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Local Laminar Flow Shear and Heat Transfer Solutions for Reduced Order Reentry Simulation

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Multi-Fidelity Reentry Model

- **A multi-fidelity tool kit for reentry flows is under development at Sandia National Laboratories**
- **Why?**
  - **Time to build an aerothermal design data set order of years**
  - **Considerable human intervention required**
  - **More quantities of interest required**
  - **Required coupling of trajectory model.. Iterative approach**
  - **Uncertainty quantification… multiple realizations**
- **SAND2019-13632**


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Development of a Multi-fidelity Toolkit for Rapid Aerothermal Model Development

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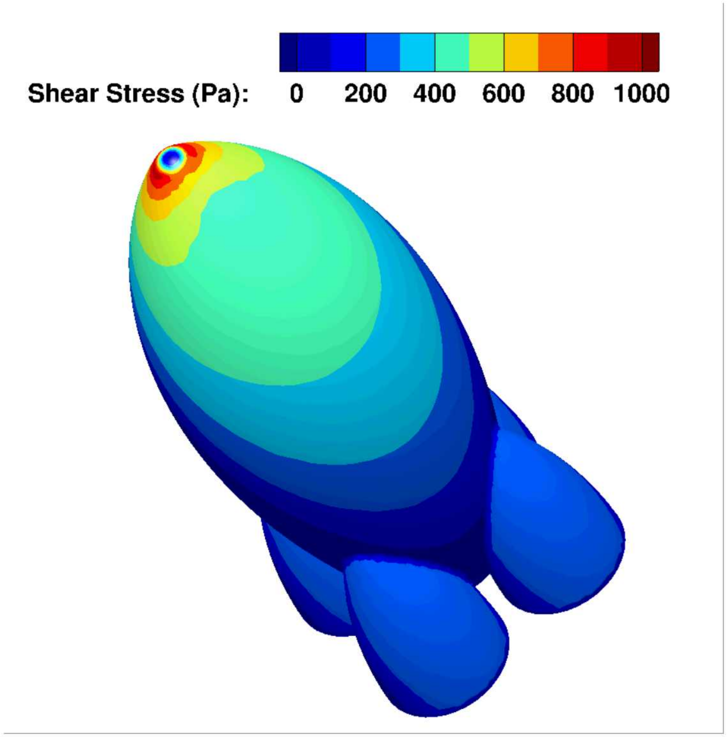
Multi-Fidelity Reentry Model

- A multi-fidelity tool kit
  - High-fidelity=RANS
  - Mid-Fidelity=Euler +Momentum/Energy Integral
  - Low-Fidelity=Modified Newtonian Aerodynamics, Local boundary layer solution/correlations
  - Overarching control strategy
  - Solution interpolation: Kriging
  - Body fitted gridding; local streamline field

Multi-Fidelity Reentry Model

- The reduced order capability depends on inviscid Newtonian local viscous skin friction/heat transfer models
- Two limiting canonical laminar viscous flows are:
  - Stagnation flow
  - Flat plate (zero pressure gradient)
- Local model should be able to locally blend these solutions
- Concentrate on high altitude/low density laminar flow

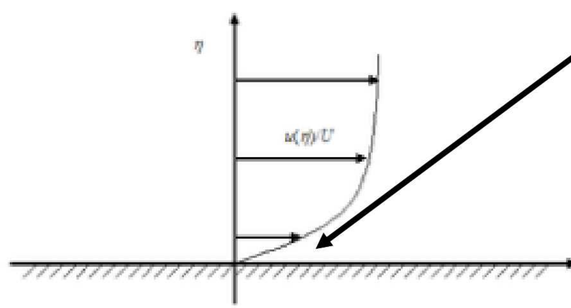


Analytical Solutions

- **Self-similar laminar flows (both flat plate and stagnation) are given by:**

$$f''' + \alpha f f'' + \beta(1 - f'^2) = 0$$

- **Numerical solutions are always possible…**
- **Finite difference BVP/IVP**
- **However… we only need near wall information e.g. f'(0) whereby the full solution field is extraneous**



