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A stochastic general equilibrium approach to an infrastructure planning problem

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Albuquerque, NM

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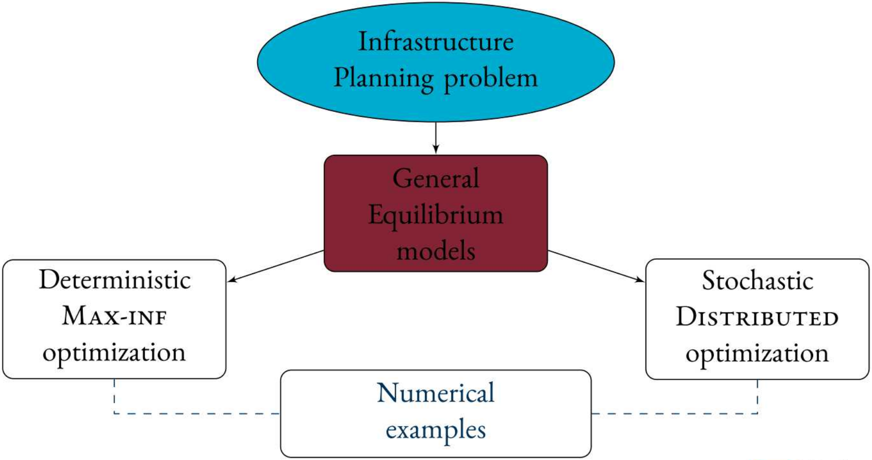
About this talk



Joint work with Dr.Walter Guo (UCF),
and Dr.Yueyue Fan (UCDavis)

Infrastructure planning problems

Participants We have a multi-agent system, formed by Investors/Producers and Consumers

Interaction They interact independently by trade, using a price system that indicates the market value of a commodity in a given time-space situation

Network structure Interaction between agents are constrained to a network structure, reflecting the physical connection between points of trade/interaction

Problem

Need to find the system state where the total production satisfies the total demand

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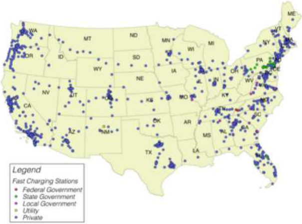
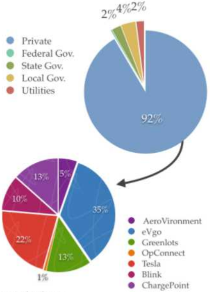
EV-charging station problem

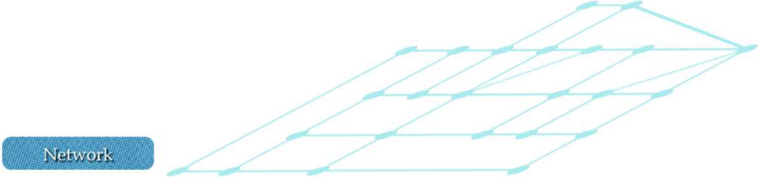
- Electric vehicle (EV) adoption is growing fast (EV sales data from Argonne)
- Charging infrastructure needs to catch up with the increasing charging demand
- Understanding the pattern of charging infrastructure investment is critical to evaluate the impacts of EV on both power and transportation systems





Data Source: http://www.afdc.energy.gov/locator/stations/, accessed on 02/17/2016

EV-charging station problem



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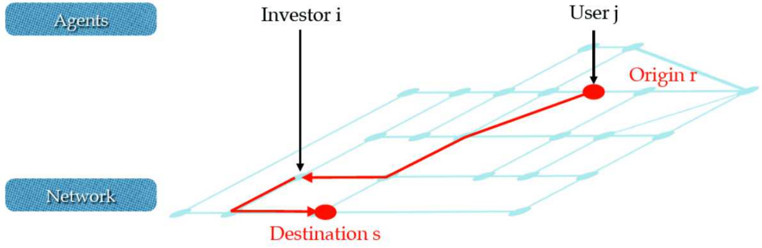
EV-charging station problem



