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# Agenda

## Introduction

## Detection Models

- Mathematical Basis
- Intuition
- Research Challenges

## Waveform Correlation and Linear Discriminant Functions

- Comparison
- Time dependence

## Alternative Null Hypothesis Correlation Method

- Training Templates
- Matched Filter and ANCorr Template Example
- Experimental Setup and Results

## Research Contributions

Waveform correlation uses waveform templates to detect and identify repeating seismic events.

High false alarm rates for global and regional monitoring of sparse networks, where false alarms are caused by geographically non-collocated sources, not noise.

Alternate Null Hypothesis Correlation (ANCorr) applies machine learning algorithms to generate alternative templates that increase the detection/false alarm ratio.

Conventional template matching is based on matched filter (MF)

- Optimal linear filter for maximizing the signal-to-noise ratio (SNR)
- In the presence of additive white Gaussian noise (AWGN).

The matched filter detection model consists of binary hypotheses:

$$\mathcal{H}_0: \mathbf{x} = \sigma \mathbf{n}$$

Null hypothesis  $\mathcal{H}_0$ :  
the data contains only noise<sup>†</sup>.

$$\mathcal{H}_1: \mathbf{x} = A\mathbf{w}_{\text{MF}} + \sigma \mathbf{n}$$

Signal present hypothesis  $\mathcal{H}_1$ :  
the data contains an amplitude-scaled  
copy of the template waveform and noise.

<sup>†</sup>Noise vector is drawn from a multivariate normal distribution  $\mathbf{n}: \mathcal{N}(\mathbf{0}, \mathbf{I})$  and scaled by constant  $\sigma$



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- Optimal linear filter for maximizing the signal-to-noise ratio (SNR)
- In the presence of additive white Gaussian noise (AWGN).

The matched filter detection model consists of binary hypotheses:

$$\mathcal{H}_0: \mathbf{x} = \sigma \mathbf{n}$$

*Problem: Matched filter detection model does not account for the presence of realistic alternative signals, such as other seismic waveforms.*

$$\mathcal{H}_1: \mathbf{x} = A\mathbf{w}_{\text{MF}} + \sigma \mathbf{n}$$

†Noise vector is drawn from a multivariate normal distribution  $\mathbf{n}: \mathcal{N}(\mathbf{0}, \mathbf{I})$  and scaled by constant  $\sigma$

## Alternate Null-Hypothesis Correlation (ANCorr)

ANCorr detection model consists of binary hypotheses, but with an alternate null hypothesis,  $\mathcal{H}_0^*$ :

$$\mathcal{H}_0^*: \mathbf{x} = B\mathbf{v} + \sigma\mathbf{n}$$

$\mathcal{H}_0^*$  recognizes that in addition to noise, other seismic arrival waveforms may be present in the data.

$$\mathcal{H}_1: \mathbf{x} = A\mathbf{w}_{\text{MF}} + \sigma\mathbf{n}$$

We develop an ANCorr template to maximize the SNR ratio between  $\mathcal{H}_0^*$  and  $\mathcal{H}_1$ . In other words, an ANCorr template is synthesized to detect collocated events but minimize false alarms from seismic arrivals with a different event location.

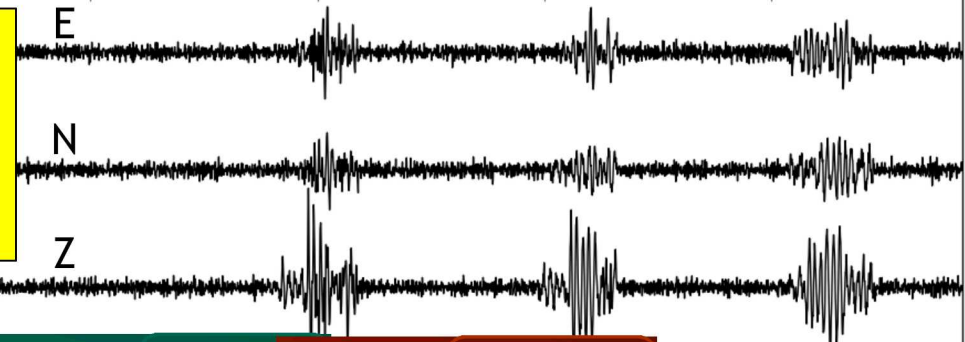
# 7 Defining Geodistance for 3 Seismic Arrivals at a Three-component Station

Template location  
(-7.07, 129.41)



**Geodistance** definition:  
Epicentral separation of the  
template's seismic event and  
arrival's seismic event.

Seismic Arrival Waveforms



0.26°  
arrival

21°  
arrival 2

127°  
arrival 3

Arrival 1 location  
(-6.85, 129.56)



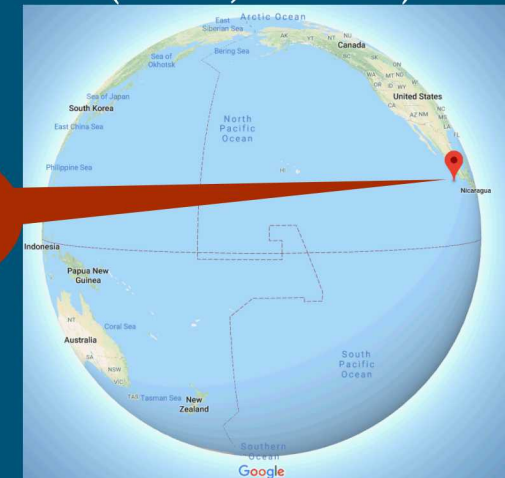
Detection  
desired

Arrival 2 location  
(11.08, 140.46)



