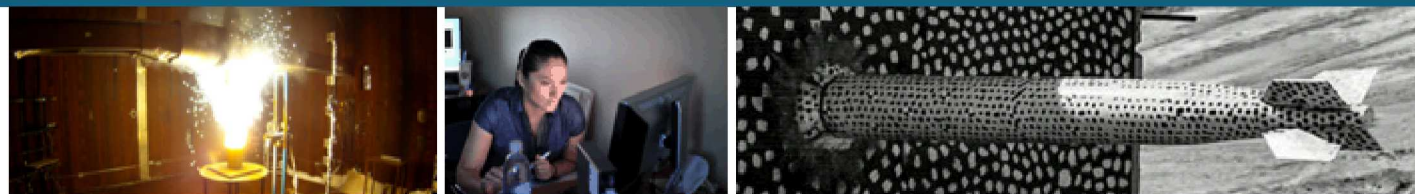


Biological and Chemical Sensors and Modeling Methods for Acoustic Devices



Darren W. Branch, Distinguished Member of Technical Staff
Biological and Chemical Sensors Department, 08634
Sandia National Laboratories, Albuquerque, NM 87122

Presentation: Oregon Health Science University (OHSU)

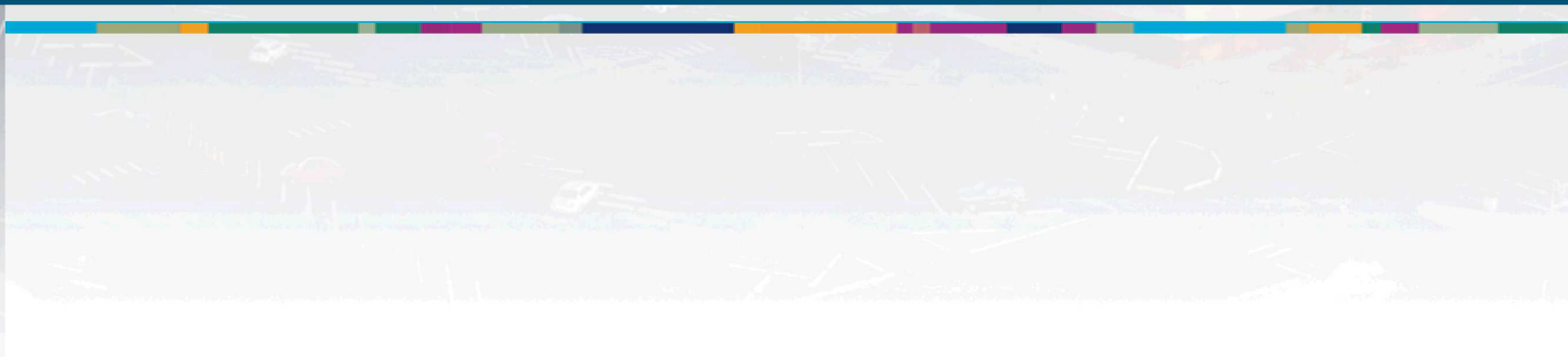
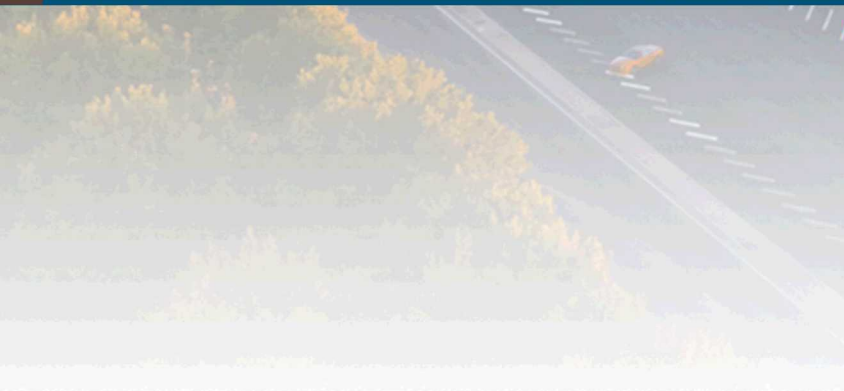
dwbranc@sandia.gov



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Biological and Chemical Sensors



Volatile Organic Compounds (VOCs): Human Health Monitoring



Breath



Blood



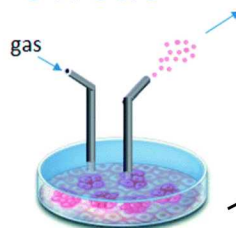
Skin



Saliva



Sweat



Cell culture

Gas Phase Sensing

Volatile organic compound (VOC) biomarkers for:

- Stress
- Infection- flu, TB, smallpox
- Performance degradation (fatigue)
- Disease- cancer, IBS, asthma
- Drug use
- Ketosis
- Hypoglycemia
- Hypoxia- *J. Breath Res.* 9 (2015) 047103
- Human identification



Urine



Feces

Variety of sample substrates available for analysis.

Bacterial VOCs for pathogen ID

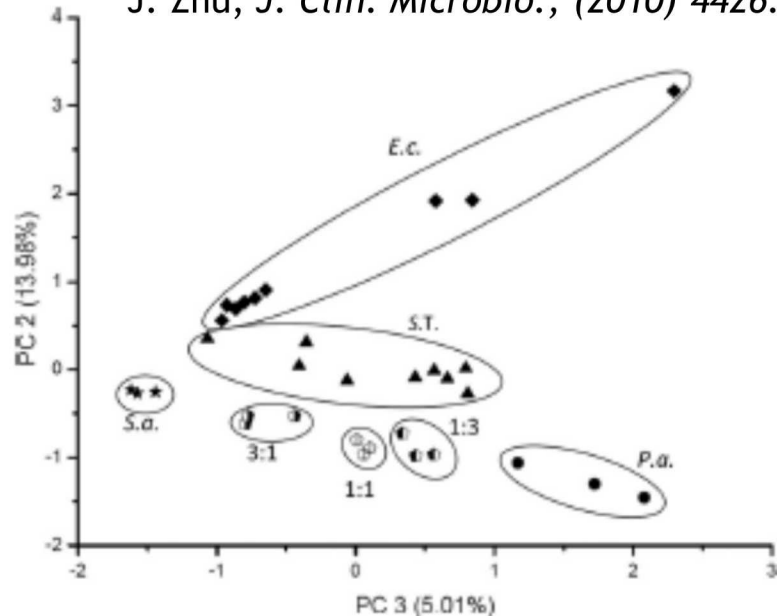
Bacterial VOCs specific to the species, strain, and serovar levels

- ~10-30 VOC specific markers and patterns from volatilome
- E.g. distinguish Shiga Toxin Producing E. coli (STEC) from non-STEC by ppm-sensitive e-nose with 16 hours of culturing

Large database of VOCs diagnostic for diseases, pathogens

- Bos et al, 2013; Lemfack et al, 2014
- Shnayderman *et al*, 2005; Zhu *et al*, 2010; Thorn *et al*, 2011

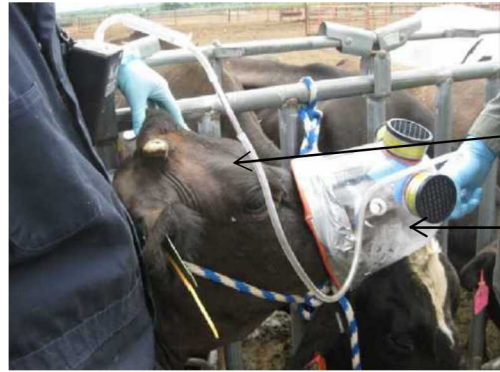
J. Zhu, *J. Clin. Microbio.*, (2010) 4426.



- GC-MS and statistics to identify bacteria – markers and patterns

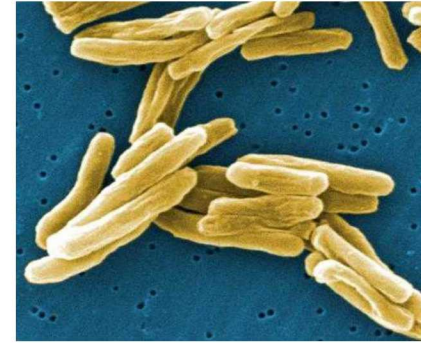
Pathway to diagnostics that are complementary to, but orthogonal from, traditional liquid-based assays.

In vivo detection of *M. bovis* in breath or blood/serum headspace



Cow

VOC collection mask

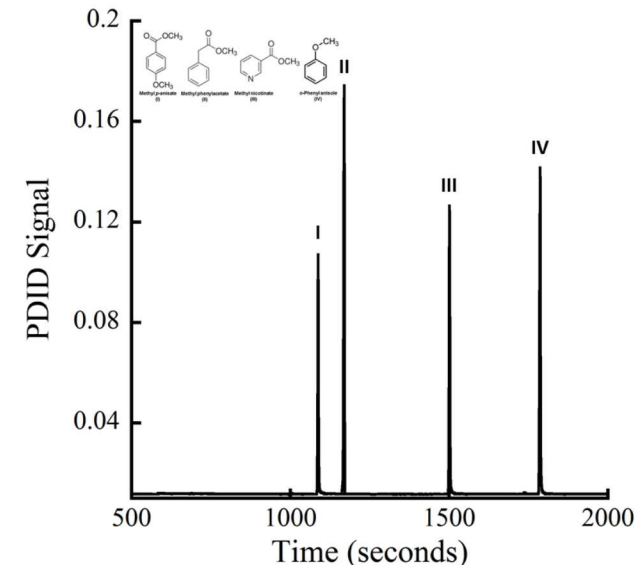


- C.K. Ellis *et al*, PLoS ONE, 9: e892280, 2014:
Exhaled cattle breath by GC-MS, PCA, LDA to distinguish healthy calves from calves with *M. bovis* infection with sensitivity and selectivity of 97% and 99%
- R. Fend *et al*, J. Clin. Microbiol., 43: 1745-1751, 2005:
Badgers, Cattle blood/serum headspace VOCs by e-Nose and PCA DFA identified all infected animals correctly

Our team has demonstrated detection of four characteristic VOC biomarkers of TB* at a 0.02 ppb LOD.

*J. Breath Res. 7 (2013) 037107.

Allows rapid assessment of potential infections and disease states.



VOC Sampling of Water-Stressed Adult Plants



VOC
sampling



Soil VOC
sampling

- Water stress experiments showed, VOC markers arising at different times: blue- < 24 hrs., green- 24 – 48 hrs., and red- > 48 hrs.

VOC markers diagnostic of health conditions exist for plants, animals, and humans.

Analyte
3-methyl-2-Heptanone
2,4,4-trimethyl-Cyclopentanone
2,3-Butanedione
2-Pentanone
Levulinic acid
2,6,6-trimethyl-1-Cyclohexene-1-carboxaldehyde
2-Octanol
3-Octen-2-ol
4-Oxohept-2-enal
2-methyl-butanoic acid
Isophorone
3-Pentanone
3-methyl-Butanoic acid
3-Octanone
2-methyl-Butanal
3-methyl-Butanal
2,2,6-trimethyl-Cyclohexanone
2-ethyl-Furan
Pentanal
Acetic acid
Azulene
1-methyl-4-(1-methylethenyl)-, trans-Cyclohexene

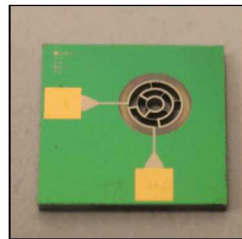
MicroChemlab Gas Phase Detection Technologies



Sandia's proven MicroChemlab sensor system:

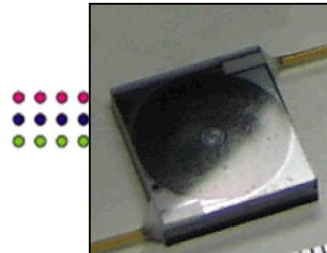
- Field-tested/rugged
- Inexpensive
- 1-2 minute analysis time
- New detector now allows for medical applications
- Hand-portable
- Push button automation

Preconcentration Stage



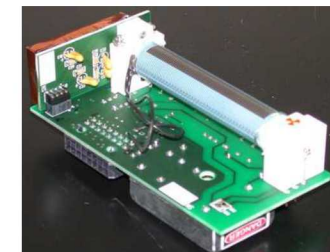
- Collects analyte
- Reduces false alarms

Chromatography Stage



- Separates complex chemical mixtures
- Increases analysis confidence

IMS Detection Stage

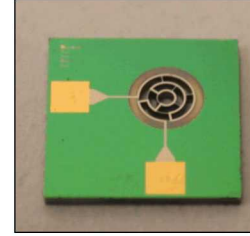
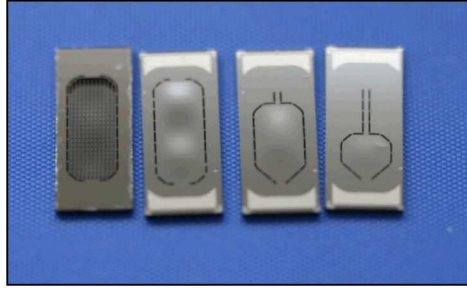


- Sub ppb sensitivity
- Wide applicability to VOC detection

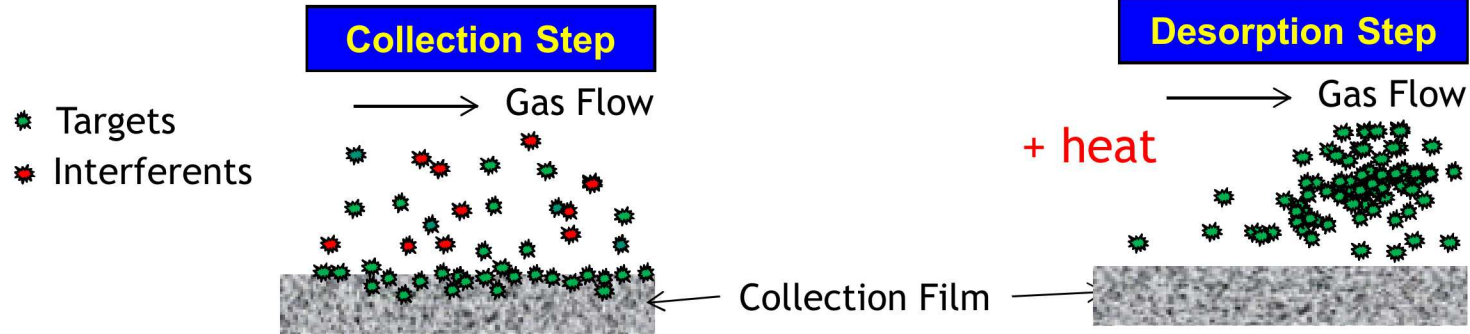
This technology enables portable, VOC-based diagnostics.

Sandia's Chemical Sensing Methodology: Preconcentration

Microfabricated
Preconcentrators



The Preconcentrator accumulates species of interest in a chemical film and rejects interferences.



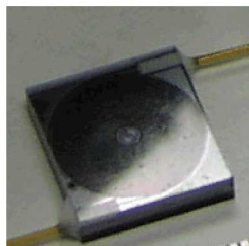
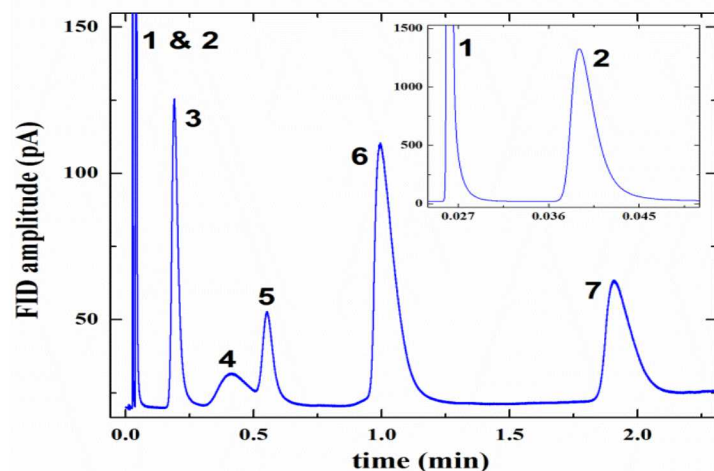
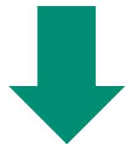
A high-surface area collection film is exposed to air flow. Targets preferentially absorb into the film due to their chemical affinity. Chemically-incompatible interferences are not captured.

- Tailoring the proper collection film for the target compounds is a critical system design detail.

The component is rapidly heated, causing the absorbed target chemicals to leave the collection film in a narrow, highly-concentrated pulse.

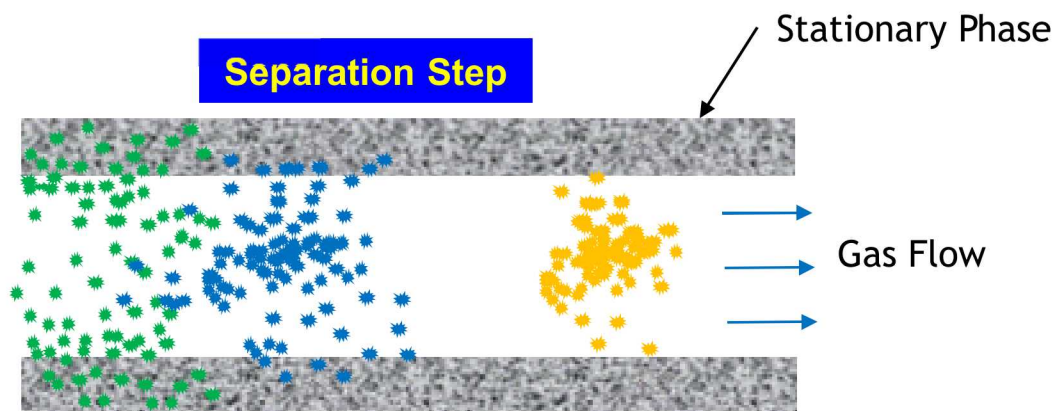
Sandia's Chemical Sensing Methodology: Separation

Example Chromatogram



Gas Chromatography:

- Separates chemical species passing through a channel based upon their affinity for a chemical coating.



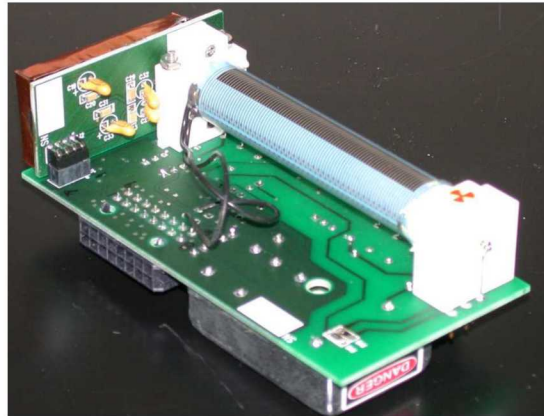
Analytes passing thru the channel have differing affinities for a chemical coating on the channel wall called a stationary phase. Chemicals with greater affinity for the stationary phase (★) are more readily retained than those with a lesser affinity for the stationary phase (☆).

- The time it takes analytes to transit the channel helps identify the compound.
- In this way a complex mixture can be separated into simpler mixtures or single components which simplifies the detector's task.

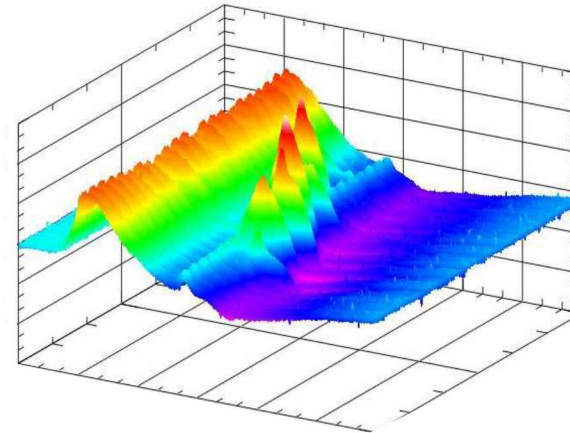
Sandia's Chemical Sensing Methodology: Detection

Sandia has recently developed an ion-mobility spectrometer (IMS) capable of detecting positive ions.

- Allows us to detect VOC targets.
- Sensitivity in the hundreds of picograms.



Sandia's Miniature IMS



Detection of a VOC compound mix

Allows us to detect many VOCs that would otherwise have been undetectable before.

Other Technology Applications: Synthetic Opioid Detection

Synthetic opioids, fentanyl being the most common, are an acutely toxic class of compounds.

- Thousands of suspected structural variants, or analogs, of this molecule class and < 200 have been synthesized and even fewer characterized.
- Carfentanil, an analog of fentanyl, is more toxic than VX gas nerve agent.

Current detection technology has limitations.

- No DEA-approved technology to detect synthetic opioids in the field at trace levels.
- Current technologies cannot detect unknown analogs and have high false alarm rates.
- Better detection technology is being requested by DHS, CTTSO, JPEO, and DoJ.



2-3 mg lethal dose of fentanyl



1 kg "brick" of fentanyl

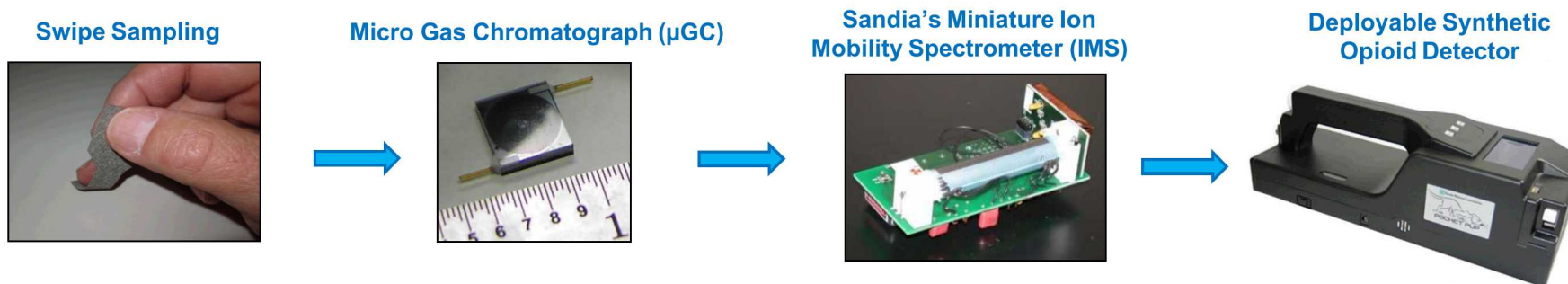
Other Technology Applications: Sandia's Unique Approach

Sandia is developing a novel solution that will be amenable to portable detection by leveraging our current portfolio of microsystem technologies.

- Amenable to swipe or particulate sampling.
- Low false alarm rates.
- Portable instrumentation being designed.

Results:

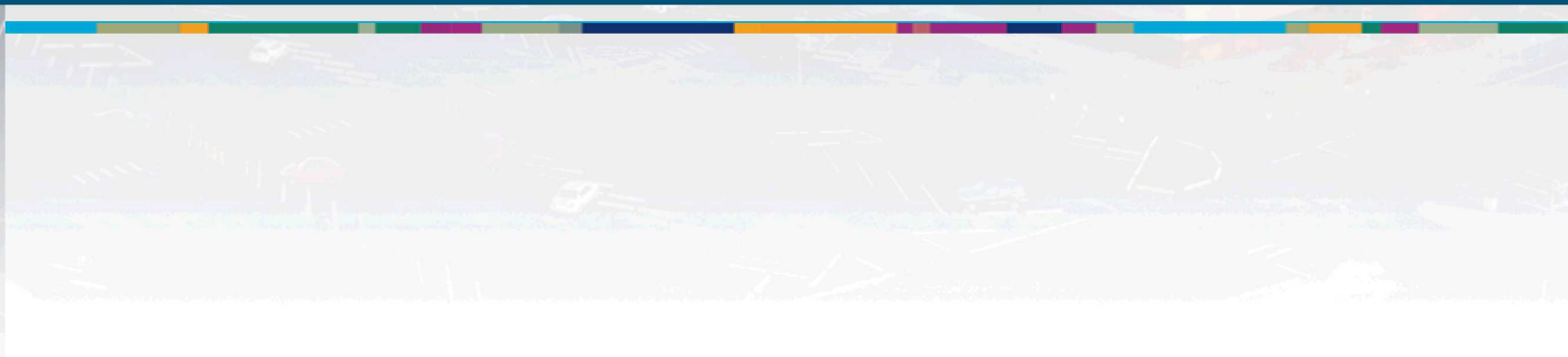
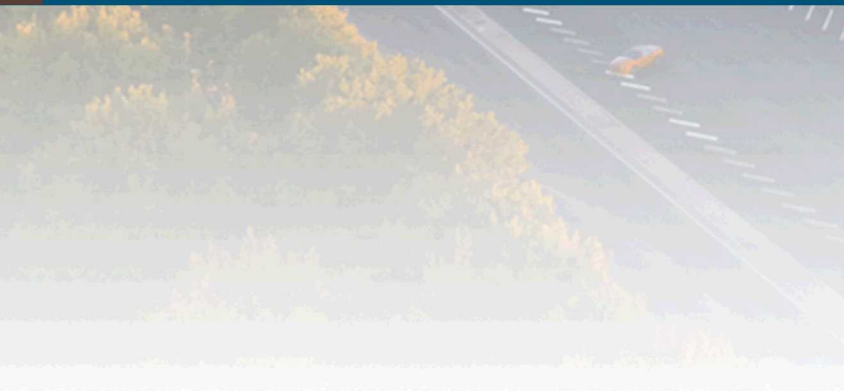
- Able to identify all fentanyl analogs tested thus far (12/12).
- Detected 3 μg of furanyl fentanyl even when mixed with 3 μg of five different cutting agents (lactose, baby formula, baking soda, acetaminophen, and procaine- total of 15 μg of interferences).



No equivalent technical solutions on the horizon from industry or other government agencies.