Wall Shear/Heat Transfer

- **Access to wall velocity gradient f''(0) is directly related to skin friction:**

$$C_f = \frac{2\tau_w}{\rho U^2} = \frac{2}{\rho U^2}\left(\frac{\mu U f''(0)}{\left(\frac{2\nu x}{U}\right)^{1/2}}\right) = \sqrt{2} f''(0)\,\mathrm{Re}_x^{-1/2}$$

- **Heat transfer is related through Reynolds analogy**

$$Nu_x = \frac{-g'(0)}{\sqrt{2}}\mathrm{Re}_x^{1/2} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

Solution Parameters

- **Previous solution**

$$f''' + \alpha f f'' + \beta(1 - f'^2) = 0$$

- **Has two parameters, α and β which parameterize several flows:**

Blasius Flat Plate: α=1, β=0
Planar Stagnation: α=1, β=1
Axisymmetric Stagnation: α=2, β=1
Faulkner-Skan: α=1, 0<β<∞

Analytical Approximate Solutions; Blasius Example

- **Approximate analytical solution to ODE are possible.**
- **Consider (classical!) Blasius equation solution process (Weyl, Boyd, White):**

$$ff'' + f''' = 0$$

- **Using** $f'' \equiv w$ **rewrite as:**

$$w' + fw = 0 \rightarrow w = w(0)\exp\left(-\int_0^{\eta} f(\xi)\,d\xi\right)$$

- **Estimate f(ξ):**

$$f(\eta) = f(0) + \eta f'(0) + \frac{1}{2} f''(0)\eta^2 + \ldots$$

$$= \frac{1}{2} f''(0)\eta^2 + \ldots$$

Analytical Approximate Solutions; Blasius Example

- **Substitute:**

$$1 = f''(0)[f''(0)]^{-1/3}\int_0^\infty \exp(-\frac{1}{6}\bar{\xi}^3)d\bar{\xi}$$

- **Where the integral yields:**

$$\int_0^\infty \exp(-\frac{1}{6}\bar{\xi}^3)d\bar{\xi} = \frac{2}{9}\frac{6^{1/3}\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}$$

- **So as to give (within about 3% of exact):**

$$f''(0) = 0.48380$$

$$f''(0)\big|_{exact} = 0.46960$$

Analytical Approximate Solutions; Heat Transfer

- **Readily compute heat transfer. Energy equation:**

$$g'' + \Pr f g' = 0 \qquad g = \frac{T - T_e}{T_w - T_e}$$

- **Which can be written**

$$\int_0^\infty g' d\eta = -1 = g'(0) \int_0^\infty \exp\left(-\Pr \int_0^\eta f(\xi) d\xi\right) d\eta$$

- **So as to give (justifying the Reynolds analogy):**

$$Nu_x = \frac{-g'(0)}{\sqrt{2}} \mathrm{Re}_x^{1/2} = 0.332 \, \mathrm{Re}_x^{1/2} \Pr^{1/3}$$

Analytical Approximate Solutions; General Problem

- **Consider full expression:**

$$f''' + \alpha f f'' + \beta(1 - f'^2) = 0$$

- **Rewrite as:**

$$w' + \alpha f w = \beta(f'^2 - 1)$$

Approximate:

$$f' = 1 - \exp(-f''(0)\eta)$$
$$f'' = f''(0)\exp(-f''(0)\eta)$$

- **Estimate f'²-1…**

$$\begin{aligned} 1 - f'^2 &= [2 - \exp(-f''(0)\eta)]\exp(-f''(0)\eta) \\ &\approx \exp(-f''(0)\eta) \\ &\approx \frac{1}{f''(0)} f'' \end{aligned}$$

To give:

$$w' + \left(\alpha f + \frac{\beta}{f''(0)}\right) w = 0$$

Analytical Approximate Solutions; General Problem

- **Solving:**

$$w' + (\alpha f + \frac{\beta}{f''(0)})w = 0 \rightarrow w = w(0)\exp(-\alpha \int_0^{\eta} f(\xi) + \frac{\beta}{f''(0)} d\xi)$$

- **To give the expression:**

$$1 = f''(0)\int_0^{\infty} \exp[-\frac{\alpha}{6} f''(0)\eta^3 + \frac{\beta}{f''(0)}\eta] d\eta = 0$$

