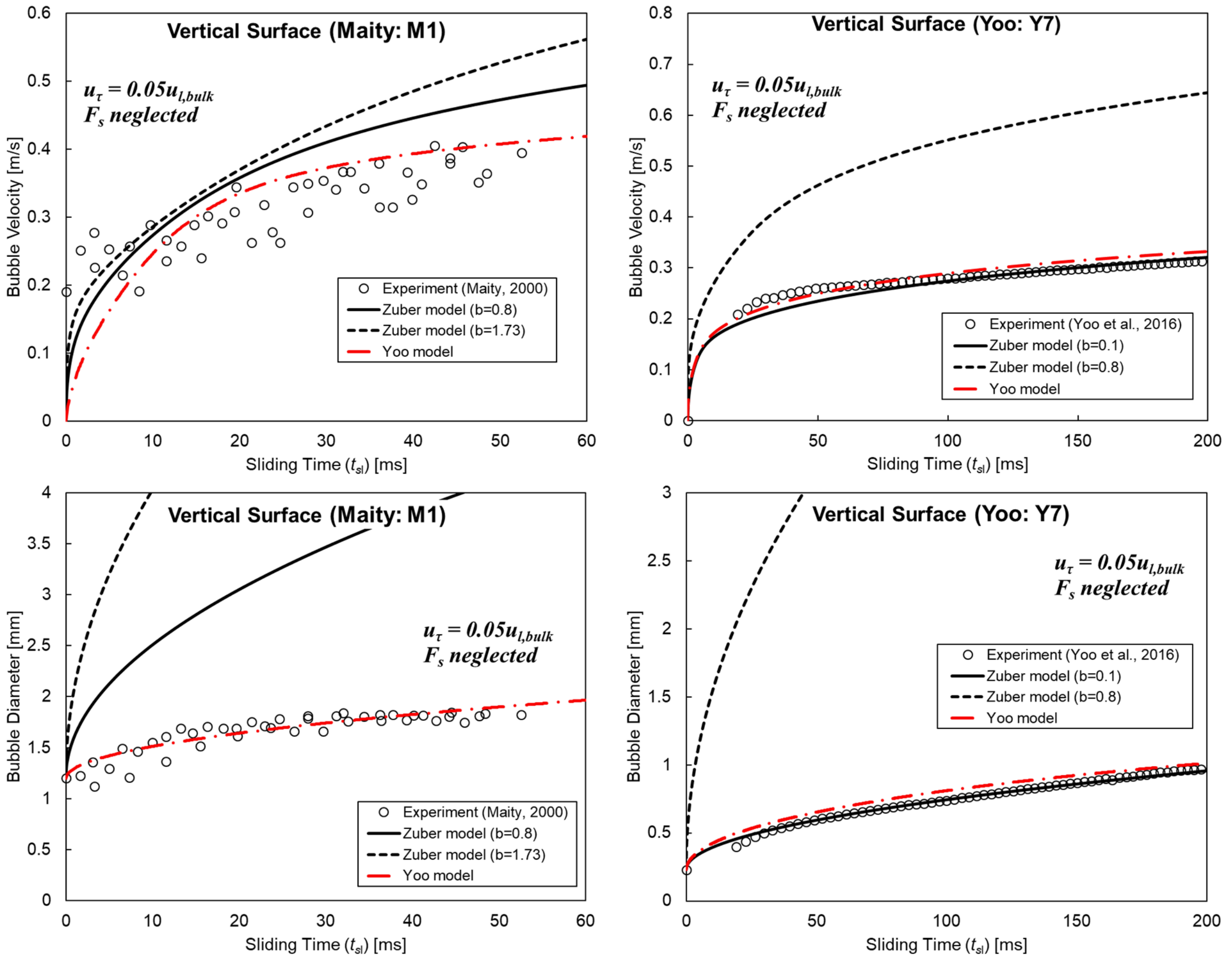


Figure 4. Bubble growth model sensitivity on predicting the sliding bubble's velocity (top row) and growth (bottom row) [Experimental data: Maity (M1, left column), Yoo (Y7, right column)]