Microneedles



Microneedles

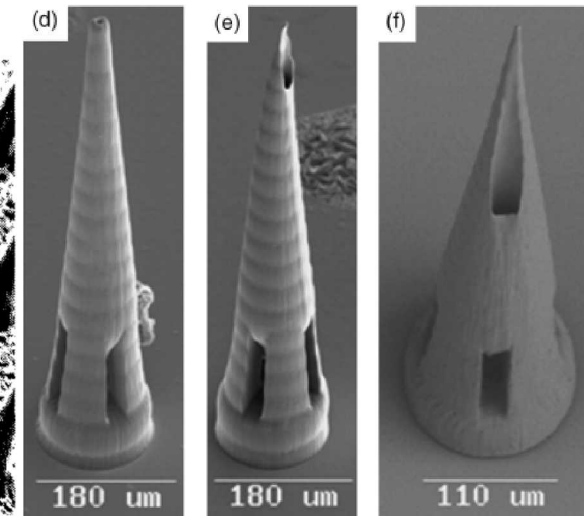
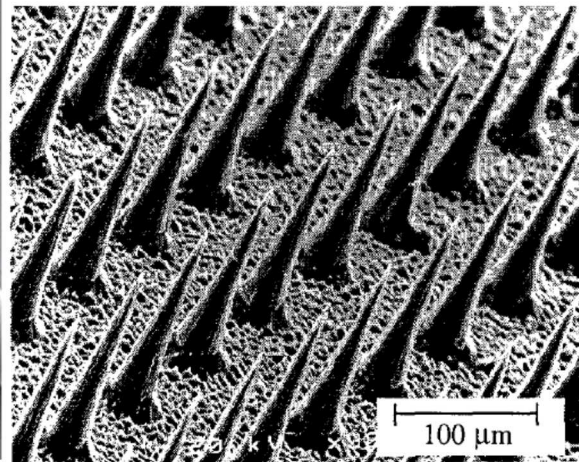
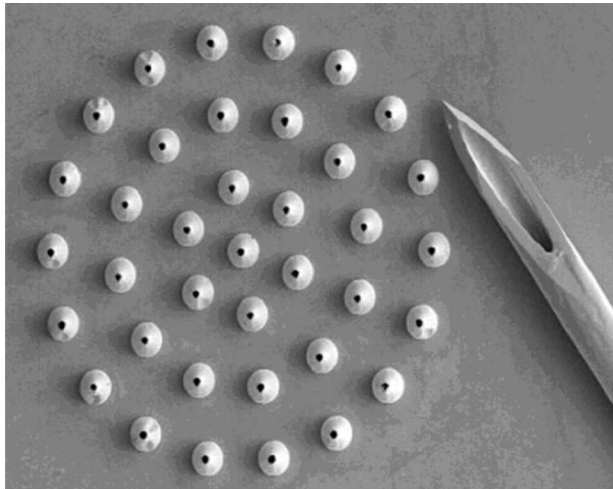
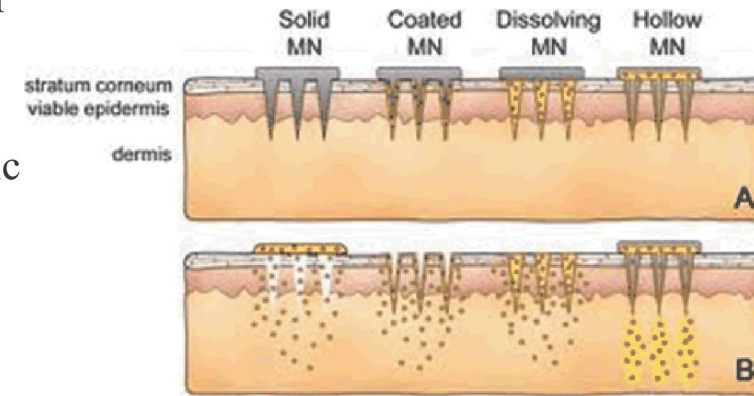
Initially developed for drug delivery to overcome stratum corneum barrier

General requirements, one feature less than 500 μ m

MN's are associated with significantly less pain than 26 gauge hypodermic needles

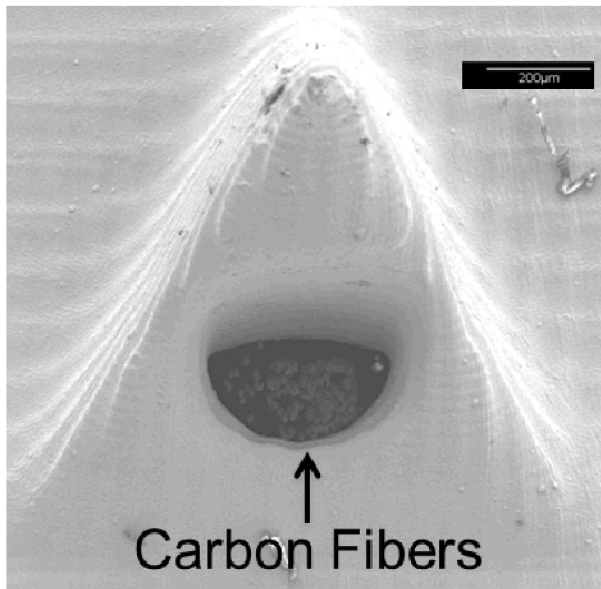
Considered minimally invasive relative to hypodermic syringe

Can be self-administered



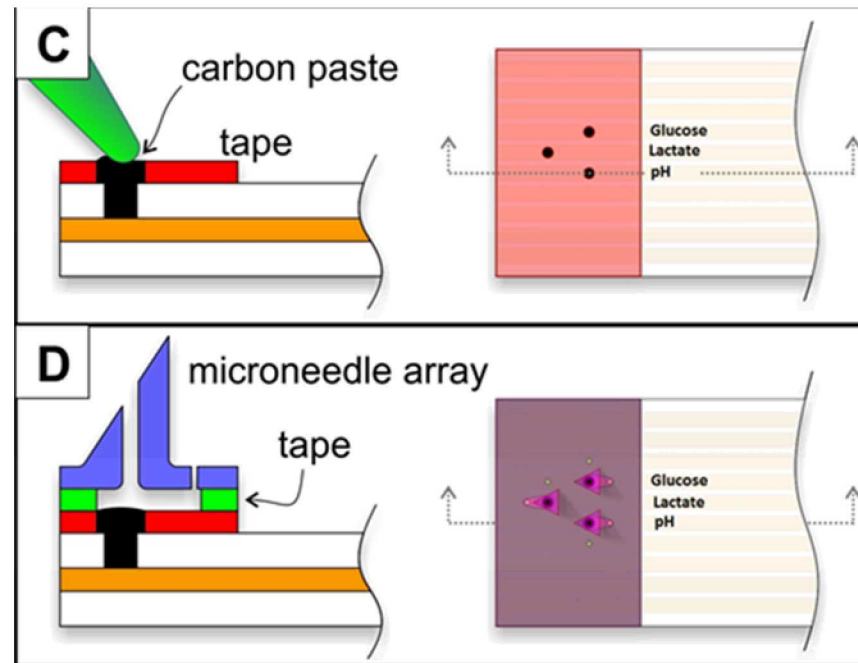
Early Work on Microneedle: Small Molecule Detection

Microneedle



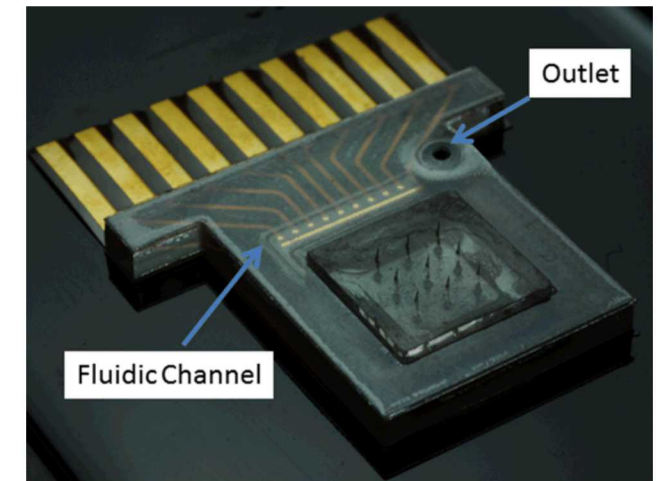
Fabricated with Microstereolithography

Enzymatic Sensors



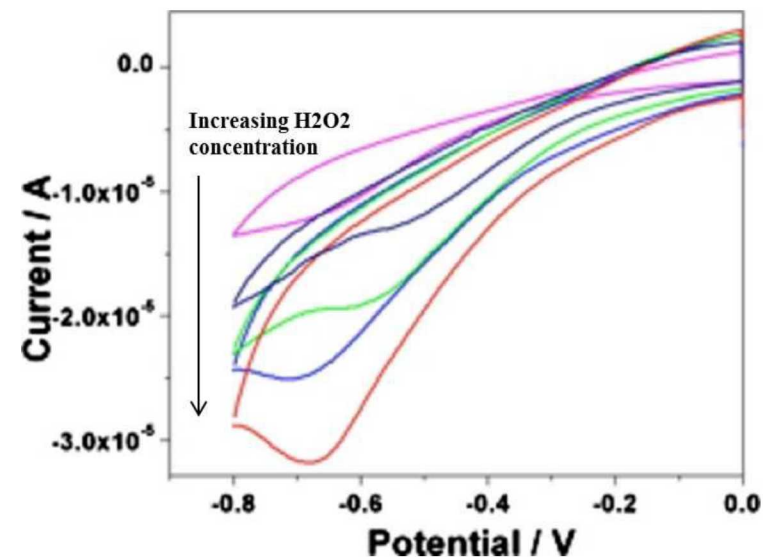
Schematic of device

Protein Detection Chip

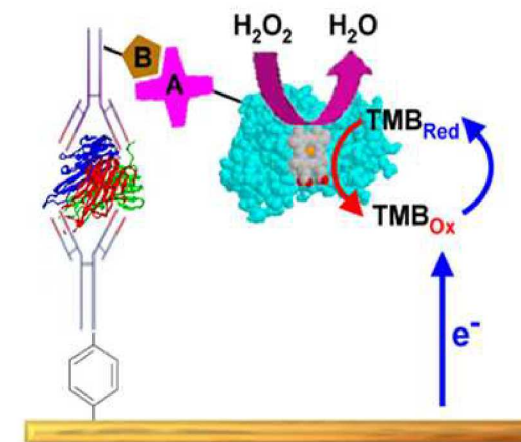
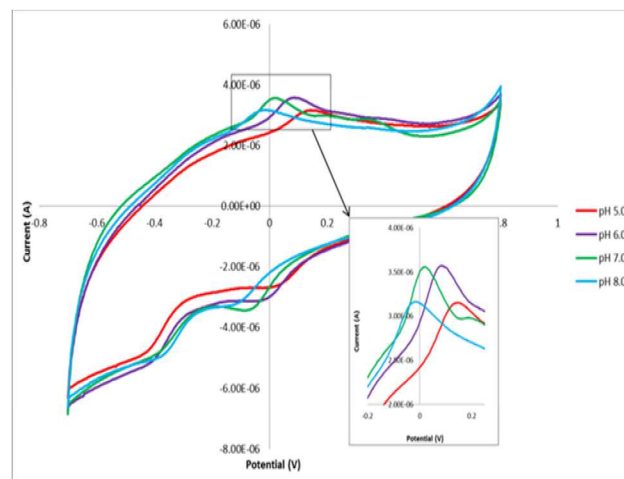


Early Work on Microneedle: Small Molecule Detection

Detecting reactive oxygen species (H_2O_2)

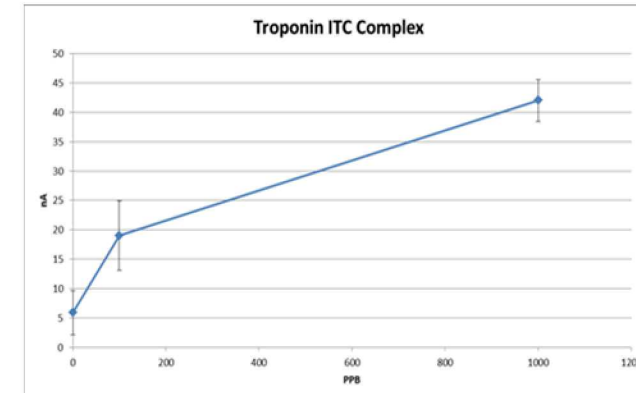
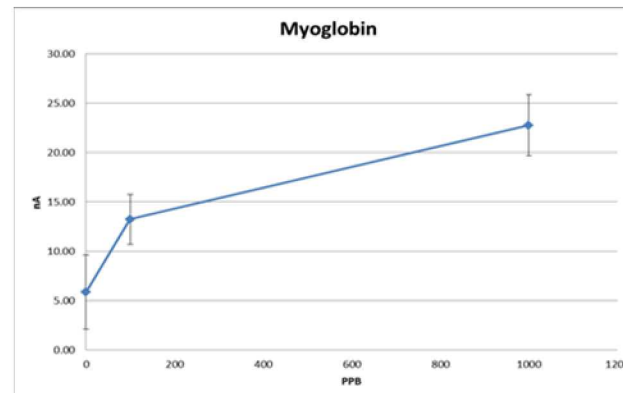
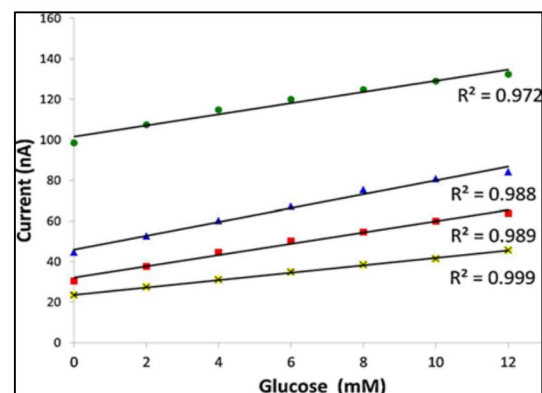
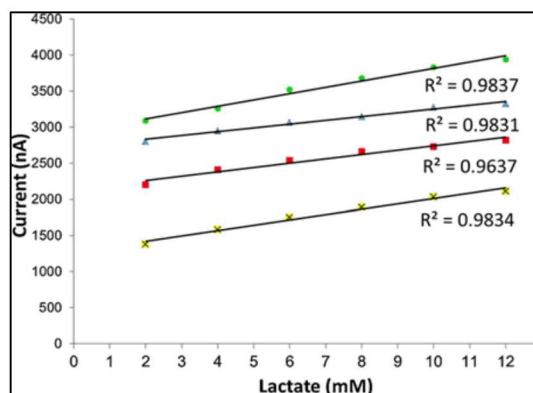


Multiplexed detection of lactate, pH, and glucose



PR Miller, Biomicrofluidics 5.013415 (2011)

Electrochemical detection of cardiac biomarkers

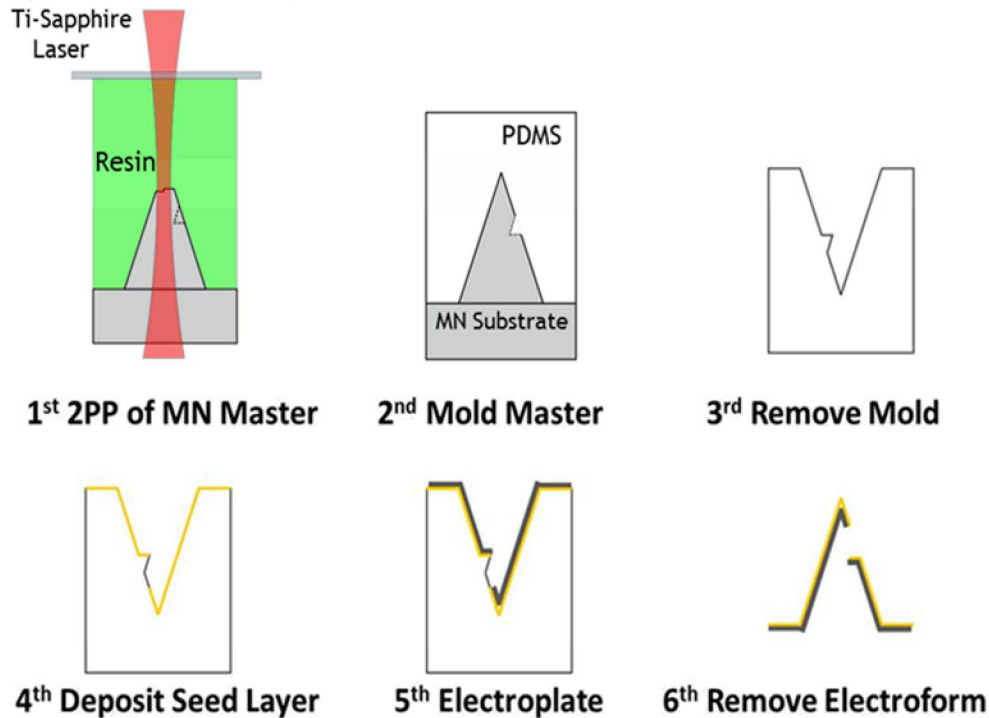


P.R. Miller et al., Talanta 88 (2012)

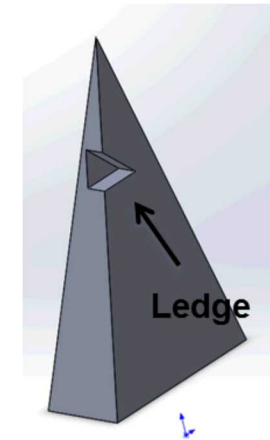
PR Miller, Electroanalysis, 28,6 (2016)

Microneedle Fabrication

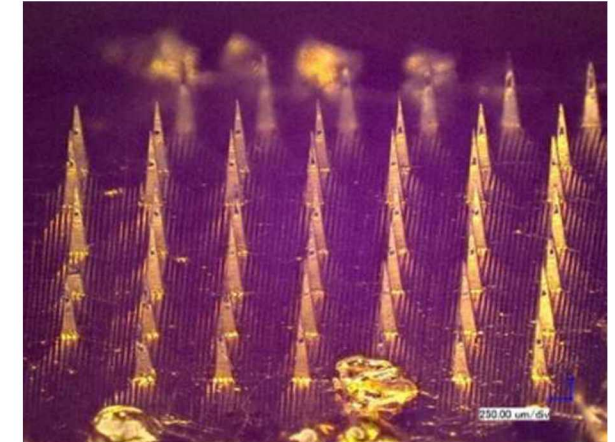
Fabrication Process



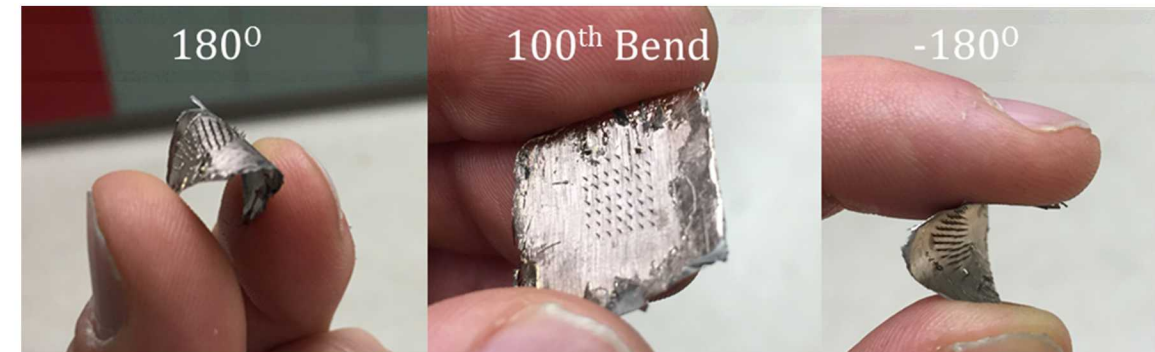
Cross-section of Microneedle Master



Hollow Metal Microneedle Array



Flexibility of Metal Microneedle Array Substrate

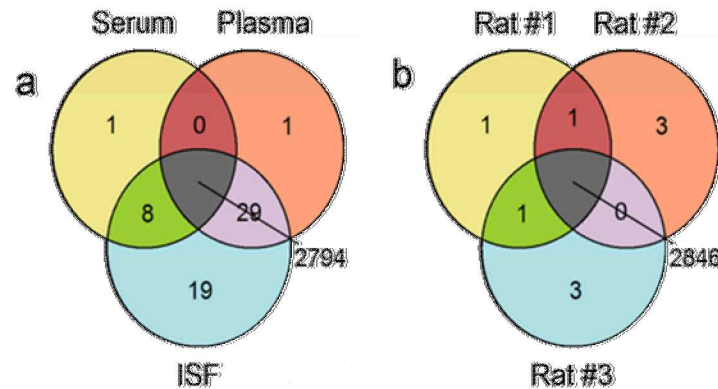


- Larger scalability, one master can make dozens of molds
- Molds may be reusable
- Can electroplate numerous arrays at once with dimensions produced from 2PP

Clinical Studies

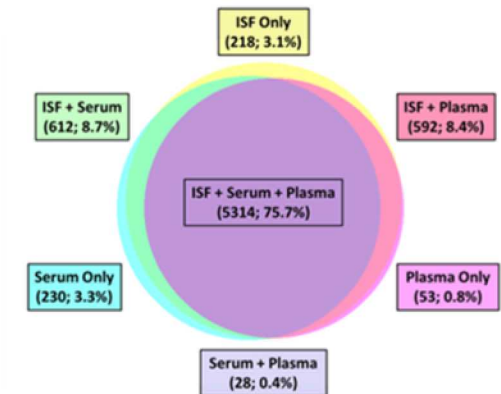
- Optimized needle geometries for interstitial fluid extraction in humans and rats
- Needles and holders were designed to extract enough fluid for analysis in <20min
- Biochemical analysis of glucose, lactate, total protein, pH, proteomic and genomic profiles, and exosome

Protein Expression in ISF



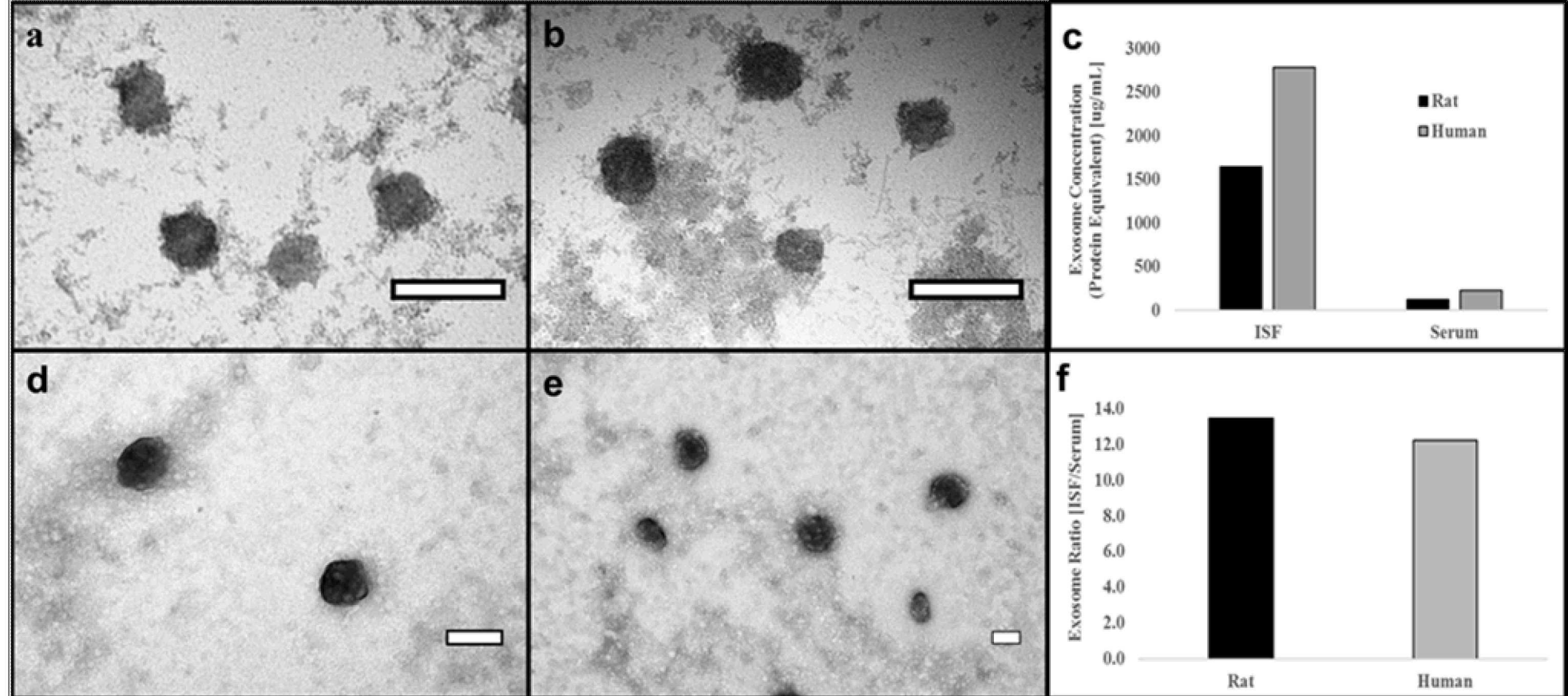
RNA Profiles in ISF, Serum, and Plasma

a	ISF	Serum	Plasma-1	Plasma-2
Mapped Reads (M)	1.9 (1.7 - 4.8)	2.2 (1.3 - 5.6)	4.9 (2.3 - 6.8)	3.7 (2.5 - 4.8)
CDS (%)	63.2 (22.3 - 84.2)	70.6 (41.1 - 87.9)	78.9 (71.5 - 88.8)	71.4 (62.1 - 76.9)
lncRNA (%)	26.9 (7.4 - 74.4)	21.8 (7.1 - 52.3)	18.6 (7.2 - 26.6)	22.9 (18.8 - 28.0)
RNase (%)	5.4 (1.4 - 11.3)	3.9 (0.4 - 5.3)	1.2 (0.4 - 2.6)	2.6 (0.5 - 5.1)
tRNA (%)	3.5 (1.3 - 5.7)	2.4 (1.4 - 4.2)	0.7 (0.4 - 1.4)	1.9 (0.7 - 3.0)
SRP (%)	0.9 (0.2 - 2.6)	1.2 (0.2 - 3.2)	0.5 (0.3 - 0.7)	1.2 (0.6 - 1.7)
miRNA (%)	0.05 (0.01 - 0.09)	0.08 (0.04 - 0.13)	0.03 (0.01 - 0.05)	0.05 (0.01 - 0.10)
piRNA (%)	0.04 (0.00 - 0.07)	0.03 (0.01 - 0.06)	0.01 (0.00 - 0.01)	0.01 (0.00 - 0.02)



- Protein expression and comparison with serum and plasma with fluid collected from rats.
- Some ISF specific proteins were found however fluid composition mimics blood composition.
- RNA profiles in interstitial fluid extracted from rats compared with serum and plasma with Venn diagram distribution.
- Protein coding transcripts (CDS) were detected in all examples and proportion of transcripts detected in each sample is reflected in the size of the circle.

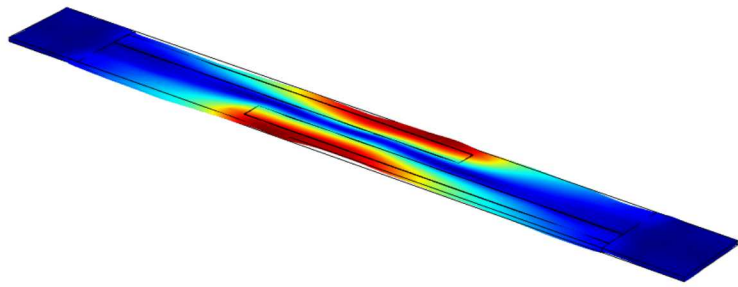
Exosome Extraction



Exosome concentrations were higher in both rats and humans compared to serum. TEM images of rat (a and b) and human (d and e) exosomes purified from ISF (a and d) and serum (b and e). Concentrations (protein equivalent) were measured using a nanoDrop spectrophotometer (c). The exosome ratio of ISF vs. serum in rat and human samples is shown in f. Scale bars; a, b, d, and e: 100 nm.

