

Low Frequency Stable and Accurate Potential-Based Time Domain Integral Equations for Dielectric Regions

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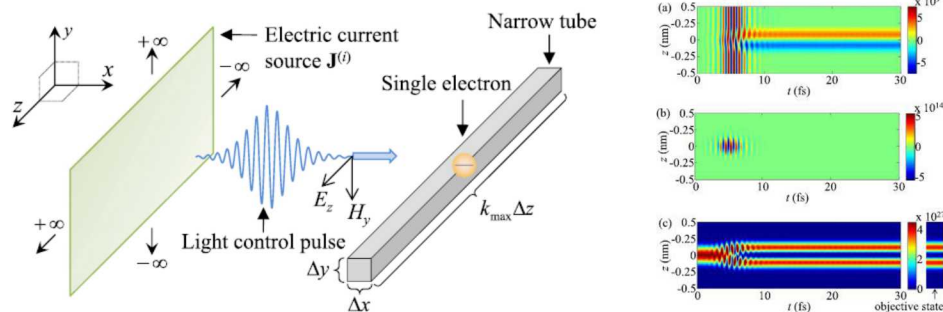
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Quantum Physics & CEM

Semiclassical

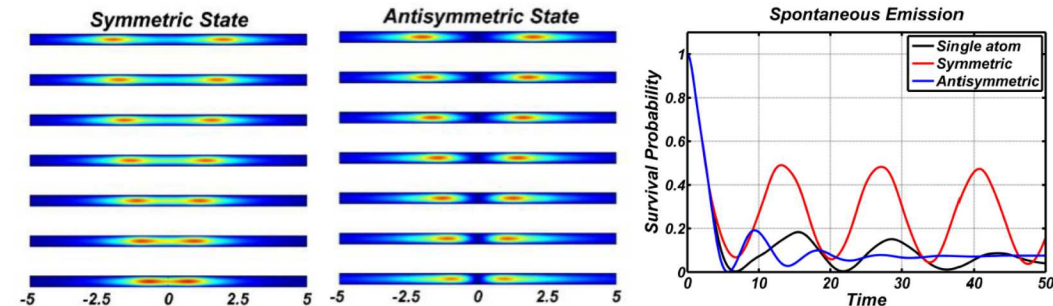
Maxwell-Schrödinger System



T. Takeuchi *et al.*, DOI: 10.1103/PhysRevA.91.033401

Fully Quantized

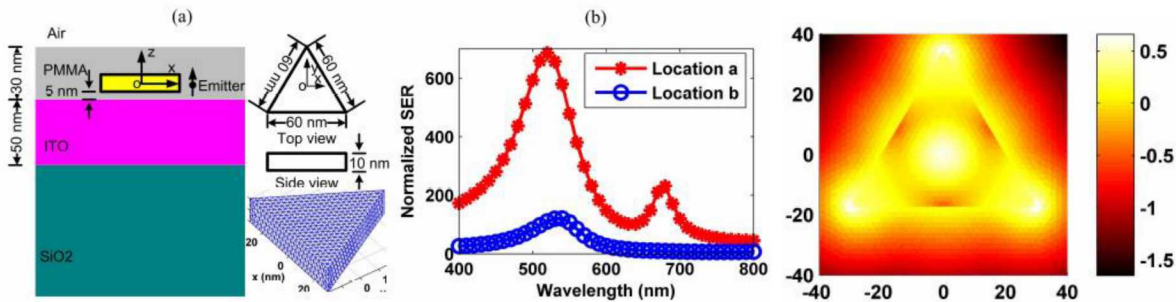
Waveguide-QED



W. C. Chew *et al.*, DOI: 10.1109/PIERS.2017.8262143

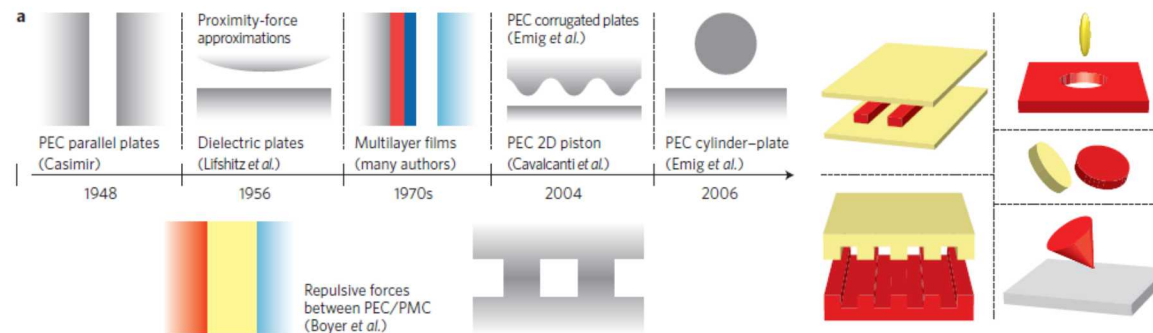
Requires Wider BW

Nanoantenna Spontaneous Emission Rate



W. C. Chew *et al.*, DOI: 10.1364/OE.20.020210

Casimir Force



F. Capasso *et al.*, DOI: 10.1038/NPHOTON.2011.39

Emerging quantum applications require *classical* CEM tools for *increasingly* broadband analysis of multiscale, subwavelength, and lossy dielectrics

Potential-Based CEM

- Maxwell's equations are valid over vast length scales and broad frequency ranges
 - Subatomic to galactic; static to ultraviolet
 - Valid in quantum regime
- Discretized equations typically are not!
- Field-based formulation
 - Breaks down for low frequency or long wavelength
 - Not multiscale
- Potential-based formulation
 - Reinterpretation of how to solve Maxwell's equations numerically
 - Discrete equations do not break down
 - Good for analyzing multiscale/subwavelength structures
 - Potentials are typically quantities of interest for quantum applications

Field-based Formulation

Non-unique solution as $\omega \rightarrow 0$

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - k^2 \mathbf{E}(\mathbf{r}) = i\omega\mu\mathbf{J}(\mathbf{r}) - \nabla \times \mathbf{M}(\mathbf{r})$$

$$\nabla \times \nabla \times \mathbf{H}(\mathbf{r}) - k^2 \mathbf{H}(\mathbf{r}) = i\omega\epsilon\mathbf{M}(\mathbf{r}) + \nabla \times \mathbf{J}(\mathbf{r})$$

Potential-based Formulation

No break down as $\omega \rightarrow 0$

$$\nabla^2 \mathbf{A}(\mathbf{r}) + k^2 \mathbf{A}(\mathbf{r}) = -\mu\mathbf{J}(\mathbf{r})$$

$$\nabla^2 \Phi(\mathbf{r}) + k^2 \Phi(\mathbf{r}) = -\rho(\mathbf{r})/\epsilon$$