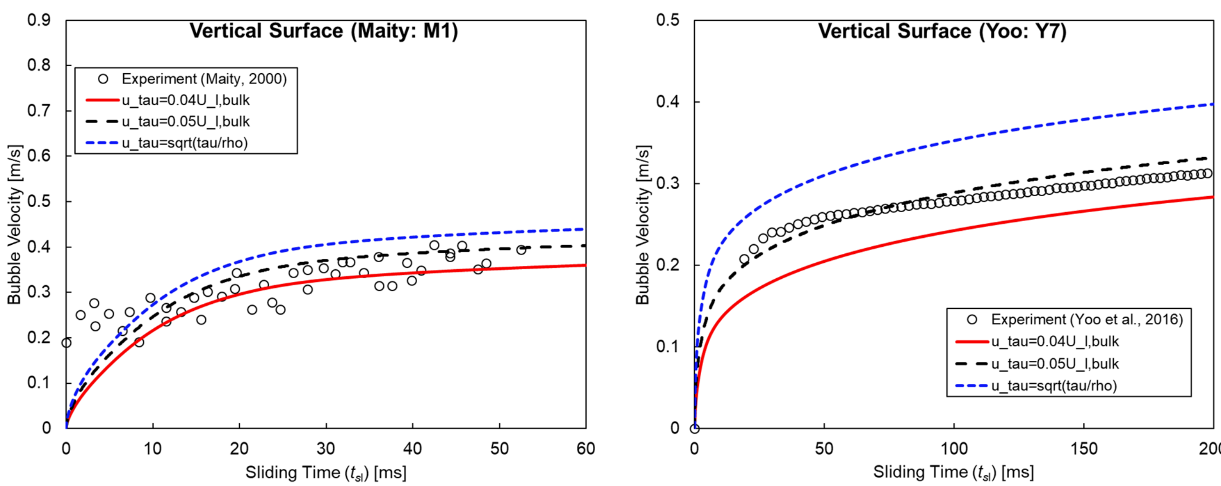


Figure 5. Effect of near-wall liquid velocity modeling on the prediction of sliding bubble velocity [Experimental data: Maity (M1, left side), Yoo et al. (Y7, right side)]

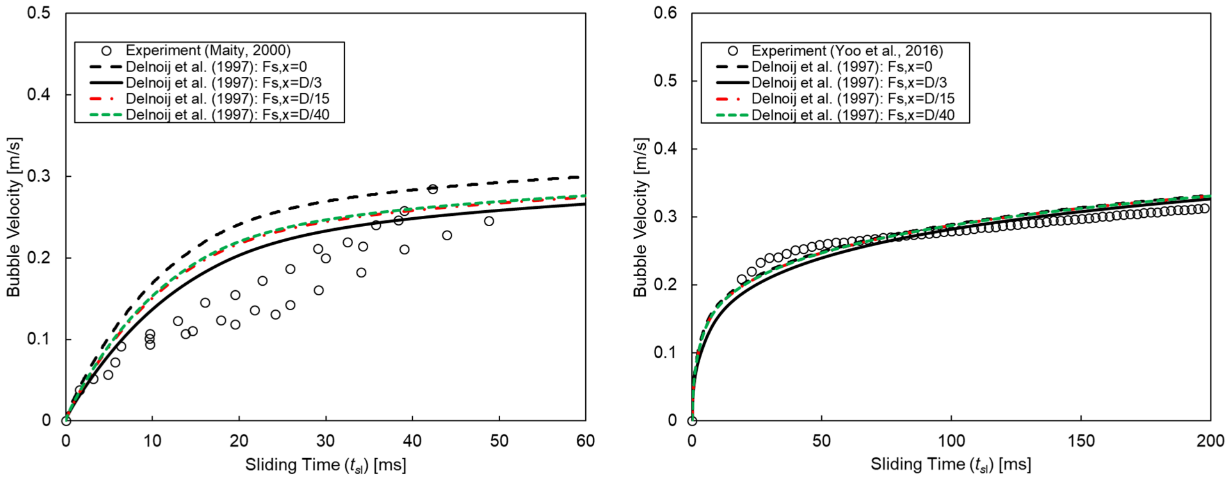


Figure 6. Effect of bubble contact diameter modeling on the prediction of sliding bubble velocity [Experimental data: Maity (M3, left side), Yoo et al. (Y7, right side)]