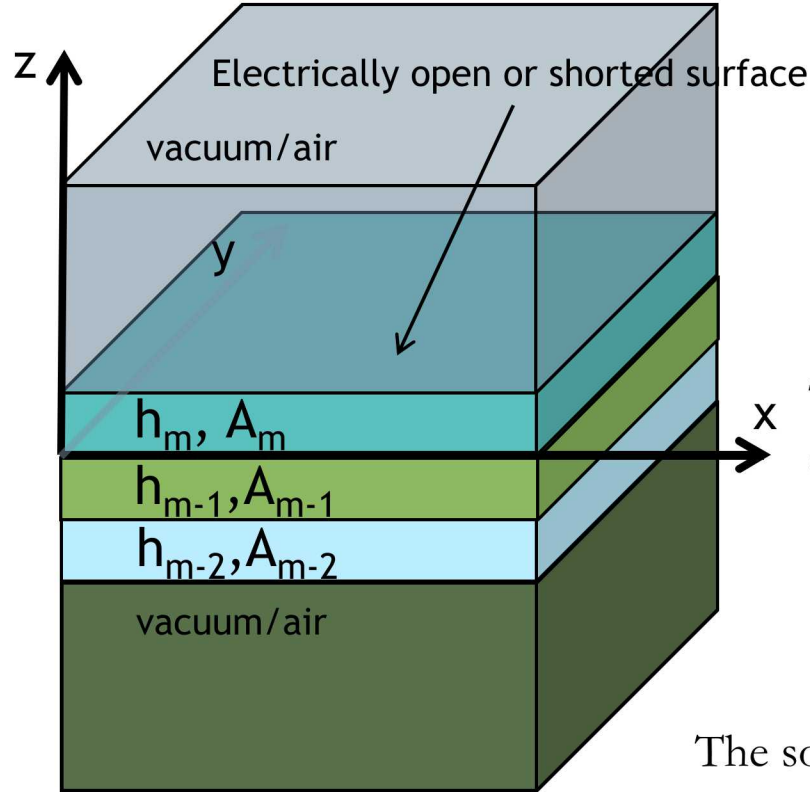


Modeling Acoustic Wave Devices



Simulating Acoustic Wave Behavior from the Material Properties

Classic Approach:



$$\frac{d}{dz} \begin{bmatrix} \mathbf{U}(z) \\ \mathbf{T}(z) \end{bmatrix} = i\mathbf{A} \begin{bmatrix} \mathbf{U}(z) \\ \mathbf{T}(z) \end{bmatrix} \quad \text{Eigenvalue Problem}$$

$$\mathbf{U} = [u, \phi]^T \quad \mathbf{u} = [u_1, u_2, u_3] \quad \text{displacement}$$

$$\mathbf{T} = [\sigma, D_3]^T \quad \sigma = [\sigma_{13}, \sigma_{23}, \sigma_{33}] \quad \text{stress}$$

The general anisotropic electro-elastic media acoustic Stroh tensor (\mathbf{A}) describes all wave behavior in each layer, (8x8):

$$\mathbf{A} = \begin{bmatrix} k_x \mathbf{X} \Gamma_{31} & -i\mathbf{X} \\ -i(\Gamma_{11} - \Gamma_{13} \mathbf{X} \Gamma_{31}) k_x^2 + i\rho\omega^2 \mathbf{I}' & k_x \Gamma_{13} \mathbf{X} \end{bmatrix} \quad \Gamma_{ik} = \begin{bmatrix} c_{1i1k} & c_{1i2k} & c_{1i3k} & e_{k1i} \\ c_{2i1k} & c_{2i2k} & c_{2i3k} & e_{k2i} \\ c_{3i1k} & c_{3i2k} & c_{3i3k} & e_{k3i} \\ e_{i1k} & e_{i2k} & e_{i3k} & -\epsilon_{ik} \end{bmatrix}$$

The solution above can be expressed by a transfer matrix \mathbf{B} :

$$\xi(z+h) = \mathbf{B} \xi(z)$$

$$\mathbf{B} = e^{i\mathbf{A}h}$$

The total transition matrix is product of individual layers:

$$\mathbf{B}_J = \mathbf{B}_j \prod_{i=1}^{J-1} \mathbf{B}_i$$

Simulating Acoustic Wave Behavior from the Material Properties, cont'd

To avoid numerical instabilities, we convert the Transfer matrix (**B**) to a Stiffness matrix (**K**) then add up the thin layers recursively:

$$B_J = B_j \prod_{i=1}^{J-1} B_i = \left[B_{II}(h) \right]^N \Big|_{h/\lambda \ll 1} = \mathbf{B}_{II} = \left(I - i \frac{h}{2} \mathbf{A} \right)^{-1} \left(I + i \frac{h}{2} \mathbf{A} \right)$$

Transforms the Asymptotic Transfer Matrix (**B**) to an Asymptotic Stiffness Matrix (**K**):

$$\mathbf{K} = \begin{bmatrix} \mathbf{B}_{22}(\mathbf{B}_{12})^{-1} & \mathbf{B}_{21} - \mathbf{B}_{22}(\mathbf{B}_{12})^{-1} \mathbf{B}_{11} \\ (\mathbf{B}_{12})^{-1} & -(\mathbf{B}_{12})^{-1} \mathbf{B}_{11} \end{bmatrix} = \mathbf{K}_{II} = \begin{bmatrix} \frac{\Gamma_{33}}{h} - (\Gamma_{31} - \Gamma_{13}) \frac{ik_x}{2} + \frac{\Gamma_{11} h k_x^2}{4} - \frac{\rho h \omega^2}{4} I' & -\frac{\Gamma_{33}}{h} - (\Gamma_{31} + \Gamma_{13}) \frac{ik_x}{2} + \frac{\Gamma_{11} h k_x^2}{4} - \frac{\rho h \omega^2}{4} I' \\ \frac{\Gamma_{33}}{h} - (\Gamma_{31} + \Gamma_{13}) \frac{ik_x}{2} - \frac{\Gamma_{11} h k_x^2}{4} + \frac{\rho h \omega^2}{4} I' & -\frac{\Gamma_{33}}{h} - (\Gamma_{31} - \Gamma_{13}) \frac{ik_x}{2} - \frac{\Gamma_{11} h k_x^2}{4} + \frac{\rho h \omega^2}{4} I' \end{bmatrix}$$

K relates the displacement at layer m-1 to layer m:

$$\begin{bmatrix} T(z+h) \\ T(z) \end{bmatrix} = \mathbf{K} \begin{bmatrix} U(z+h) \\ U(z) \end{bmatrix} \quad \begin{bmatrix} \mathbf{u} \\ \varphi \end{bmatrix} = \mathbf{G}_S \begin{bmatrix} \sigma \\ \gamma \end{bmatrix}, \mathbf{G}_S = \begin{bmatrix} S_S^f + \alpha \eta S_S^{fe} S_S^{ef} & -\alpha S_S^{fe} \\ \alpha S_S^{ef} & -\alpha S_S^e \end{bmatrix}$$

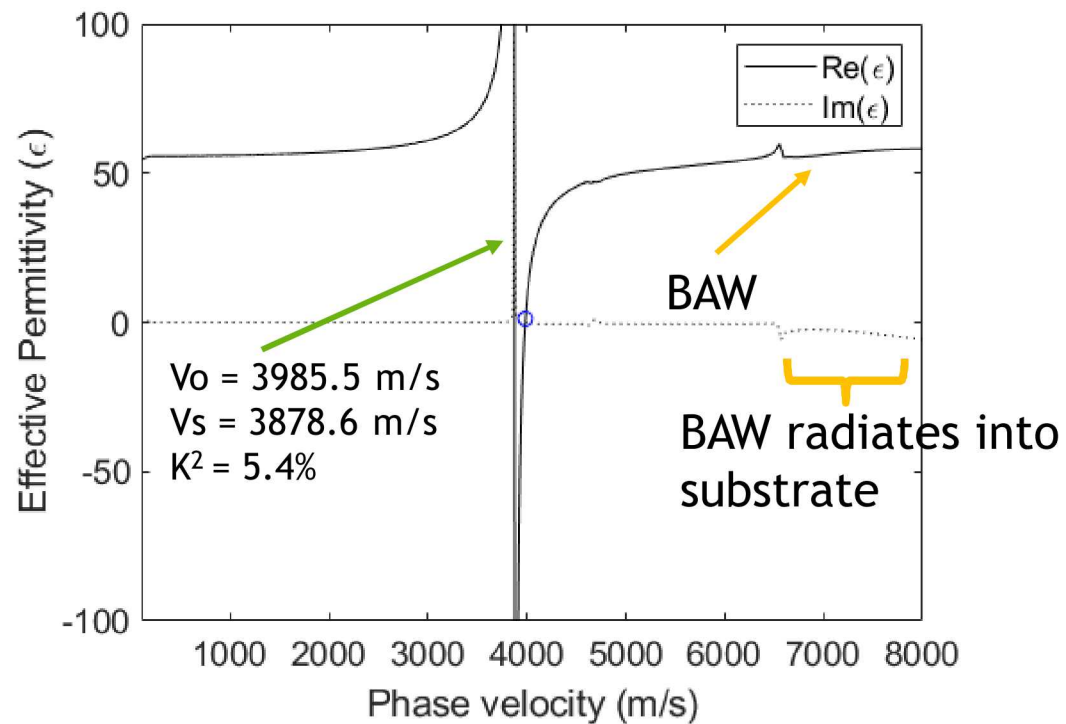
This is convenient since it allows easy calculation of the “Green’s Function” and “Effective Permittivity” which tells us where piezoelectric or non-piezoelectric modes exist:

$$\varepsilon_{eff}(k_x, \omega) = \frac{D_3^{top}(k) - D_3^{bot}(k)}{|k_x| \cdot \phi(k_x, \omega)} = \frac{\bar{\sigma}(k_x, \omega)}{|k_x| \cdot \bar{\phi}(k_x, \omega)} = \varepsilon_o - \frac{1}{|k_x| G_s(4, 4)}$$

Simulating Acoustic Wave Behavior from the Material Properties, cont'd

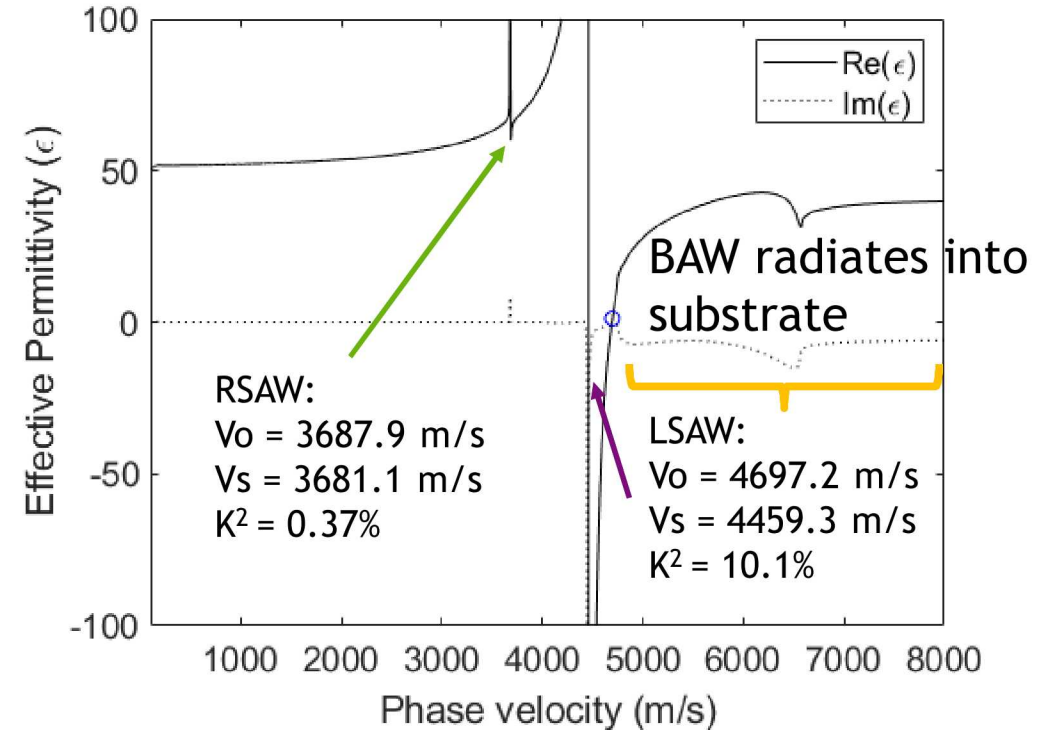
Example: 128° Y-rotated X-propagating Lithium Niobate (0,38,0), $\langle z, x, z \rangle$

Rayleigh SAW:



Example: 64° Y-rotated X-propagating Lithium Niobate (0,-26,0), $\langle z, x, z \rangle$

Rayleigh SAW, Leaky SAW, and BAW



Ambiguity in Calculating Electromechanical Coupling

Formal Definition of Electromechanical Coupling:

$$K^2 = -2\varepsilon_o \operatorname{Res}\left(\bar{G}_{4,4}(k,0)\right) = -2\varepsilon_o G_{4,4}\Big|_{k=k_m} \neq$$

R.F. Milsom et al. 1977, Y. Jin et al. 1991

Approximate Calculation of Electromechanical Coupling:

$$K^2 \approx 2 \left(\frac{V_{open} - V_{short}}{V_{open}} \right)$$

$\frac{h}{\lambda} > 1$ For certain modes (e.g. Lamb), variation of the particle displacement is highly perturbed for the metallized case

Thickness modes (BAW):

$$k_t^2 = \frac{\frac{\pi f_r}{2 f_a}}{\tan\left(\frac{\pi f_r}{2 f_a}\right)} \quad \text{Warner et al. 1966}$$

$$k_t^2 = \frac{\pi}{2} \frac{f_r}{f_a} \tan\left(\frac{\pi}{2} \frac{f_a - f_r}{f_a}\right) \approx \frac{\pi^2}{4} \frac{(f_a - f_r)}{f_a} \quad \text{D. Royer pg. 27 vol I}$$

$$K^2 = \frac{\pi^2}{4} \frac{f_r (f_a - f_r)}{f_a^2}$$

N. F. Naumenko et al. e.g. Author intended kt^2 and not K^2

Ambiguity in Calculating Electromechanical Coupling, cont'd

$$K^2 = \frac{e^2}{\epsilon^S c^E} \quad k_t^2 = \frac{e^2}{\epsilon^S \left(c^E + \frac{e^2}{\epsilon^S} \right)} = \frac{\frac{e^2}{\epsilon^S c^E}}{1 + \frac{e^2}{\epsilon^S c^E}} = \frac{K^2}{1 + K^2} \quad k_t^2 = \frac{K^2}{1 + K^2} \quad \text{J.F. Rosenbaum pg. 146}$$

B.A. Auld pg. 307 vol I
J. F. Rosenbaum pg. 146

Thus, $k_t^2 < K^2$

$$k_t^2 = \frac{e^2}{\epsilon^S c^D} \quad \text{B.A. Auld pg. 322 vol I}$$

Subscript “t” requires that the electric field is applied across the thickness of the crystal.

Note: In weak piezoelectric materials k_t^2 and K^2 are nearly identical, such as ZnO, $K^2 = 0.08$ implies $k_t^2 = 0.074$

If the device is not excited through the thickness the proper designation is to use big K^2 .

Quick Proof: K^2 interpretation is the ratio of the **electrical** to **mechanical energy**:

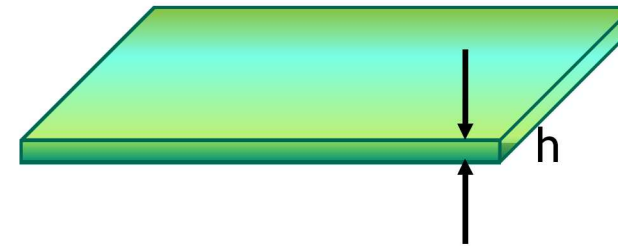
$$\frac{u_E}{u_M} = \frac{\epsilon^S E^2}{c^E S^2}, \because D = 0 \text{ inside piezoelectric, } D = \epsilon^S : E + e : S = 0 \rightarrow E = -\frac{eS}{\epsilon^S} \rightarrow \frac{u_E}{u_M} = \frac{e^2}{c^E \epsilon^S} = K^2$$

Using the Green's function Method to Sweep an Euler Angle

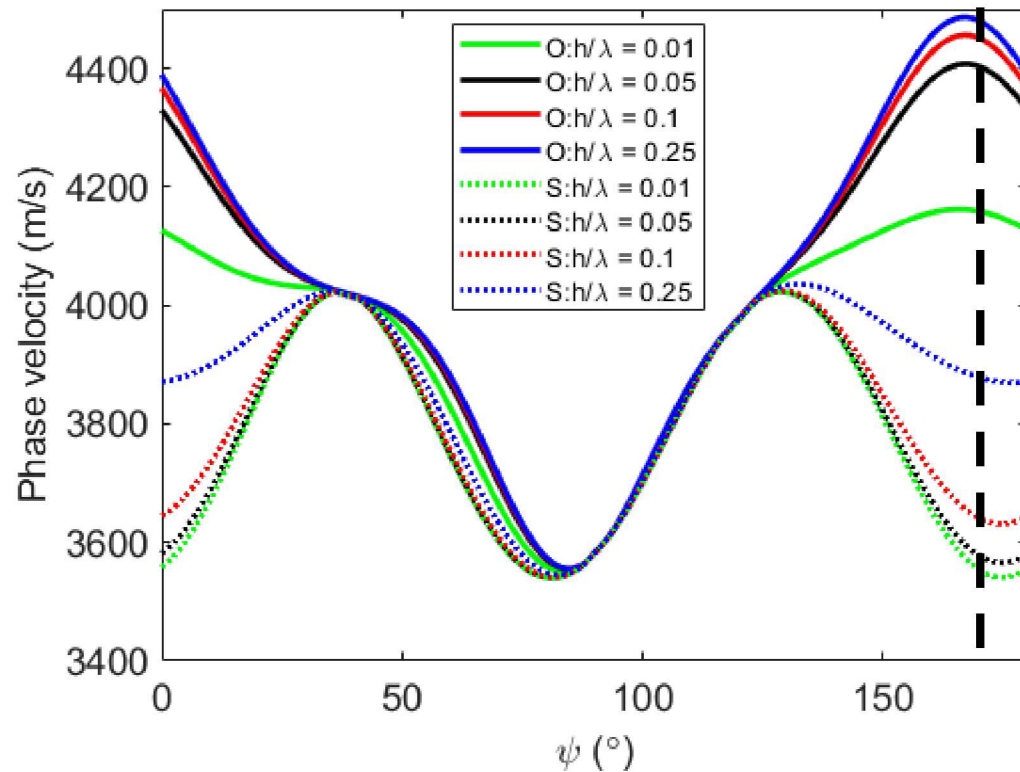
X-cut LN as a thin plate with stress-free boundary conditions on top and bottom:

$\sigma = 0$ and $\phi = 0$ (short) or $\gamma = 0$ (open)

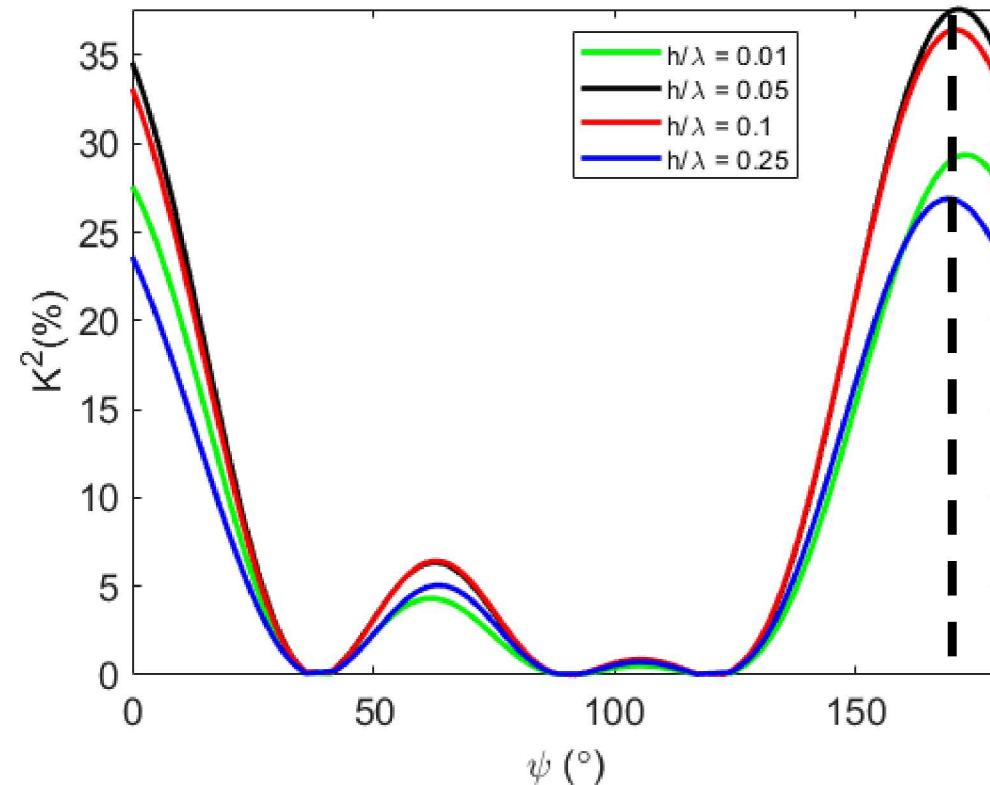
Advantages: Fast (10 min), no mode sorting



SH mode



SH mode

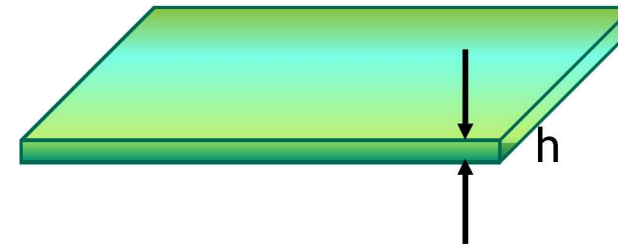


Using the Green's function Method to Sweep an Euler Angle

Y-cut LN as a thin plate with stress-free boundary conditions on top and bottom:

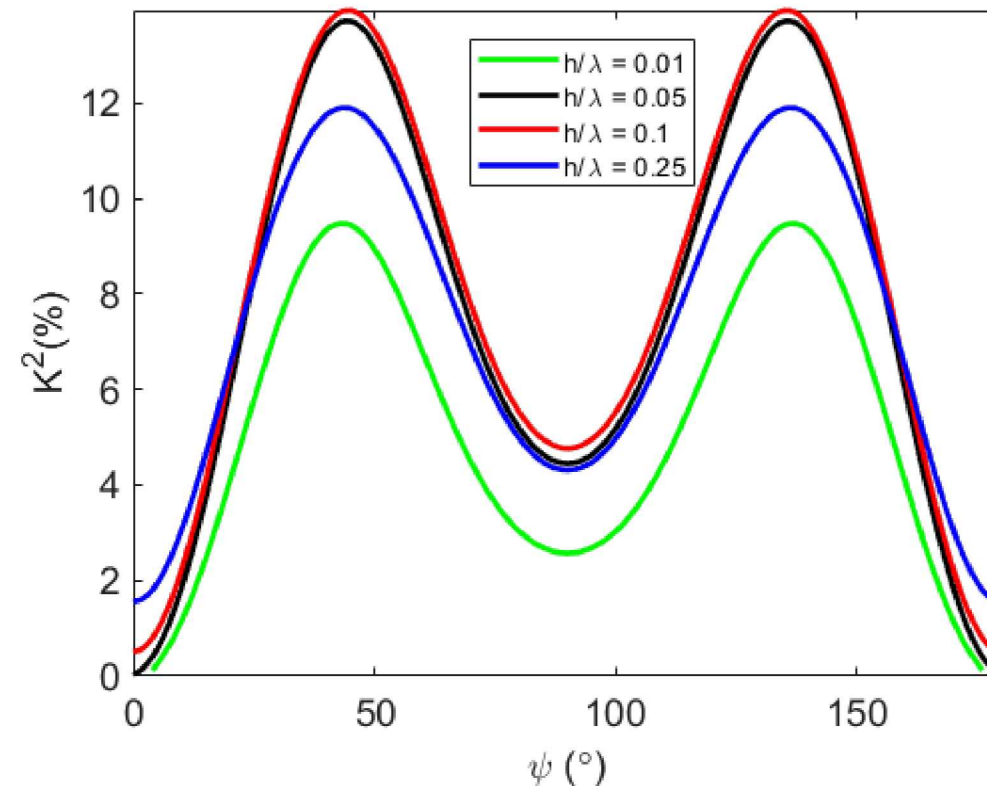
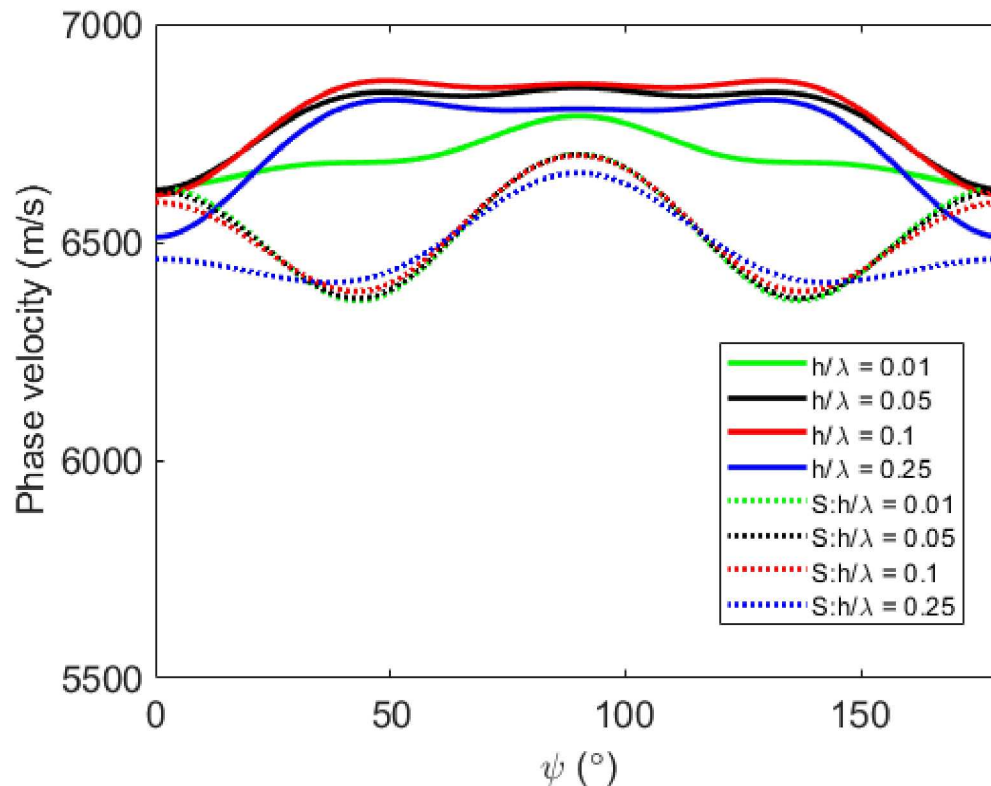
$\sigma = 0$ and $\phi = 0$ (short) or $\gamma = 0$ (open)