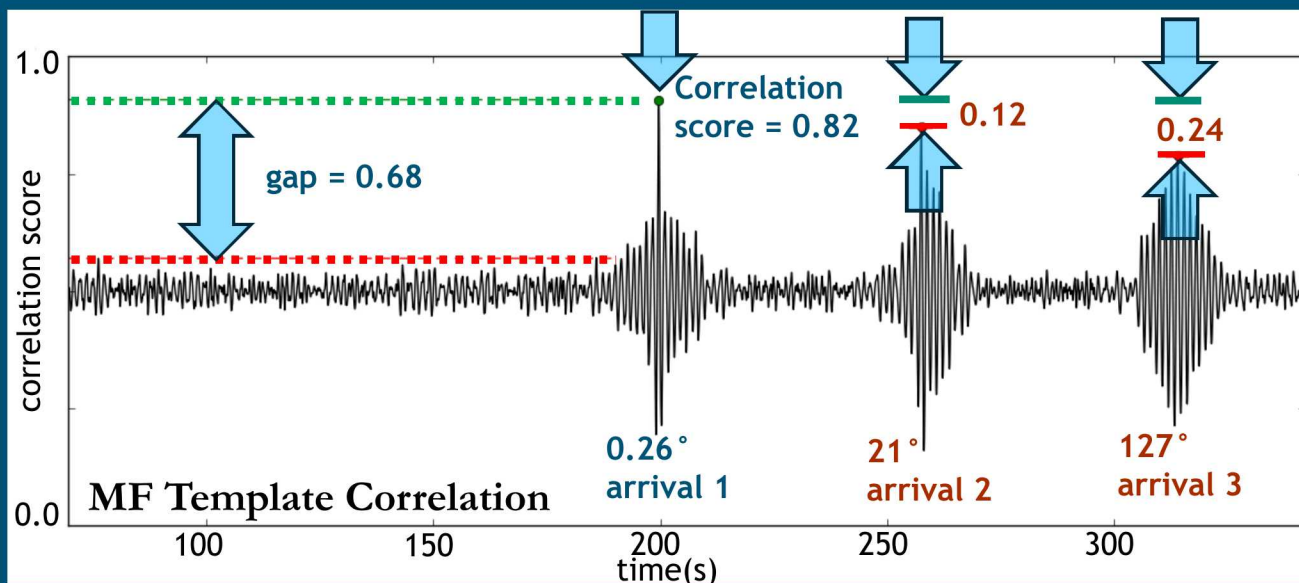
Detection  
not desired

Arrival 3 location  
(15.32, -104.57)



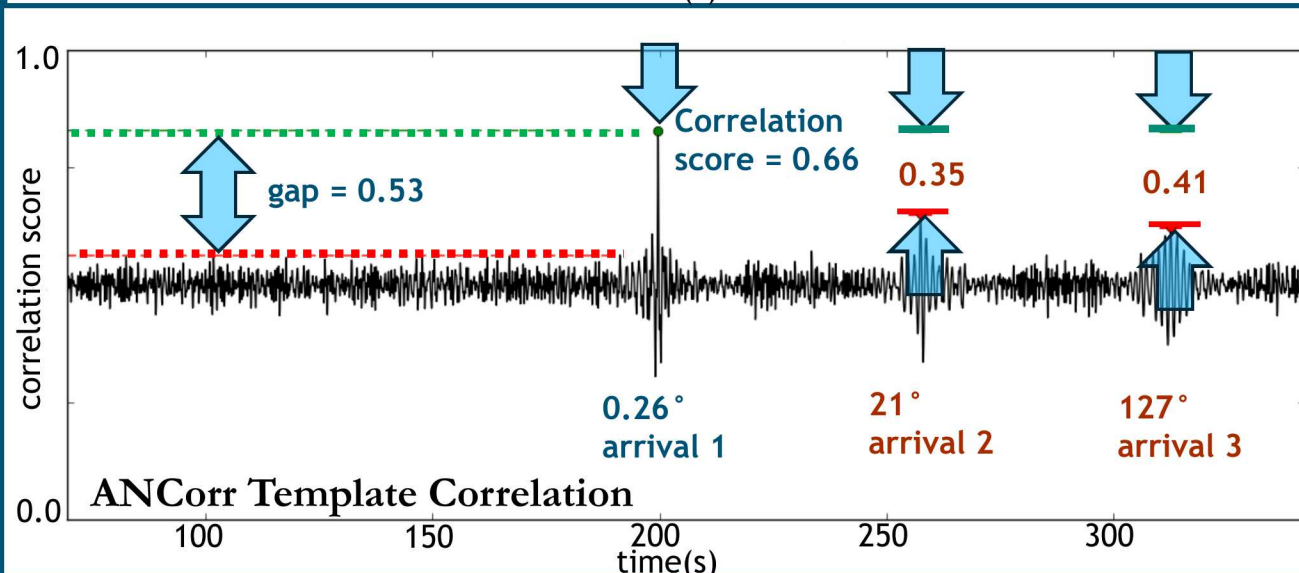


# Correlation Functions: Matched Filter and ANCorr Templates



Arrival	MF Score
1	0.82
2	0.7
3	0.58
Noise	0.14

*False  
Detections  
Likely*



Arrival	ANCorr Score
1	0.66
2	0.31
3	0.25
Noise	0.13

*Reduced  
Chance of  
False  
Detections*



How to make an ANCorr template to maximize the SNR ratio between  $\mathcal{H}_0^*$  and  $\mathcal{H}_1$ ?

$$\mathcal{H}_0^*: \mathbf{x} = B\mathbf{v} + \sigma\mathbf{n}$$

$$\mathcal{H}_1: \mathbf{x} = A\mathbf{w}_{\text{MF}} + \sigma\mathbf{n}$$

Research Challenges:

1. Unlike white Gaussian noise, seismic arrivals are nonstationary.



Stationary Noise: Swapping windows of noise will not lead to a different answer from the detection algorithm.