Potential-Based CEM – PEC Formulation

- Potential-based TDIEs have been derived and shown to be stable for PEC objects
- Simple modifications to remove interior resonances have been demonstrated
- Resulting systems perform well over broad frequency range, including very low frequencies
 - \mathbf{A} equation captures quasi-magnetostatic physics
 - Φ equation captures quasi-electrostatic physics
 - Equations naturally decouple as frequency lowers, maintaining performance
- Unknowns and equations are carefully selected based on a well-known rigorous functional framework for retarded potential boundary integral equations

Differentiated \mathbf{A} - Φ Integral Equations (D-APIE)

$$\hat{\mathbf{n}} \times \int_S \left[\mu \frac{\dot{\mathbf{J}}(\mathbf{r}', t - R/c)}{4\pi R} - \nabla \frac{\Pi(\mathbf{r}', t - R/c)}{4\pi R} \right] dS' = \dot{\mathbf{A}}^{\text{inc}}(\mathbf{r}, t) \times \hat{\mathbf{n}}$$

$$\int_S \left[\epsilon^{-1} \frac{\nabla' \cdot \mathbf{J}(\mathbf{r}', t - R/c)}{4\pi R} - \frac{\dot{\Pi}(\mathbf{r}', t - R/c)}{4\pi R} \right] dS' = \dot{\Phi}^{\text{inc}}(\mathbf{r}, t)$$

Unknowns: $\mathbf{J}(\mathbf{r}', t)$, $\Pi(\mathbf{r}', t) = \hat{\mathbf{n}}' \cdot \dot{\mathbf{A}}(\mathbf{r}', t)$

\mathbf{A} - Φ Integral Equations (APIE)

$$\hat{\mathbf{n}} \times \int_S \left[\mu \frac{\mathbf{J}(\mathbf{r}', t - R/c)}{4\pi R} - \nabla \int_{-\infty}^{t-R/c} \frac{\Pi(\mathbf{r}', t')}{4\pi R} dt' \right] dS' = \mathbf{A}^{\text{inc}}(\mathbf{r}, t) \times \hat{\mathbf{n}}$$

$$\int_S \left[\epsilon^{-1} \int_{-\infty}^{t-R/c} \frac{\nabla' \cdot \mathbf{J}(\mathbf{r}', t')}{4\pi R} dt' - \frac{\Pi(\mathbf{r}', t - R/c)}{4\pi R} \right] dS' = \Phi^{\text{inc}}(\mathbf{r}, t)$$

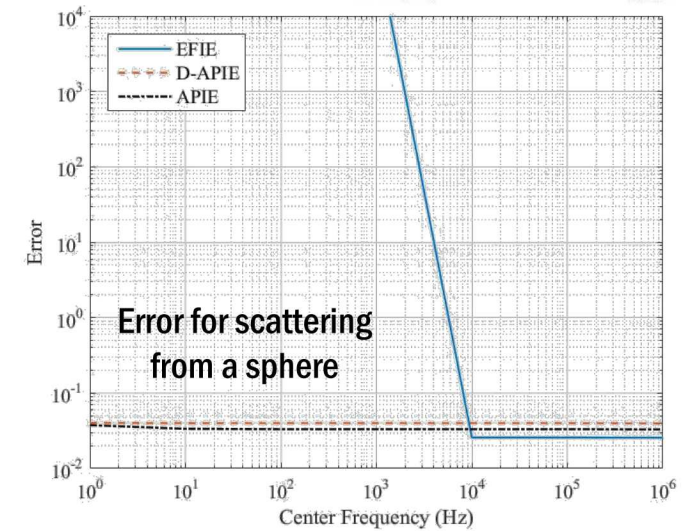
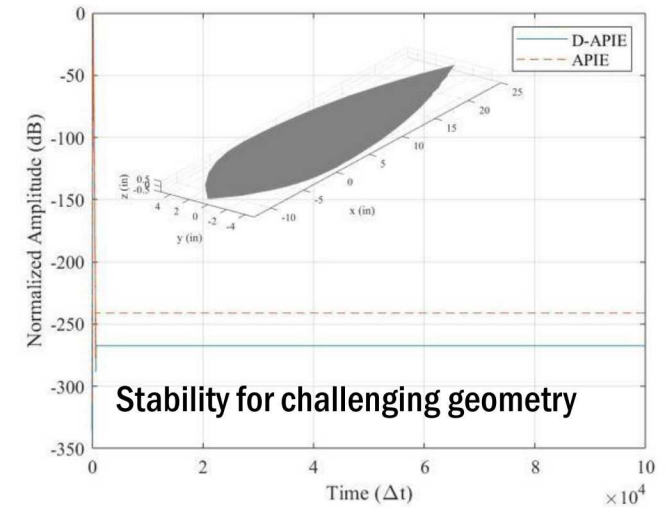
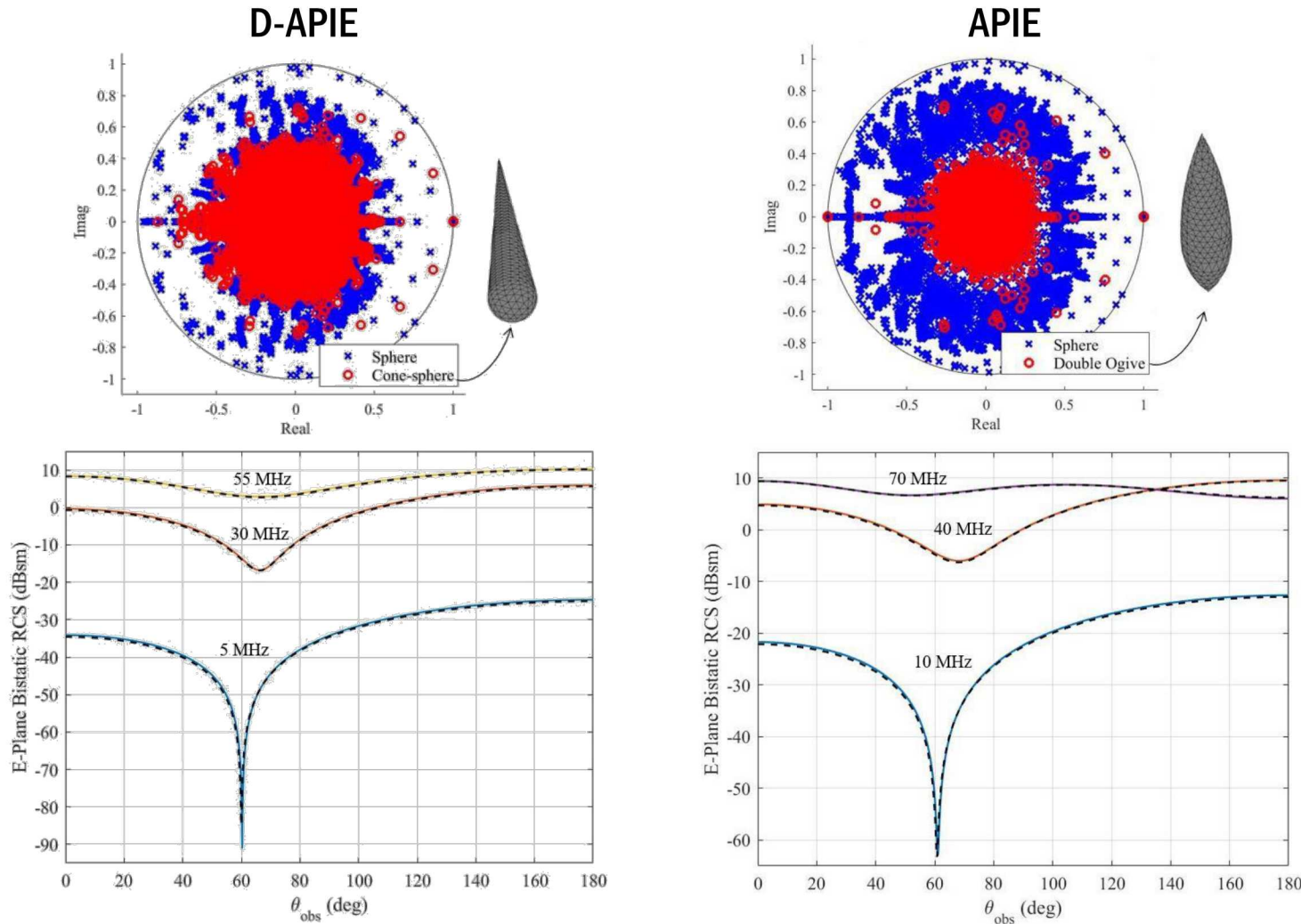
Unknowns: $\mathbf{J}(\mathbf{r}', t)$, $\Pi(\mathbf{r}', t) = \hat{\mathbf{n}}' \cdot \dot{\mathbf{A}}(\mathbf{r}', t)$

