

# Predicting System Response at Unmeasured Locations using a Laboratory Pre-Test



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## Motivation for improving system dynamic response estimates

- Components need to be qualified to survive dynamic environments in the system.
- Often one or more system level field/flight tests are conducted with some accelerometers at some convenient locations.
- These locations are usually not the attachment locations of the components.
- Ideally, all six DoF at each attachment would be measured to provide an accurate input motion to the component. This is never provided.
- Enveloping of the few measurements at the nearest locations is performed and defined as the component input environment – this is usually much different than the real flight environment.
- No one knows the uncertainty or conservatism of this component environment.
- This work with the AWE attempts to take a step toward providing an estimate of enough unmeasured critical responses to define a measured field/flight environment.
- This work focuses on an experimental approach in which a laboratory pre-test is performed to determine relationships between measured field/flight responses and critical responses which can be measured in the laboratory (but will not be measured in field/flight).

## Experimental approach to estimate unmeasured system responses from field/flight measurements

- Assume one can extract ortho-normal singular vectors,  $\mathbf{U}$ , from a laboratory pre-test using the singular value decomposition such that



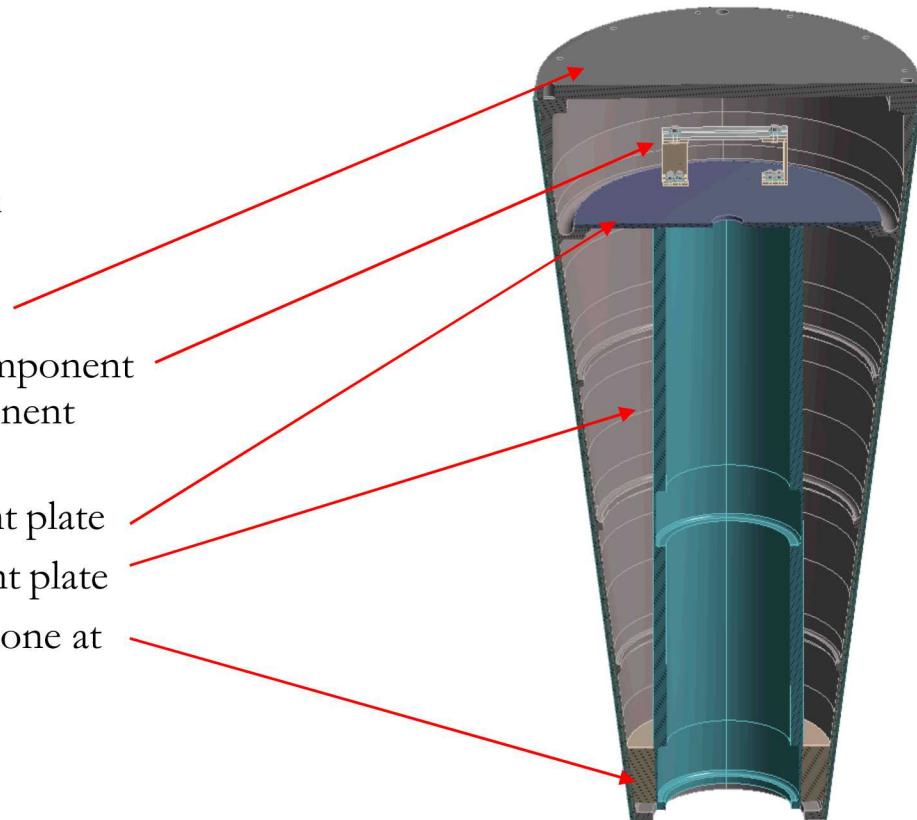
$$\begin{Bmatrix} \ddot{\mathbf{x}}_{meas} \\ \ddot{\mathbf{x}}_{unmeas} \end{Bmatrix} \approx \begin{bmatrix} \mathbf{U}_{meas} \\ \mathbf{U}_{unmeas} \end{bmatrix} \{ \ddot{\mathbf{p}} \} \quad (1)$$

$$\{ \ddot{\mathbf{p}} \} = \mathbf{U}_{meas}^{-1} \{ \ddot{\mathbf{x}}_{meas} \} \quad (2)$$

- Here the response quantity measured is given as  $\ddot{\mathbf{x}}$ , and the singular vectors are multiplied by generalized DoF,  $\ddot{\mathbf{p}}$ , to approximate the response.
- The unmeasured responses are those which need to be predicted in the field/flight test to define the component response of interest.
- We pose an over-determined problem with more field acceleration measurements than generalized DoF in order to average out experimental errors.
- The laboratory pre-test will be explained later, but first consider the proof-of-concept hardware.