## Research Challenges of Alternate Null-Hypothesis Correlation Model

How to make an ANCorr template to maximize the SNR ratio between  $\mathcal{H}_0^*$  and  $\mathcal{H}_1$  ?

$$\mathcal{H}_0^*: \mathbf{x} = B\mathbf{v} + \sigma\mathbf{n}$$

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# Research Challenges of Alternate Null-Hypothesis Correlation Model

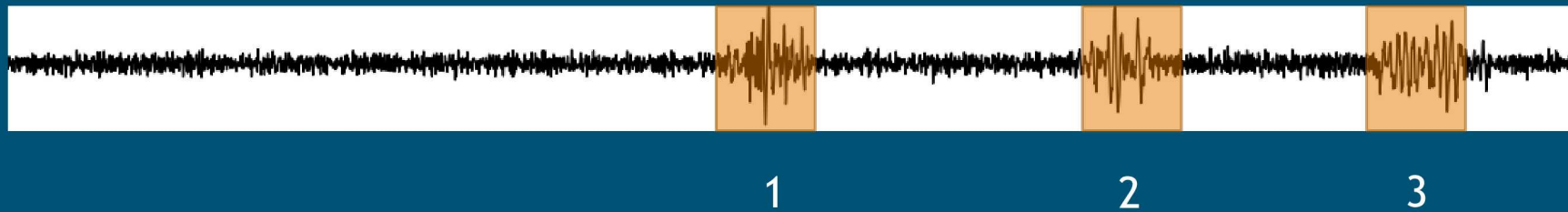
How to make an ANCorr template to maximize the SNR ratio between  $\mathcal{H}_0^*$  and  $\mathcal{H}_1$  ?

$$\mathcal{H}_0^*: \mathbf{x} = B\mathbf{v} + \sigma\mathbf{n}$$

$$\mathcal{H}_1: \mathbf{x} = A\mathbf{w}_{\text{MF}} + \sigma\mathbf{n}$$

Research Challenges:

1. Unlike white Gaussian noise, seismic arrivals are nonstationary.
2. Nondeterministic: Each possible event and seismic phase that can generate  $B\mathbf{v}$  must be considered.





How to make an ANCorr template to maximize the SNR ratio between  $\mathcal{H}_0^*$  and  $\mathcal{H}_1$  ?

$$\mathcal{H}_0^*: \mathbf{x} = B\mathbf{v} + \sigma\mathbf{n}$$

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Confluence of three factors enable the ANCorr method:

1. the new model of  $\mathcal{H}_0^*$  and  $\mathcal{H}_1$ ;
2. the large sampling of labeled  $\mathbf{v}$ 's (i.e., seismic arrivals) available in seismic bulletins; and
3. the similarity between correlation-based detection and linear discriminant functions (LDFs).

*These three factors provide sufficient conditions for supervised machine learning.*

*We will exploit machine learning to create ANCorr templates.*

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Alternative Null Hypothesis Correlation Method

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Research Contributions

# Comparison: Waveform Correlation and LDF equations

## Waveform Correlation

The correlation function for data  $\mathbf{x}$  and template  $\mathbf{w}$  at time  $n$  has correlation score:

$$R(\mathbf{x}, n) = \mathbf{w}^T \mathbf{x}_n$$

Incorporate the template threshold to make new function:

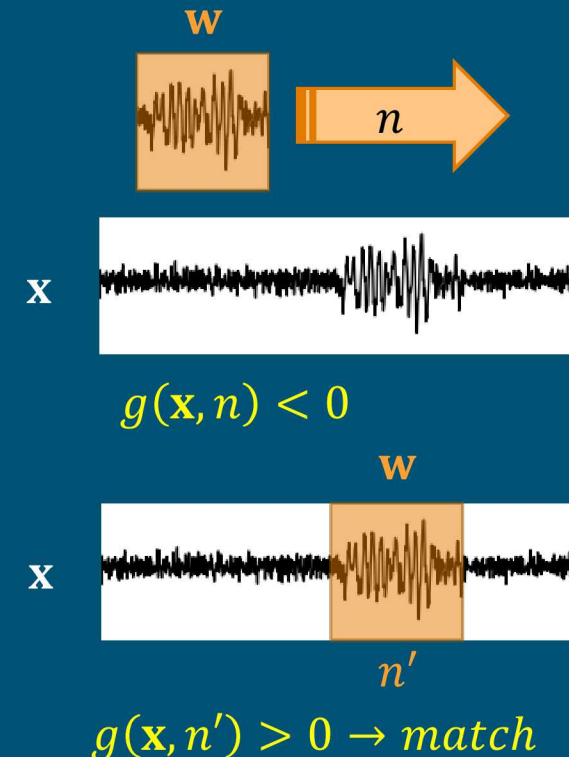
$$g(\mathbf{x}, n) = R(\mathbf{x}, n) - \omega_0 = \mathbf{w}^T \mathbf{x}_n - \omega_0$$

With match criterion at the time when maximum correlation occurs:

$$g(\mathbf{x}, n') > 0 \rightarrow \text{match}$$

$$g(\mathbf{x}, n') < 0 \rightarrow \text{no match}$$

We interpret correlation as a series of dot products by incrementing  $n$  and sliding the template  $\mathbf{w}$  over the data stream  $\mathbf{x}$ .





## Comparison: Waveform Correlation and LDF equations

### Waveform Correlation

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### Linear Discriminant Functions

A **discriminant function** is a linear combination of the components of the data vector  $\mathbf{u}$  where  $\mathbf{w}^T$  is the weight vector and  $\omega_0$  is the bias (Duda, *et al.*, 2001):

$$g(\mathbf{u}) = \mathbf{w}^T \mathbf{u} - \omega_0$$

In the two-category case, classification of  $\mathbf{u}$  goes by the decision rule

$$g(\mathbf{u}) > 0 \rightarrow \text{class } 1$$

$$g(\mathbf{u}) < 0 \rightarrow \text{class } -1$$

# Comparison: Waveform Correlation and LDF equations

## Waveform Correlation

The correlation function for data  $\mathbf{x}$  and template  $\mathbf{w}$  at time  $n$  has correlation score:

$$R(\mathbf{x}, n) = \mathbf{w}^T \mathbf{x}_n$$

Incorporate the template threshold to make new function:

$$g(\mathbf{x}, n) = R(\mathbf{x}, n) - \omega_0 = \boxed{\mathbf{w}^T \mathbf{x}_n - \omega_0}$$

With match criterion at the time when maximum correlation occurs:

$$\begin{aligned} g(\mathbf{x}, n') &> 0 \rightarrow \text{match} \\ g(\mathbf{x}, n') &< 0 \rightarrow \text{no match} \end{aligned}$$

## Linear Discriminant Functions

Modified correlation function looks like linear discriminant function:

$$g(\mathbf{u}) = \boxed{\mathbf{w}^T \mathbf{u} - \omega_0}$$

Match criterion looks like LDF classification rules:

$$\begin{aligned} g(\mathbf{u}) &> 0 \rightarrow \text{class 1} \\ g(\mathbf{u}) &< 0 \rightarrow \text{class } -1 \end{aligned}$$