4.2. Base Line Model and Its Predictive Performance

This section proposes a base line force balance model based on the model parametric study discussed in the previous sections. The constituent sub-models and specific model assumptions introduced are described in Table IV. It is important to note that the predictive performance of the new base line model was demonstrated in terms of predicting both sliding bubble growth and sliding bubble velocity (only part of the results is presented in this paper). This is an important step toward the future research to properly

Table IV. Constitutive models and assumptions employed for the present base line model

Force components or parameters to be modeled	Models/equations
Quasi-steady drag force (F_{qs})	(i) F_{qs}=0.5C_D·ρ_l· (u_l-u_b) |u_l-u_b|πR^2 (ii) C_D is determined as Delnoij et al. (1997): $\begin{cases} C_D = 240 \quad for\ Re_b \le 0.1 \\ C_D = (24/Re_b)(1+0.15 Re_b^{0.687}) \quad for\ 0.1 \le Re_b \le 1000 \\ C_D = 0.44 \quad for\ Re_b \ge 1000 \end{cases}$ (where, Re_b=2·$R(t)$|$u_l(y)$-$u_b(y)$|/ν_l)
Liquid acceleration (for determining F_{AM})	Neglected (i.e., $\frac{du_l}{dt}$=0)
Bubble contact diameter (for determining F_s)	Neglected (i.e., F_s is simply neglected for the base line model described in this section)
Friction velocity (for near-wall local liquid velocity modeling)	u_τ=0.05u_l
Near-wall local liquid velocity	Reichardt's expression for single-phase turbulent flow [13]
Sliding bubble growth	Yoo et al. (2018) [23]: $D(t_{sl}) = \dfrac{2A' t_{sl}^{0.5}\left(1+\frac{B' t_{sl}}{3}\right)+D_0}{1+B' t_{sl}}$

predict the bubble lift-off diameter in a mechanistic fashion using force balance model. The exemplary validation results for the present base line force balance model are shown in Figure 7.

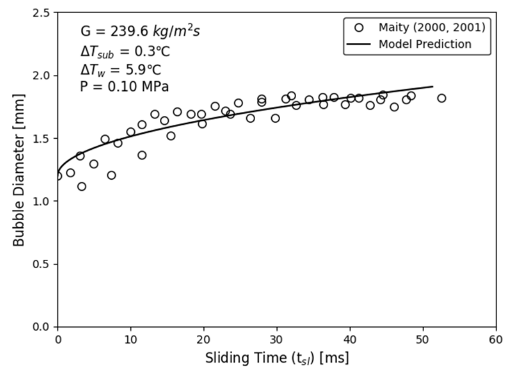
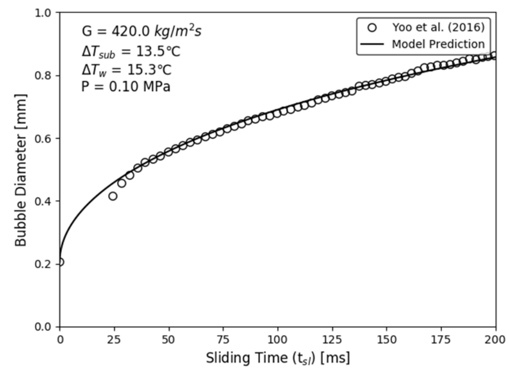
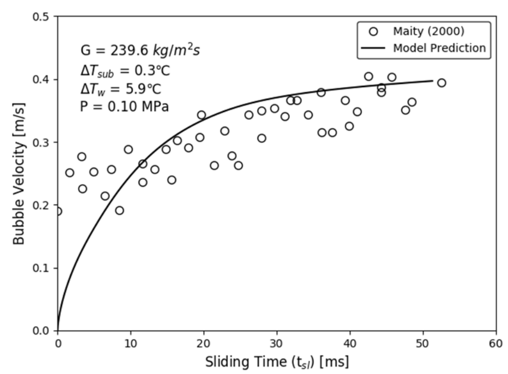
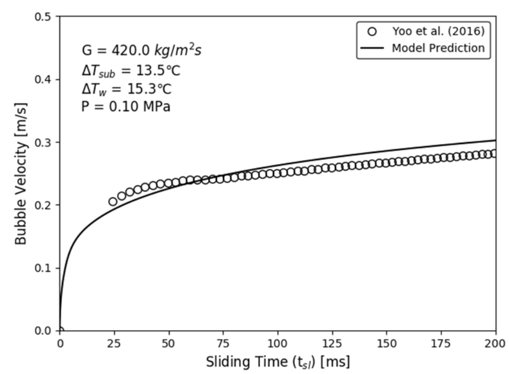
<table>
<tr><th></th><th>Comparison with Maity data (M1)</th><th>Comparison with Yoo data (Y6)</th></tr>
<tr><td>Sliding bubble growth</td><td>
</td><td>
</td></tr>
<tr><td>Sliding bubble velocity</td><td>
</td><td>
</td></tr>
</table>