- **Or** $$1 = \alpha^{-1/3}(f''(0))^{2/3}\int_0^{\infty} \exp[-\frac{1}{6}\xi^3 - \frac{\beta}{\alpha^{1/3}(f''(0))^{4/3}}\xi] d\xi = 0$$
- **The integral is possible but complex… consider <u>approximation</u>:**

$$\int_0^{\infty} \exp\left(-\frac{1}{6}\xi^3 - \xi\right) d\xi \approx \exp(-\xi_0)\int_0^{\infty} \exp\left(-\frac{1}{6}\xi^3\right) d\xi \qquad \xi_0 = 0.74961$$

Analytical Approximate Solutions; General Problem

- **Yields the implicit solution:**

$$1 = 1.62265\alpha^{-1/3}(f''(0))^{2/3}\exp\left(-\frac{\beta}{\alpha^{1/3}(f''(0))^{4/3}}0.74961\right)$$

- **Or the explicit solution (special function):**

$$f''(0) = 1.35487\alpha^{1/2}\beta^{3/4}\left(\alpha LambertW\left(\frac{3.9455\beta}{\alpha}\right)\right)^{-3/4}$$

Solution Parameters

- **Previous solution**

$$1 = 1.62265\alpha^{-1/3}(f''(0))^{2/3}\exp(-\frac{\beta}{\alpha^{1/3}(f''(0))^{4/3}}0.74961)$$

- **Has two parameters, α and β which parameterize several flows:**

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Faulkner-Skan: α=1, 0<β<∞

Solutions…

Flow	α	β	f''(0) approx..	f''(0) exact	Rel. err.
Blasius	1	0	0.4838	0.4696	3%
Plane Stag	1	1	1.1856	1.2326	4%
Axi-stag	2	1	1.2909	1.3119	2%
Faulkner-Skan	1	2	1.6037	1.6872	5%
Faulkner-Skan	1	10	3.6300	3.6752	1%

Solution Parameters… relation to flow geometry

- **Base parameters on local flow angle: $y_p(x)$ function describing body surface**

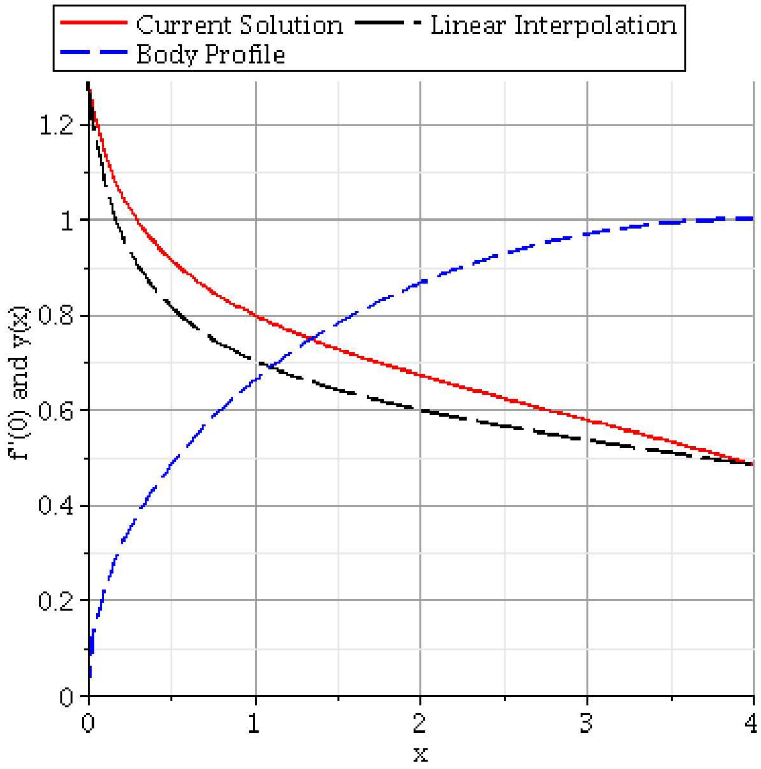
$$\sin\theta = \frac{\frac{dy_p}{dx}}{\sqrt{1+(\frac{dy_p}{dx})^2}} \equiv G(\theta)$$

- **Where**

$$\alpha = 1 + G(\theta) \quad ; \quad \beta = G(\theta)$$

- **And freestream velocity**

$$U = U_0(1 - G(\theta)) + BxG(\theta)$$



Compressibility..