Advantages: Fast (10 min), no mode sorting



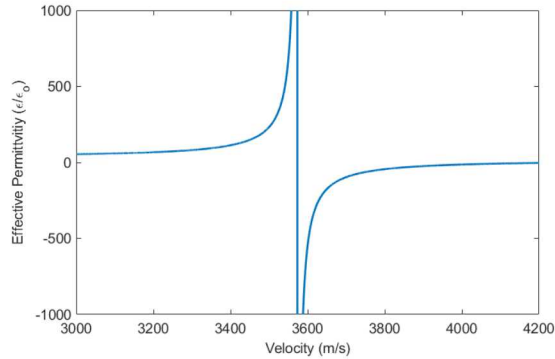
S0 mode

S0 mode

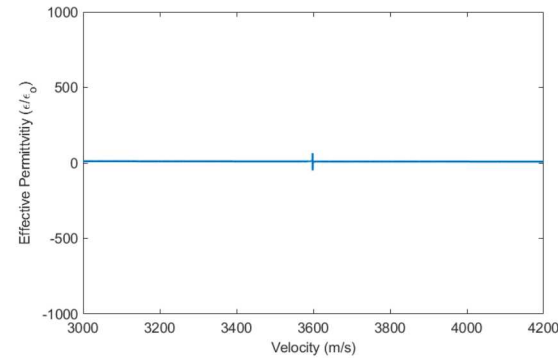


X-cut LN Thin Plate: SH mode

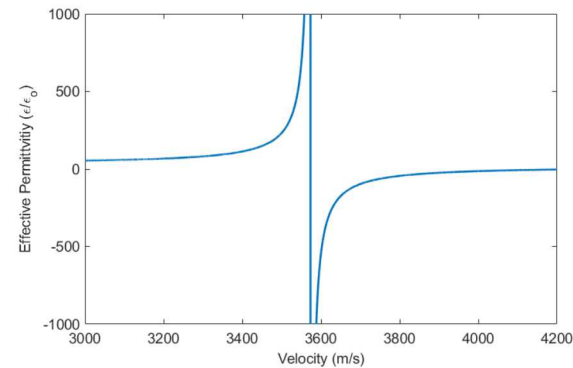
Lithium niobate. free-bottom, no bottom electrode, $h = 1.2 \text{ } \mu\text{m}$, $h/\lambda = 0.1$



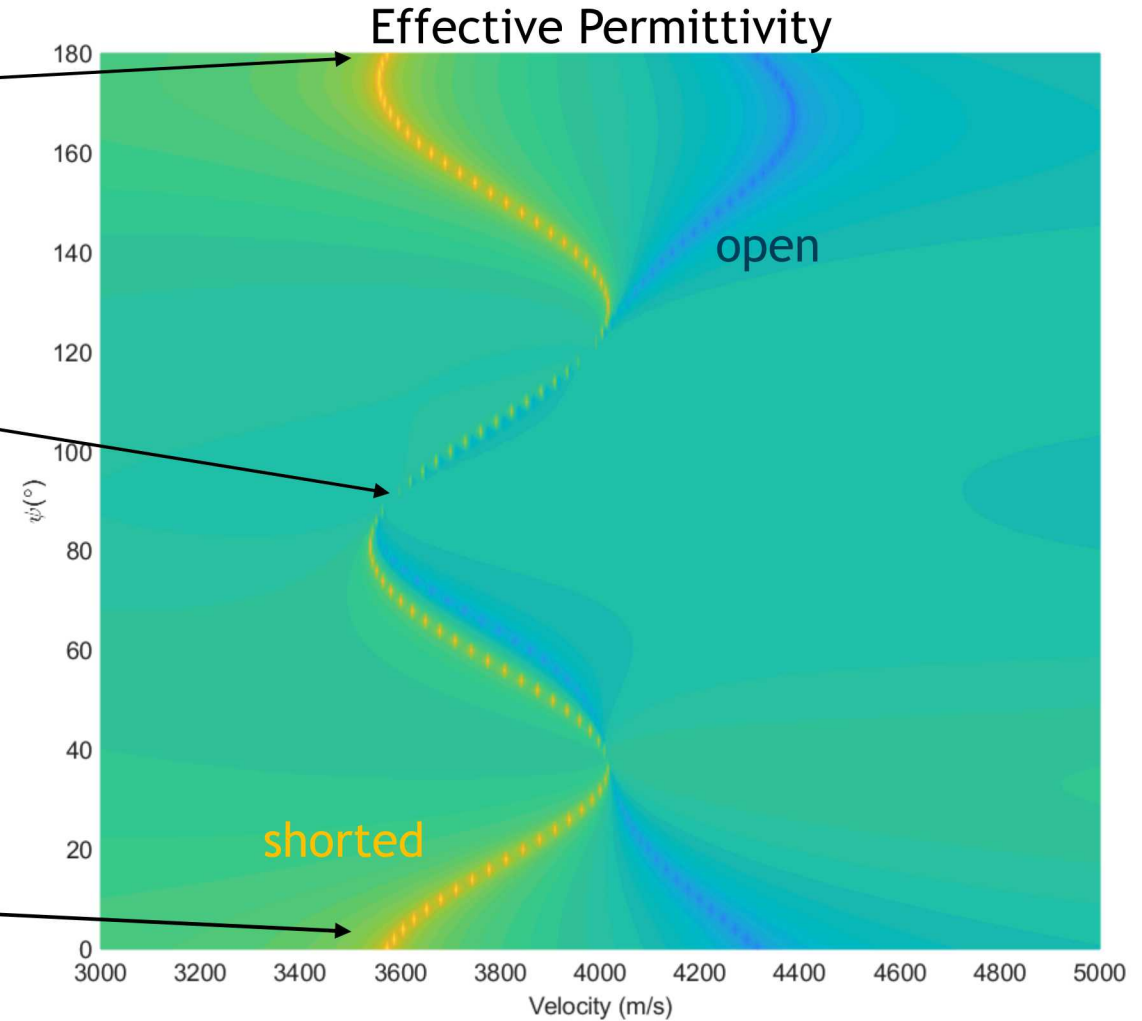
$(90^\circ, 90^\circ, \psi = 180^\circ)$

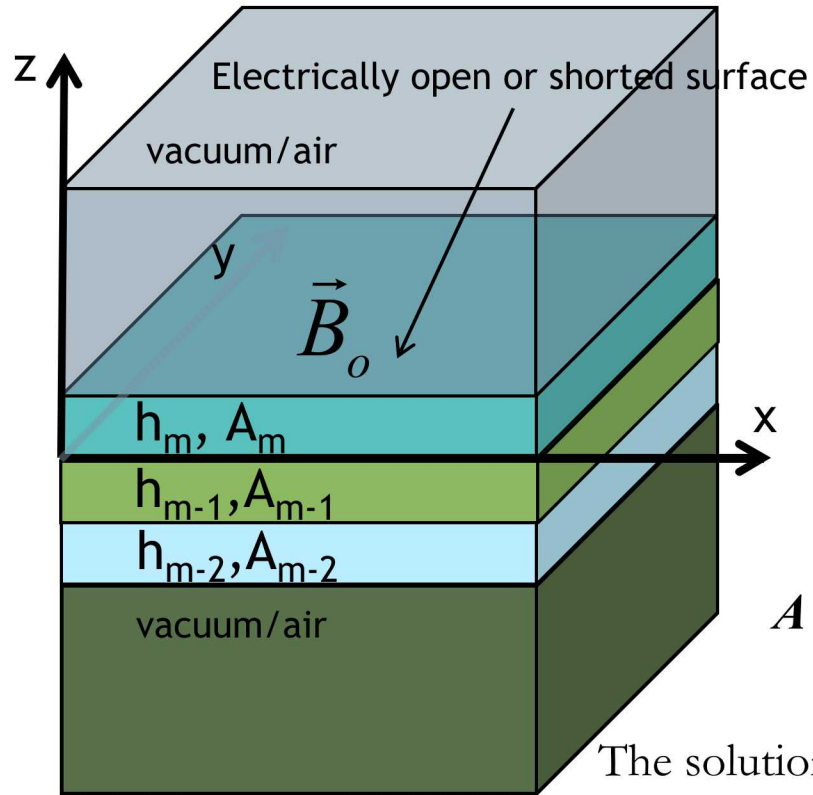


$(90^\circ, 90^\circ, \psi = 94^\circ)$



$(90^\circ, 90^\circ, \psi = 0^\circ)$





Approach:

$$\frac{d}{dz} \begin{bmatrix} \mathbf{U}(z) \\ \mathbf{T}(z) \end{bmatrix} = i\mathbf{A} \begin{bmatrix} \mathbf{U}(z) \\ \mathbf{T}(z) \end{bmatrix} \quad \text{Eigenvalue Problem}$$

$$\mathbf{U} = [u, \phi, \psi]^T \quad \mathbf{u} = [u_1, u_2, u_3] \quad \text{displacement}$$

$$\mathbf{T} = [\sigma, D_3, B_3]^T \quad \sigma = [\sigma_{13}, \sigma_{23}, \sigma_{33}] \quad \text{stress}$$

The general anisotropic electro-elastic media acoustic Stroh tensor (\mathbf{A}) describes all wave behavior in each layer, (8x8):

$$\mathbf{A} = \begin{bmatrix} k_x \mathbf{X} \Gamma_{31} & -i\mathbf{X} \\ -i(\Gamma_{11} - \Gamma_{13} \mathbf{X} \Gamma_{31}) k_x^2 + i\rho\omega^2 \mathbf{I}' & k_x \Gamma_{13} \mathbf{X} \end{bmatrix}$$

The solution can be expressed as a transfer matrix \mathbf{B} :

$$\xi(z+h) = \mathbf{B} \xi(z)$$

$$\mathbf{B} = e^{i\mathbf{A}h}$$

$$\mathbf{B}_J = \mathbf{B}_J \prod_{i=1}^{J-1} \mathbf{B}_i$$

$$\Gamma_{ik} = \begin{bmatrix} c_{11k} & c_{12k} & c_{13k} & e_{k1i} & q_{k1i} \\ c_{21k} & c_{22k} & c_{23k} & e_{k2i} & q_{k2i} \\ c_{31k} & c_{32k} & c_{33k} & e_{k3i} & q_{k3i} \\ e_{i1k} & e_{i2k} & e_{i3k} & -\varepsilon_{ik} & -\alpha_{ik} \\ q_{i1k} & q_{i2k} & q_{i3k} & -\alpha_{ik} & -\mu_{ik} \end{bmatrix}$$

Piezoelectric/Piezomagnetic Layered Modeling

In the Quasi-Static approximation for magneto-electro-elastic solid:

$$T_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}, D_{i,j} = 0, B_{j,j} = 0$$

$$E_i = -\phi_i, H_i = -\psi_i \quad \text{Electric and Magnetic Potentials}$$

Write solution as stiffness matrix:

$$K = \begin{bmatrix} B_{22}(B_{12})^{-1} & B_{21} - B_{22}(B_{12})^{-1}B_{11} \\ (B_{12})^{-1} & -(B_{12})^{-1}B_{11} \end{bmatrix}$$

Then,

$$T_{ij,j} = c_{ijkl} u_{k,l} + e_{lij} \phi_l + q_{lij} \psi_l$$

$$D_i = e_{ikl} u_{k,l} - \epsilon_{il} \phi_l - \alpha_{il} \psi_l$$

$$B_i = q_{ikl} u_{kl} - \alpha_{il} \phi_l - \mu_{il} \psi_l$$

Units:

c: N/m ²	magnetoelectric,	α : C/A · m
e: C/m ²	magnetic permeability, μ : N/A ²	
ϵ : F/m	piezomagnetic, q: N/A · m	

Define a Piezoelectric/Magnetic Green's function:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \phi \\ \psi \end{bmatrix} = \mathbf{S} \begin{bmatrix} T_{13} \\ T_{23} \\ T_{33} \\ D_3 \\ B_3 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_S^f & \mathbf{S}_S^{fe} & \mathbf{S}_S^{fm} \\ \mathbf{S}_S^{ef} & \mathbf{S}_S^e & \mathbf{S}_S^{em} \\ \mathbf{S}_S^{mf} & \mathbf{S}_S^{me} & \mathbf{S}_S^m \end{bmatrix} \begin{bmatrix} \mathbf{T}_n \\ D_{3n} \\ B_3 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_S^f & \mathbf{S}_S^{fe} & \mathbf{S}_S^{fm} \\ \mathbf{S}_S^{ef} & \mathbf{S}_S^e & \mathbf{S}_S^{em} \\ \mathbf{S}_S^{mf} & \mathbf{S}_S^{me} & \mathbf{S}_S^m \end{bmatrix} \begin{bmatrix} \mathbf{T}_n \\ \eta \phi_n - \gamma_n \\ B_3 - B_o \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \phi \\ \psi \end{bmatrix} = \mathbf{G} \begin{bmatrix} T_{13} \\ T_{23} \\ T_{33} \\ \gamma_n \\ B_o \end{bmatrix}$$

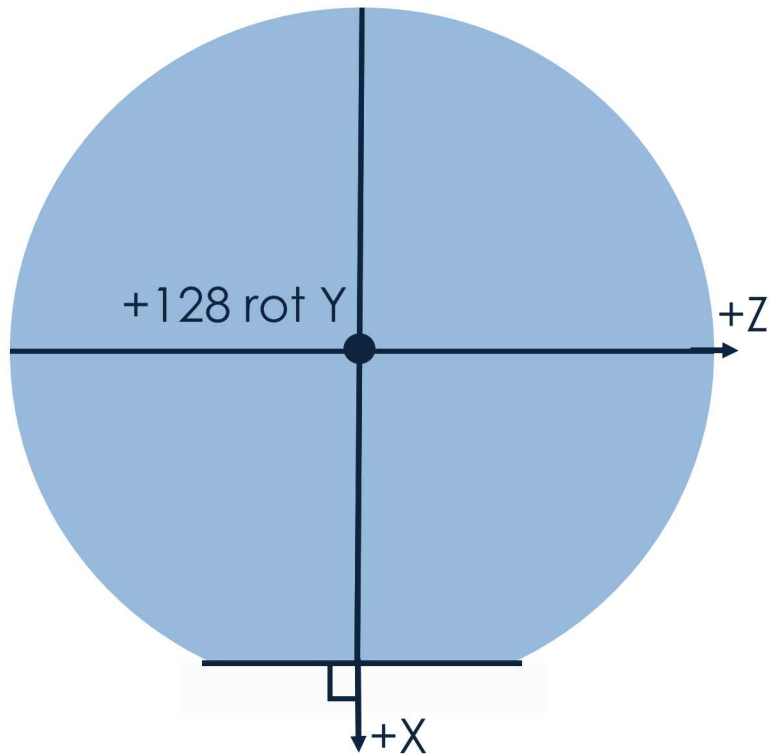
Comsol Approach: couple Solid, Electrostatics, Magnetostatics:

e.g. top boundary conditions:

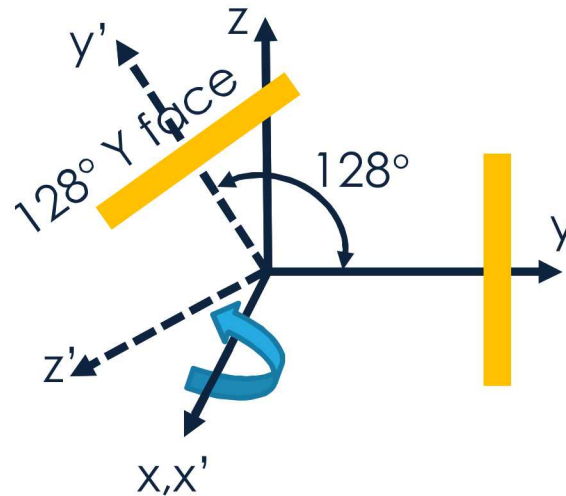
$$T_{13} = T_{23} = T_{33} = D_3 = B_3 = 0$$

$$T_{13} = T_{23} = T_{33} = \phi_n = \psi_n = 0$$

Comsol: Simulating Surface Acoustic Wave (SAW) on 128° Y – rotated X propagating Lithium Niobate



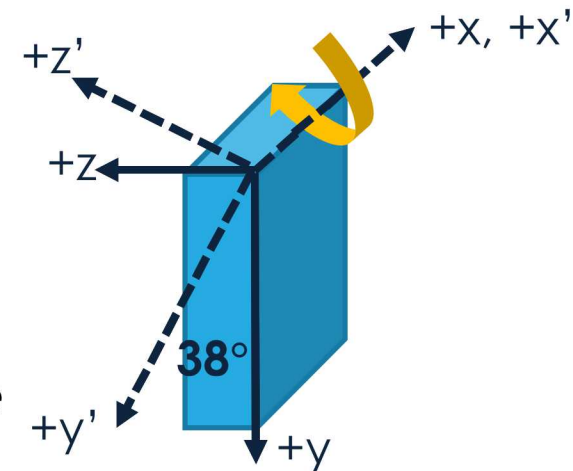
128° YX LN: $\langle z, x, z \rangle (0, 38, 0)$



Comsol rotated axis: $z'y'$ plane

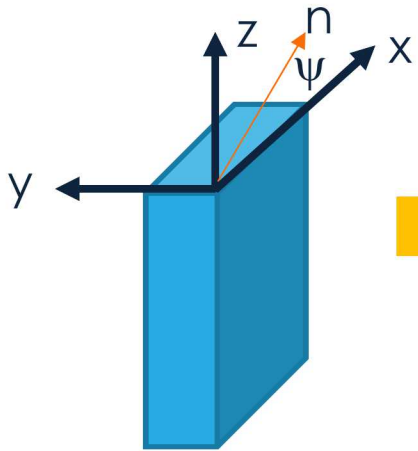
Comsol rotates the coordinate system not the material tensors

1. Draw a slab in Comsol then apply rotation using the rotated system: $(0, 38, 0)$ for 128YX LN
2. Comsol rotates the coordinate axis from the drawn axis and which is viewed by plotting the coordinate system:



Comsol: Getting the Rotation Correct: YZ Lithium Niobate (0,90,90), $\langle z, x, z \rangle$

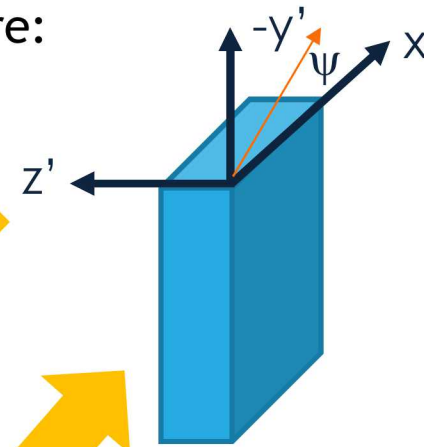
Comsol drawn block:



For Y-cut, Z-propagating we desire:

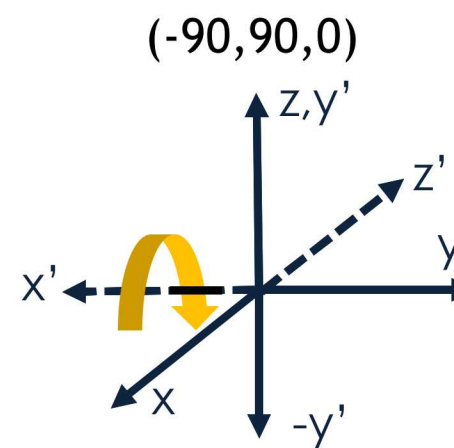
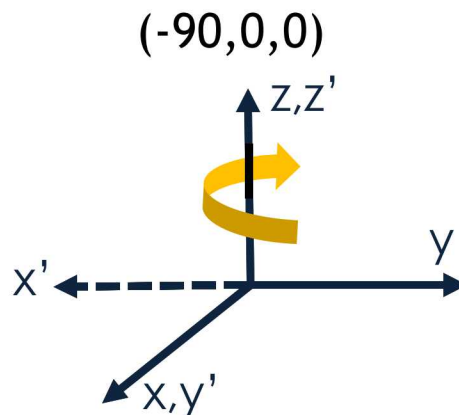
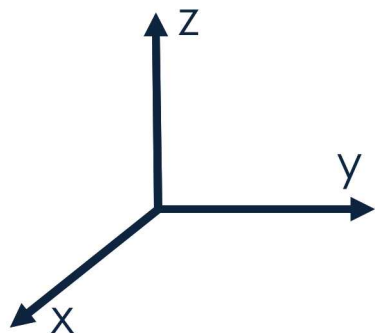
$(-90, 90, 0)$ rotation in Comsol

Definition of Y-cut



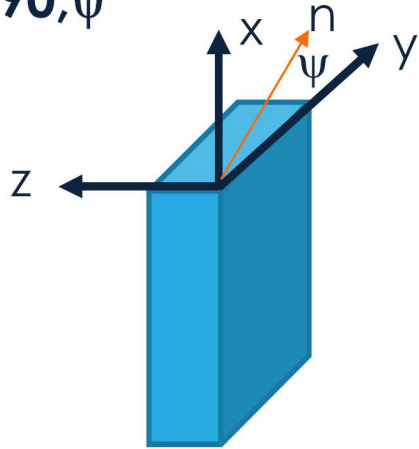
Three Options:

1. Define the model in the unrotated axis then apply Euler rotation to obtain desired cut and orientation.
2. Draw the model in the rotated cut if possible then apply additional rotation if needed.
3. Draw the model in the unrotated axis and rotate the material tensors directly using Euler rotation matrix.

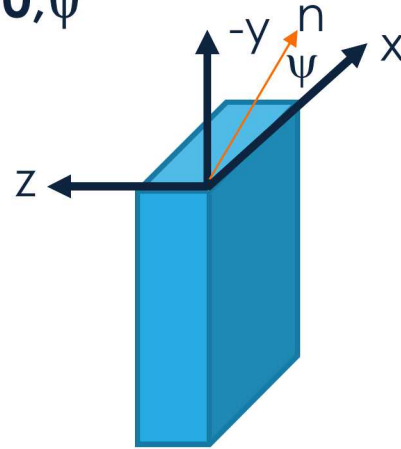


Comsol: Getting the Rotation Correct, cont'd

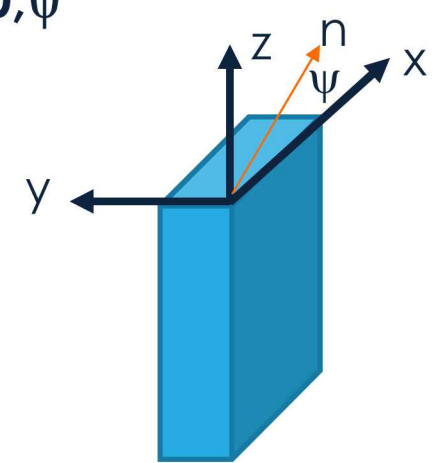
X-cut:
90,90, ψ



Y-cut:
0,90, ψ



Z-cut:
0,0, ψ



$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix (Auld vol I, 1990)

$$A = \begin{matrix} & \mathbf{z} & & \mathbf{x} \end{matrix} \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} & \mathbf{z} & & \mathbf{x} \end{matrix} \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} & \mathbf{z} & & \mathbf{x} \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

Comsol: Getting the Rotation Correct, cont'd

Using X-cut rotation (90,90,170) for large SH mode where $K^2 = 35\%$ @ $h/\lambda = 0.1$

$$A = \begin{bmatrix} 0 & -0.9848 & 0.1736 \\ 0 & -0.1736 & -0.9848 \\ 1 & 0 & 0 \end{bmatrix}$$

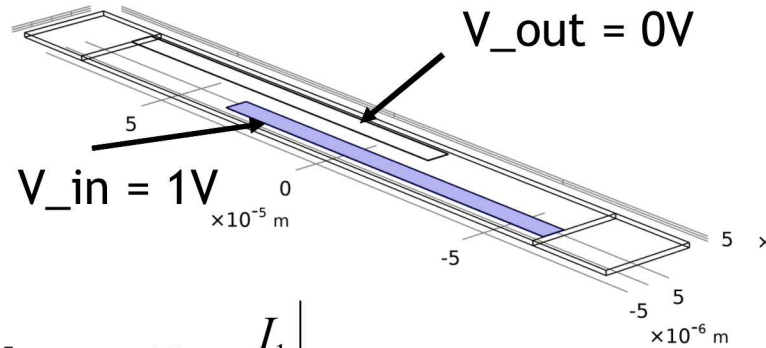
Base vector rotation is: $M / A = \begin{bmatrix} 0 & 0 & 1 \\ -0.9848 & -0.1736 & 0 \\ 0.1736 & -0.9848 & 0 \end{bmatrix}$

Enter into Comsol's "base vector system"

At least Four Options to perform rotations:

1. Define the model in the unrotated axis then apply Euler rotations to obtain desired orientation.
2. Draw the model in the rotated cut if possible then apply additional rotation if needed.
3. Draw the model in the unrotated axis and rotate the material tensors directly using Euler rotation matrix using MATLAB. Downside of this method is can't perform rotations using the sweep option.
4. Draw the model in the unrotated axis and use base vector system to define the rotation. Need to perform rotations in another tool to obtain the "base vector system."