Stable discretizations are possible because the temporal Sobolev spaces of the domain and range of integral operators are matched between equations

T. Roth, DOI: 10.1109/JMMCT.2018.2889535 and 10.1016/J.JCP.2019.109102



Potential-Based CEM – PEC Formulation

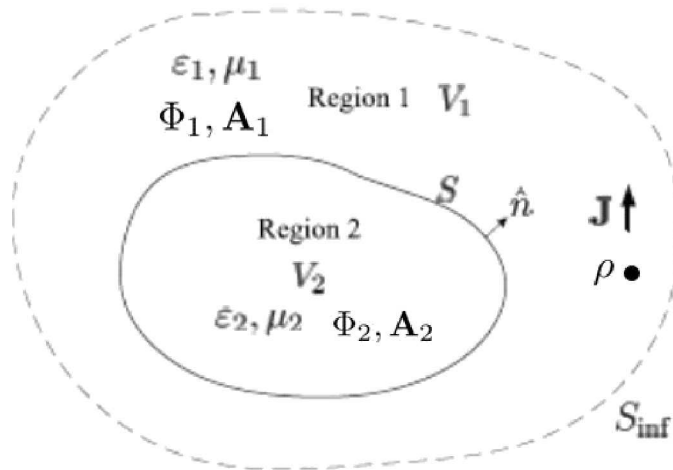


T. Roth, DOI: 10.1109/JMMCT.2018.2889535 and 10.1016/J.JCP.2019.109102



Dielectric Formulation – Two-Region Problem

- Problem Statement:
 - Simple two-region problem is assumed in order to derive surface TDIEs
 - Incident potentials have not reached the surface for $t < 0$
 - Total potentials (incident + scattered) are denoted as \mathbf{A}_j and Φ_j in region j
- Derivation:
 - Auxiliary exterior and interior problems are formulated
 - Integral representations are substituted into auxiliary problem boundary conditions to derive TDIEs
 - Actual boundary conditions are used to tie auxiliary problems together and lead to a solvable system



Boundary Conditions

$$\left\{ \begin{array}{l} \hat{n} \times \mathbf{A}_1 = \hat{n} \times \mathbf{A}_2 \\ \Phi_1 = \Phi_2 \\ \hat{n} \times \mu_1^{-1} \nabla \times \mathbf{A}_1 = \hat{n} \times \mu_2^{-1} \nabla \times \mathbf{A}_2 \\ \hat{n} \cdot \epsilon_1 \mathbf{A}_1 = \hat{n} \cdot \epsilon_2 \mathbf{A}_2 \\ \hat{n} \cdot \epsilon_1 \nabla \Phi_1 = \hat{n} \cdot \epsilon_2 \nabla \Phi_2 \\ \hat{n} \cdot \nabla \times \mathbf{A}_1 = \hat{n} \cdot \nabla \times \mathbf{A}_2 \end{array} \right. \rightarrow \left\{ \begin{array}{l} \hat{n} \times \mathbf{E}_1 = \hat{n} \times \mathbf{E}_2 \\ \hat{n} \times \mathbf{H}_1 = \hat{n} \times \mathbf{H}_2 \\ \hat{n} \cdot \mathbf{D}_1 = \hat{n} \cdot \mathbf{D}_2 \\ \hat{n} \cdot \mathbf{B}_1 = \hat{n} \cdot \mathbf{B}_2 \end{array} \right.$$

Dielectric Formulation – Previous Work

Definitions

$$R = |\mathbf{r} - \mathbf{r}'|$$

$$\tau_j = t - \frac{R}{c_j} \quad \hat{R} = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

Unknowns

$$\mathbf{J} = -\mu^{-1} [\hat{\mathbf{n}}' \times \nabla' \times \mathbf{A}_{\text{inc}} + \hat{\mathbf{n}}' \times \nabla' \times \mathbf{A}_e]$$

$$\mathbf{m} = -\hat{\mathbf{n}}' \times \dot{\mathbf{A}}_{\text{inc}} - \hat{\mathbf{n}}' \times \dot{\mathbf{A}}_e$$

$$\Psi = -\dot{\Phi}_{\text{inc}} - \dot{\Phi}_e \quad \Pi = -\hat{\mathbf{n}}' \cdot \dot{\mathbf{A}}_{\text{inc}} - \hat{\mathbf{n}}' \cdot \dot{\mathbf{A}}_e$$