Linear Discriminant Functions

$$g(\mathbf{u}) = \mathbf{w}^T \mathbf{u} - \omega_0$$

$$g(\mathbf{u}) > 0 \rightarrow \text{class } 1$$

$$g(\mathbf{u}) < 0 \rightarrow \text{class } -1$$

Waveform Correlation

$$g(\mathbf{x}, n) = R(\mathbf{x}, n) - \omega_0 = \mathbf{w}^T \mathbf{x}_n - \omega_0$$

Time-dependent linear discriminant function.

$$g(\mathbf{x}, n') > 0 \rightarrow \text{match}$$

$$g(\mathbf{x}, n') < 0 \rightarrow \text{no match}$$

LDF classification is equivalent if

$$\mathbf{u} = \mathbf{x}_{n'}$$

Insight: Overcome the time dependence by evaluating the waveform correlation functions for a single time index that is known prior to training the ANCorr templates.



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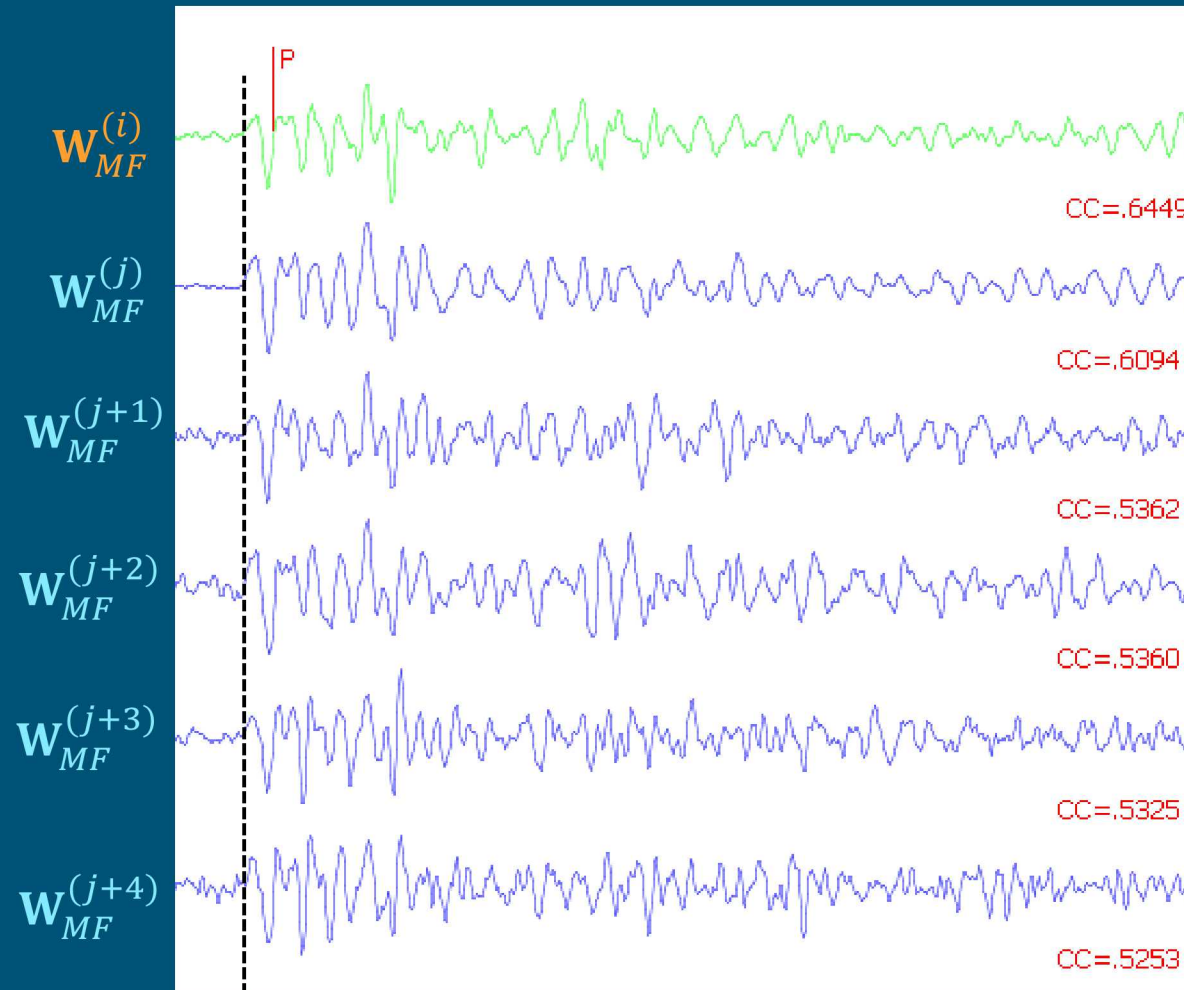
## Alternative Null Hypothesis Correlation Method

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## Research Contributions

# Training: Determine time of maximal correlation $n'$

1. Start with a matched filter template library of arrival waveforms at a station.
2. Choose the  $i$ th template.
3. Circularly shift every other template in the matched filter library to align with the time lag of maximal correlation.