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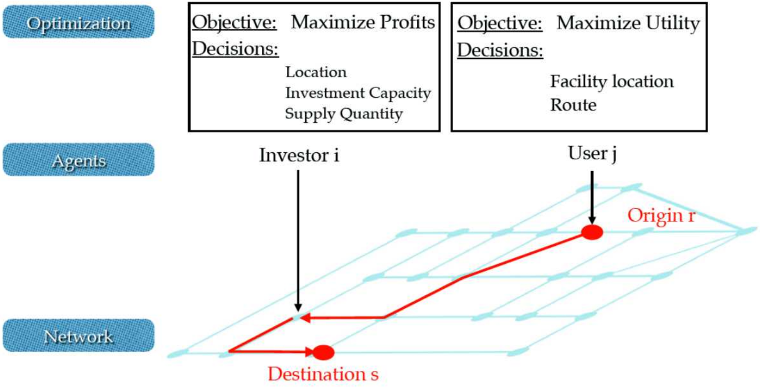
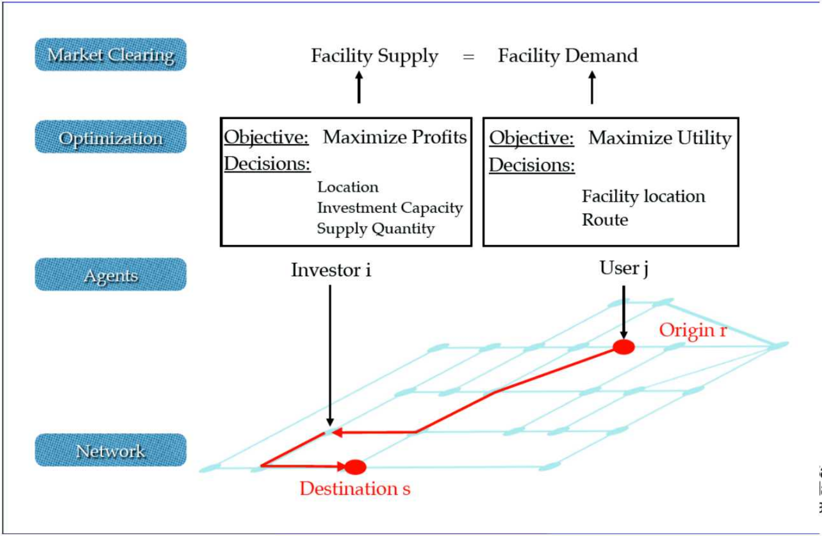
EV-charging station problem



EV-charging station problem



Mathematical modeling: General Setting

Markets We assume that interactions are made using markets

Perfect competition We assume that markets are perfectly competitive: there is a market for each desired commodity traded

Investors Multiple heterogeneous investors, maximizing profit (depending on capacity investment costs and generation costs

Consumers Drivers maximize utility, and ther facility choice depends on charging price, travel/charging time, and locational attractiveness

Traffic Assignment Wardrop user equilibrium

Perfect information Both investors and users have perfect information

Supply side: Investors' problem

$$\underset{c_i^s, g_i^s}{\text{maximize}} \quad \underbrace{\sum_{s \in S_i} \left[\varrho^s g_i^s - \varphi_g(g^s) \right]}_{\textit{operational net profit}} - \underbrace{\sum_{s \in S_i} \varphi_c(c_i^s)}_{\textit{investment costs}}$$

$$\text{subject to} \quad (\gamma_i^s) \quad \underbrace{g_i^s - c_i^s \leq 0}_{\textit{capacity constraint}}, \ \forall s \in S_i; g_i^s \geq 0, \ c_i^s \geq 0, \ \forall s \in S_i.$$

where:

- S_i : set of investment locations of firm i;
- c_i^s : charging capacity [kW] at location s by firm i;
- g_i^s : energy supply [kWh] at location s by i-firm;
- ϱ^s : unit charging price at location s (market determined);
- $\varphi_c(\cdot)$: capital cost wrt charging capacity;
- $\varphi_g(\cdot)$: operational cost wrt supply quantity and scenario;
- I : set of investors.