- Our previous work developed a system of four equations using exterior and interior equations for two of the six possible boundary conditions – showing only exterior equations on this slide for brevity
- Tangential component of vector potential equation

$$\int_S \left(\mu_1 \frac{\mathbf{J}(\mathbf{r}', \tau_1)}{4\pi R} - \nabla \int_{-\infty}^{\tau_1} \frac{\Pi(\mathbf{r}', t')}{4\pi R} dt' + \hat{\mathbf{n}}' \mu_1 \epsilon_1 \frac{\Psi(\mathbf{r}', \tau_1)}{4\pi R} \right) dS' + \frac{1}{2} \int_{-\infty}^t \hat{\mathbf{n}}' \times \mathbf{m}(\mathbf{r}, t') dt' - \int_S \hat{R} \times \left[\int_{-\infty}^{\tau_1} \frac{\mathbf{m}(\mathbf{r}', t')}{4\pi R^2} dt' + \frac{\mathbf{m}(\mathbf{r}', \tau_1)}{4\pi R c_1} \right] dS' = -\hat{\mathbf{n}} \times [\mathbf{A}_{\text{inc}}(\mathbf{r}) \times \hat{\mathbf{n}}]$$

- Scalar potential equation

$$\int_S \left(- \int_{-\infty}^{\tau_1} \frac{\nabla \cdot \mathbf{J}(\mathbf{r}', t')}{4\pi R \epsilon_1} dt' + \frac{\Pi(\mathbf{r}', \tau_1)}{4\pi R} \right) dS' + \frac{1}{2} \int_{-\infty}^t \Psi(\mathbf{r}, t') dt' + \int_S \hat{\mathbf{n}}' \cdot \hat{R} \left(\int_{-\infty}^{\tau_1} \frac{\Psi(\mathbf{r}', t')}{4\pi R^2} dt' + \frac{\Psi(\mathbf{r}', \tau_1)}{4\pi R c_1} \right) dS' = -\Phi_{\text{inc}}$$

- The discretized system was found to be stable at low and medium frequencies, but was only accurate at middle frequencies
- Focus of this work was to develop new potential-based formulation to achieve accurate results down to very low frequencies for dielectric regions

T. Roth and W. Chew, 2019 PIERS – SPRING in Rome



Dielectric Formulation – New System

- To achieve low frequency accuracy equations corresponding to additional boundary conditions are enforced
 - Ensures all boundary conditions are met – only need 4 since remaining two are redundant
 - Provides more complete physics-based decomposition of the magnetic current density into (approximately) solenoidal and non-solenoidal subspaces

Müeller-type Potential-Based Integral Equations

$$\hat{n} \times [\dot{\mathbf{A}}_1\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t) \times \hat{n}] - \hat{n} \times [\dot{\mathbf{A}}_2\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t) \times \hat{n}] = -\hat{n} \times [\dot{\mathbf{A}}_{\text{inc}}(\mathbf{r}, t) \times \hat{n}]$$

$$\dot{\Phi}_1\{\mathbf{J}, \Pi, \Psi\}(\mathbf{r}, t) - \dot{\Phi}_2\{\mathbf{J}, \Pi, \Psi\}(\mathbf{r}, t) = -\dot{\Phi}_{\text{inc}}(\mathbf{r}, t)$$

$$\hat{n} \times [\mu_1^{-1} \nabla \times \mathbf{A}_1\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t)] \times \hat{n} - \hat{n} \times [\mu_2^{-1} \nabla \times \mathbf{A}_2\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t)] \times \hat{n} = -\hat{n} \times [\mu_1^{-1} \nabla \times \mathbf{A}_{\text{inc}}(\mathbf{r}, t)] \times \hat{n}$$

$$\hat{n} \cdot \epsilon_1 \dot{\mathbf{A}}_1\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t) - \hat{n} \cdot \epsilon_2 \dot{\mathbf{A}}_2\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t) = -\hat{n} \cdot \epsilon_1 \dot{\mathbf{A}}_{\text{inc}}(\mathbf{r}, t)$$

- Müeller-type combination of exterior and interior integral equations is used to have a square matrix system with a perturbation of identity operators along the block diagonal (i.e., second kind integral equations)
- Temporal derivatives applied to equations based on rigorous functional framework so that temporal Sobolev spaces are the same for all equations – allows for stable marching-on-in-time (MOT) discretization

See T. Roth, 10.1016/J.JCP.2019.109102 for details on functional framework



Dielectric Formulation – New System

- Historically, low frequency stability has been achieved by projecting current densities onto solenoidal and non-solenoidal subspaces (e.g., loop-tree and loop-star decompositions)
 - Separates the contributions from the vector and scalar potentials so they can be accurately computed
- Similar effect naturally occurs in formulating potential-based integral equations – provides a *physics-based* decomposition of the vector and scalar potential contributions
 - One direct benefit of this is that simpler spatial basis functions can be used in discretization (e.g., no searching for global loops)
- Physics-based decomposition for some electric effects can be seen in equations below considering relationships of \mathbf{J} and Π (i.e., a component of the electric charge density)

Detailed Definitions of Integral Equations

$$\hat{n} \times [\dot{\mathbf{A}}_i\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t) \times \hat{n}] = \int_S \left(\underbrace{\mu_i \frac{\dot{\mathbf{J}}(\mathbf{r}', \tau_i)}{4\pi R}}_{\text{Vector potential contribution}} - \underbrace{\nabla \frac{\Pi(\mathbf{r}', \tau_i)}{4\pi R}}_{\text{Scalar potential contribution}} + \hat{n}' \mu_i \epsilon_i \frac{\dot{\Psi}(\mathbf{r}', \tau_i)}{4\pi R} \right) dS' + \frac{1}{2} \hat{n}' \times \mathbf{m}(\mathbf{r}, t) - \int_S \hat{R} \times \left[\frac{\mathbf{m}(\mathbf{r}', \tau_i)}{4\pi R^2} + \frac{\dot{\mathbf{m}}(\mathbf{r}', \tau_i)}{4\pi R c_i} \right] dS'$$

Vector potential contribution
(e.g., loop-like contribution)

Scalar potential contribution
(e.g., tree-like contribution)

$$\dot{\Phi}_i\{\mathbf{J}, \Pi, \Psi\}(\mathbf{r}, t) = \int_S \left(- \frac{\nabla' \cdot \mathbf{J}(\mathbf{r}', \tau_i)}{4\pi R \epsilon_i} dt' + \frac{\dot{\Pi}(\mathbf{r}', \tau_i)}{4\pi R} \right) dS' + \frac{1}{2} \Psi(\mathbf{r}, t) - \int_S \hat{n}' \cdot \hat{R} \left[\frac{\Psi(\mathbf{r}', \tau_i)}{4\pi R^2} + \frac{\dot{\Psi}(\mathbf{r}', \tau_i)}{4\pi R c_i} \right] dS'$$