- **Introduce Chapman-Rubesin parameter**

$$C_w = \frac{\rho_w \mu_w}{\rho_e \mu_e} \approx \left(\frac{T_w}{T_e} \right)^{-1/3}$$

- **Solution takes form:**

$$f''(0) = 1.35487 \alpha^{1/2} \beta^{3/4} C_w^{1/2} \left(\alpha \, LambertW \left(\frac{3.9455 \beta}{\alpha C_w^2} \right) \right)^{-3/4}$$

Comparison to other methods..

- **Traditionally, mapping from stagnation to flat plate is done via:**

$$\frac{q}{q_{stag}} = func(X) \qquad X = \frac{\int_0^x \rho^* \mu^* u_e r^{2j} dx}{\rho^* \mu^* u_e r^{2j}} \qquad \beta_F = \frac{2X}{u_e}\frac{du_e}{dX}$$

- **Where:**

Flow	X	β_F
Flat plate	x	0
Planar stagnation	x/2	1
Axisymmetric stagnation	x/4	½

- **There a wide range of models** $func(X)$ **an asymptotically correct empirical expression is:**

$$Nu = 0.332(1 + 0.1853\beta_F^{1/2})\,\mathrm{Re}^{-1/2}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re} = \frac{\rho^* X u_e}{\mu^*}$$

Comparison to other methods..

- **For a known body… we can directly compare these models**
- **Consider an axi-symmetric spherical body:**

$$G(x) = \sqrt{1-\left(\frac{x}{r}\right)^2} \qquad G(\theta) = \frac{\frac{dy_p}{dx}}{\sqrt{1+(\frac{dy_p}{dx})^2}}$$

- **We can try the two models**
- **Kemp, Rose, Detra (1959) (empirical)**

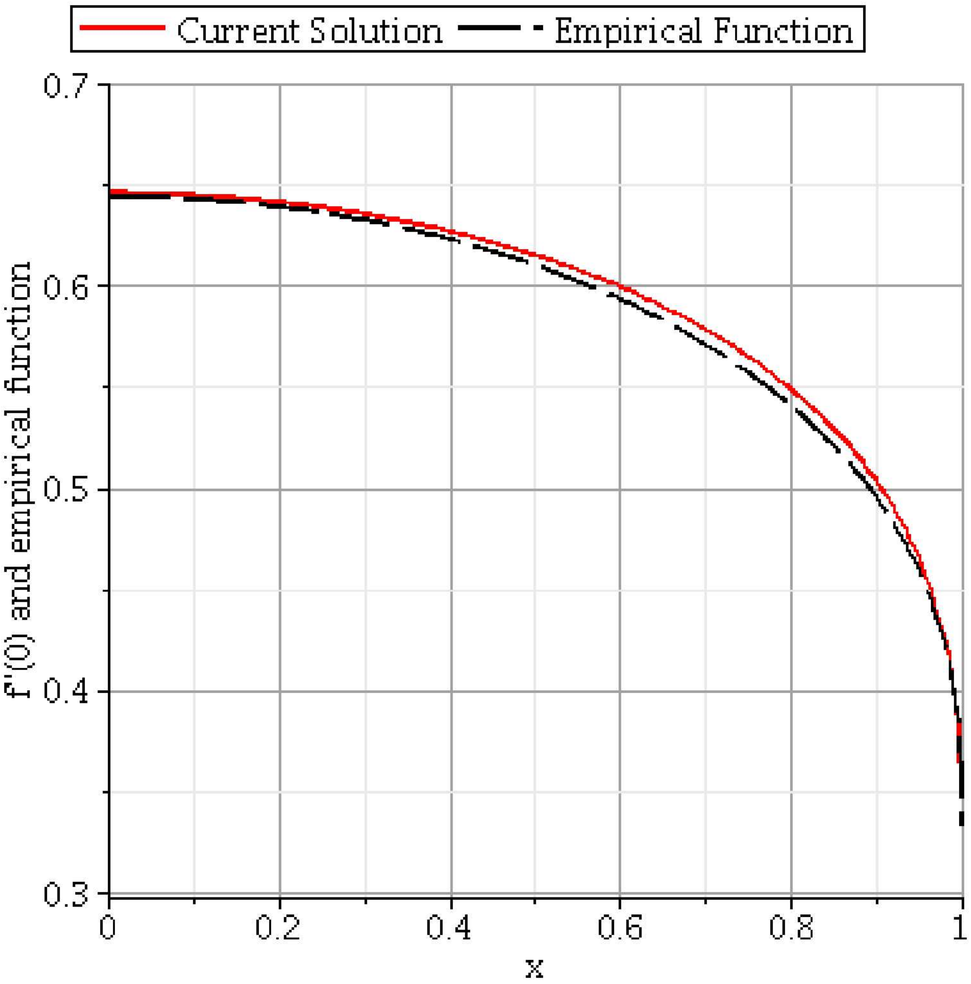
$$Nu\,\mathrm{Re}^{1/2} = 0.332[1+0.1853(4G(x))^{1/2}](1+G(x))^{1/2}\,\mathrm{Pr}^{1/3}$$