Figure 7. Sliding bubble growth and sliding bubble size predicted by the base line model proposed from the present work

5. CONCLUSIONS

This study investigated the parametric effect of key sub-models used in the force balance model as well as the various modeling assumptions that have long been employed by previous researchers. Discussion was made by comparing the force balance model predictions with experimental data measured under various subcooled boiling conditions of both water and refrigerant flowing upward through vertical channels. The comparative analyses revealed the better choice of key sub-models, such as bubble growth and drag force models, and the modeling assumptions. Based on this study, a base line force balance model has been proposed to predict both the sliding bubble velocity and sliding bubble growth, and the model performance was proven by comparison with experimental data.

It is important to note that the present study is a critical step toward the future research to predict the bubble lift-off diameter in a mechanistic fashion using force balance model. Meanwhile, there is still uncertainty in the modeling of surface tension force as well as near-wall local liquid velocity in two-phase flow, and thus further study is required from this perspective.

NOMENCLATURE

A_{flow}	Flow area [m^2]
$A^{'}$	Model parameter used in Yoo et al. bubble growth model (see [23])
b	Empirical growth constant used in Zuber model [-]
$B^{'}$	Model parameter used in Yoo et al. bubble growth model (see [23])
d_w	Bubble contact diameter [m]
D_0	Initial sliding bubble diameter or departure diameter [m]
D_b	Bubble diameter [m]
D_h	Hydraulic diameter [m]
F	Force [N]
F_b	Buoyancy force [N]
F_{qs}	Quasi-steady drag force [N]
F_s	Surface tension force [N]
F_{AM}	Added mass force [N]
F_l	Shear lift force [N]
F_h	Hydrodynamic pressure force [N]
F_{cp}	Contact pressure force [N]
m_b	Mass of bubble [kg]
n	Index of a time step [-]
R	Bubble radius [m]
t_{sl}	bubble sliding time [s]
ΔT_{sub}	Subcooling degree [K]
ΔT_{sup}	Wall superheat [K]
u_l	liquid velocity [m/s]
u_b	bubble velocity [m/s]
u_τ	Friction velocity [m/s]
$U_{l,bulk}$	Bulk liquid velocity [m/s]
ΔU	Relative velocity $[u_l(y) - u_b(y)]$ [m/s]

Greeks

χ	Vapor quality [-]
φ	Channel inclination angle [radian]
θ_i	bubble inclination angle [radian]
υ_l	Kinematic viscosity of liquid [m^2/s]

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