$n'_{MF}(i,j)$

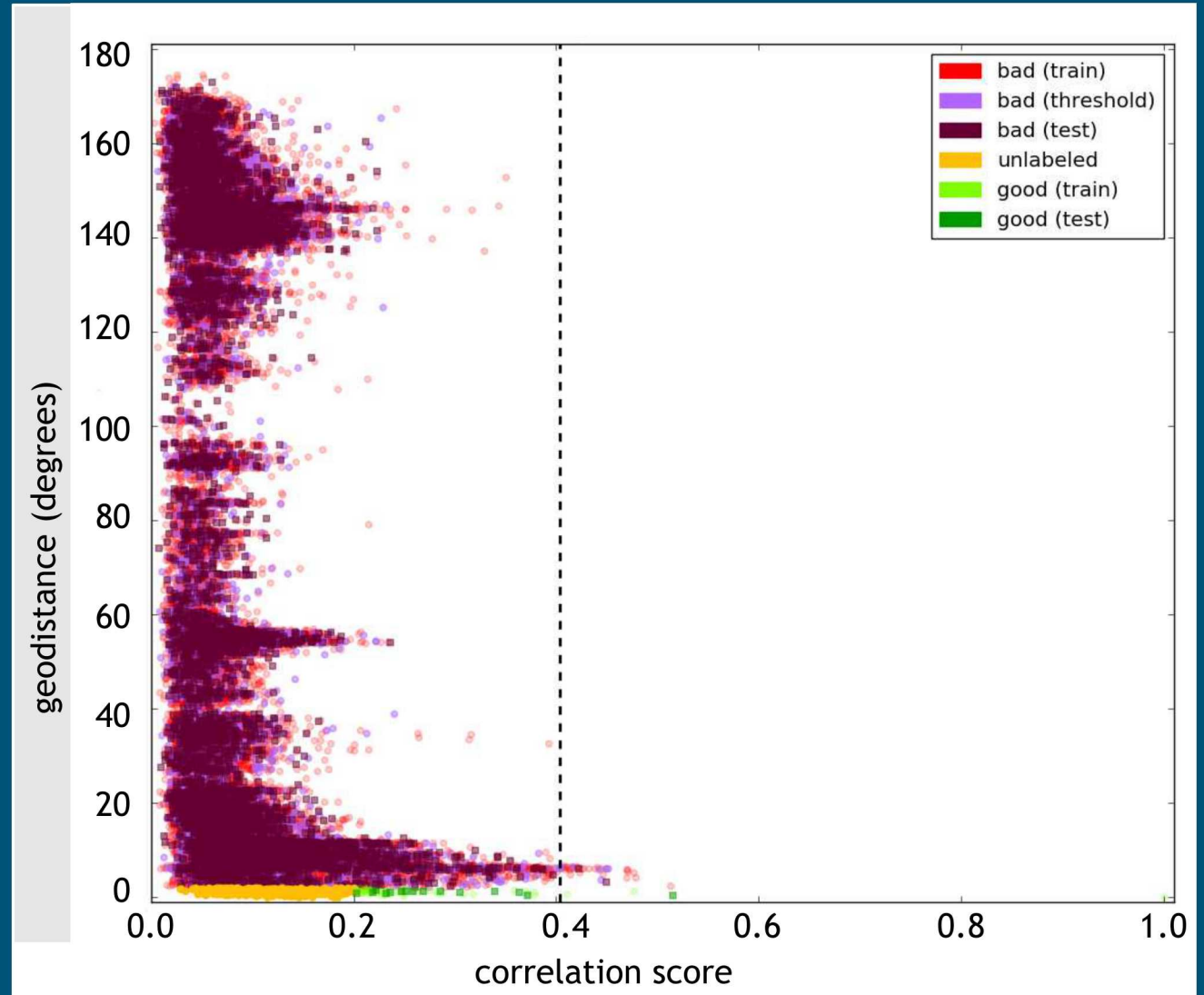
Time lag (i.e., sample index) of maximal correlation for  $i$ th template and every  $j$ th template,  $i \neq j$

## Training: Label Matched Filter Templates

Label templates to make a training set for the  $i$ th template.

- Only template waveforms from nearby sources are desired as matches
- Most templates in the library are labeled *No match* training class for a geodistance of 2 degrees.

Choose a template correlation threshold that rejects 99.9% of templates in the *No match* training class.



↑ Template Threshold



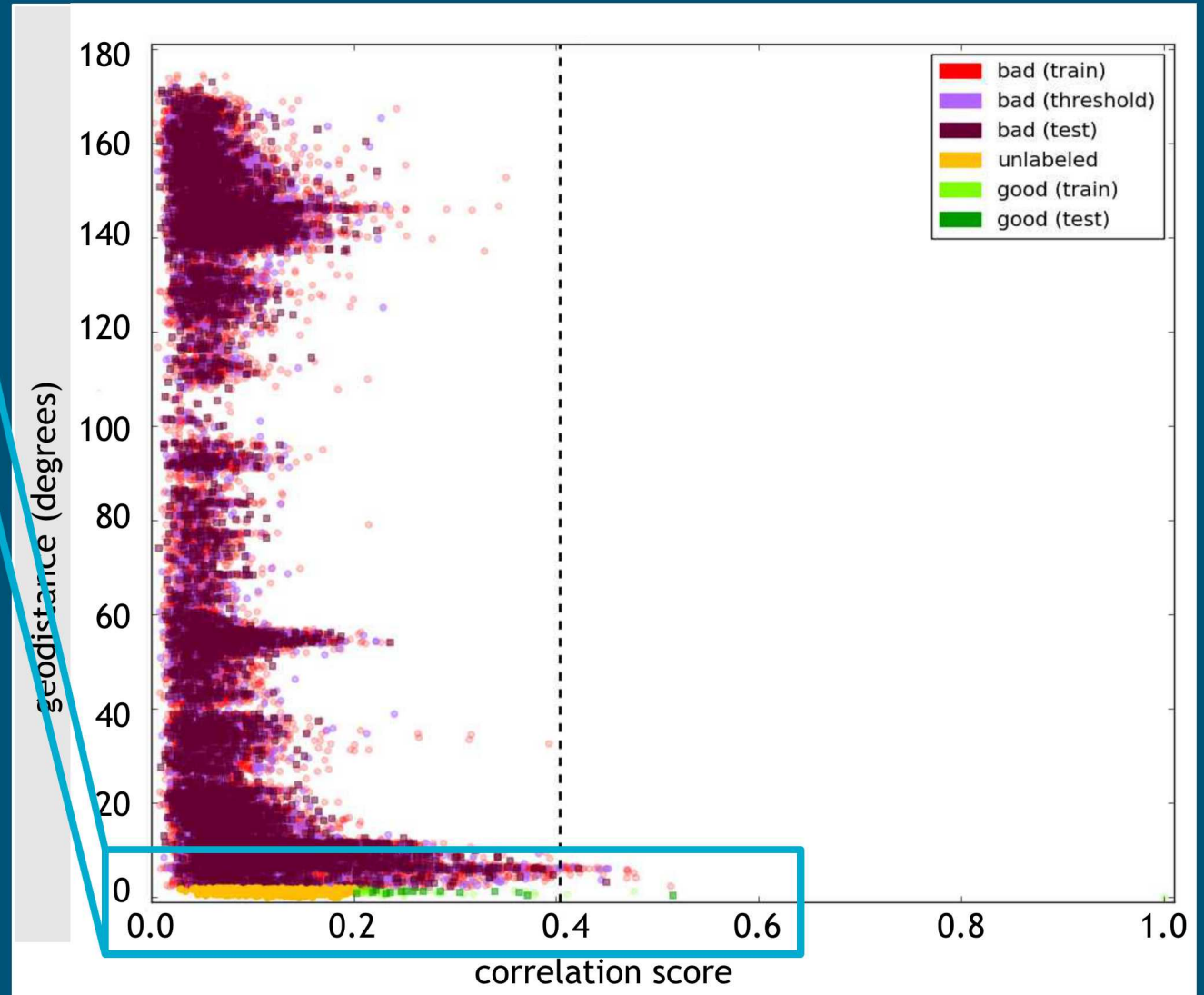
## Training: Waveform similarity as a tuning parameter