Scalar potential equation constrains non-solenoidal
behavior of electric current density



Dielectric Formulation – New System

- Physics-based decomposition for magnetic current density can be seen in equations below considering relationships of \mathbf{m} and Ψ
 - Note that \mathbf{m} is a direct component of the magnetic current density due to the vector potential
 - Tangential component of the gradient of Ψ forms scalar potential contribution to the magnetic current density – results in a solenoidal variation similar to loop basis functions
- Equations below show a physics-based decomposition of magnetic current density into approximately solenoidal and non-solenoidal contributions – a key to the low frequency accuracy of this system

Detailed Definitions of Integral Equations

$$\hat{n} \times [\mu_i^{-1} \nabla \times \mathbf{A}_i\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t)] \times \hat{n} = \frac{1}{2} \hat{n}' \times \mathbf{J}(\mathbf{r}, t) - \int_S \hat{R} \times \left[\frac{\mathbf{J}(\mathbf{r}', \tau_i)}{4\pi R^2} dt' + \frac{\dot{\mathbf{J}}(\mathbf{r}', \tau_i)}{4\pi R c_i} \right] dS'$$

Tested with approximately non-solenoidal space
Solenoidal contribution
Approximately non-solenoidal contribution

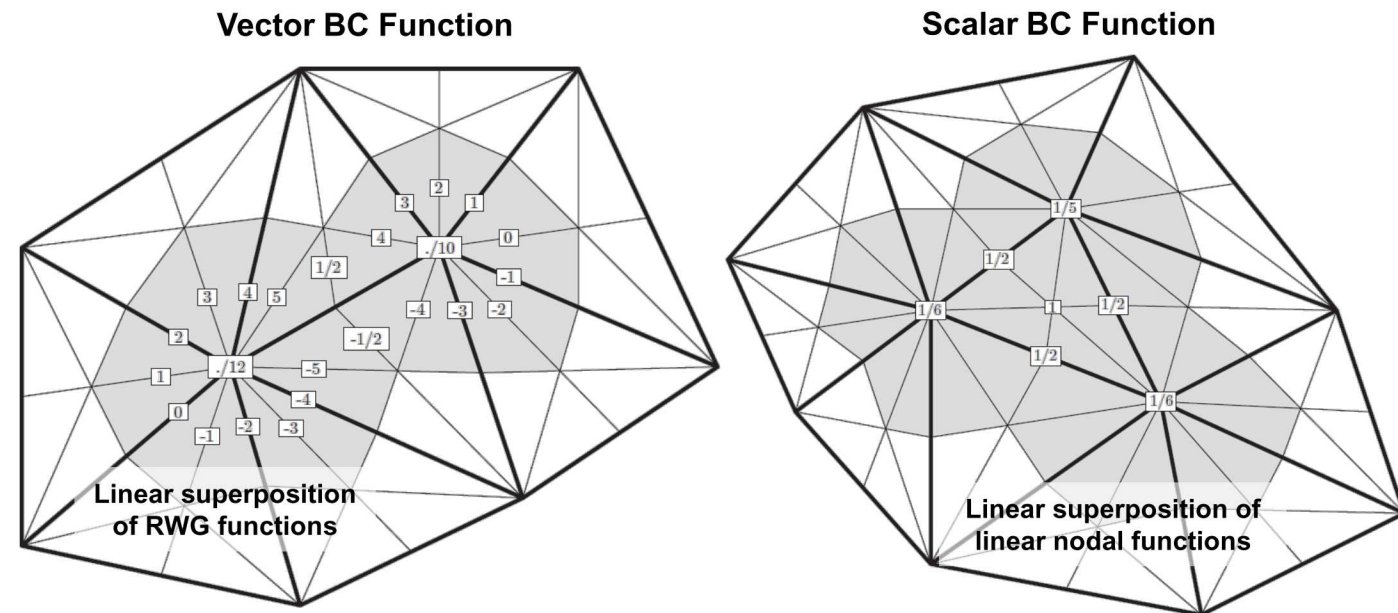
$$- \int_S \epsilon_i \frac{\hat{n}' \times \nabla' \Psi(\mathbf{r}', \tau_i)}{4\pi R} dS' - \int_S \left[\epsilon_i \frac{\dot{\mathbf{m}}(\mathbf{r}', \tau_i)}{4\pi R} - \nabla \int_{-\infty}^{\tau_i} \frac{\nabla' \cdot \mathbf{m}(\mathbf{r}', t')}{4\pi R \mu_i} dt' \right] dS'$$

$$\hat{n} \cdot \dot{\mathbf{A}}_i\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t) = \int_S \left(\mu_i \hat{n} \cdot \frac{\dot{\mathbf{J}}(\mathbf{r}', \tau_i)}{4\pi R} + \hat{n} \cdot \hat{n}' \mu_i \epsilon_i \frac{\dot{\Psi}(\mathbf{r}', \tau_i)}{4\pi R} \right) dS' + \frac{1}{2} \Pi(\mathbf{r}, t) + \int_S \hat{n} \cdot \hat{R} \left[\frac{\Pi(\mathbf{r}', \tau_i)}{4\pi R^2} dt' + \frac{\dot{\Pi}(\mathbf{r}', \tau_i)}{4\pi R c_i} \right] dS' + \hat{n} \cdot \nabla \times \int_S \frac{\mathbf{m}(\mathbf{r}', \tau_i)}{4\pi R} dS'$$

Tested with approximately solenoidal space
Solenoidal contribution
Approximately non-solenoidal contribution

MOT – Spatial Discretization

- Basis functions
 - \mathbf{J} should use div-conforming functions – use Rao-Wilton-Glisson (RWG) functions
 - \mathbf{m} is a component of magnetic current density – use vector Buffa-Christiansen (BC) functions
 - Π is a component of electric charge density – use triangular pulse functions (constant on a triangle)
 - Ψ needs to be differentiable to recover magnetic current density and also should be defined on dual mesh due to its relationship with \mathbf{m} – use scalar BC functions
- Testing functions
 - Test $\hat{n} \times [\dot{\mathbf{A}}_i\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t) \times \hat{n}]$ equation with RWG
 - Test $\dot{\Phi}_i\{\mathbf{J}, \Pi, \Psi\}(\mathbf{r}, t)$ equation with triangular pulse function
 - Test $\hat{n} \times [\mu_i^{-1} \nabla \times \mathbf{A}_i\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t)] \times \hat{n}$ equation with BC function
 - Test $\hat{n} \cdot \dot{\mathbf{A}}_i\{\mathbf{J}, \Pi, \mathbf{m}, \Psi\}(\mathbf{r}, t)$ with scalar BC function
- Discretization conforms to spatial Sobolev space properties