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Travelers and ISO problem

$$
\begin{aligned}
\underset{x,v,q}{\text{minimize}} \quad & U(\text{travel time, local attractiveness,} \\
& \quad \text{charging capacities, charging cost}) \\
\text{subject to} \quad & v_a = \sum_{r \in R} \sum_{s \in S} x_a^{rs}, \ \forall a \in \mathcal{A} \\
(\zeta^{rs}) \quad & A x^{rs} = q^{rs} E^{rs}, \ \forall r \in R, s \in S, \\
(\eta^{r}) \quad & \sum_{s \in S} q^{rs} = d^{r}, \ \forall r \in R \\
& x_a^{rs} \geq 0, \ \forall a \in \mathcal{A}, r \in R, s \in S \\
& v_a \geq 0, \ \forall a \in \mathcal{A} \\
& q^{rs} \geq 0, \ \forall r \in R, s \in S.
\end{aligned}
$$

where

$$U(x,v,q) = \underbrace{\sum_{a\in\mathcal{A}} \int_0^{v_a} t_a(u)\mathrm{d}u}_{\text{Wardrop User Equilibrium}} + \frac{1}{\beta_1} \sum_{r\in R} \sum_{s\in S} \underbrace{q^{rs} \left(\ln q^{rs} - 1 + \beta_3 \frac{\varrho^s e^{rs}}{inc^r} - \beta_2 \sum_{i\in I_s} c_i^s - \beta_0^s \right)}_{\text{Logit choice model}}$$

- $U(x,v,q)$: Consumers' utility function;
- x_a^{rs} : traffic flow on link a associated with O-D pair rs;
- v_a : traffic flow on link a;
- q^{rs} : traffic flow from r to s;
- $t_a(\cdot)$: travel time function of link a, e.g. the Bureau of Public Roads (BPR) function;
- $\mathcal{A}$: node-link incidence matrix of network (model input based on network topology);
- E^{rs} : O-D incidence vector of O-D pair rs with +1 at origin and -1 at destination (modeling input based on network topology);
- d^r : total EV travel demand from location r (model input).
- e^{rs} : average charging demand from r to s (model input);

Equilibrium

System equilibrium

The system is in equilibrium if there exists a price ϱ^* such that

- Investors maximize their utility by producing a quantity $g^i(\varrho^*)$,
- Transportation system maximizes the consumers' utility function with an optimal traffic flow of $q(\varrho^*)$,
- Charging market at each location is balanced, i.e., total supply meets total demand.

Market equilibrium Find ϱ^* such that

$$\mathrm{ES}_s(\varrho^*) := \underbrace{\sum_{i \in I_s} g_s^i(\varrho^*)}_{\textit{total generation at } s} - \underbrace{\sum_{r \in R} e^{rs} q^{rs}(\varrho^*)}_{\textit{total demand at } s} = 0, \; s \in S$$

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Solution strategy: Max-inf formulation

Walrasian bifunction

$$(\varrho, \varphi) \mapsto W(\varrho, \varphi) = -\sum_{s \in S} \varphi_s \mathrm{ES}_s^2(\varrho)$$

Maxinf characterization

ϱ^* is an equilibrium point $\iff$ $\varrho^* \in \operatorname{argmaxinf}_{(\varrho,\varphi)} W$

Main problem

find ϱ^* that maximizes $\varrho \mapsto \inf_{\varphi} W(\varrho, \varphi)$

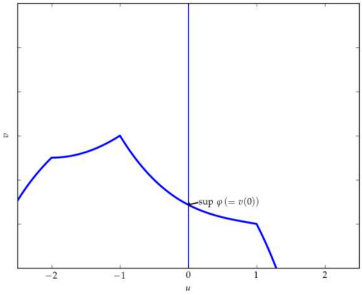
Approximation and non-concave duality

Main problem as maximization

Find ϱ^* that maximizes $\varrho \mapsto \inf_{\varphi} W(\varrho, \varphi)$

How to approximate?

Construct a sequence of bi-functions $\{W^\nu\}$ using Non-concave duality [?, 11.K]



Approximation and non-concave duality

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Find ϱ^* that maximizes $\varrho \mapsto \inf_\varphi W(\varrho, \varphi)$

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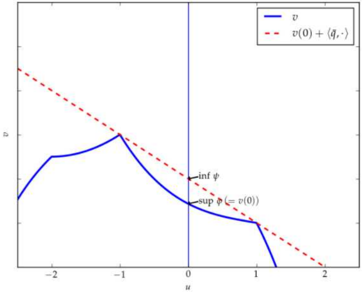

Approximation and non-concave duality

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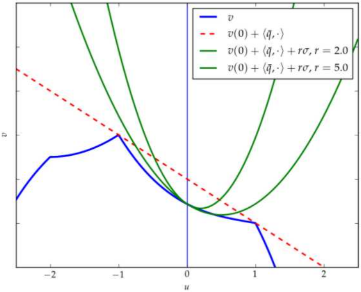