X-cut SH mode on thin LN plate at 170°deg rotation from y-axis

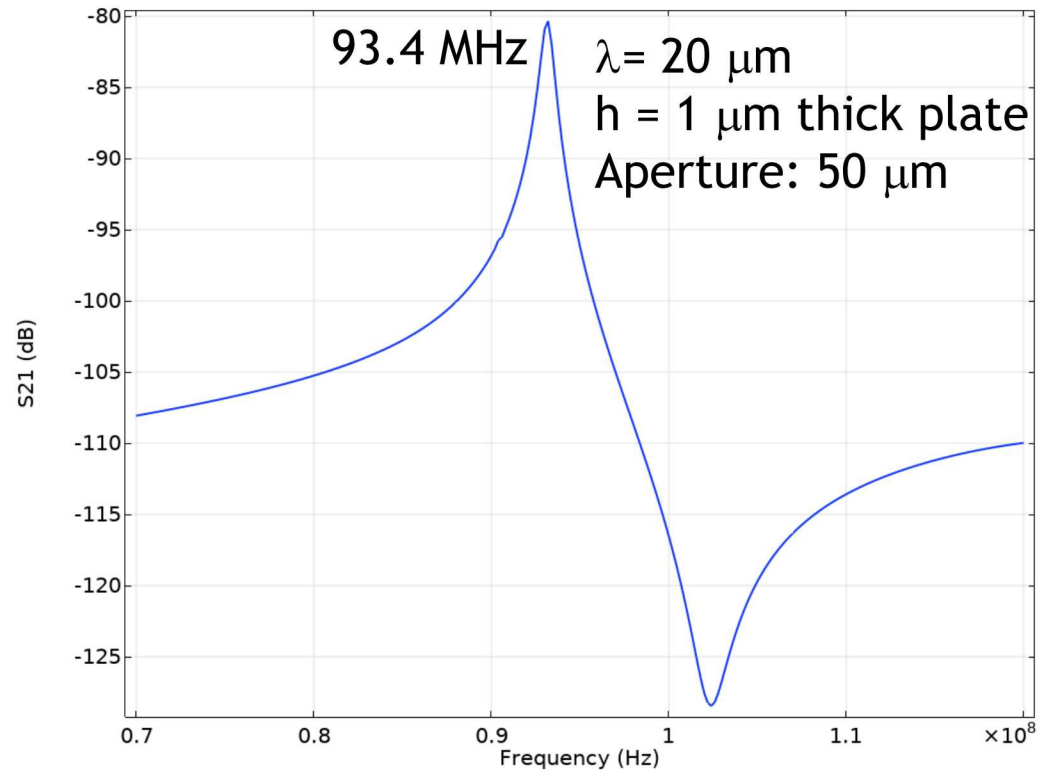


$$Y_{11} = \frac{I_1}{V_1} \bigg|_{V_2=0}$$

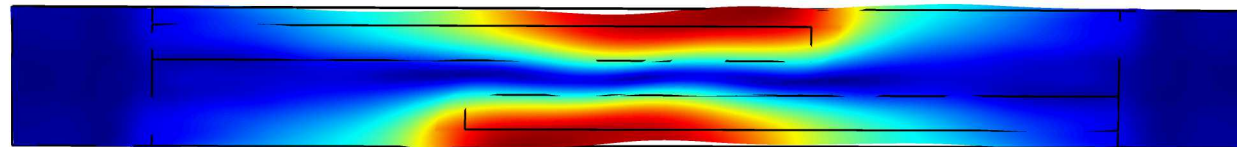
$$Y_{12} = \frac{I_1}{V_2} \bigg|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1} \bigg|_{V_2=0}$$

$$Y_{22} = \frac{I_2}{V_2} \bigg|_{V_1=0}$$

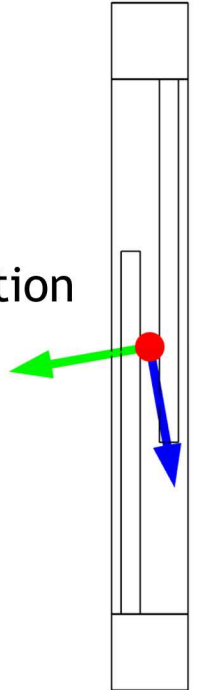


SH displacement at resonance

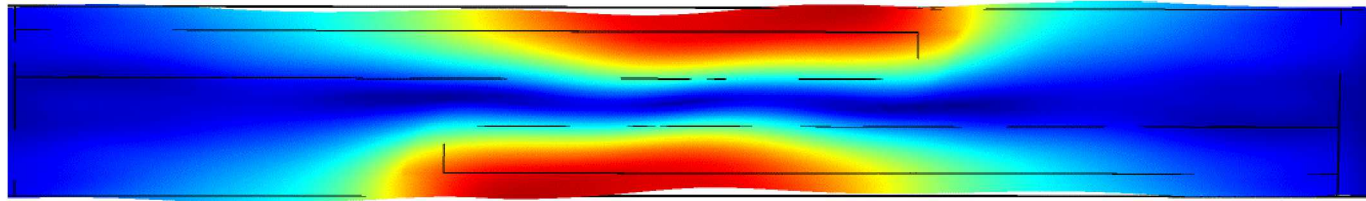


Base vector rotation

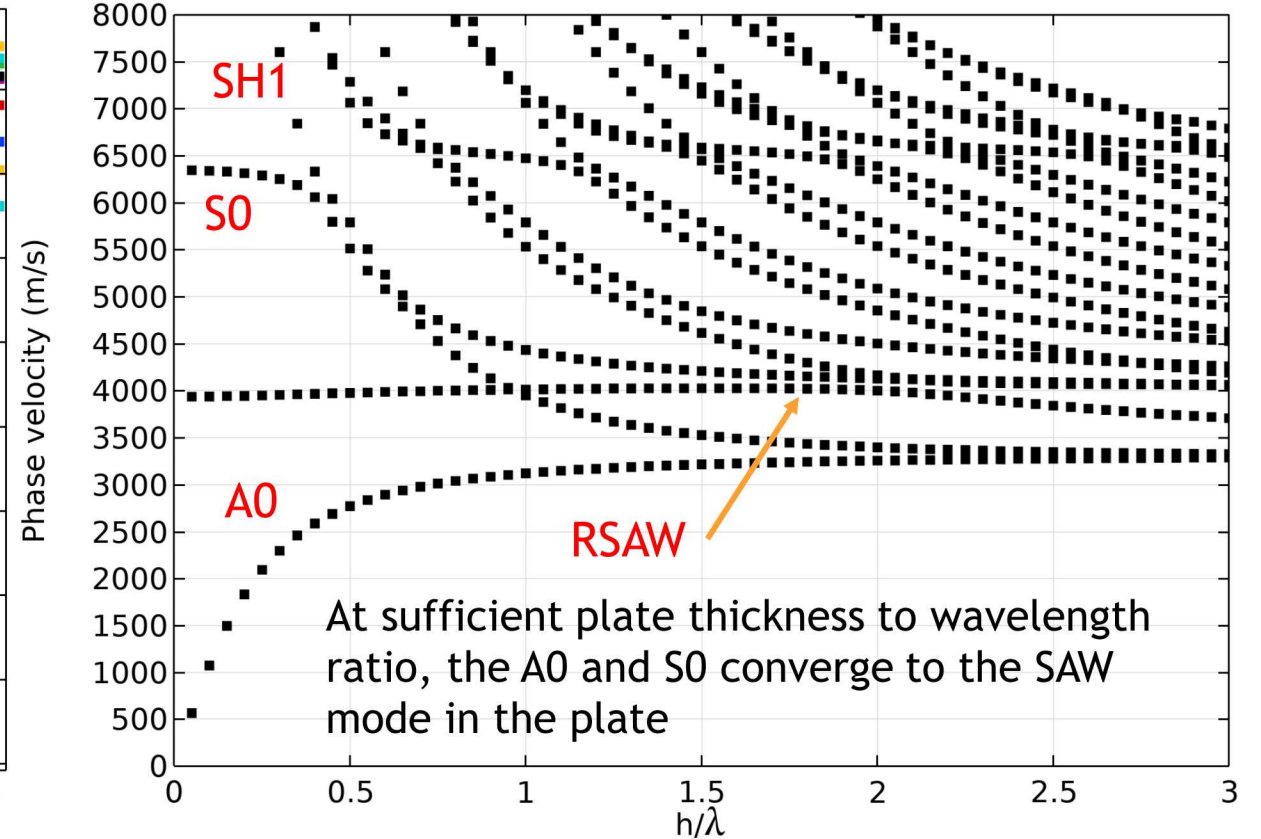
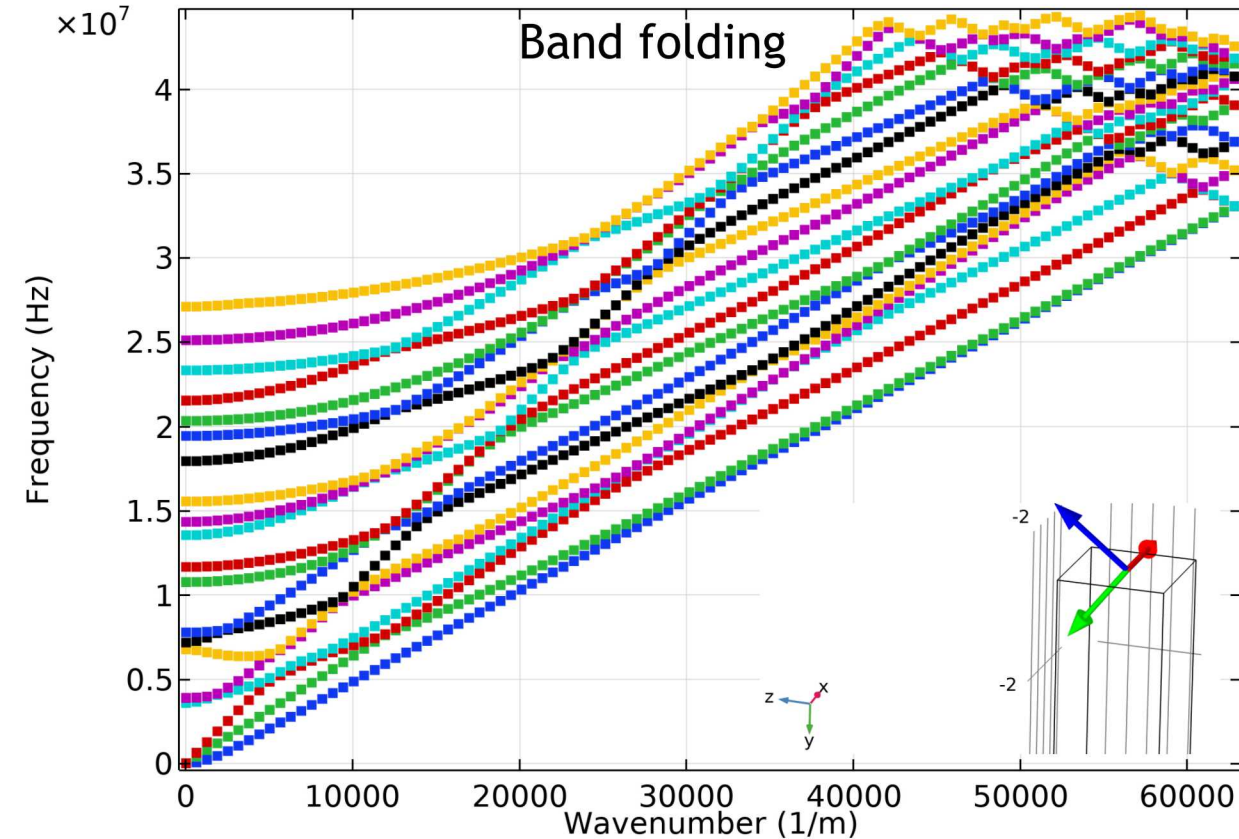
Unrotated axis



X-cut SH mode on thin LN plate at 170°deg rotation from y-axis

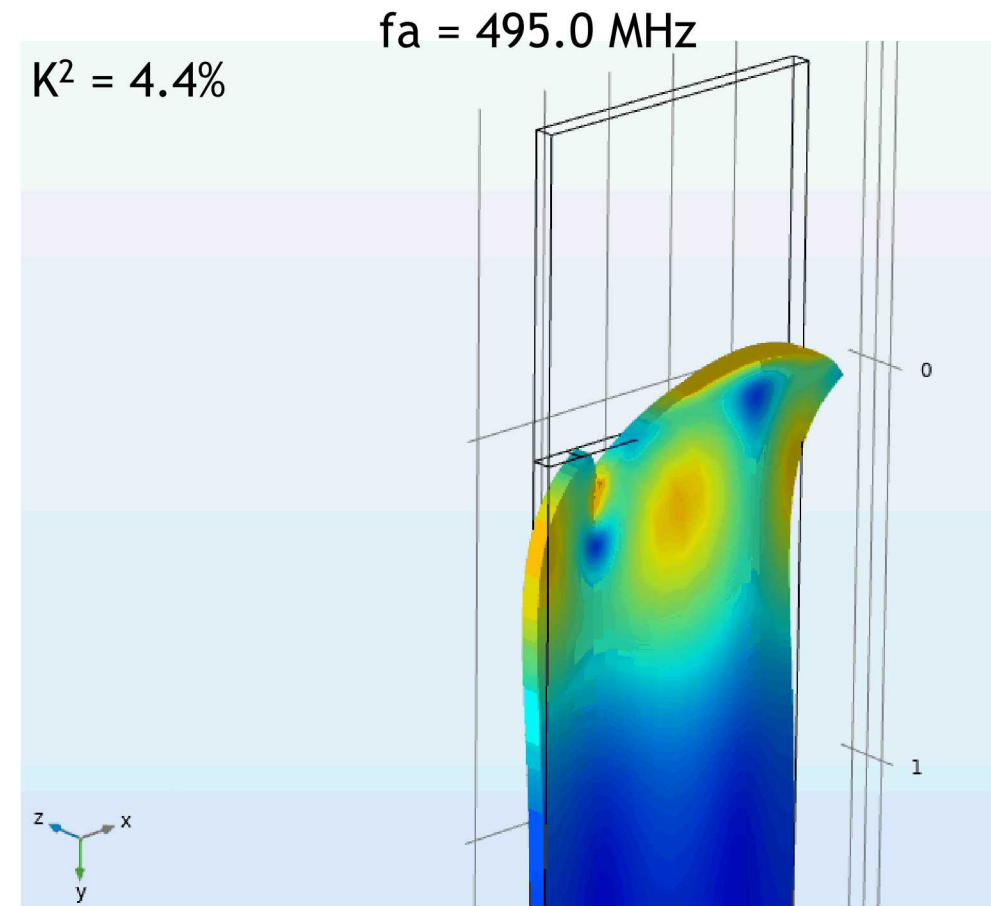
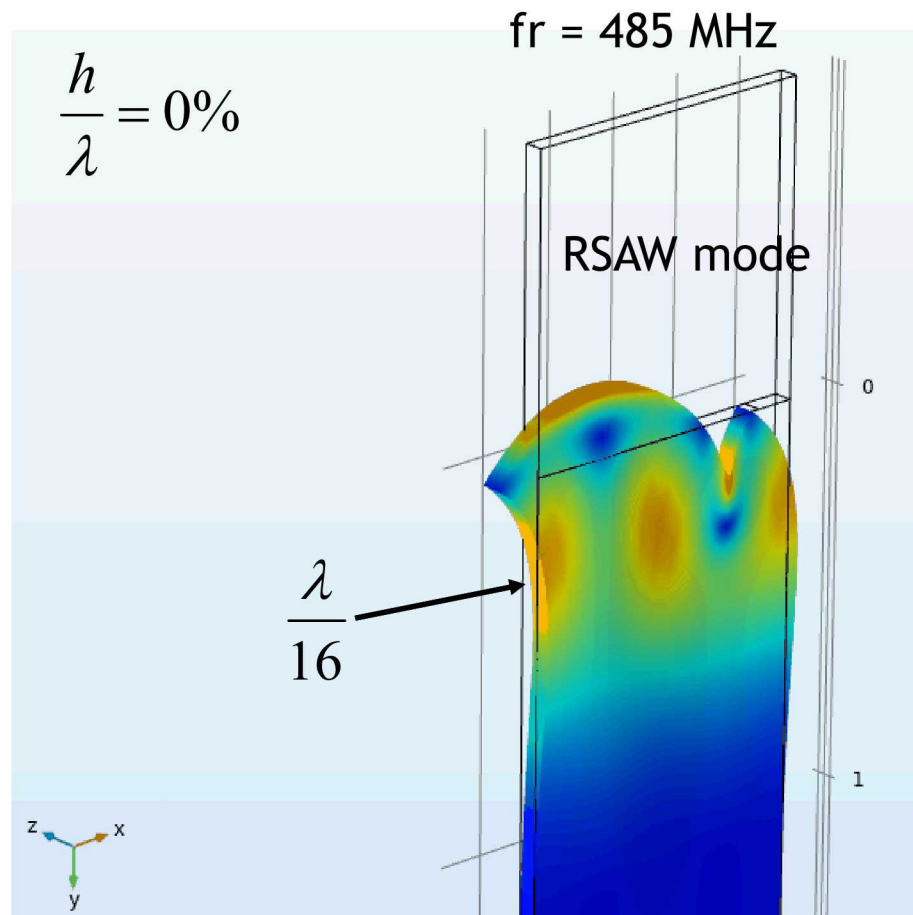


Comsol: Simulating Surface Acoustic Wave (SAW) on 128° Y – rotated X propagating Lithium Niobate: **Dispersion Plot**

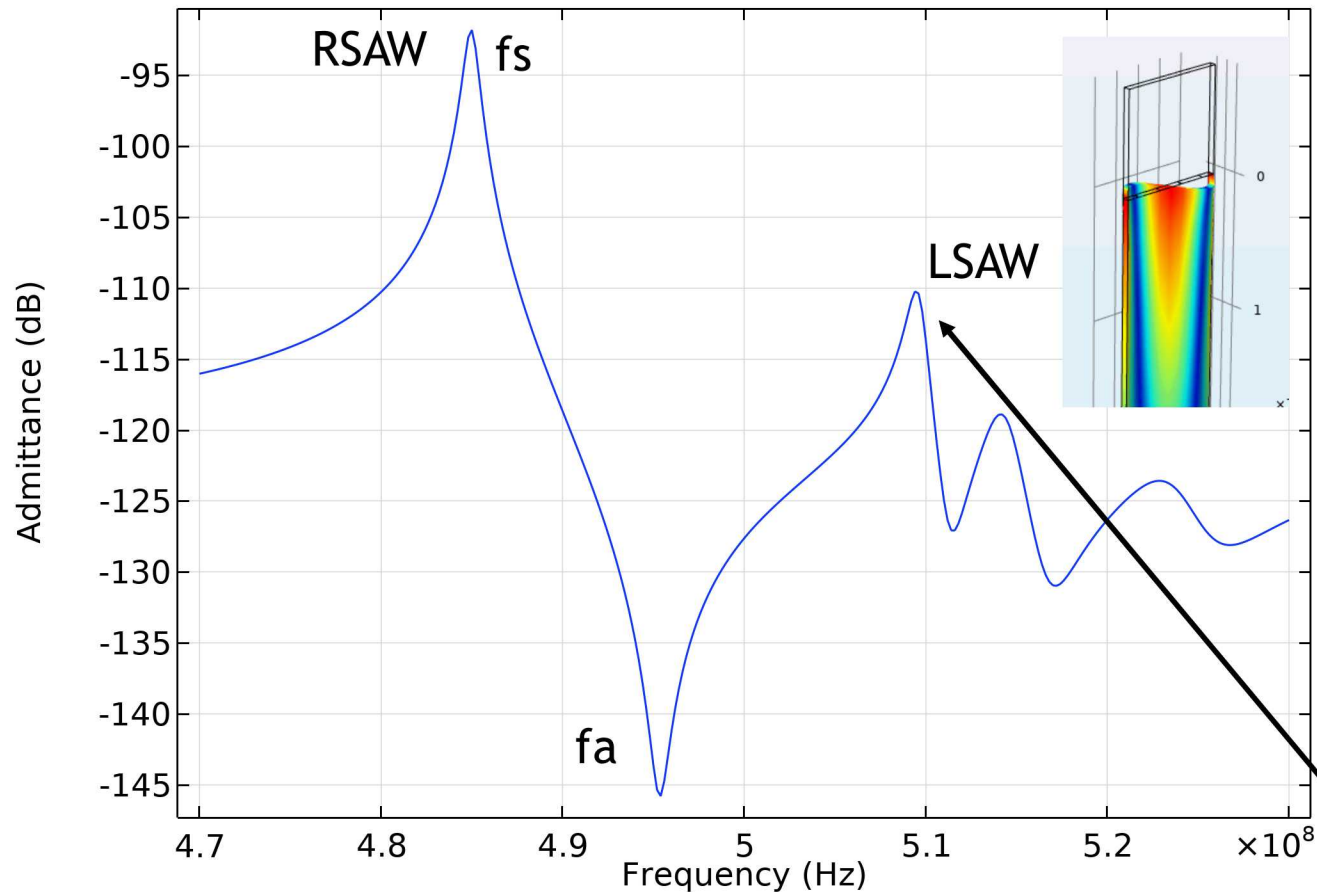


$$\frac{hk}{2\pi} = \frac{h}{\lambda}$$

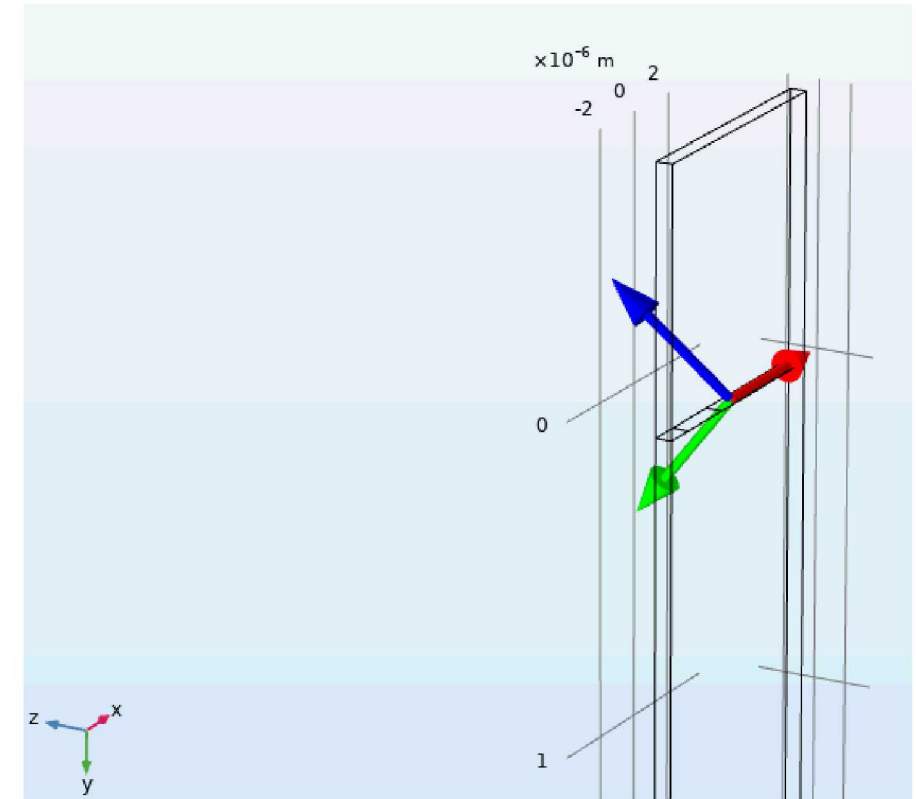
Comsol: Simulating Surface Acoustic Wave (SAW) on 128° Y – rotated X propagating Lithium Niobate: **Modes**



Comsol: Simulating Surface Acoustic Wave (SAW) on 128° Y – rotated X propagating Lithium Niobate: **Admittance**

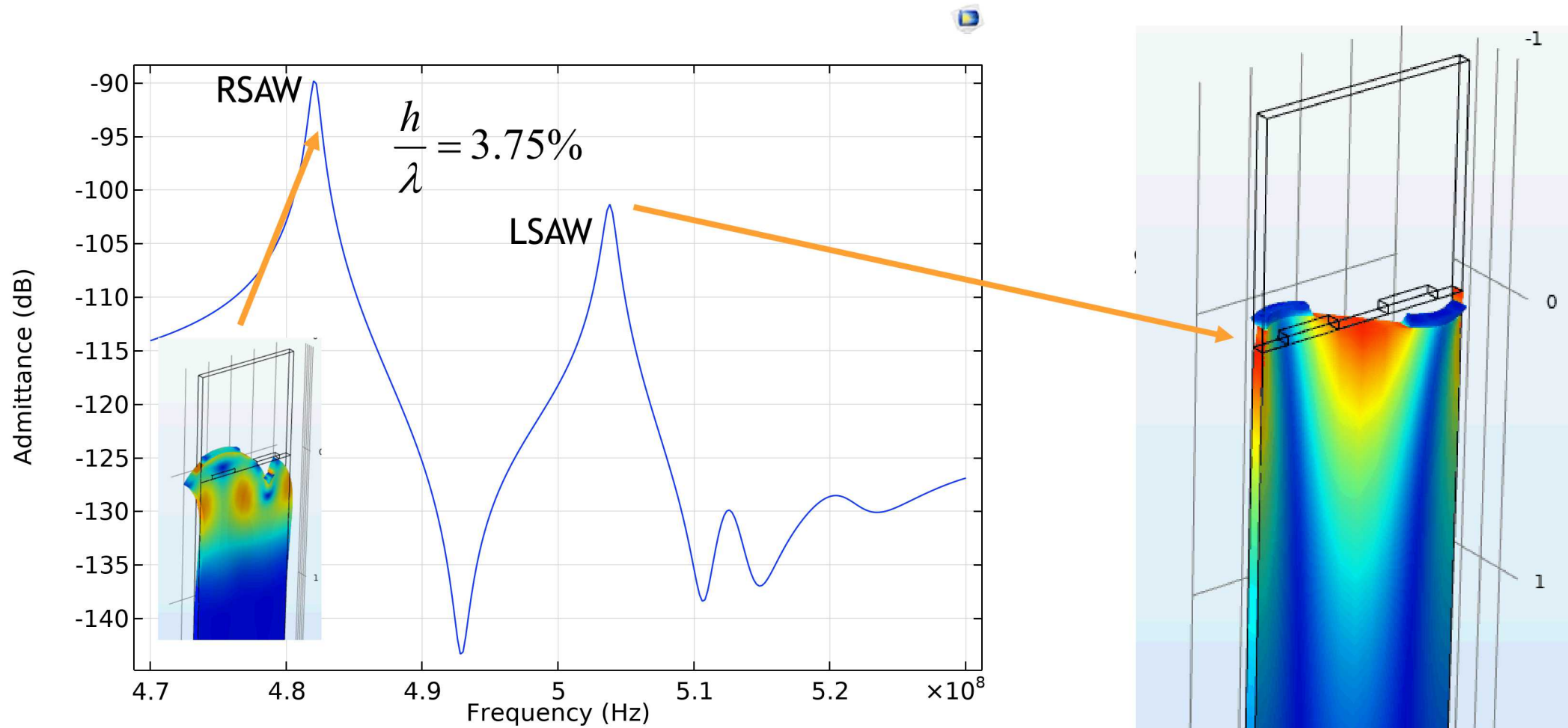


Rotated Coordinate System



Leaky Surface Acoustic Wave (LSAW).
A true surface wave is confined within one-wavelength from the top surface

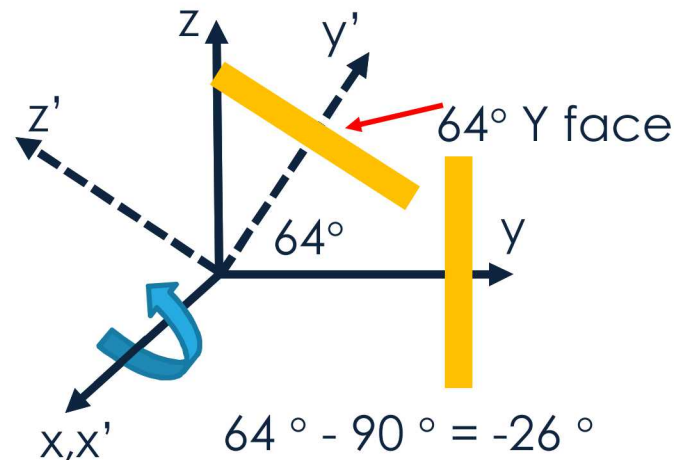
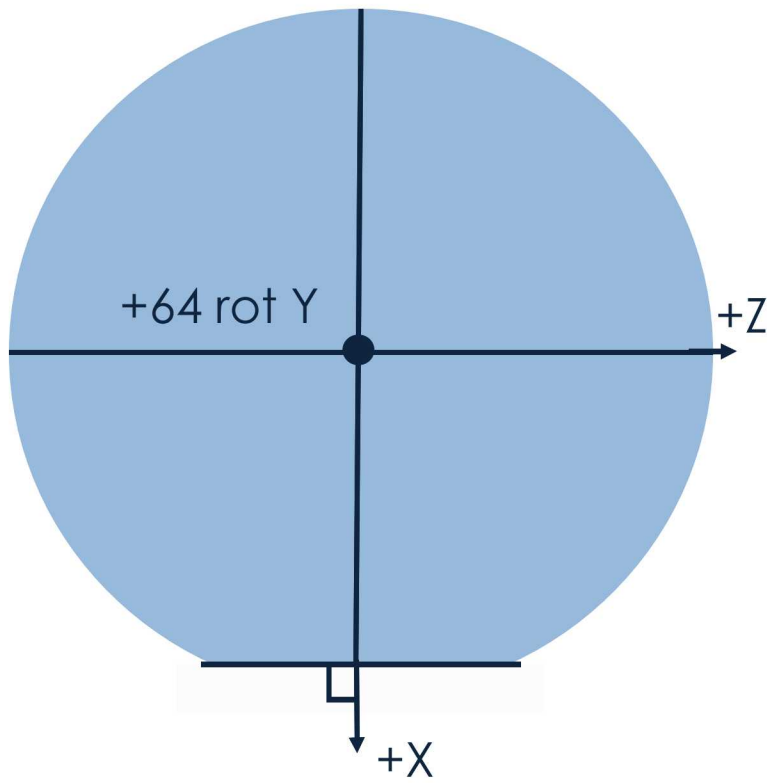
Comsol: Simulating Surface Acoustic Wave (SAW) on 128° Y – rotated X propagating Lithium Niobate: **Admittance Part #2**