- **Current Model**

$$Nu\,\mathrm{Re}^{1/2} = \frac{1}{2}f''(0)\,\mathrm{Pr}^{1/3} \qquad f''(0) = 1.35487\alpha^{1/2}\beta^{3/4}C_w^{1/2}\left(\alpha LambertW\left(\frac{3.9455\beta}{\alpha C_w^2}\right)\right)^{-3/4}$$

Comparison to other methods..

- **Spherical body**
- **Solution parameters vary by body shape**
- **Agrees well with empirical approach e.g. Kemp, Rose, Detra (1959).**



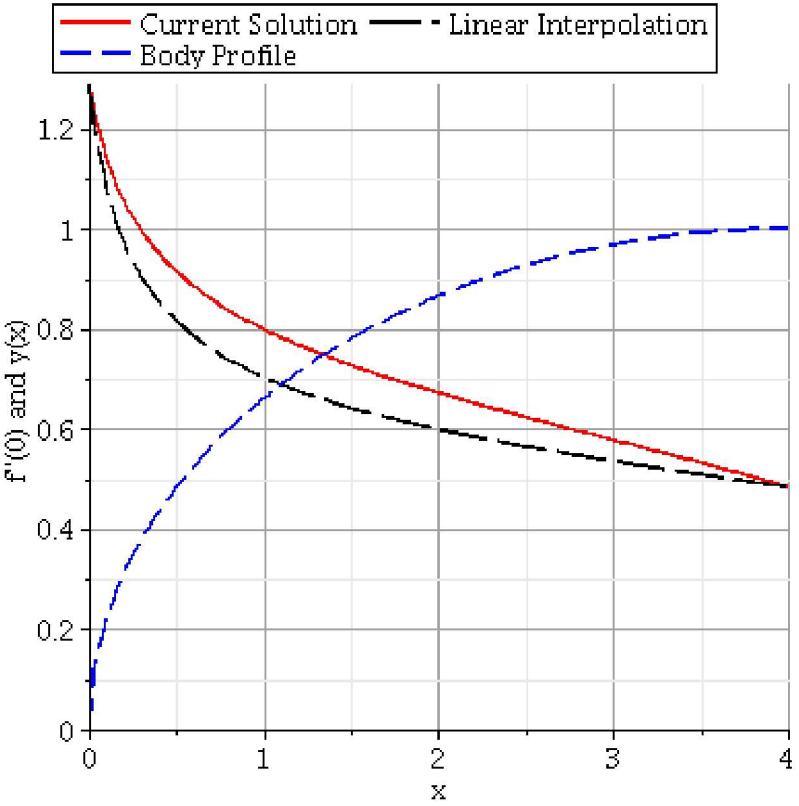
Comparison to other methods..