$\sigma : \mathbb{R}^m \to \bar{\mathbb{R}}$ proper, lsc, convex, $\min \sigma = 0$, $\operatorname{argmin} \sigma = \{0\}$

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Augmentation and Walras economic model

Augmented Walrasian

$$\tilde{W}^\nu(\varrho, \varphi) = \inf_z \{W(\varrho, \varphi - z) + r_\nu \sigma^*(r_\nu^{-1} z)\}$$

$$\begin{array}{ccccc}
\varrho^* & \in & 0 & \text{argmaxinf} & W \\
\uparrow & & \uparrow & & \uparrow \text{tight-lop} \\
\varrho^\nu & \in & \varepsilon^\nu & \text{argmaxinf} & \tilde{W}^\nu
\end{array}$$

Augmentation and Walras economic model

Augmented Walrasian

$$\tilde{W}^{\nu}(\varrho, \varphi) = \inf_{z} \{W(\varrho, \varphi - z) + r_{\nu}\sigma^{*}(r_{\nu}^{-1}z)\}$$

$$\varrho^{*} \in 0 \text{ argmaxinf } W$$

$\uparrow$ $\uparrow$ $\uparrow$ tight-lop

$$\varrho^{\nu} \in \varepsilon^{\nu} \text{ argmaxinf } \tilde{W}^{\nu}$$

Phase I (or primal)	Phase II (or dual)
$\varphi^{\nu+1} \in \varepsilon^{\nu+1}\text{-}\operatorname*{argmin}_{q\in\Delta} \tilde{W}^{\nu+1}(\varrho^{\nu}, q)$	$\varrho^{\nu+1} \in \varepsilon^{\nu+1}\text{-}\operatorname*{argmax}_{\varrho} \tilde{W}^{\nu+1}(\varrho, \varphi^{\nu+1})$
(solved with Gurobi)	(solved with BOBYQA)

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Numerical example

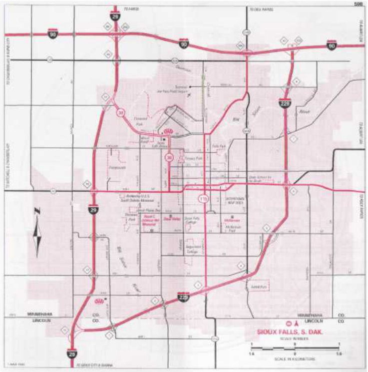

Figure: Sioux-falls network

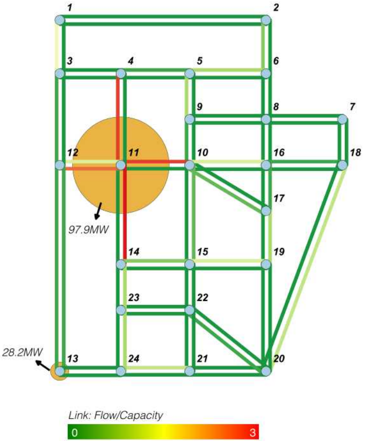

Figure: Solution

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Solution strategy: ADMM

Capacity investment problem as a MOPEC

In general terms, a capacity investment equilibrium problem can be writen as a Multiple Optimization Problems with Equilibrium Constraints (MOPEC) as follows

$$x^i \in \underset{x}{\operatorname{argmin}} f^i(x;p),\ i \in I \quad \sum_i x^i = 0$$

EV example

EV-charging station problem

In our example, we have functions of the form $f^i(x;p) = f_0^i(x^i) + \alpha^i\langle p,\ x^i\rangle$

$$f^i(x^i;p) = \underbrace{\sum_{s\in S_i} \varphi_g(g^s) + \varphi_c(c_i^s)}_{f_0^i(x^i)} + \underbrace{-\sum_{s\in S_i} \varrho^s g_i^s}_{\alpha^i\langle p,\, x^i\rangle}$$

$$f^i(x^i;p) = \underbrace{\sum_{a\in\mathcal{A}} \int_0^{v_a} t_a(u)\mathrm{d}u + \frac{1}{\beta_1}\sum_{r,s} q^{rs}\left(\ln q^{rs} - 1 - \beta_2 \sum_{i\in I_s} c_i^s - \beta_0^s\right)}_{f_0^i(x^i)}$$

$$+ \underbrace{\frac{1}{\beta_1}\sum_{r\in R}\sum_{s\in S} q^{rs}\beta_3 \frac{\varrho^s e^{rs}}{inc^r}}_{\alpha^i\langle p,\, x^i\rangle}$$