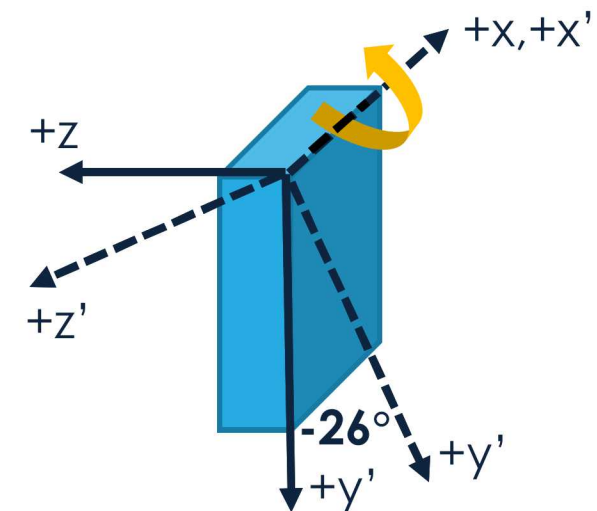
Simulating Surface Acoustic Wave (SAW) on 64° Y –rotated X propagating Lithium Niobate

Comsol rotates the coordinate system not the material tensors

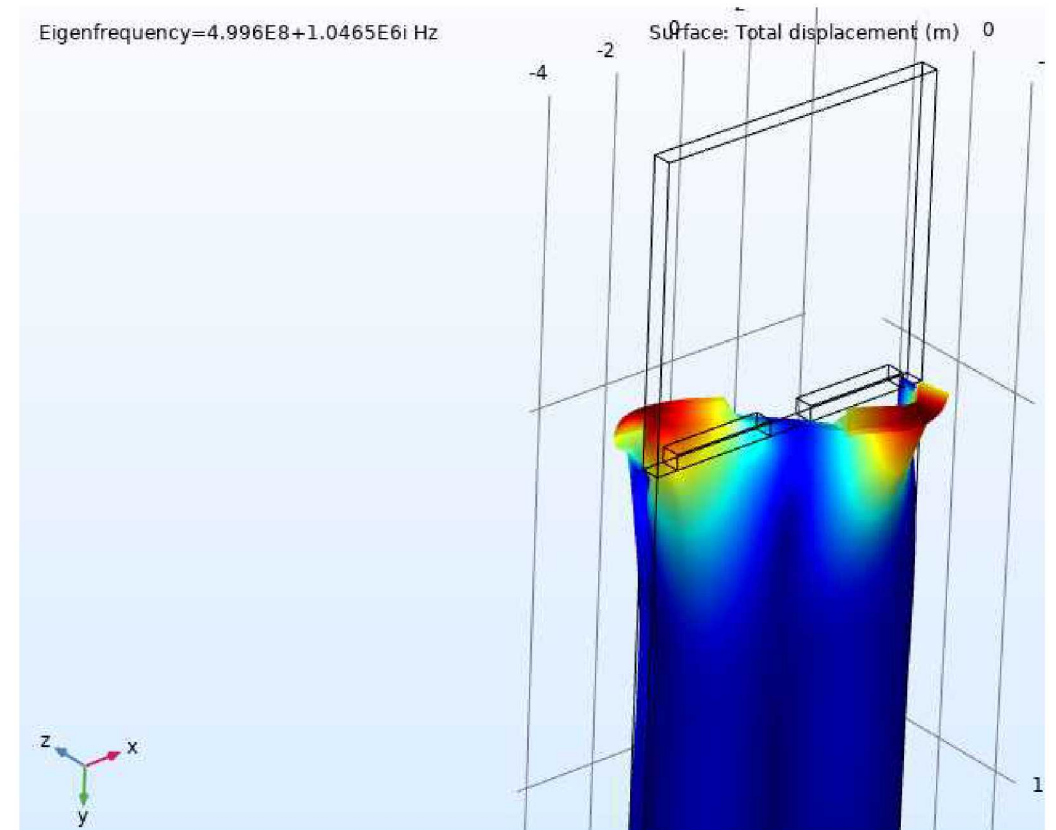
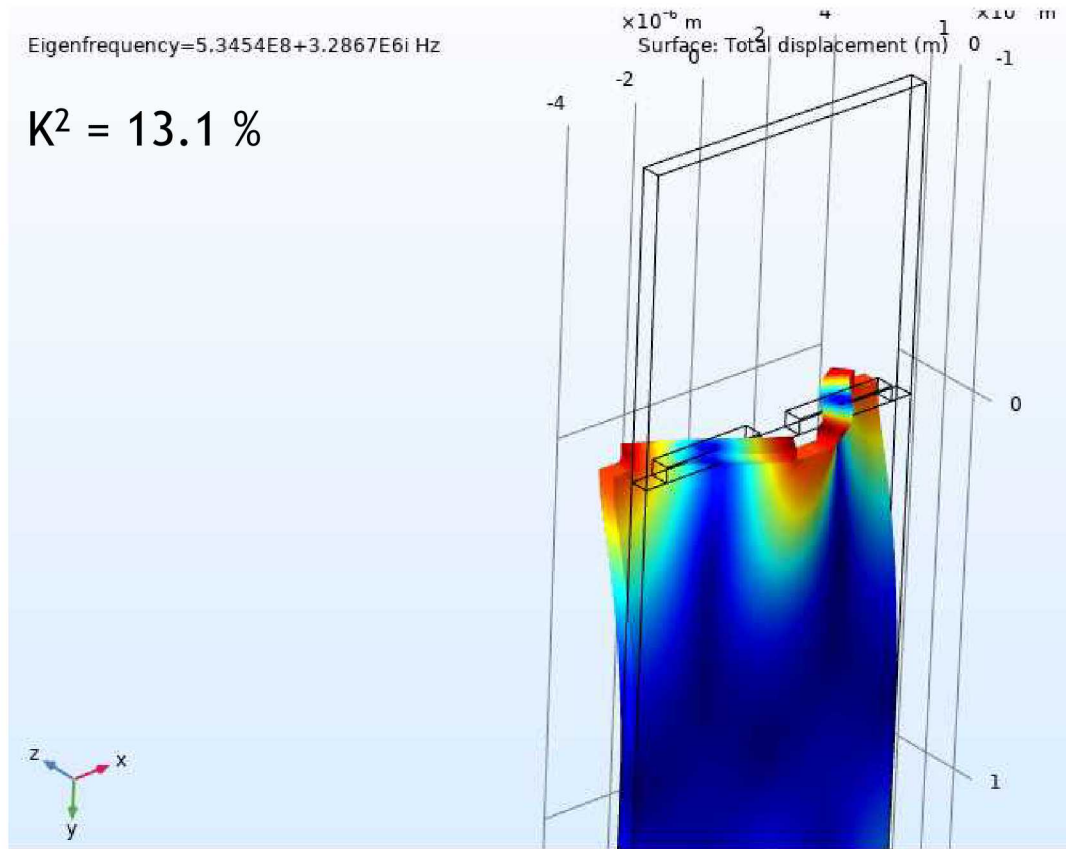
64° YX LN: $\langle z, x, z \rangle (0, -26, 0)$



1. Draw slab in Comsol then apply rotation using the rotated system: $(0, -26, 0)$ for 64° YX LN
2. Comsol rotates the coordinate axis from the drawn axis and which is viewed by plotting the coordinate system:

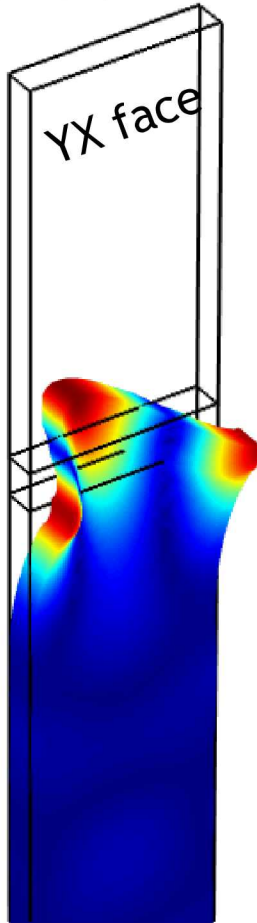
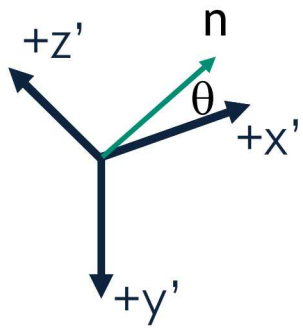


Simulating Surface Acoustic Wave (SAW) on 64° Y –rotated X propagating Lithium Niobate: Results

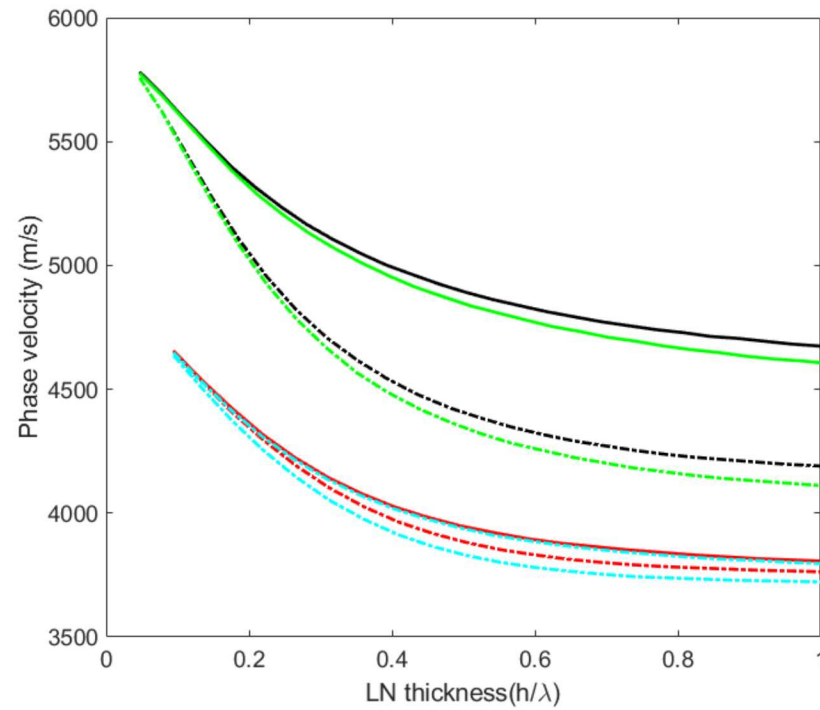


Y-cut X-prop $(0, 90, \theta)$

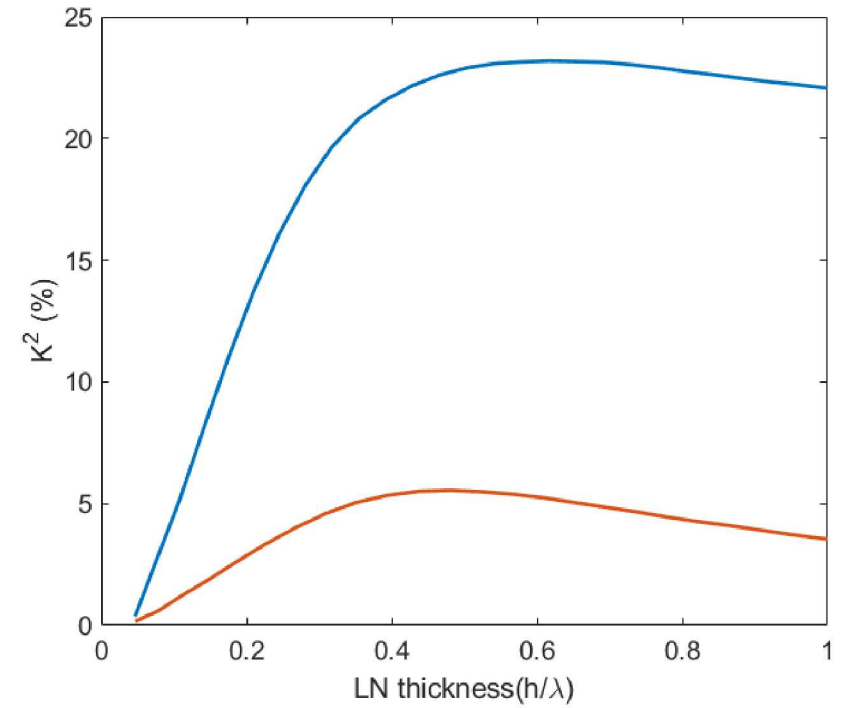
Y-cut definition



Phase velocities



Electromechanical Coupling



Comsol: Modeling Newtonian Fluid Loading on Acoustic Devices

$$u_i = \beta_i e^{j\omega t} e^{-j\omega x_1/V} e^{\alpha\omega x_3/V}, v = j\omega u$$

Hydrostatic pressure of fluid

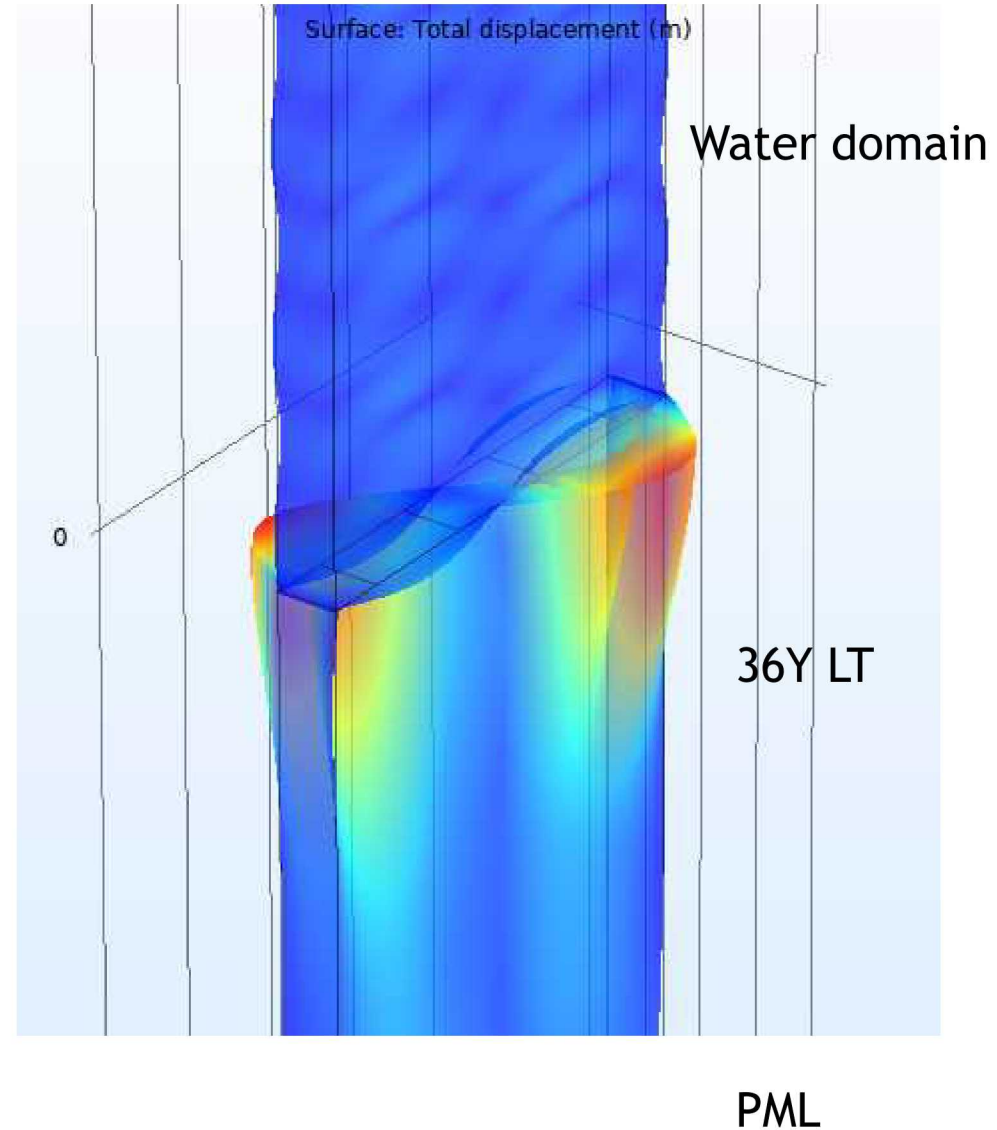
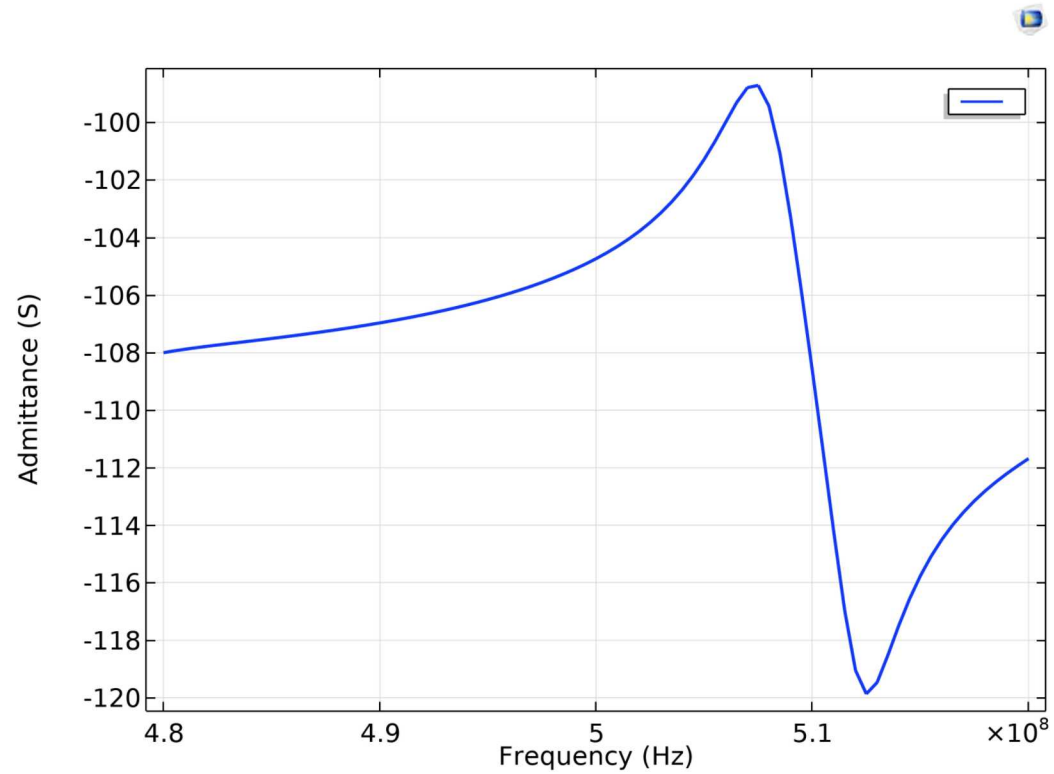
$$\begin{aligned}
 T_{11} &= -K \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) - \frac{2}{3} \eta \left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3} \right) + 2\eta \frac{\partial v_1}{\partial x_1} \\
 T_{12} &= \eta \left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} \right) \\
 T_{22} &= -K \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) - \frac{2}{3} \eta \left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3} \right) + 2\eta \frac{\partial v_2}{\partial x_2} \\
 T_{13} &= \eta \left(\frac{\partial v_1}{\partial x_3} + \frac{\partial v_3}{\partial x_1} \right) \\
 T_{33} &= -K \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) - \frac{2}{3} \eta \left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3} \right) + 2\eta \frac{\partial v_3}{\partial x_3} \\
 T_{23} &= \eta \left(\frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2} \right)
 \end{aligned}$$

$$\mathbf{T} = \begin{bmatrix} K - \frac{2j\omega\eta}{3} + 2j\omega\eta & K - \frac{2j\omega\eta}{3} & 0 & 0 & 0 & 0 \\ K - \frac{2j\omega\eta}{3} & K - \frac{2j\omega\eta}{3} + 2j\omega\eta & 0 & 0 & 0 & 0 \\ K - \frac{2j\omega\eta}{3} & K - \frac{2j\omega\eta}{3} & K - \frac{2j\omega\eta}{3} + 2j\omega\eta & 0 & 0 & 0 \\ 0 & 0 & 0 & j\omega\eta & 0 & 0 \\ 0 & 0 & 0 & 0 & j\omega\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & j\omega\eta \end{bmatrix} \mathbf{S}$$

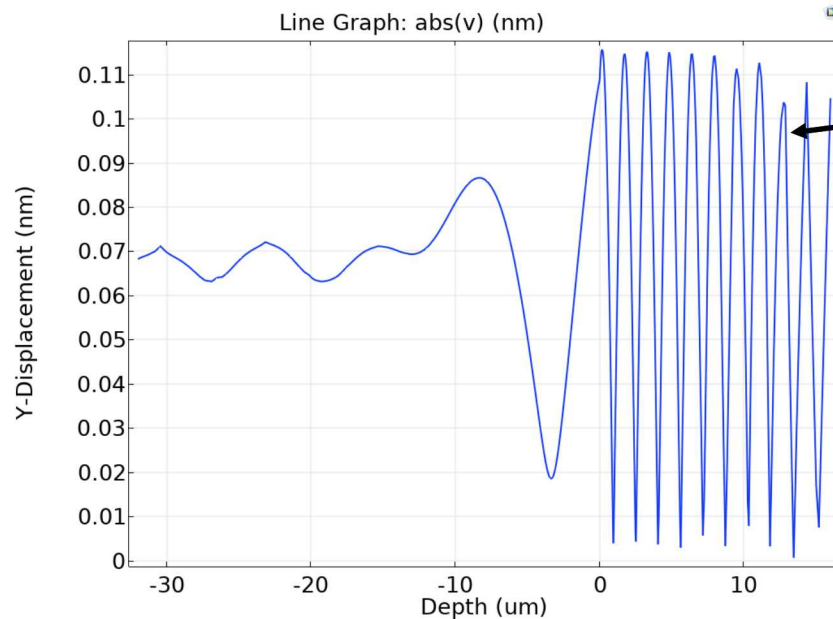
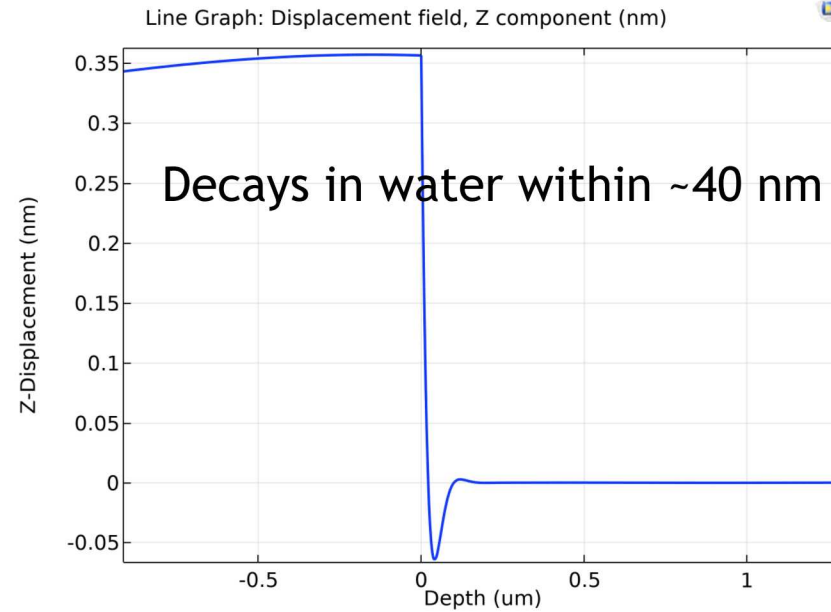
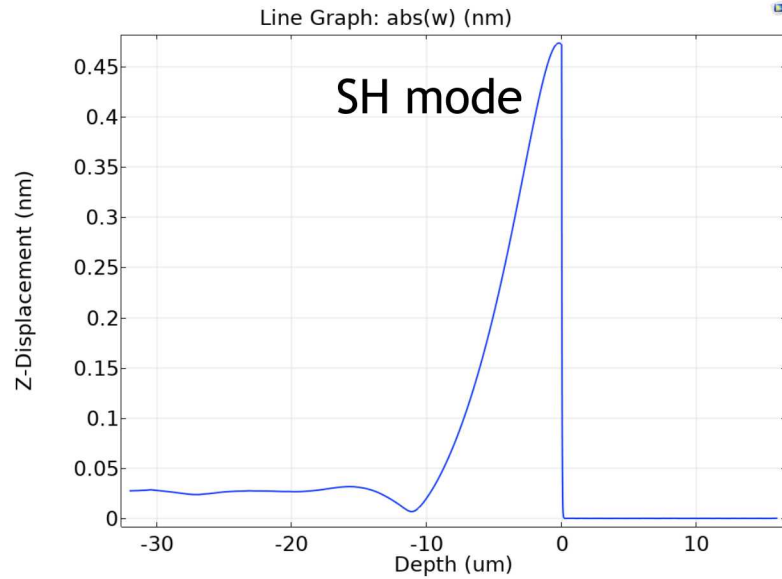
Longitudinal and Shear velocity in the Fluid:

$$V_L = \sqrt{\frac{K + \frac{4j\omega\eta}{3}}{\rho_f}}, V_S = \sqrt{\frac{j\omega\eta}{\rho_f}}$$