# Proof of concept hardware for estimating unmeasured responses MATV

- The proof of concept for the estimation of unmeasured responses utilized research hardware provided by AWE known as the Modal Analysis Test Vehicle (MATV) which would be tested in a field random acoustic environment.
- MATV Description
  - A meter long
  - 47 kg
  - Composite wrapped on aluminum substrate cone
  - Large end aluminum cover plate
  - Bracket called the Removable Component (RC) bolted to the internal component plate
  - Aluminum internal flat component plate
  - Steel pipe bolted to the component plate
  - Foam support between pipe and cone at small end



5 Field acoustic test for MATV

- A field acoustic test was run to 147 dB at the Institute of Sound and Vibration Research at Southampton University in a reverberant chamber with a horn.
- Place in corner of chamber
- Horn
- MATV suspended by bungees
- 67 total accelerometer channels recorded

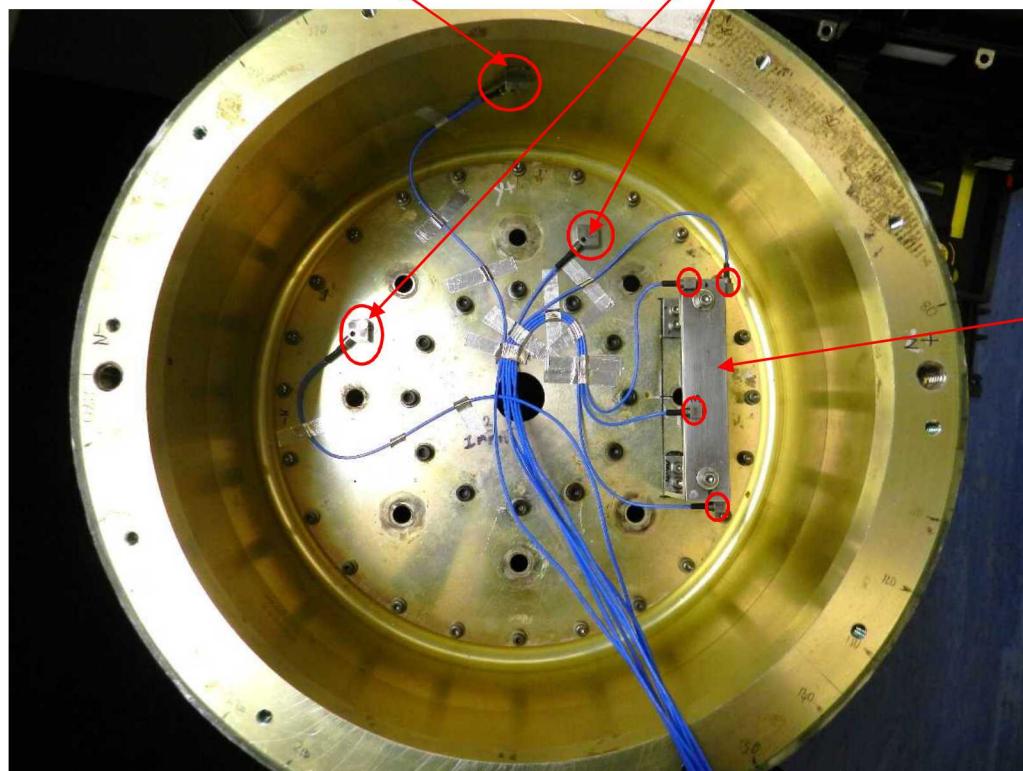


## MATV truth “unmeasured” accelerometer locations

- 14 truth accelerometer DoF were chosen either on the RC or triaxial locations at possible mounting locations for a component. These were not used in the set to calculate the  $\mathbf{p}$ 's, but were truth measurements for comparison to the estimates generated from the method using 30 optimal measurement accelerometers.