Images from Buffa and Christiansen, DOI: 10.1090/S0025-5718-07-01965-5

MOT – Temporal Discretization

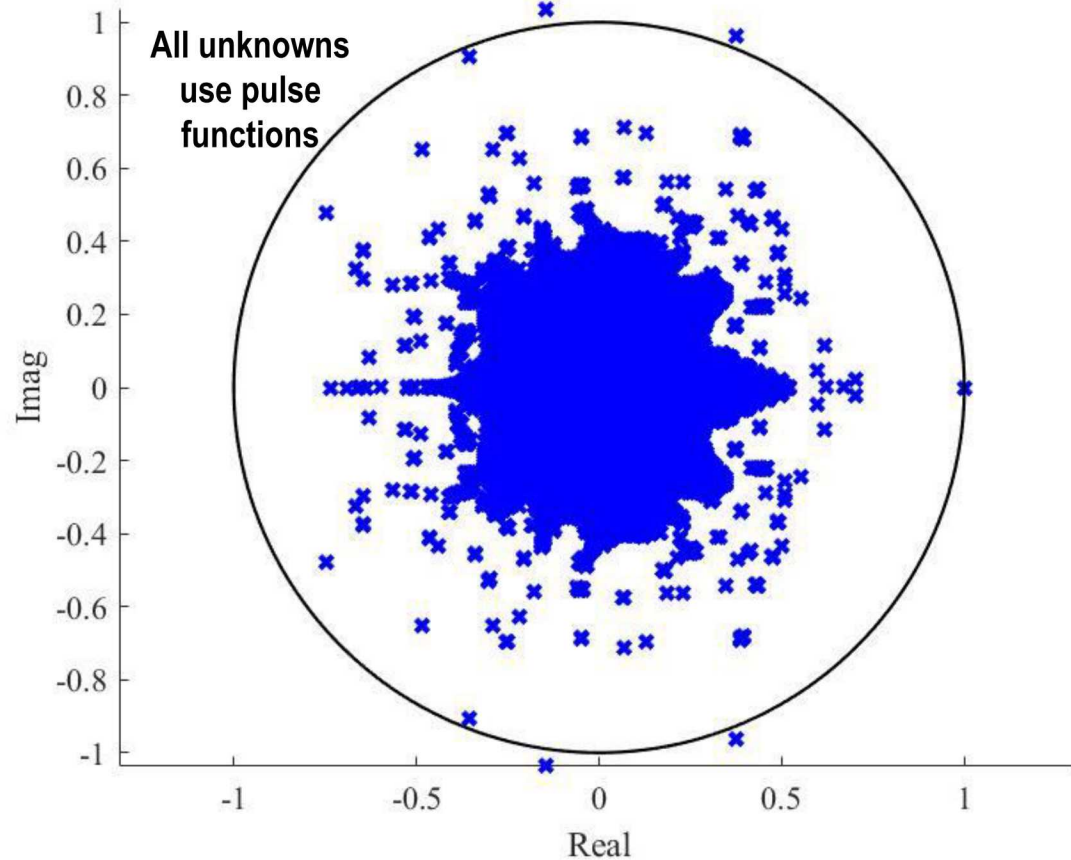
- Numerical stability is substantially improved for MOT discretizations that conform to appropriate temporal Sobolev space properties
- MOT discretization uses Dirac delta function to test each equation
- Selecting an appropriate basis function for each unknown is then key to have the overall discretization match the correct Sobolev spaces
- Using this approach gives the following basis functions
 - \mathbf{J} and \mathbf{II} use triangle functions as basis for PEC problems – same works here
 - \mathbf{m} and Ψ were specifically selected as unknowns so that the same temporal basis function could be used – i.e., triangle functions
- Discretization conforms to temporal Sobolev space properties of the individual exterior/interior integral equations

- In our numerical results we use an eigenvalue stability analysis to show if the system is stable or not – any eigenvalues outside the unit circle means the system is unstable



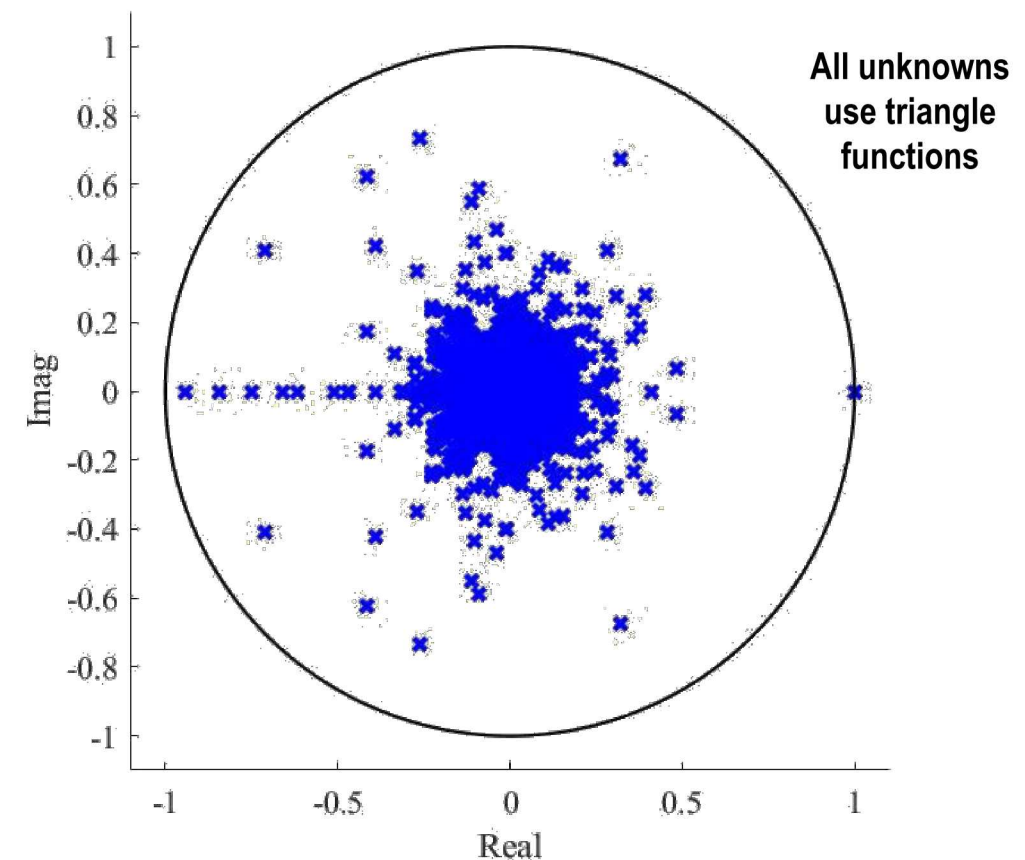
Results – Temporal Discretization Comparison