What about multiple sources or different source types that are geographically collocated?



↑ similarity constraint = 0.5

The **similarity constraint** restricts the waveforms in the *match* class according to correlation with matched filter template. Dissimilar waveforms are removed from the training set.



↑ Template Threshold

## Training: Synthesize ANCorr Template from Training Set

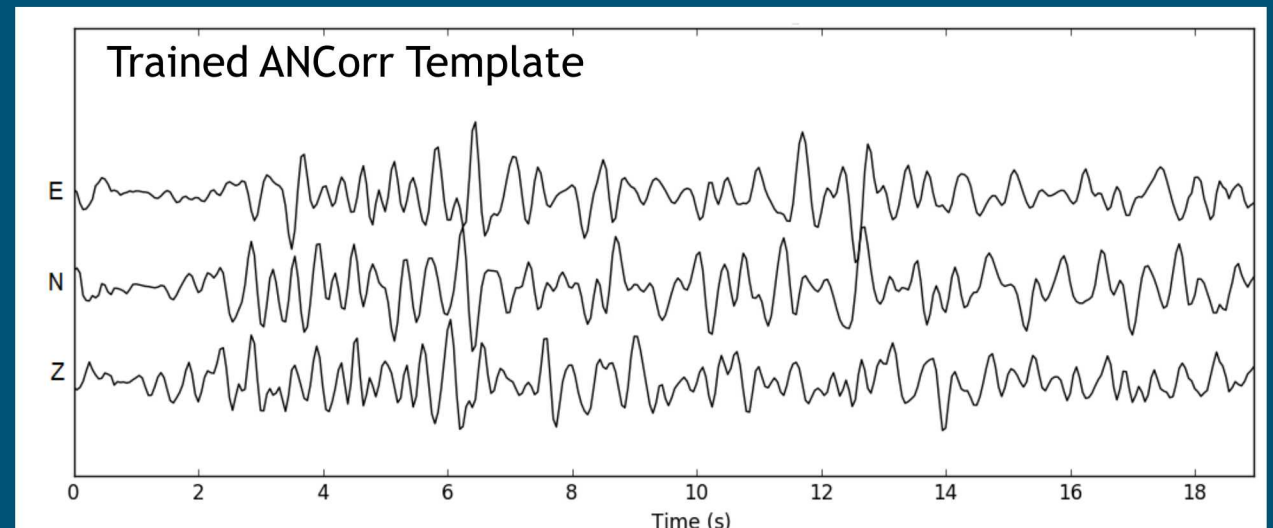
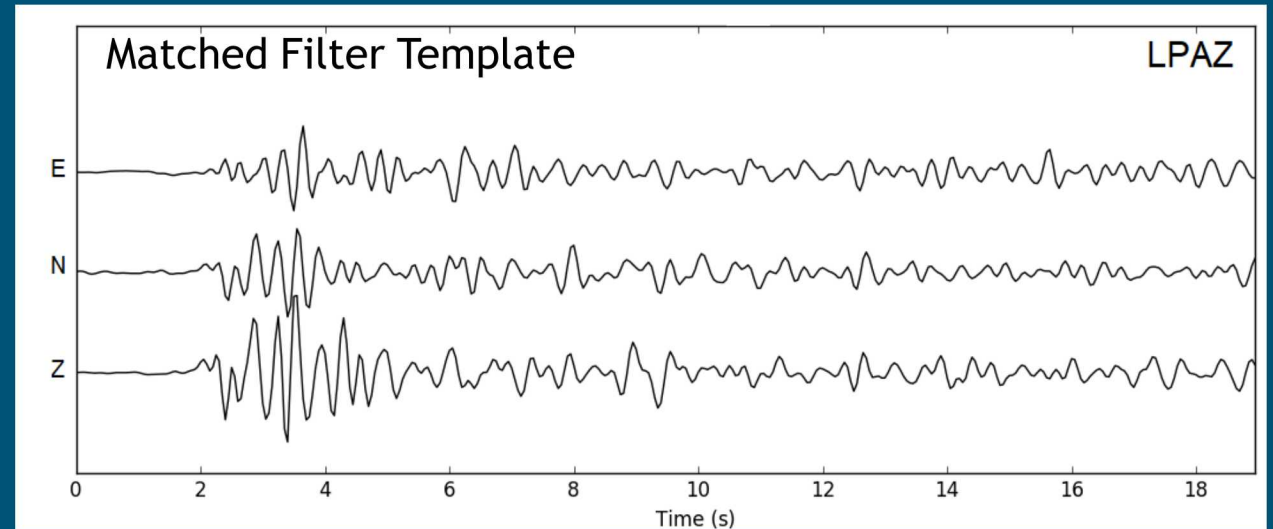
Synthesize the ANCorr template by applying machine learning algorithm to the training set of time-shifted and labeled matched filter templates.

We used a Linear Support Vector Machine (LSVM) implementation in Python.