1 Triax on Cone

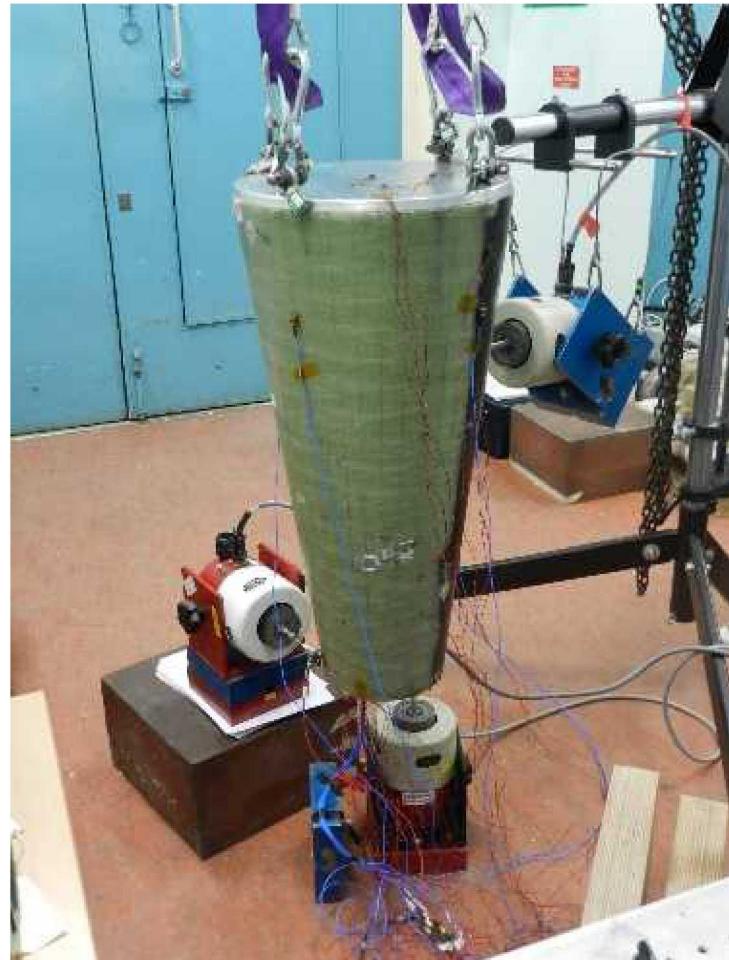
2 Triax on Component Plate



RC – 5 DoF  
chosen on 4  
Triaxes

## Laboratory pre-test

- A three shaker modal test was performed up to 2000 Hz.
- We utilized the three sets of experimental accelerance FRFs derived from the modal test.
- The FRFs from the 14 “unmeasured” responses were gathered as well as a candidate set from 53 possible measurement locations that were available in the system acoustic field test.
- The FRF bandwidth was divided into 10 equal sized 200 Hz bands.
- $\mathbf{U}$  matrices were derived from the FRFs in each 200 Hz band.



## Extraction of the **U** matrices for each frequency band

- 53 accelerometer DoF were available from which to choose 30 desired “measurement” DoF along with the 14 “unmeasured” response DoF which were also instrumented in the pre-test

$$\begin{bmatrix} \mathbf{HMAT}_m \\ \mathbf{HMAT}_u \end{bmatrix} = \begin{bmatrix} \text{real}(\mathbf{H}_{xm\_f1}) & \text{real}(\mathbf{H}_{xm\_f2}) & \text{real}(\mathbf{H}_{xm\_f3}) & \text{imag}(\mathbf{H}_{xm\_f1}) & \text{imag}(\mathbf{H}_{xm\_f2}) & \text{imag}(\mathbf{H}_{xm\_f3}) \\ \text{real}(\mathbf{H}_{xu\_f1}) & \text{real}(\mathbf{H}_{xu\_f2}) & \text{real}(\mathbf{H}_{xu\_f3}) & \text{imag}(\mathbf{H}_{xu\_f1}) & \text{imag}(\mathbf{H}_{xu\_f2}) & \text{imag}(\mathbf{H}_{xu\_f3}) \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{U}_m \\ \mathbf{U}_u \end{bmatrix}, \mathbf{S}, \mathbf{V} = \text{svd}(\begin{bmatrix} \mathbf{HMAT}_m \\ \mathbf{HMAT}_u \end{bmatrix})$$

- In each band, only 15 **U** vectors were retained to approximate the motion, giving over-determination by a factor of two with 30 measurement gages