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Representative agent formulation

MOPEC

$$x^i(p) \in \operatorname*{argmin}_{x^i \in C^i} \{f_0^i(x^i) + \mu_i \langle p, x^i \rangle\}$$

$$\sum_i x^i(p) = 0$$

Representative agent

$$\min_{\{x^i\}} \sum_i \mu^i f_0^i(x^i)$$

$$\text{such that } \sum_i x^i = 0$$

Distributed optimization approach

Split operator representation

$$\min_{\{x^i\}} \sum_i \mu^i f_0^i(x^i) + \iota_{\{0\}}\left(\sum_i z^i\right)$$
$$\text{such that} \quad x^i - z^i = 0, \; (y^i)$$

Individual modified problems Every agent-i solves

$$x^{i,\nu+1} \in \operatorname*{argmin}_{x^i \in C^i} f_0^i(x^i) + \langle y^\nu, x^i \rangle + \frac{r}{2} \left\| x^i - x^{i,\nu} + \bar{x}^{i,\nu} \right\|_2^2$$

Price update

$$y^{\nu+1} = y^\nu + r\bar{x}^{\nu+1}$$

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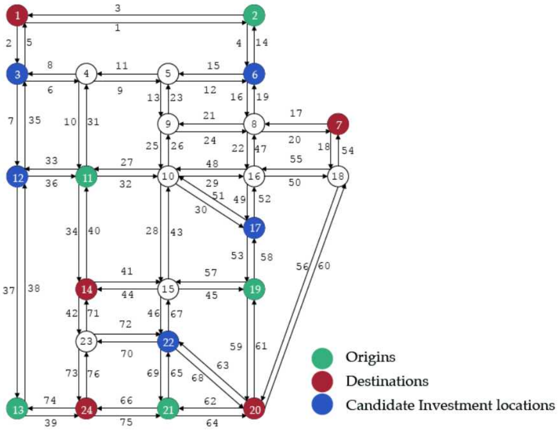
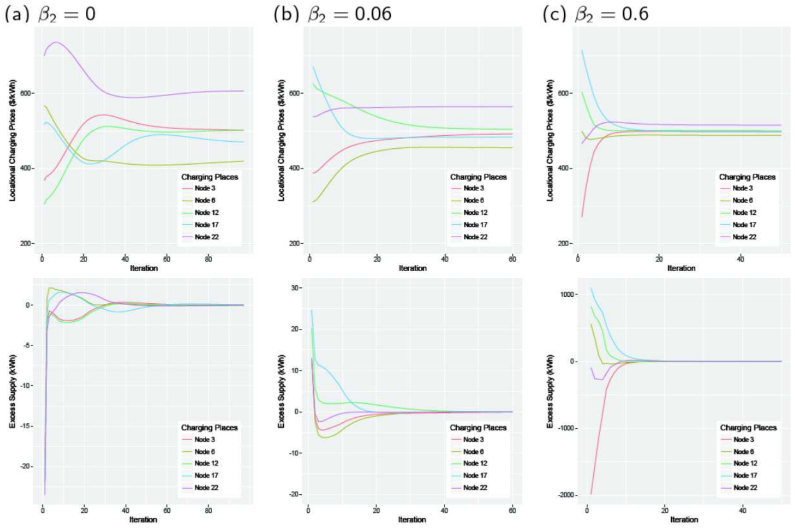
EV-charging station problem

- 24 nodes
- 76 links
- 5 origins
- 5 destinations
- 5 candidate facility locations
- BPR link cost function
- Base case:
  - $\beta_0 = 0$
  - $\beta_1 = 1$
  - $\beta_2 = 0.06$