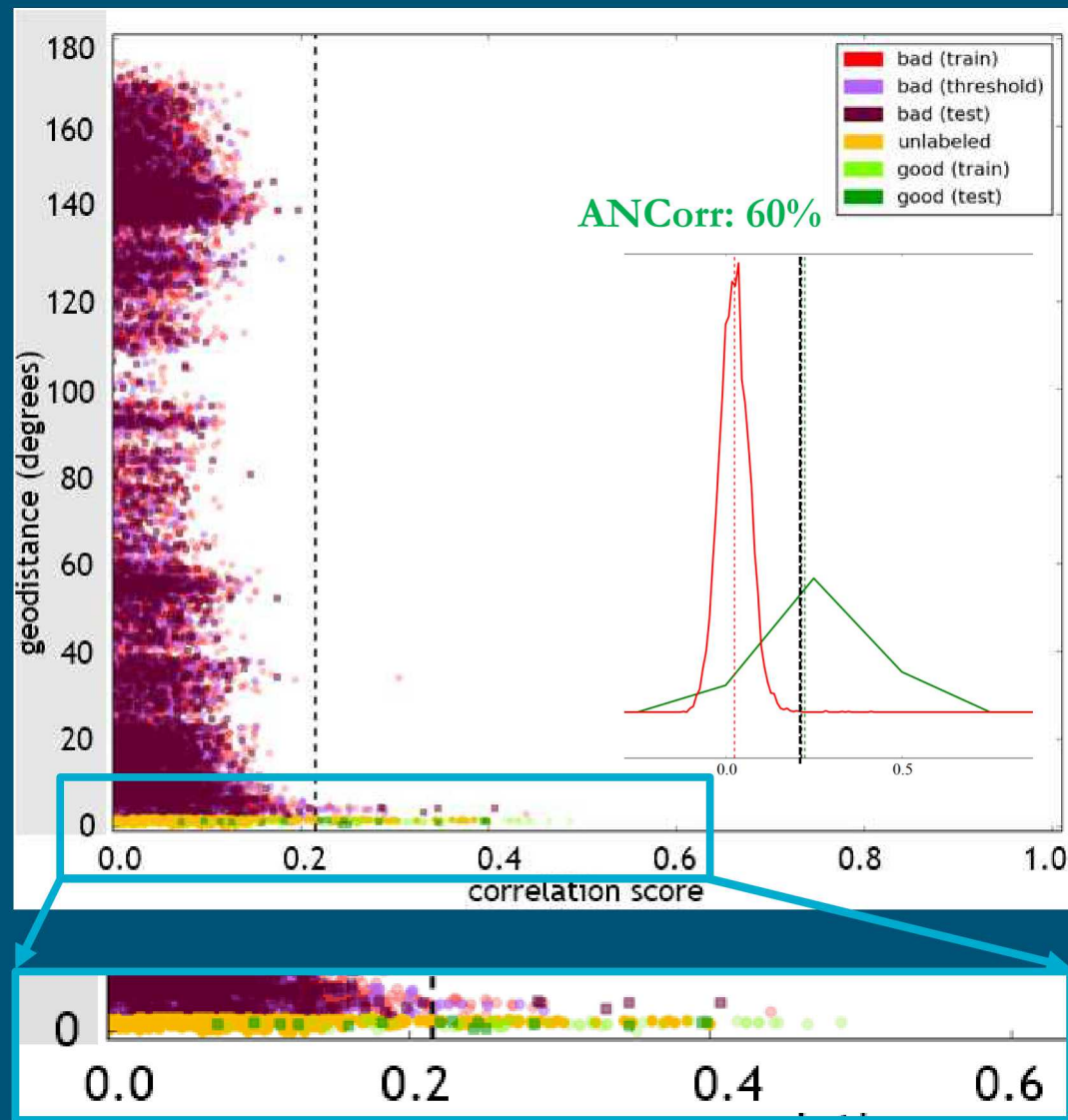
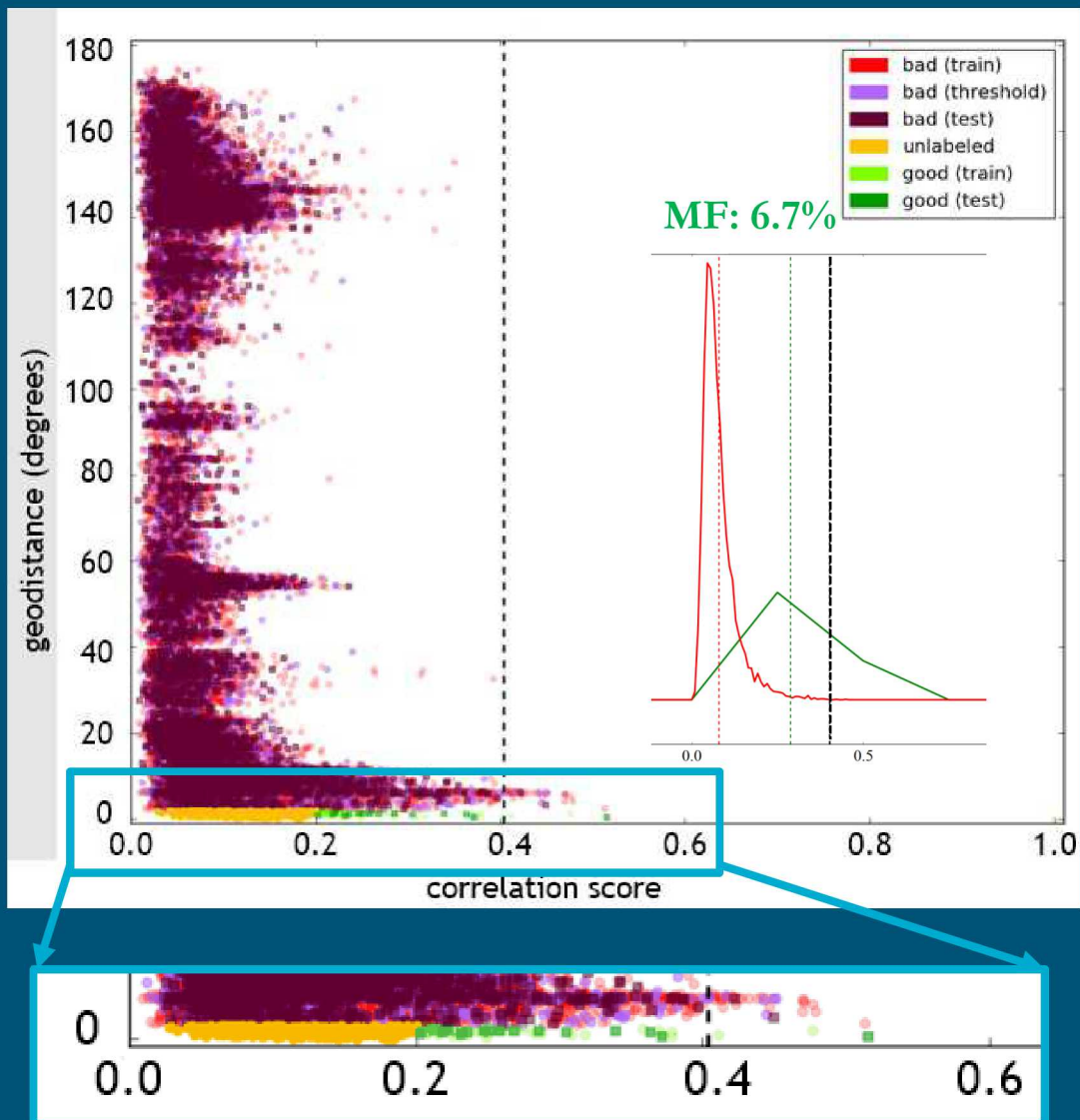
### Set ANCorr Template Threshold