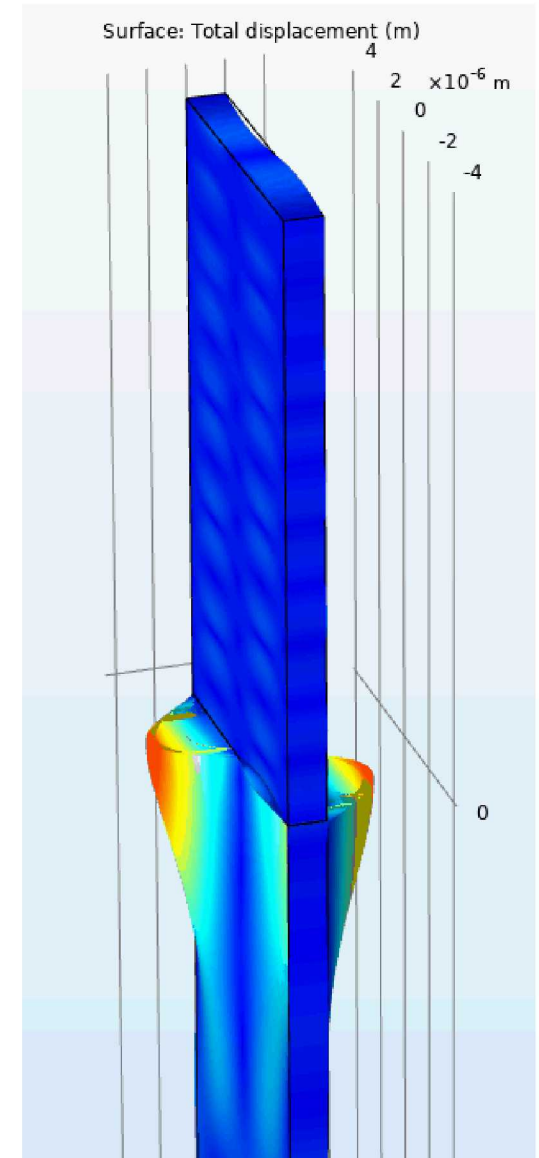
Fluid Loading on a SH-SAW Sensor (36° Y Lithium Tantalate)



Fluid Loading on a SH-SAW Sensor (36° Y Lithium Tantalate)

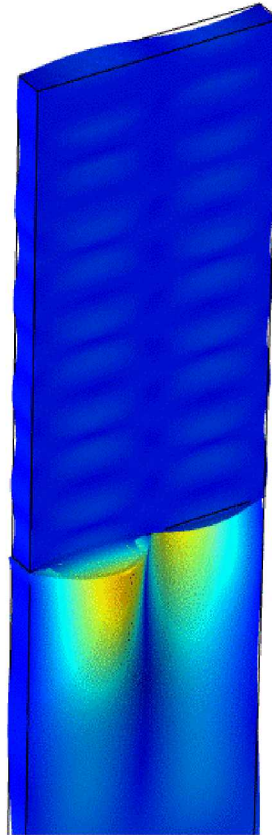


Standing wave near interface with gradual decay into fluid



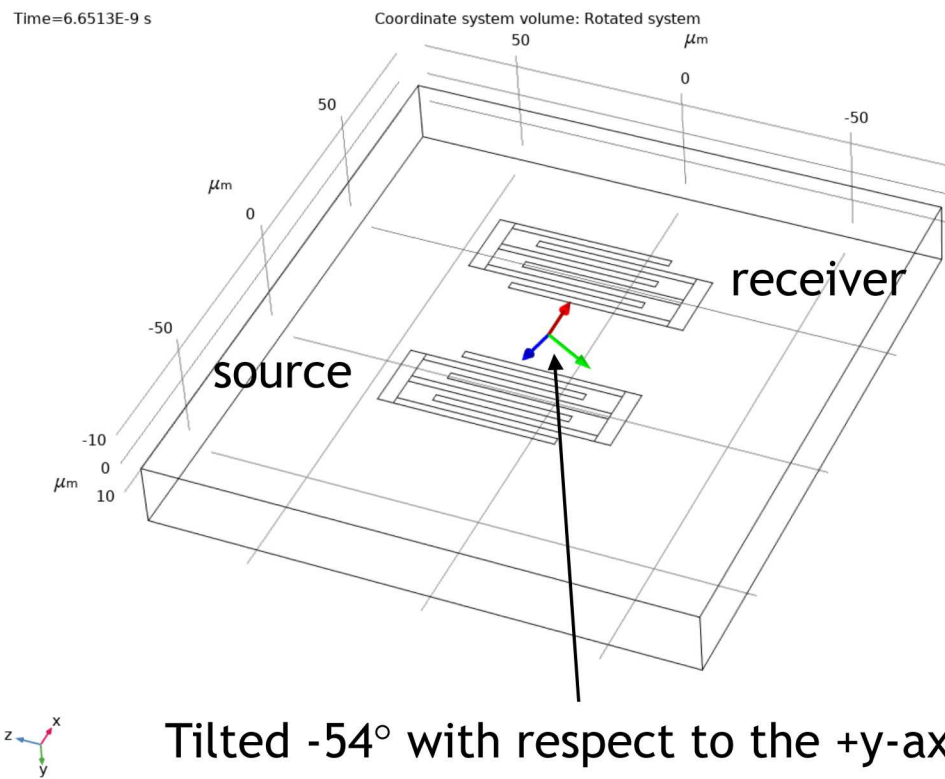
Fluid Loading on a SH-SAW Sensor (36° Y Lithium Tantalate), cont'd

Water motion at resonance



Comsol: Leaky SAW on 36° Y (0,-54°,0) Lithium Tantalate: Transient Model

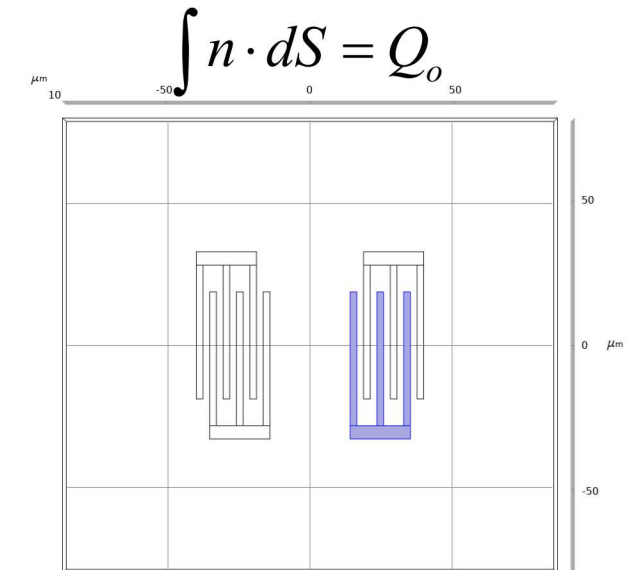
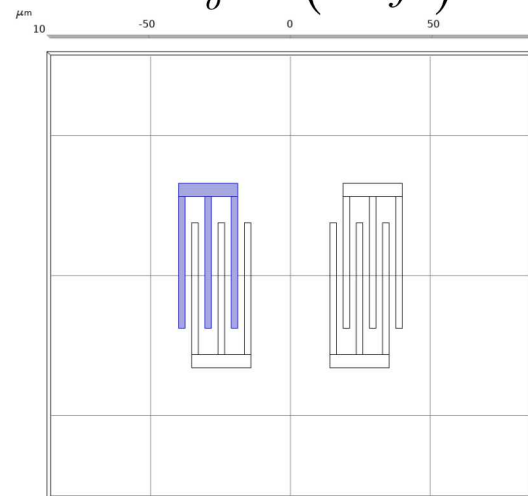
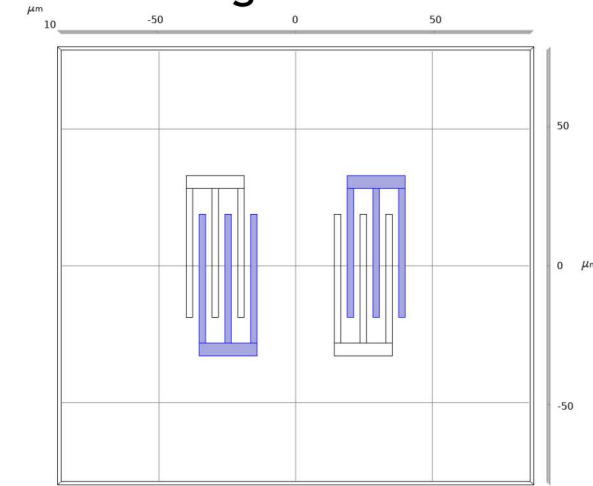
Geometry with Rotated Coordinate System set at -54° from positive y-axis



Synchronous drive:

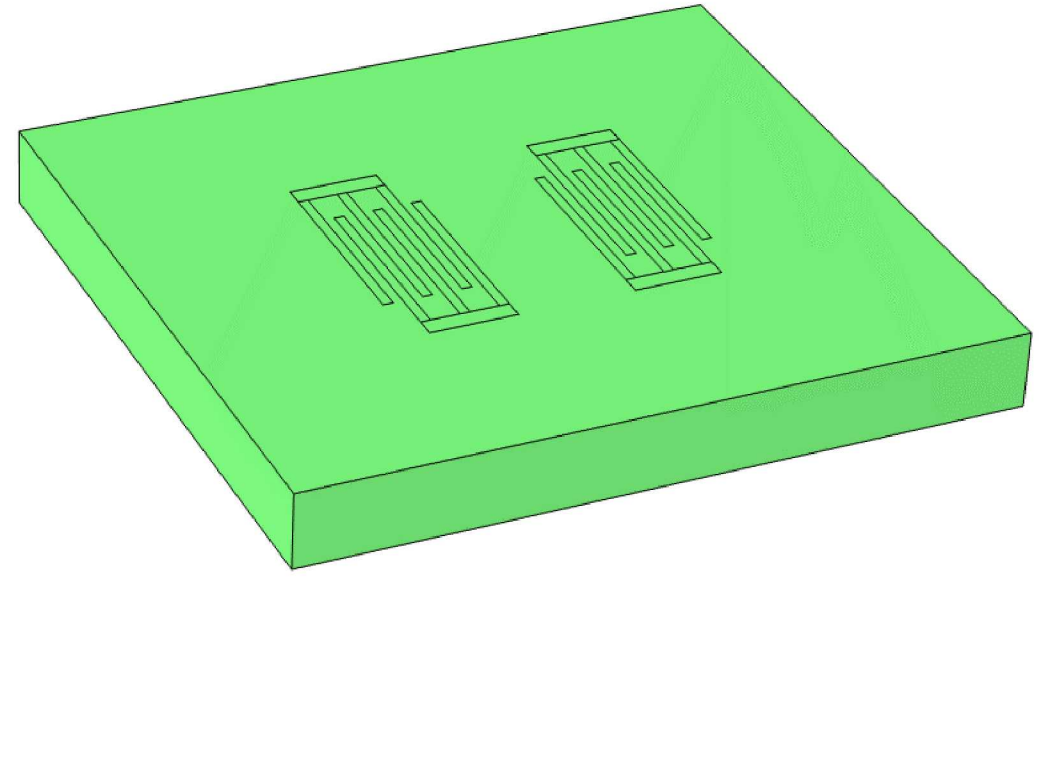
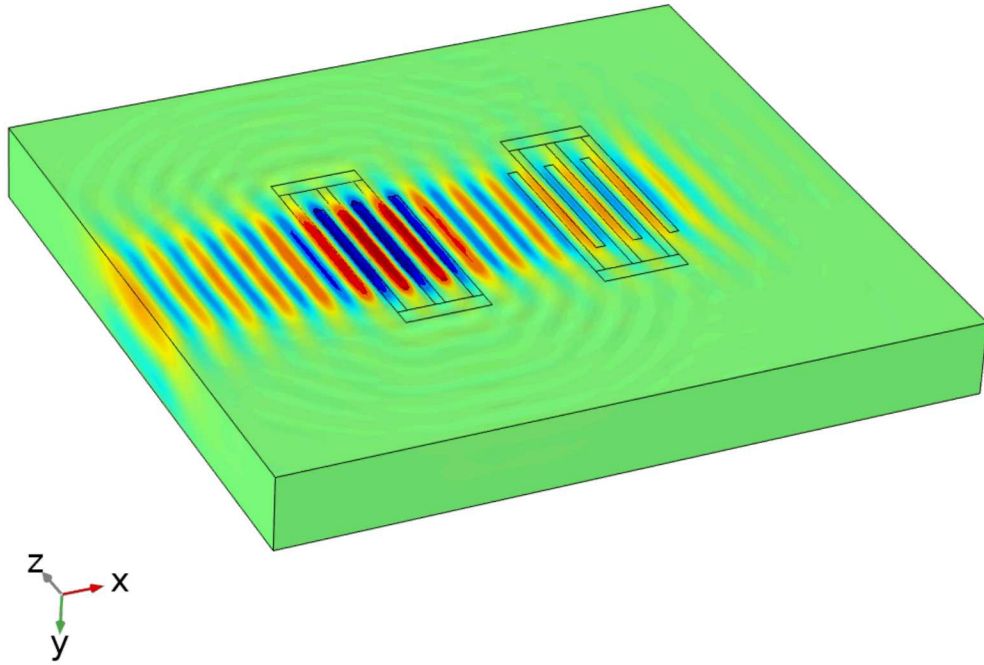
$$V = V_o \sin(2\pi ft)$$

grounds



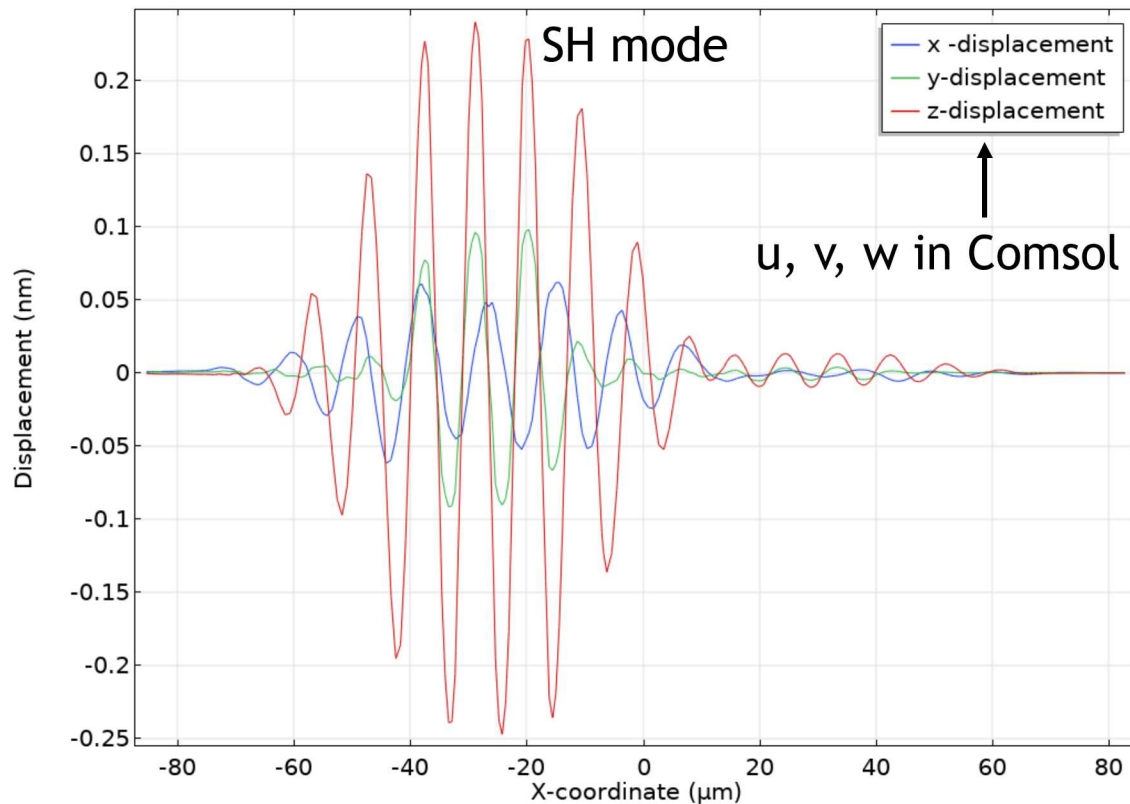
$$\int n \cdot dS = Q_o$$

Leaky SAW on 36 Y Lithium Tantalate, cont'd



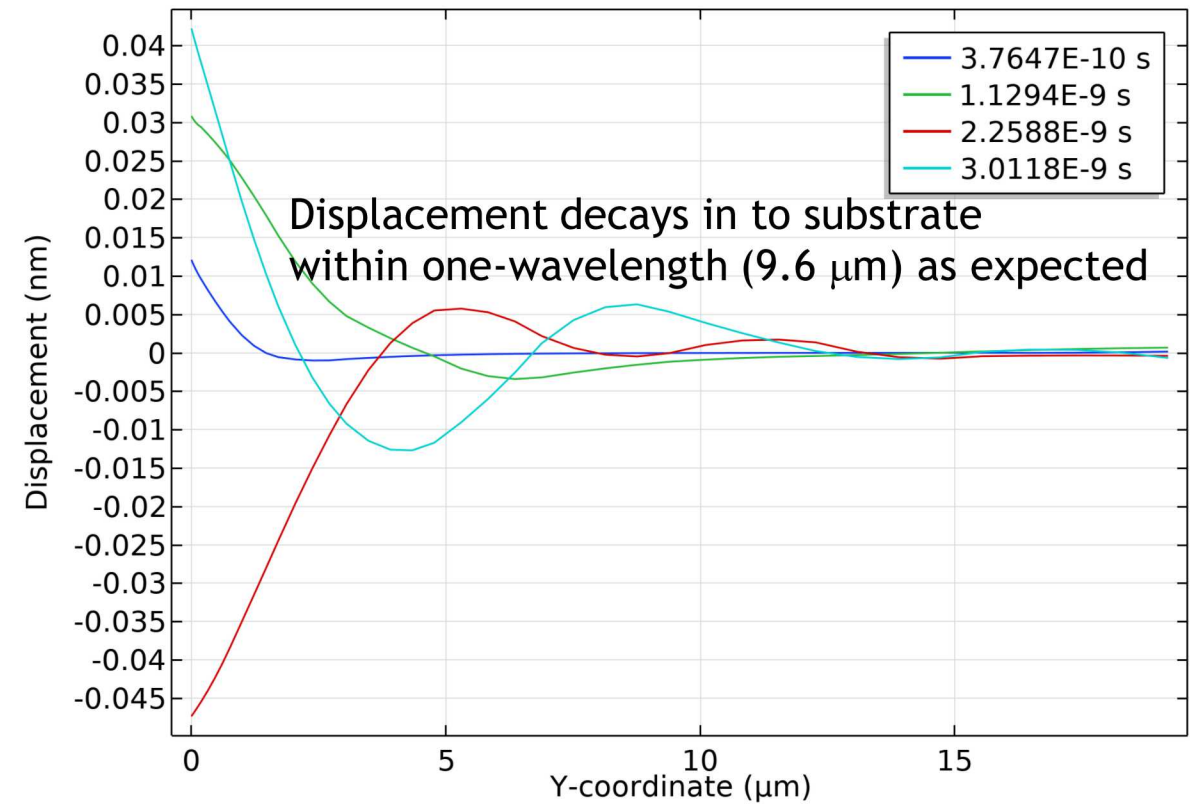
Leaky SH-SAW on 36° Y Lithium Tantalate, cont'd

Y-displacement along X-direction

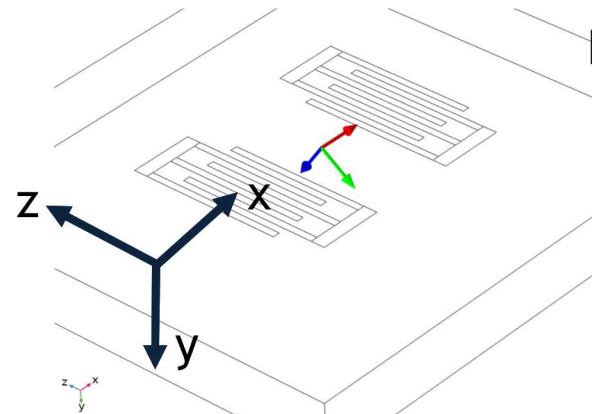


Propagation direction

Y-displacement along Y-direction



Depth into substrate

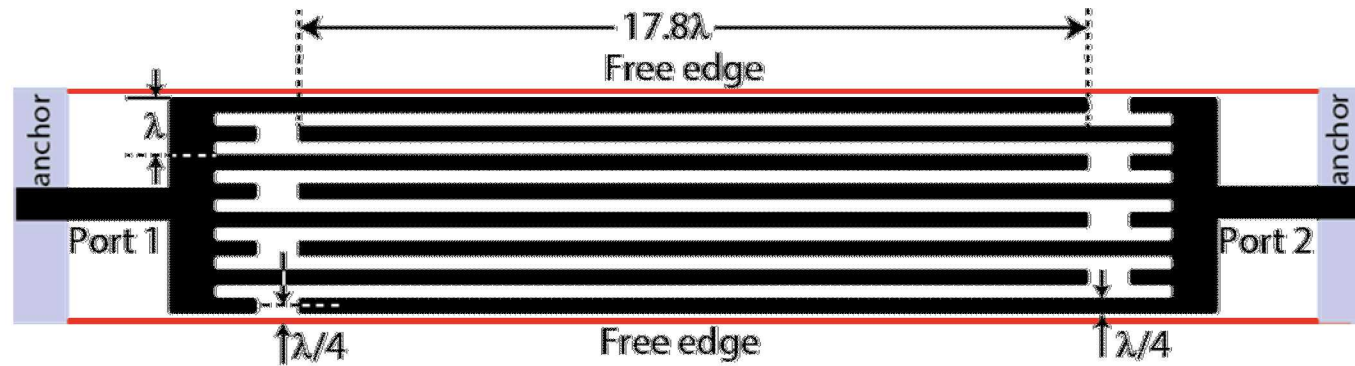


Why is y-displacement larger than expected compared to z-displacement?

A: Infinitesimal thin metal electrodes doesn't excite the shear mode adequately.
Solution: Use thicker metal electrodes

2D Coupling of Modes using Comsol: Customized Interface

In some cases, you have to develop an interface to couple specific types of physics or solve coupled PDEs not currently defined in Comsol. Consider a contour mode microresonator using the S0 mode:



What's the admittance?

A full 3D FEA simulation takes about 20 hrs. Can we speed this up without going to a very large machine (e.g. 1000 cores)? Note: FEA solve time scales more strongly with CPU clock speed than number of cores.

PDE interface in Comsol:

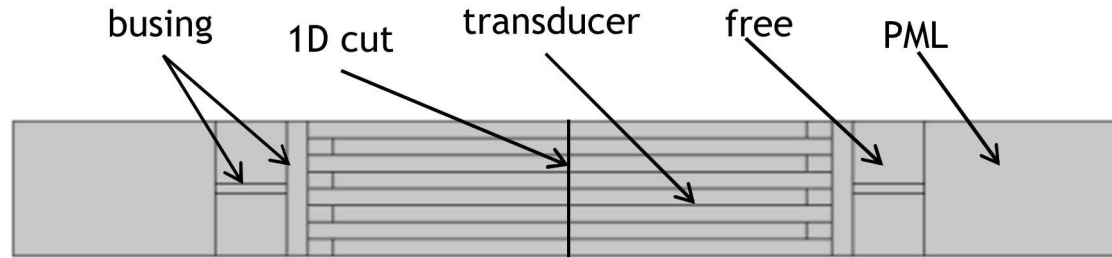
$$e \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f$$

$$\mathbf{n} \cdot (-c \nabla u + \alpha u - \gamma) = g - q u + h^T \mu$$

$$0 = R$$

$$u = r$$

2D Coupling of Modes using Comsol: Customized Interface, cont'd



2D COM Equations:

$$\nabla \cdot j \begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \frac{\gamma_a}{2k_o} \end{bmatrix} & [0] \\ [0] & \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\gamma_a}{2k_o} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{\partial A^+}{\partial y} \\ 0 \\ \frac{\partial A^-}{\partial y} \end{bmatrix} + \nabla \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A^+ \\ A^- \end{bmatrix} + j \begin{bmatrix} \delta & \kappa_{12} \\ -\kappa_{12}^* & -\delta \end{bmatrix} \begin{bmatrix} A^+ \\ A^- \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha^* \end{bmatrix} \begin{bmatrix} V \\ -V \end{bmatrix}$$

Detuning parameter: $\delta = k - k_o - j\gamma$

Currents:

$$I_i(\omega) = \iint (2\alpha^* A^+ + 2\alpha A^- + j\omega C(x, y)V_i) dx dy$$

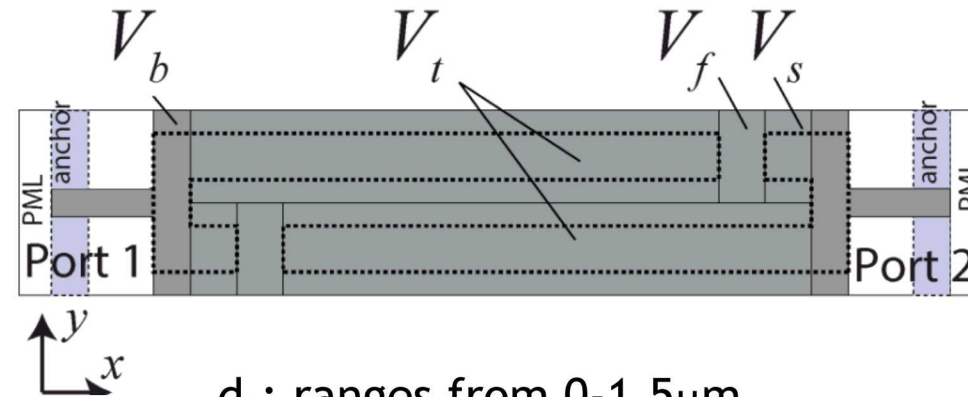
$$I_o(\omega) = \iint (2\alpha^* A^+ + 2\alpha A^-) dx dy$$

Once the fields are known, then the current can be computed

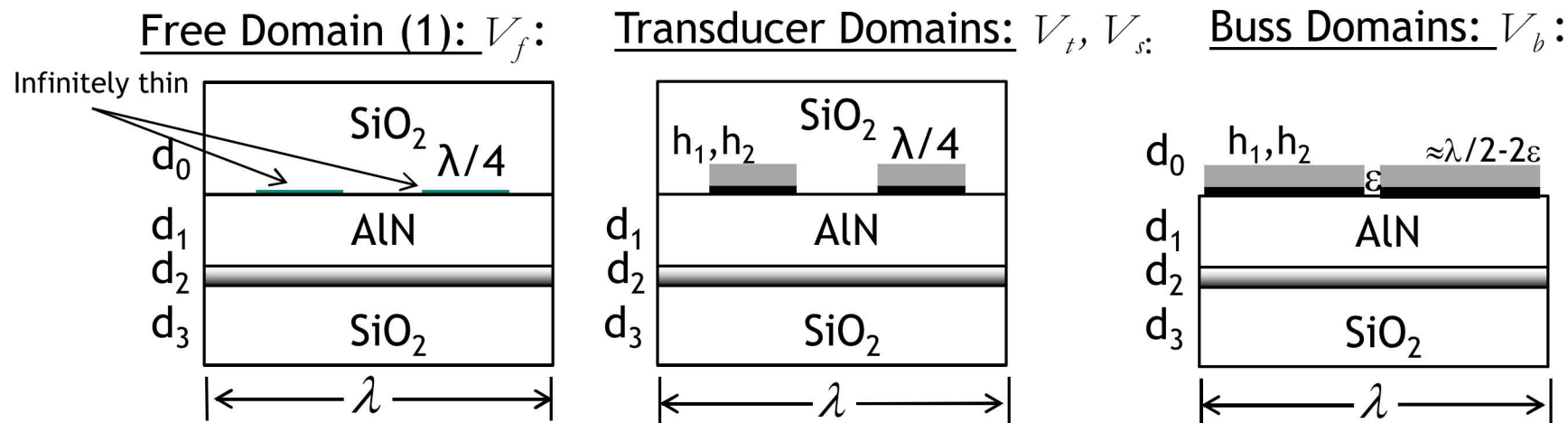
PML: $k(r) \rightarrow k_f(1 - j\eta(r)) = k_f \left(1 - j\eta_o \frac{|r - r_i|}{d} \right)$