Incorrect Temporal Discretization

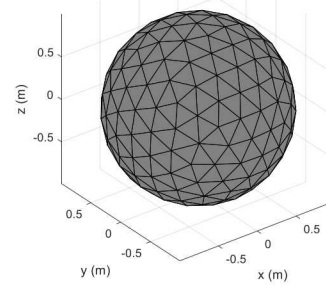


Incorrect temporal basis functions lead to instability

Correct Temporal Discretization



Correct temporal basis functions lead to improved stability



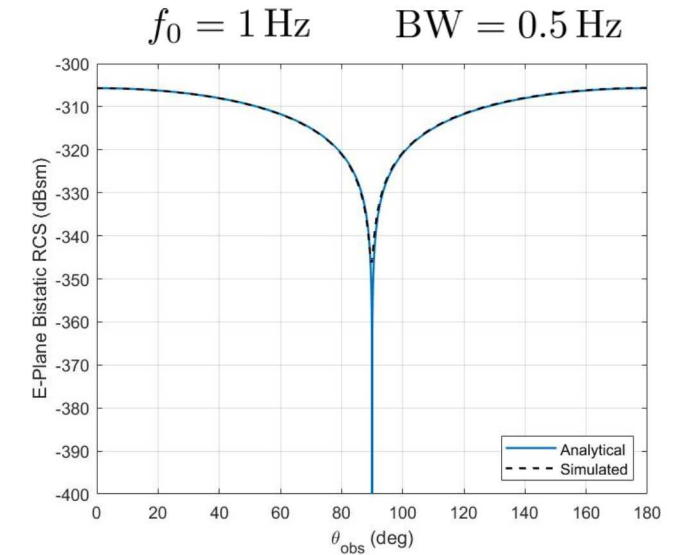
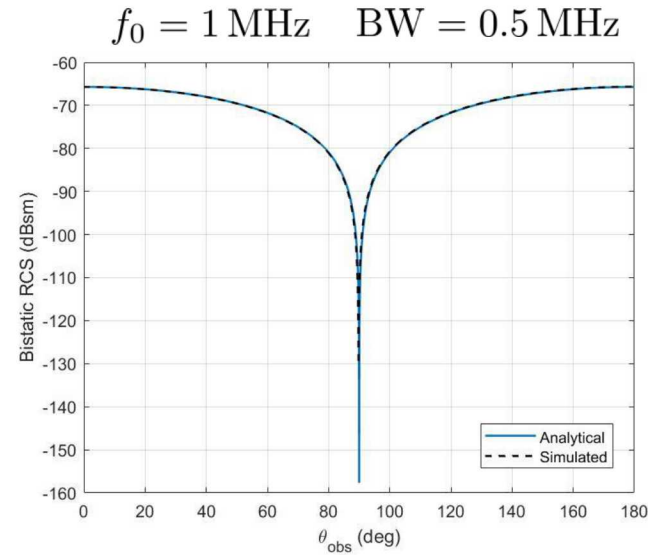
$$\epsilon_r = 2.56$$

$$f_0 = 10 \text{ MHz}$$

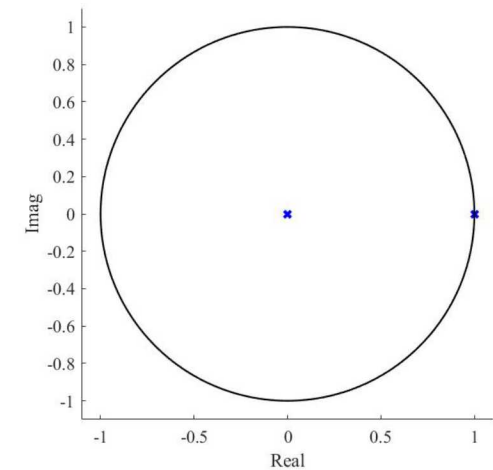
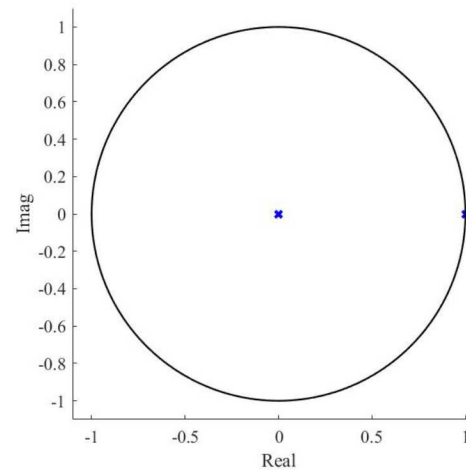
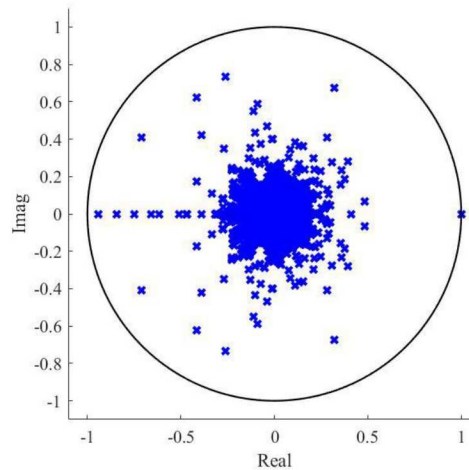
$$\text{BW} = 5 \text{ MHz}$$

Results – Sphere with $\epsilon_r = 2.56$

Accuracy



Stability



Conclusion

- Potential-based CEM formulations are of interest to address the challenging problems that need to be efficiently solved for quantum physics applications
- Coupled potential time domain integral representation formulas were used to derive integral equations for a two-region problem so that all boundary conditions are satisfied at the interface
- Numerical results demonstrate the validity and stability of the new formulation down to very low frequencies

Future Work

- Integral equations presented here are *not* stable as the frequency is increased
- Reason for instability needs to be investigated further to determine strategies to overcome this issue



Acknowledgments

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This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