- **Elliptical body**

$$y_p(x) = \sqrt{1 - \frac{1}{16}(x-4)^2}$$

- **Solution based on parameter variation**
- **Linear interpolation of solution**

$$f''(0) = f''_{stag}(0)G + f''_{plate}(0)(1-G)$$

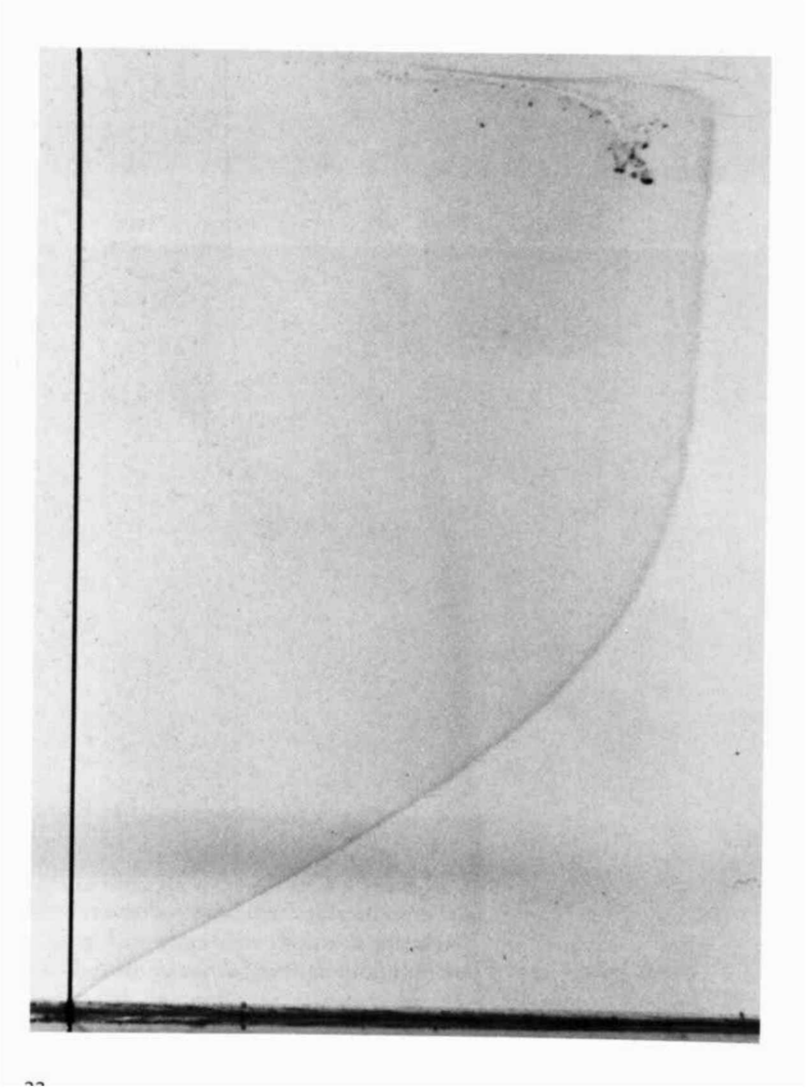


Conclusions…

- **Simple closed-form method derived to estimate laminar heat transfer**
- **Solution maps continuously between stagnation and zero pressure gradient shear (Blasius)**
- **Good agreement in comparison with empirical methods (Kemp et. al. 1959)**
- **Models implemented in Sandia Multi-Fidelity Aerothermal Toolkit (2019)**

Next Steps…

- **Examine streamline based mapping to solution parameters α and β**
- **Consider extension to simplified turbulent flow behavior with intermittency parameter mapping**
- **Connection of approximate integral solution to other (semi) analytical methods (Catal 2012), e.g. Homotopy, Adomian Decomposition etc.**