## Down selection to 30 measurement gages

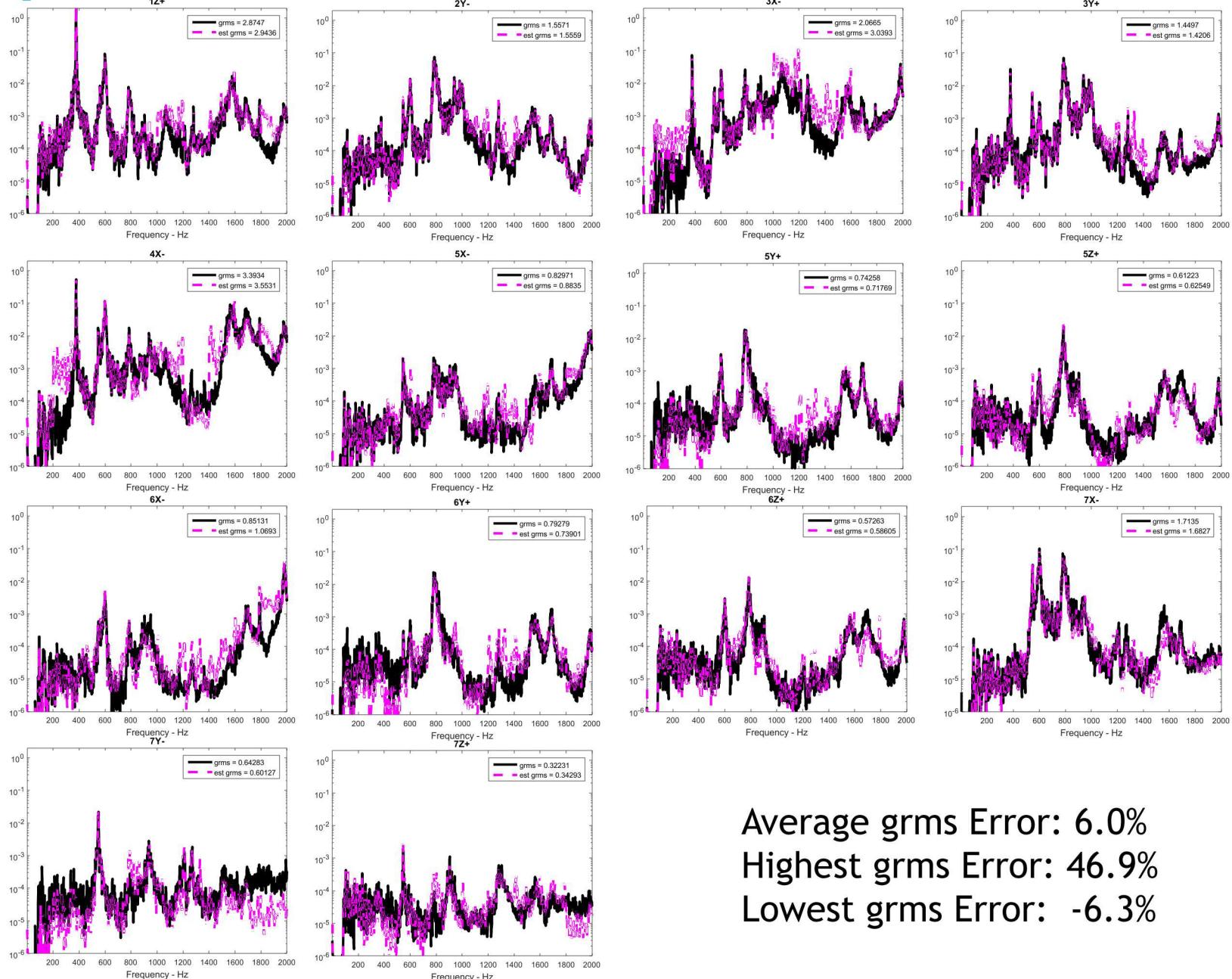
- 53 accelerometers were available from which to choose “measurement” gages
- The maximum condition number of any of the 10  $\mathbf{U}_{meas}$  matrices with 53 gages was 14.7
- A suboptimal approach to minimize the condition number of the measured  $\mathbf{U}$  matrices was performed, beginning with all 53 candidate measurement responses
  1. For the current candidate field response DoF set, remove one candidate DoF and calculate the sum of the condition numbers for all ten  $\mathbf{U}$  matrices. Save the sum associated with removal of this DoF. Replace the candidate DoF.
  2. Repeat step 1 for each of the rest of the candidate field response DoF.
  3. Discard from the candidate DoF set the one DoF whose removal produced the smallest sum of condition numbers for the  $\mathbf{U}$  matrices.
  4. Repeat steps 1-3 until one has reduced to the 30 measurement field sensor goal.
- The maximum condition number of any of the  $\mathbf{U}$  matrices after reducing down to 30 gages was 15.5
- Then equations 1 and 2 were applied to the acoustic vibration data to predict the “unmeasured” responses as below (converted to cross spectral density matrix form)