EV-charging station problem



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Stochastic model: Investors' problem

$$\underset{c_i^s, g_{i,\xi}^s}{\text{maximize}} \quad \underbrace{\sum_{s \in S_i} \left[\mathbb{E}\{\varrho_\xi^s g_{i,\xi}^s\} - \varphi_g(g^s) \right]}_{\textit{operational net profit}} - \underbrace{\sum_{s \in S_i} \varphi_c(c_i^s)}_{\textit{investment costs}}$$

$$\text{subject to} \quad (\gamma_i^s) \quad \underbrace{g_{i,\xi}^s - c_i^s \leq 0}_{\textit{capacity constraint}}, \ \forall s \in S_i; \ g_{i,\xi}^s \geq 0, \ c_i^s \geq 0, \ \forall s \in S_i, \forall \xi \in \Xi$$

where:

- $g_{i,\xi}^s$: energy supply [kWh] at location s by i-firm and scn ξ;
- ϱ_ξ^s : unit charging price at location s and scn ξ(market determined);
- Ξ : set of scenarios

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Travelers and ISO problem

$$
\begin{aligned}
\underset{x,v,q}{\text{minimize}} \quad & U(\text{travel time, local attractiveness,} \\
& \quad \text{charging capacities, charging cost, } \xi) \\
\text{subject to} \quad & v_{a,\xi} = \sum_{r\in R}\sum_{s\in S} x^{rs}_{a,\xi}, \; \forall a \in \mathcal{A} \\
(\zeta^{rs}) \quad & A x_{\xi}^{rs} = q^{rs}_{\xi} E^{rs}, \; \forall r \in R, s \in S, \forall \xi \in \Xi \\
(\eta^{r}) \quad & \sum_{s\in S} q_{\xi}^{rs} = d^{r}_{\xi}, \; \forall r \in R, \forall \xi \in \Xi \\
& x^{rs}_{a,\xi} \geq 0, \; \forall a \in \mathcal{A}, r \in R, s \in S, \xi \in \Xi \\
& v_{a,\xi} \geq 0, \; \forall a \in \mathcal{A}, \xi \in \Xi \\
& q_{\xi}^{rs} \geq 0, \; \forall r \in R, s \in S, \xi \in \Xi.
\end{aligned}
$$

Solution strategy

Equilibrium

$$\sum_{i\in I_s} g^s_{i,\xi} - \sum_{r\in R} e^{rs} q_\xi^{rs} = 0, \forall s \in S, \xi \in \Xi$$

Scheme

Decompose individual agents' problems using
Progressive Hedging algorithm
Decompose the equilvalent state-defined equilibrium problem using
ADMM algorithm

General formulation

AGENT PROBLEM

$$(x_0^i, x_{1,\xi}^i) \in \operatorname{argmin} F(x) := \left(f_0^i(x_0^i) + \mathbb{E}\{f_{1,\xi}^i(x_{1,\xi}^i)\} \right)$$

PROGRESSIVE HEDGING $\forall \xi \in \Xi$

$$(x_{0,\xi}^i, x_{1,\xi}^i) \in \operatorname{argmin} \left\{ f_0^i(x_{0,\xi}^i) + f_{1,\xi}^i(x_{1,\xi}^i) + \langle \omega_\xi^i, x_{0,\xi}^i \rangle + \frac{\varrho_{PH}}{2} \|x_{0,\xi}^i - \bar{x}_{0,\cdot}^i\|_2^2 \right\}$$

Remark

The market clearing conditions of the EV-charging station station infrastructure planning problem only depend on the second stage variables.

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Incorporating representative agent and ADMM decomposition

Combining the ADMM approach to the decompose individual problem:

Step 0. Initialize the algorithm, $\bar{x}^{i,0}$, $\varrho_{PH} > 0$, $\varrho_{ADMM} > 0$, $w_\xi^{i,0}$, y_ξ^0.