Traditional Galerkin Method…

- **Traditional Galerkin e.g. Fletcher (1984) and method-of-weighted residuals**

$$f''' + ff'' + (1 - f'^2) = 0$$

- **Propose full domain basis function**

$$f = \eta + \frac{1}{f''(0)}\left(\exp(-f''(0)\eta) - 1\right)$$

$$f' = 1 - \exp(-f''(0)\eta)$$

$$f'' = f''(0)\exp(-f''(0)\eta)$$

$$f''' = -f''(0)^2 \exp(-f''(0)\eta)$$

Traditional Galerkin Method…

- **Substitute into governing equation to give residual:**

$$-f''(0)^2\exp(-f''(0)\eta)+\left(\eta+\frac{1}{f''(0)}\left(\exp(-f''(0)\eta)-1\right)\right)f''(0)\exp(-f''(0)\eta)+$$
$$-\left(1-\exp(-f''(0)\eta)\right)^2+1$$

- **Nonlinear expression for f''(0). Demand satisfaction..**
- **Near wall result:** $-f''(0)^2+1=0\rightarrow f''(0)=1$
- **Integrate residual over domain**

$$\frac{-f''(0)^2+2}{f''(0)}=0\rightarrow f''(0)=\sqrt{2}$$

- **But which is correct? Neither… consider simple average..**

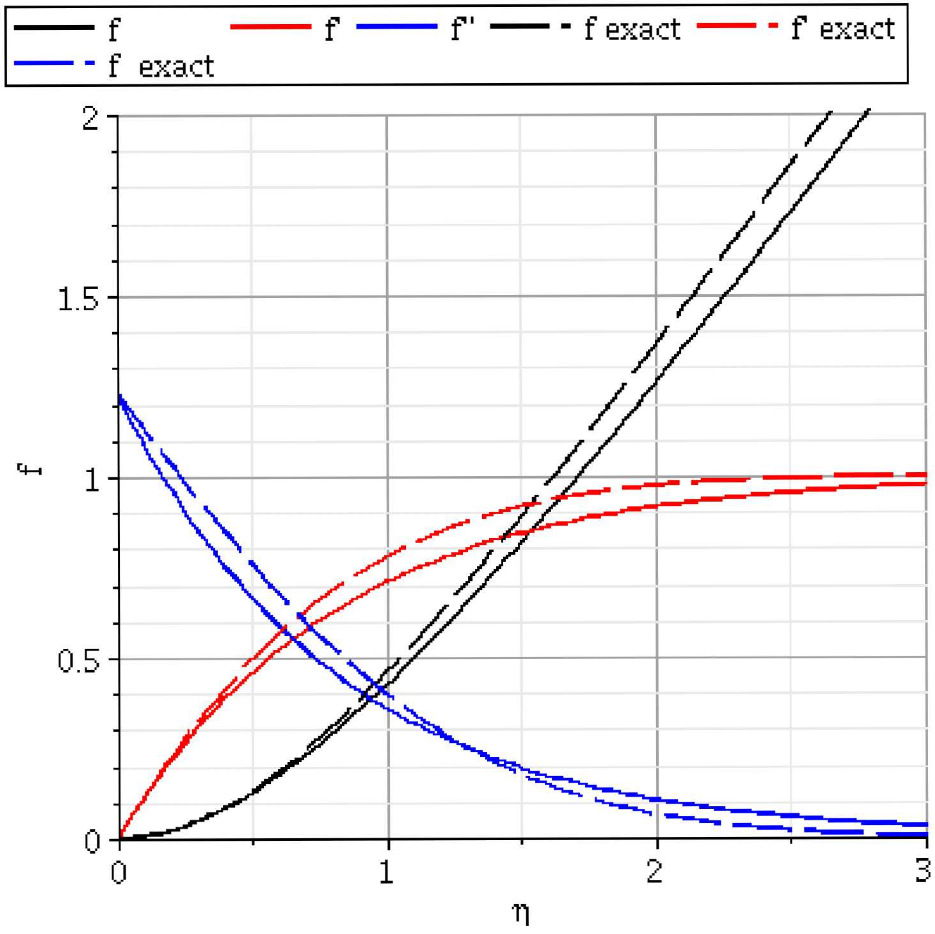
Traditional Galerkin Method…

- **Approximate (average)**

$$f''(0) = \frac{1}{2}(\sqrt{2}+1) \approx 1.21$$

- **Exact:**

$$f''(0)_{exact} = 1.23259$$



$$f''' + ff'' + (1 - f'^2) = 0$$