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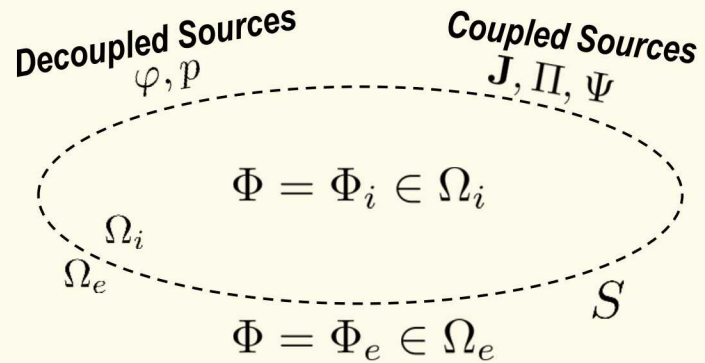
BACKUP



Dielectric Formulation – Scalar Potential Integral Representation

- Integral equations are derived using integral representations for the solutions of wave equations for the potentials
- Integral representations are derived in the frequency domain and then transformed to the time domain
- Lorenz gauge is used to “tie” the scalar and vector potentials together

Scalar Potential Surface Sources



Surface Source Definitions

$$\varphi = \Phi_i - \Phi_e \quad p = \hat{n}' \cdot \nabla' \Phi_i - \hat{n}' \cdot \nabla' \Phi_e$$

$$\mathbf{J} = \mu^{-1} [\hat{n}' \times \nabla' \times \mathbf{A}_i - \hat{n}' \times \nabla' \times \mathbf{A}_e]$$

$$\Pi = \hat{n}' \cdot \dot{\mathbf{A}}_i - \hat{n}' \cdot \dot{\mathbf{A}}_e \quad \Psi = \dot{\Phi}_i - \dot{\Phi}_e$$

Scalar Potential Integral Representation Formulas

Decoupled Potential FD Integral Representation

$$\Phi(\mathbf{r}, \omega) = \int_S \left(g(\mathbf{r}, \mathbf{r}') p(\mathbf{r}') - \hat{n}' \cdot \nabla' g(\mathbf{r}, \mathbf{r}') \varphi(\mathbf{r}') \right) dS'$$



Coupled Potential FD Integral Representation

$$\Phi(\mathbf{r}, \omega) = \int_S \left(\frac{i}{\omega \epsilon} g(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}(\mathbf{r}') - g(\mathbf{r}, \mathbf{r}') \Pi(\mathbf{r}') + \hat{n}' \cdot \nabla' g(\mathbf{r}, \mathbf{r}') \frac{1}{i\omega} \Psi(\mathbf{r}') \right) dS'$$



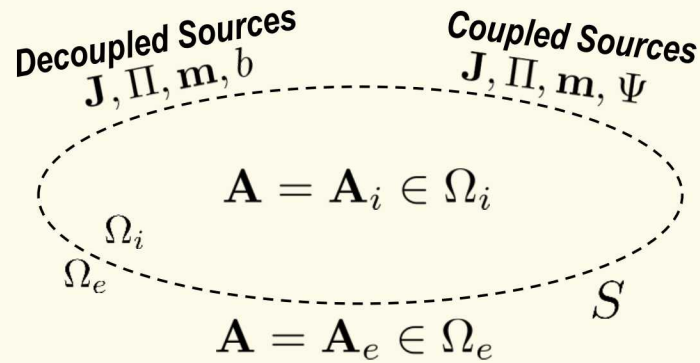
Coupled Potential TD Integral Representation

$$\Phi(\mathbf{r}, t) = \int_S \left(\int_{-\infty}^{\tau} \frac{\nabla \cdot \mathbf{J}(\mathbf{r}', t')}{4\pi R \epsilon} dt' - \frac{\Pi(\mathbf{r}', \tau)}{4\pi R} - \hat{n}' \cdot \nabla' \int_{-\infty}^{\tau} \frac{\Psi(\mathbf{r}', t')}{4\pi R} dt' \right) dS'$$

Dielectric Formulation – Vector Potential Integral Representation

- Integral equations are derived using integral representations for the solutions of wave equations for the potentials
- Integral representations are derived in the frequency domain and then transformed to the time domain
- Lorenz gauge is used to “tie” the scalar and vector potentials together

Vector Potential Surface Sources



Surface Source Definitions

$$\Psi = \dot{\Phi}_i - \dot{\Phi}_e \quad b = \nabla' \cdot \mathbf{A}_i - \nabla' \cdot \mathbf{A}_e$$

$$\mathbf{J} = \mu^{-1} [\hat{n}' \times \nabla' \times \mathbf{A}_i - \hat{n}' \times \nabla' \times \mathbf{A}_e]$$

$$\Pi = \hat{n}' \cdot \dot{\mathbf{A}}_i - \hat{n}' \cdot \dot{\mathbf{A}}_e \quad \mathbf{m} = \hat{n}' \times \dot{\mathbf{A}}_i - \hat{n}' \times \dot{\mathbf{A}}_e$$

Vector Potential Integral Representation Formulas

Decoupled Potential FD Integral Representation

$$\mathbf{A}(\mathbf{r}, \omega) = - \int_S \left(\mu g(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') - \nabla' g(\mathbf{r}, \mathbf{r}') \frac{\Pi(\mathbf{r}')}{i\omega} - \hat{n}' g(\mathbf{r}, \mathbf{r}') b(\mathbf{r}') + \nabla' g(\mathbf{r}, \mathbf{r}') \times \frac{\mathbf{m}(\mathbf{r}')}{i\omega} \right) dS'$$



Coupled Potential FD Integral Representation

$$\mathbf{A}(\mathbf{r}, \omega) = - \int_S \left(\mu g(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') - \nabla' g(\mathbf{r}, \mathbf{r}') \frac{\Pi(\mathbf{r}')}{i\omega} + \hat{n}' g(\mathbf{r}, \mathbf{r}') \mu \epsilon \Psi(\mathbf{r}') + \nabla' g(\mathbf{r}, \mathbf{r}') \times \frac{\mathbf{m}(\mathbf{r}')}{i\omega} \right) dS'$$



Coupled Potential TD Integral Representation

$$\mathbf{A}(\mathbf{r}, t) = - \int_S \left(\mu \frac{\mathbf{J}(\mathbf{r}', \tau)}{4\pi R} - \nabla \int_{-\infty}^{\tau} \frac{\Pi(\mathbf{r}', t')}{4\pi R} dt' + \hat{n}' \mu \epsilon \frac{\Psi(\mathbf{r}', \tau)}{4\pi R} + \nabla \times \int_{-\infty}^{\tau} \frac{\mathbf{m}(\mathbf{r}', t')}{4\pi R} dt' \right) dS'$$