1. Correlate the ANCorr template with the training set.
2. Set the ANCorr template threshold to reject 99.9% of templates in the *No match* training class.

Late Event Bulletin [LEB] P (-15.97, -75.74), 1 January 2006 03:01:39



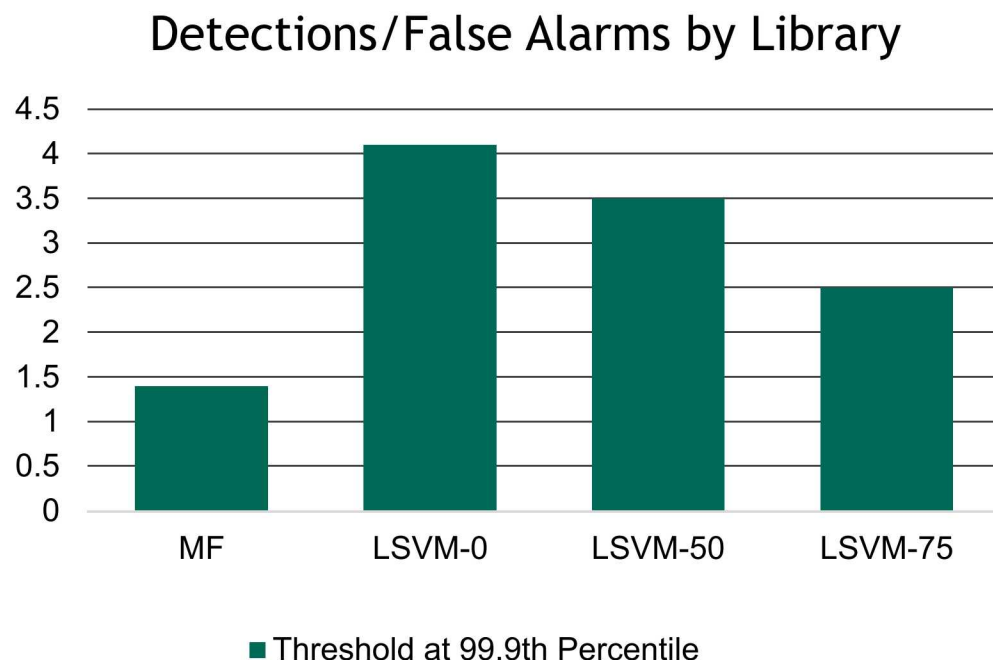
# Matched Filter and ANCorr Template Example





- We created template libraries from the three-component station LPAZ, located in La Paz, Bolivia.
- We chose a set of 51,690 template arrivals from the International Data Centre (IDC) Late Event Bulletin (LEB) for the time range of 2006 to 2016 as the training set.
- The month of May 2010 was reserved for a labeled test set, not part of training set.
- The template libraries were applied to 663 arrivals from the labeled test set.
- Application Method:
  1. Correlate with MF template to detect matches exceeding threshold.
  2. Correlate with ANCorr template at time of maximal correlation to reject false alarms.
- The detection/false alarm ratio was chosen as the metric for comparison.
- Four ANCorr libraries were trained, with *Match* class parameterized by similarity constraint.





Library	Description
MF	Matched Filter Library from seismic arrivals at station LPAZ.
All ANCorr template libraries were trained by LSVM where the <i>Match</i> training class membership included MF waveforms for events within 2 degrees geodistance and:	
LSVM-0	no similarity constraint.
LSVM-50	similarity constraint 0.5 (correlation score exceeded 50% of template threshold).
LSVM-75	similarity constraint 0.75 (correlation score exceeded 75% of template threshold).

Overall, the ANCorr templates demonstrated an improved ratio of detections to false alarms over the matched filter library for this dataset.

- The best performing library (LSVM-0) used all waveforms within 2 degree geodistance of template event.
- Removing dissimilar waveforms from the *Match* class decreased the overall detection/false alarm ratio.

- Defined an alternative detection model for waveform correlation that improves upon the conventionally applied matched filter method by modifying the null hypothesis to account for non-collocated seismicity.
- Recognized the mathematical similarity of linear discriminant functions to waveform correlation equations, justifying the use of machine learning for template synthesis.
- Simplified the time dependence of waveform correlation so that linear discriminant functions can be used for template synthesis.
- Developed a method of template synthesis that uses the new detection model.

Ganter, T., A. Sundermier, and S. Ballard (2018). Alternate Null Hypothesis Correlation: A New Approach to Automatic Seismic Event Detection. *Bulletin of the Seismological Society of America*, <https://doi.org/10.1785/0120180074>.

Scharf, L. L. (1991). *Statistical Signal Processing: Detection, Estimation, and Time Series Analysis*, Chapter 4: Neyman-Pearson Detectors, First Ed., Addison-Wesley Publishing Company, Reading, Mass.

Duda, R. O., P. E. Hart, and D. G. Stork (2001). *Pattern Classification*, Chapter 5: Linear Discriminant Functions, Second Ed., John Wiley & Sons, Inc., New York, New York.