Step 1. For every agent i, and every scenario ξ solves

$$
\begin{aligned}
(x_{0,\xi}^{i,\nu+1}, x_{1,\xi}^{i,\nu+1}) \in \operatorname{argmin} \{ & f_0^i(x_0) + f_{1,\xi}^i(x_1) \\
& + \langle \omega_\xi^{i,\nu}, x_0 \rangle + \frac{\varrho_{PH}}{2} \| x_0^i - \bar{x}_{0,\cdot}^{i,\nu} \|_2^2 \\
& + \langle y_\xi^{i,\nu}, x_1 \rangle + \frac{\varrho_{ADMM}}{2} \| x_1 - x_{1,\xi}^{i,\nu} + \bar{x}_{1,\xi}^{\cdot,\nu} \}
\end{aligned}
$$

Step 2. Update the multipliers

$$
\begin{aligned}
y_\xi^{\nu+1} &= y_\xi^{\nu} + \varrho_{ADMM} \bar{x}_{1,\xi}^{\cdot,\nu+1} \\
\omega_\xi^{i,\nu+1} &= \omega_\xi^{i,\nu} + \varrho_{PH} (x_{0,\xi}^{i,\nu+1} - \bar{x}_{0,\cdot}^{i,\nu+1})
\end{aligned}
$$

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Numerical example

Preliminary example

Network Sioux-Falls

Charging station Nodes 6,12,17,22.

Scenarios We consider $|\Xi| = 20$ scenarios

Parameters Consumers' utility function $\beta_0 = 0.0$, $\beta_1 = 1$, $\beta_2 = 0.0$, $\beta_3 = 0.06$.

Demand $d_\xi = 100 + \xi, \xi \in [0, 2]$

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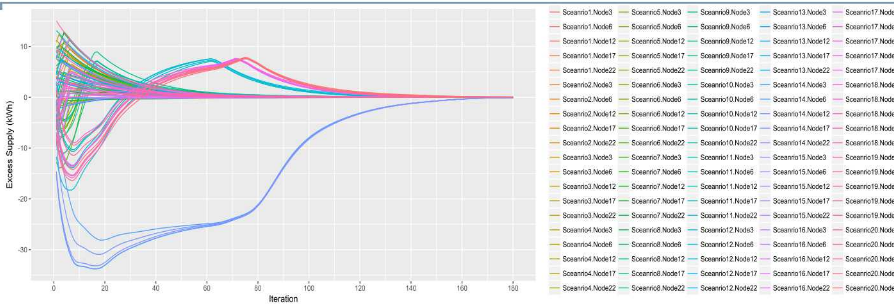
Excess Supply (kWh)
10
0
-10
-20
-30
0
20
40
60
80
100
120
140
160
180
Iteration
Sceanrio1.Node3
Sceanrio1.Node6
Sceanrio1.Node12
Sceanrio1.Node17
Sceanrio1.Node22
Sceanrio2.Node3
Sceanrio2.Node6
Sceanrio2.Node12
Sceanrio2.Node17
Sceanrio2.Node22
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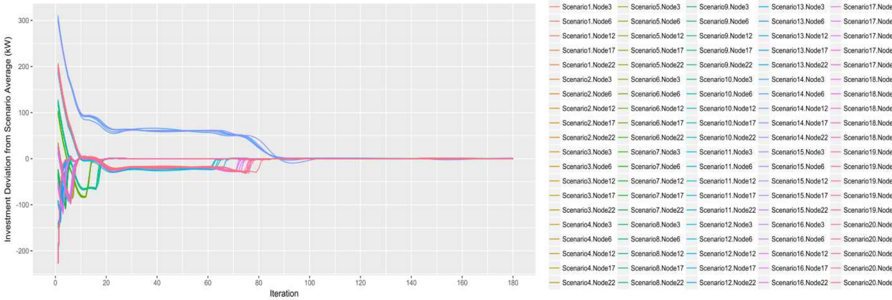
Investment Deviation from Scenario Average (kW)
300
200
100
0
-100
-200
0
20
40
60
80
100
120
140
160
180
Iteration
Scenario1.Node3
Scenario1.Node6
Scenario1.Node12
Scenario1.Node17
Scenario1.Node22
Scenario2.Node3
Scenario2.Node6
Scenario2.Node12
Scenario2.Node17
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Scenario15.Node17
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Scenario16.Node3
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Thank you for your attention
Questions?

Articles

(in progress) *Facility Location in a Competitive and Stochastic Market: incorporating complex user-network interactions*, Deride, J., Fan, Y, Guo, Z.

(2016) *Infrastructure Planning for Fast Charging Stations in a Competitive Market*, Deride, J., Fan, Y, Guo, Z., Transportation Research Part C: Emerging Technologies. vol.68, pp.215-227.

Sandia National Laboratories

A stochastic general equilibrium approach to an infrastructure planning problem

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NPS OPTIMIZATION SEMINAR
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