$$\{\ddot{\mathbf{p}}\} = \mathbf{U}_{meas}^+ \{\ddot{\mathbf{x}}_{meas}\}$$

$$\ddot{\mathbf{x}}_{unmeas} \approx \mathbf{U}_{unmeas} \{\ddot{\mathbf{p}}\}$$

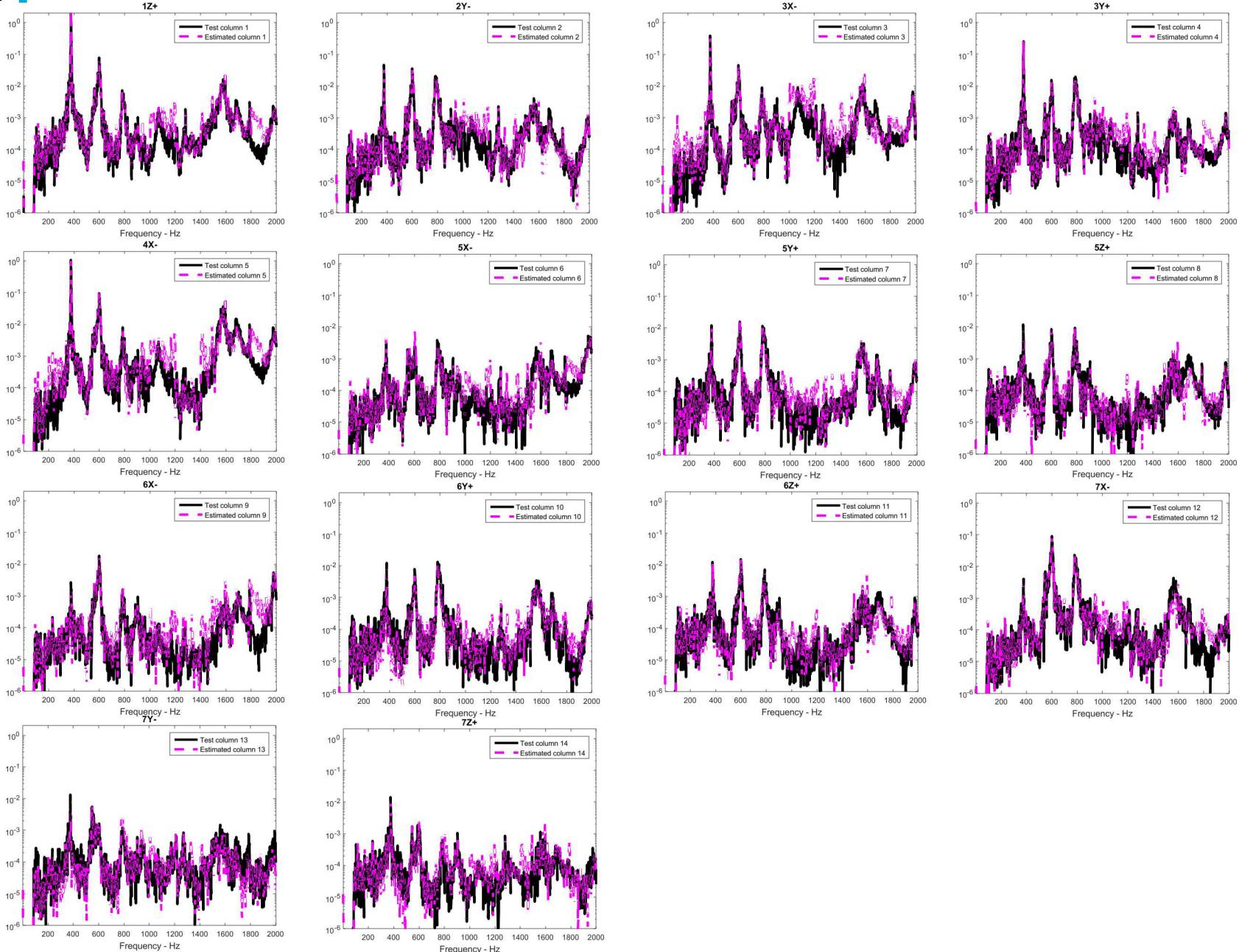
# I4 Truth PSDs– Black=Measured, Magenta=Algorithm Estimate

10



Average grms Error: 6.0%  
 Highest grms Error: 46.9%  
 Lowest grms Error: -6.3%

# Row I CPSDs– Black=Measured, Magenta=Algorithm Estimate



## Conclusions

- NOTE: This algorithm was conceived after the modal test and system level acoustic test had been performed. In the future, one would specify some additional shaker input locations to help guarantee that all the experimental modes were excited and ensure all experimental shapes would be represented in the FRF matrices.
- Considering that we only had FRFs from three shakers, the predictions expanded from the measured gages were quite encouraging.
- By dividing the bandwidth into multiple bands and optimizing to keep the sum of the condition numbers of the **U** matrices as low as possible, we were able to make predictions for a system known to have more than 70 modes with only 30 gages.
- The prediction DoF must be included in the laboratory pre-test.
- More measurement DoF are required than **U** vectors in a single bandwidth.
- No finite element model is required for this approach. Hardware and a laboratory pre-test are required along with the measured responses in the field test.