2D Coupling of Modes using Comsol: Customized Interface, cont'd

Next, we use Comsol to determine the COM parameters (κ , α , and C_s) for each domain and store them:



d_0 : ranges from 0-1.5 μm

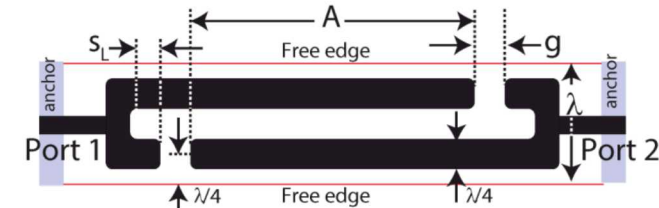


2D Coupling of Modes using Comsol: Customized Interface, cont'd

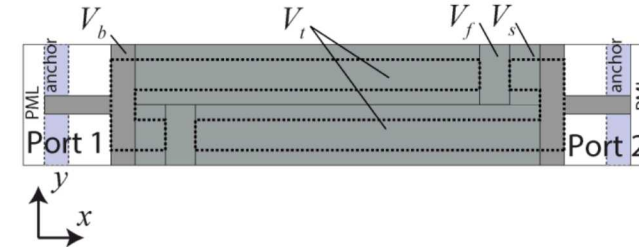
COM Parameters

Domain	δ	v (m/s)	$\kappa_{12}p$ (%)	$ \zeta p ^2/\omega C_s$ ($m^2\Omega^{-1/2}$)	C_o (pF/p/m)
Transducer	$k_t - k_o - j\gamma$	7873	-0.08	0.004	512.3
Buss	$k_m - k_o - j\gamma$	7828	0	0	512.3
Free Space	$k_f - k_o - j\gamma$	7910	0	0	0
Stub	$k_s - k_o - j\gamma$	7873	-0.08	0.004	512.3

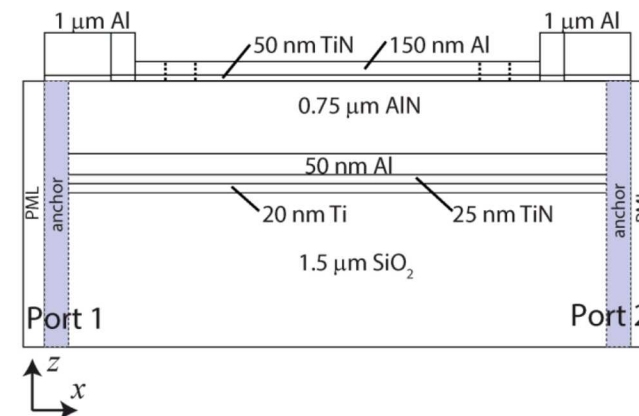
Layout



Equivalent Field Model Layout

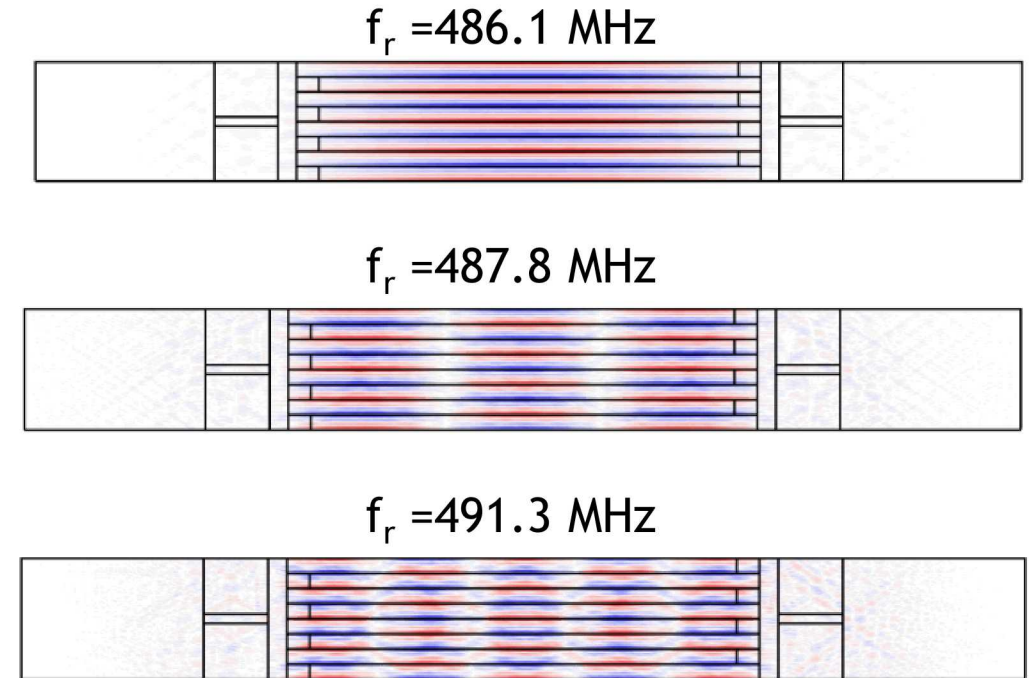
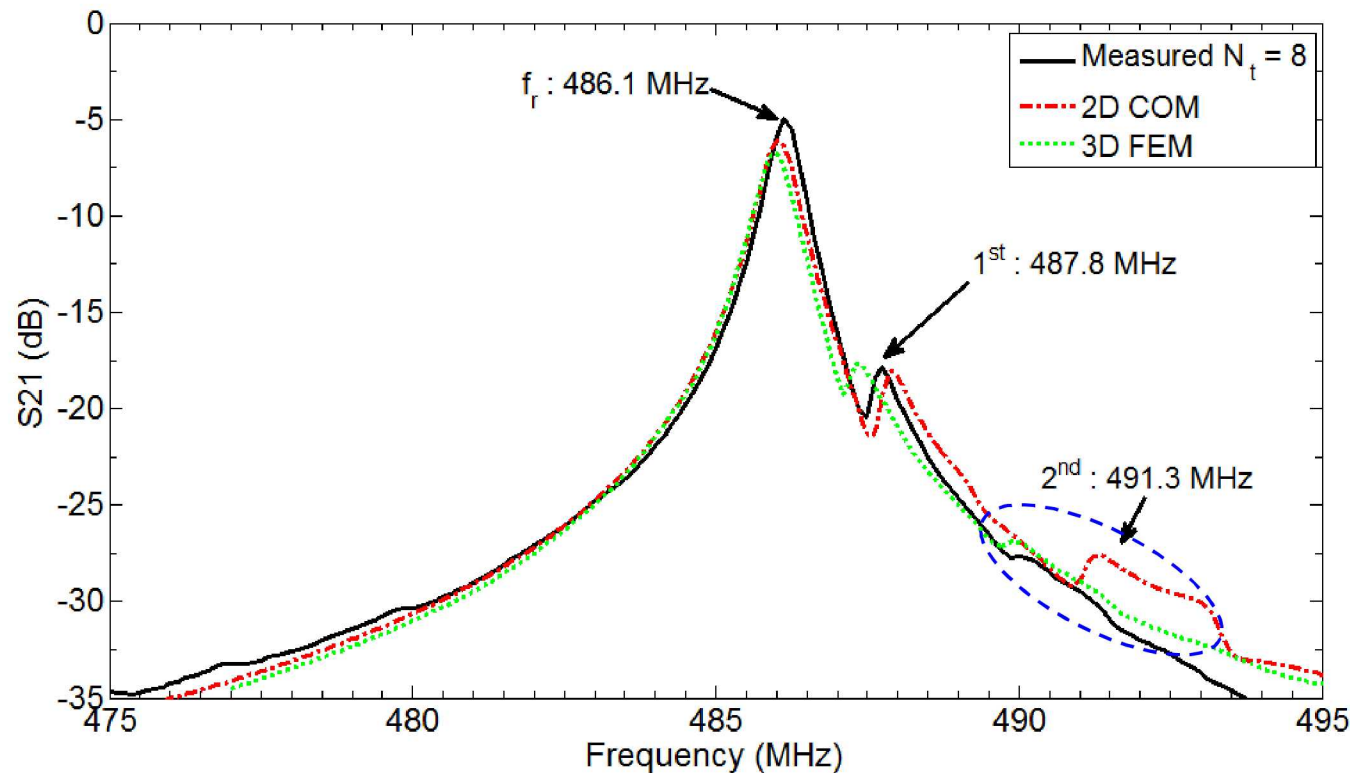


Cross-Section



2D Coupling of Modes using Comsol: Customized Interface, cont'd

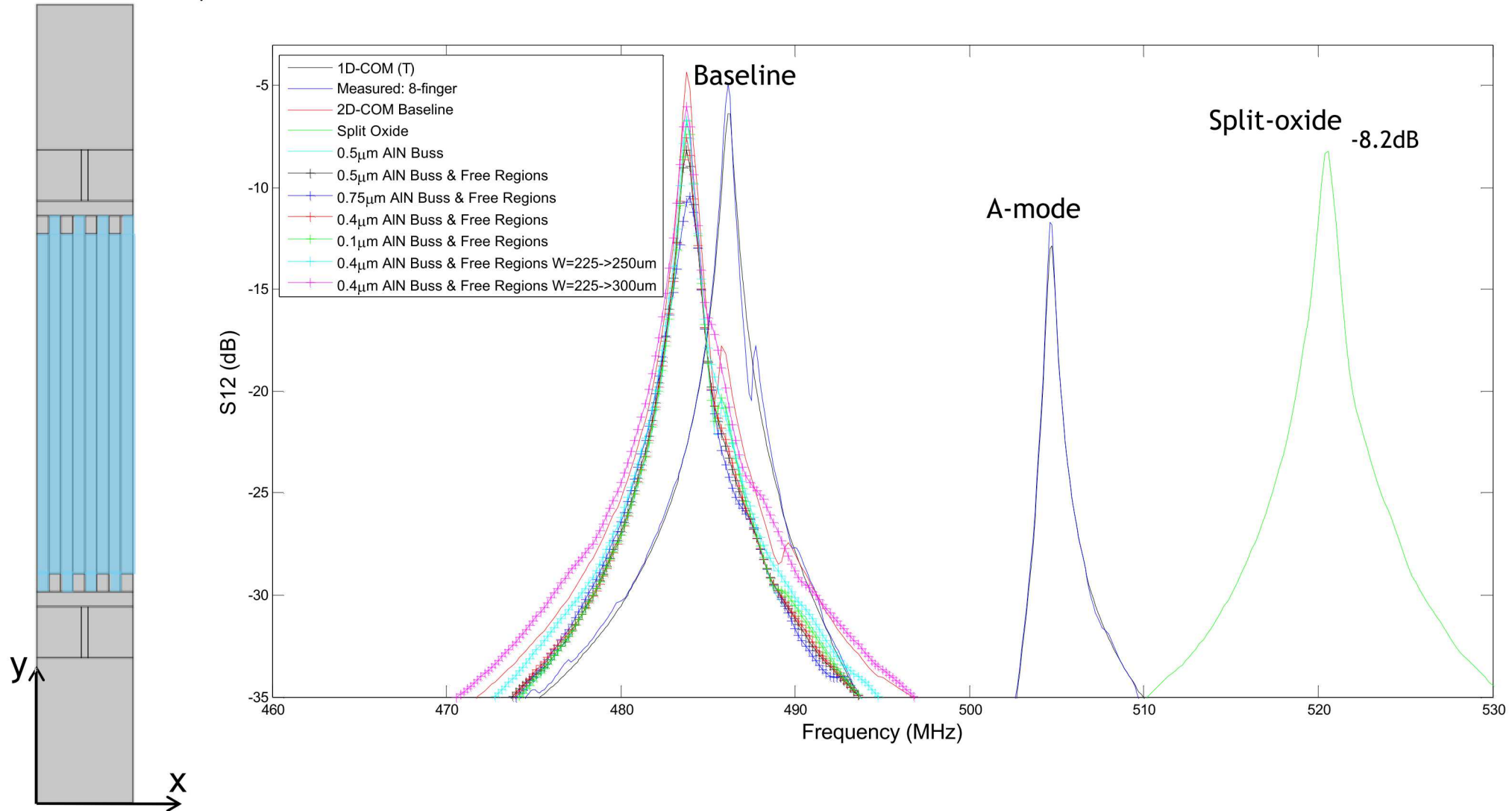
The 2D COM simulation predicted two distinct spurious modes at 487.8 MHz and a second at 491.3 MHz and tracked the measurement



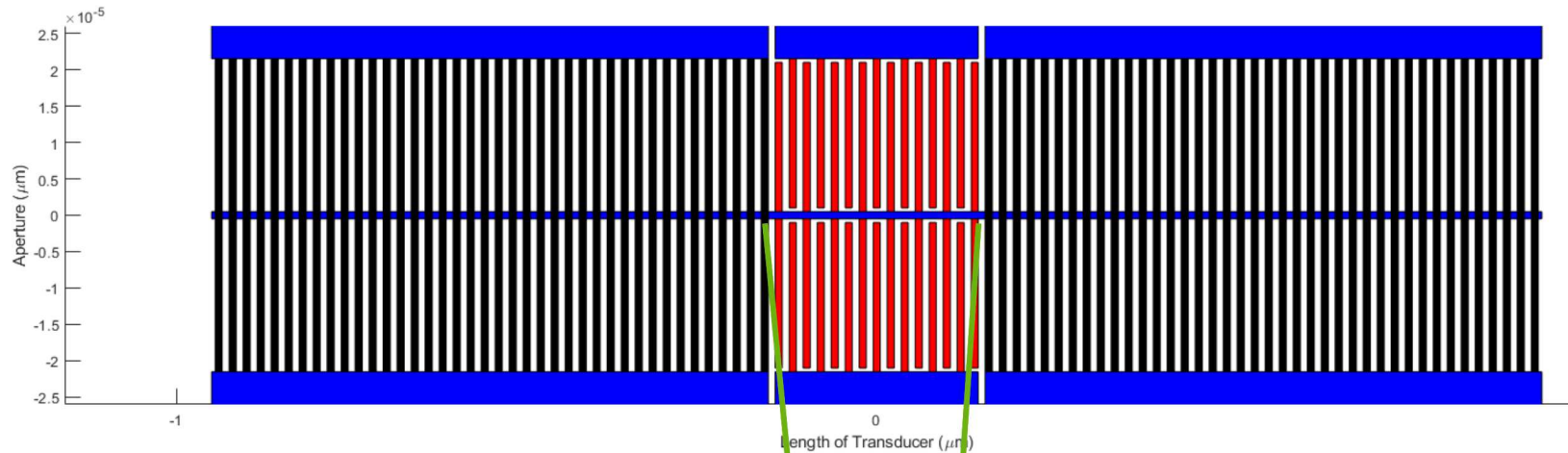
Useful Method for Spurious Mode Control

Blue Region:

Oxide film: $0.75\mu\text{m}$

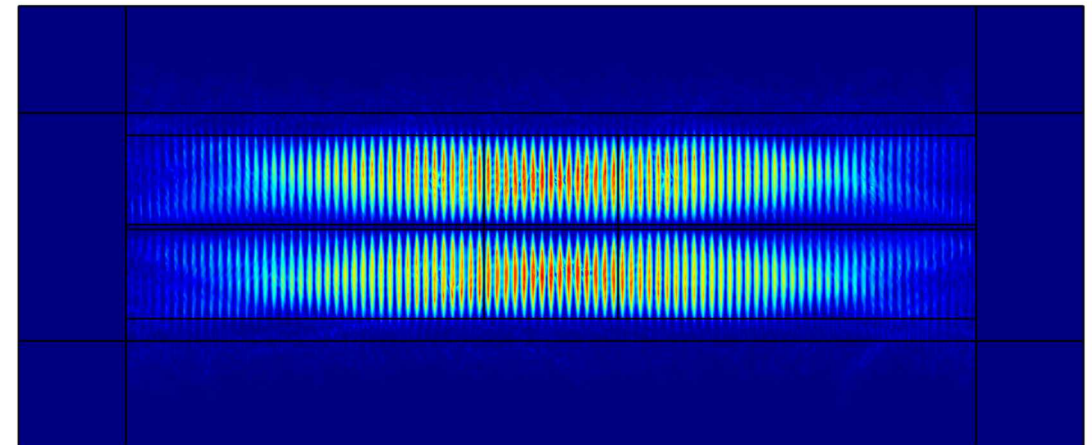
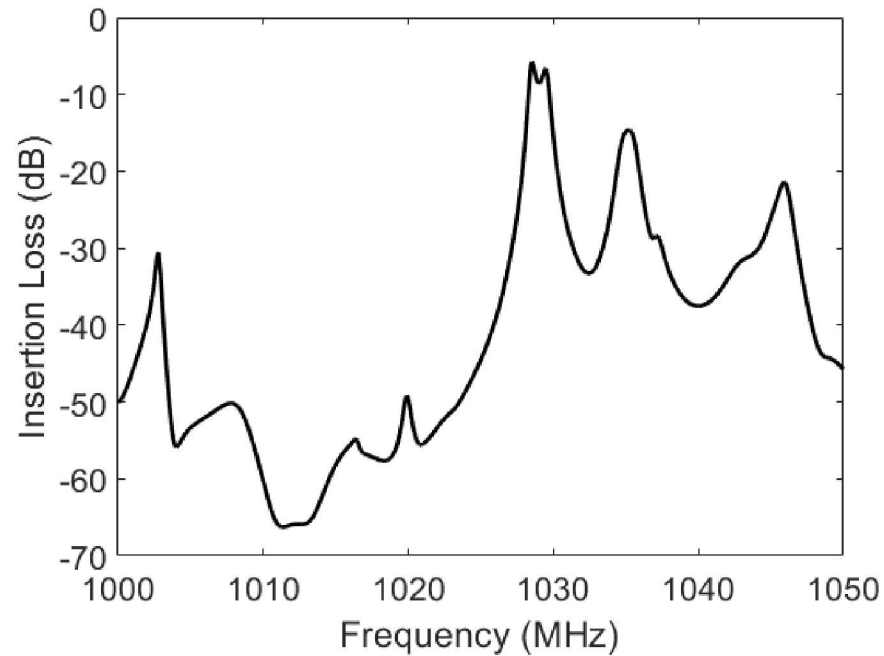
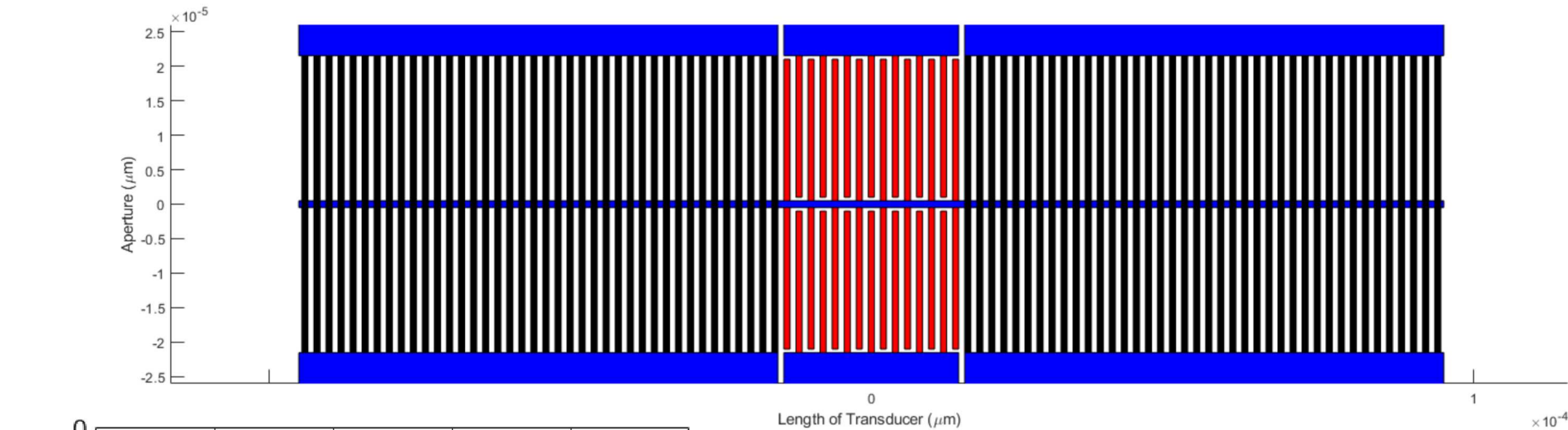


Coupled Resonant Filter in Comsol (2D COM): 2-Port, Bragg, $\lambda=4\mu\text{m}$



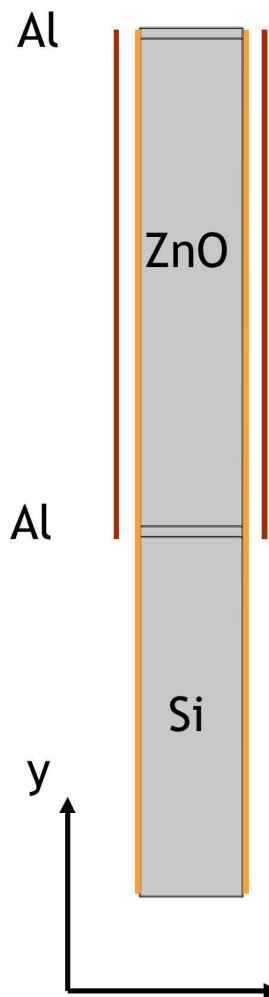
3	10				19
2	9				18
	8	13	16		
	7	12	15		
	6	11	14		
1	5				17
	4				

Coupled Resonant Filter in Comsol (2D COM): 2-Port, Bragg, $\lambda=4\mu\text{m}$



BAW Dispersion

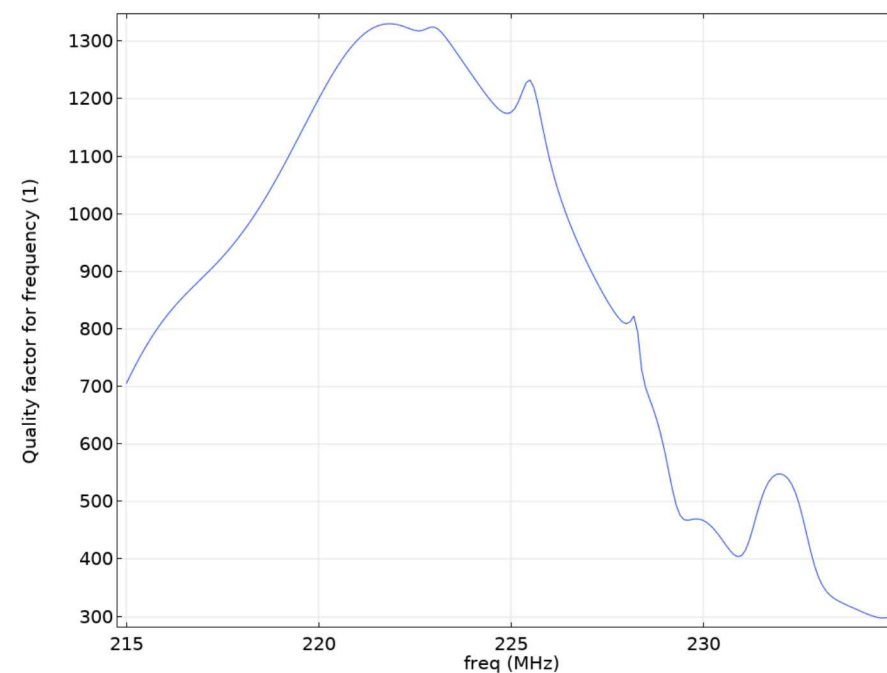
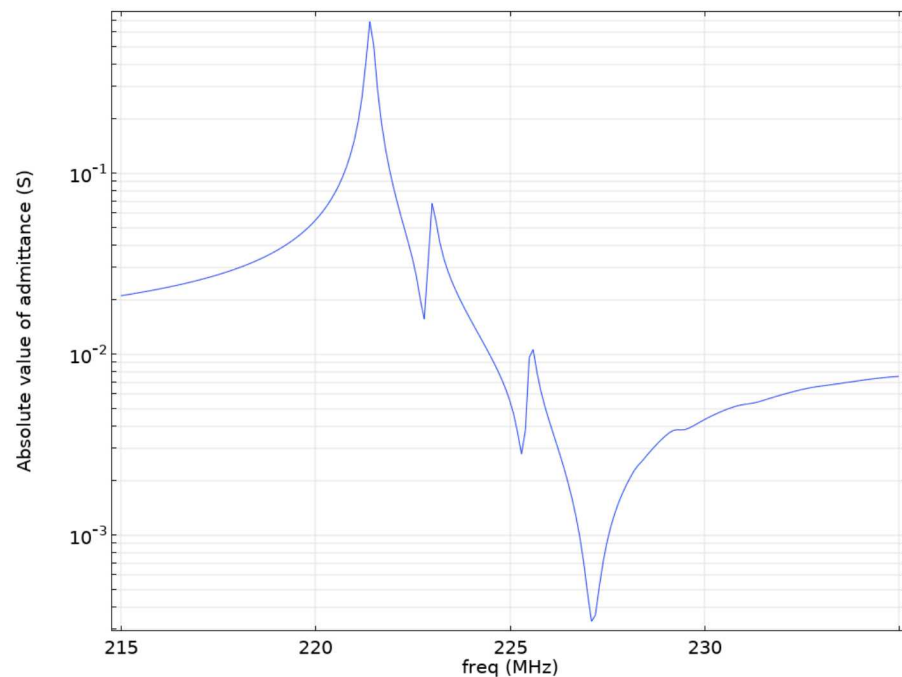
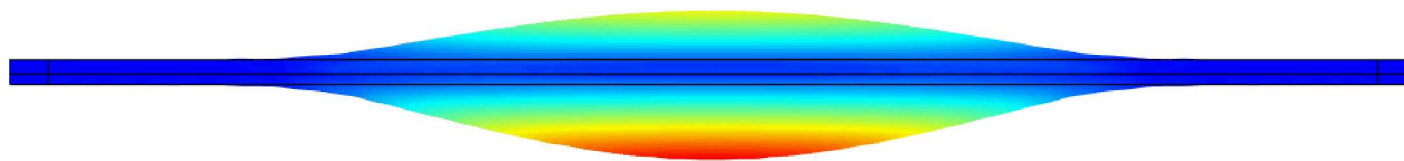
Model:



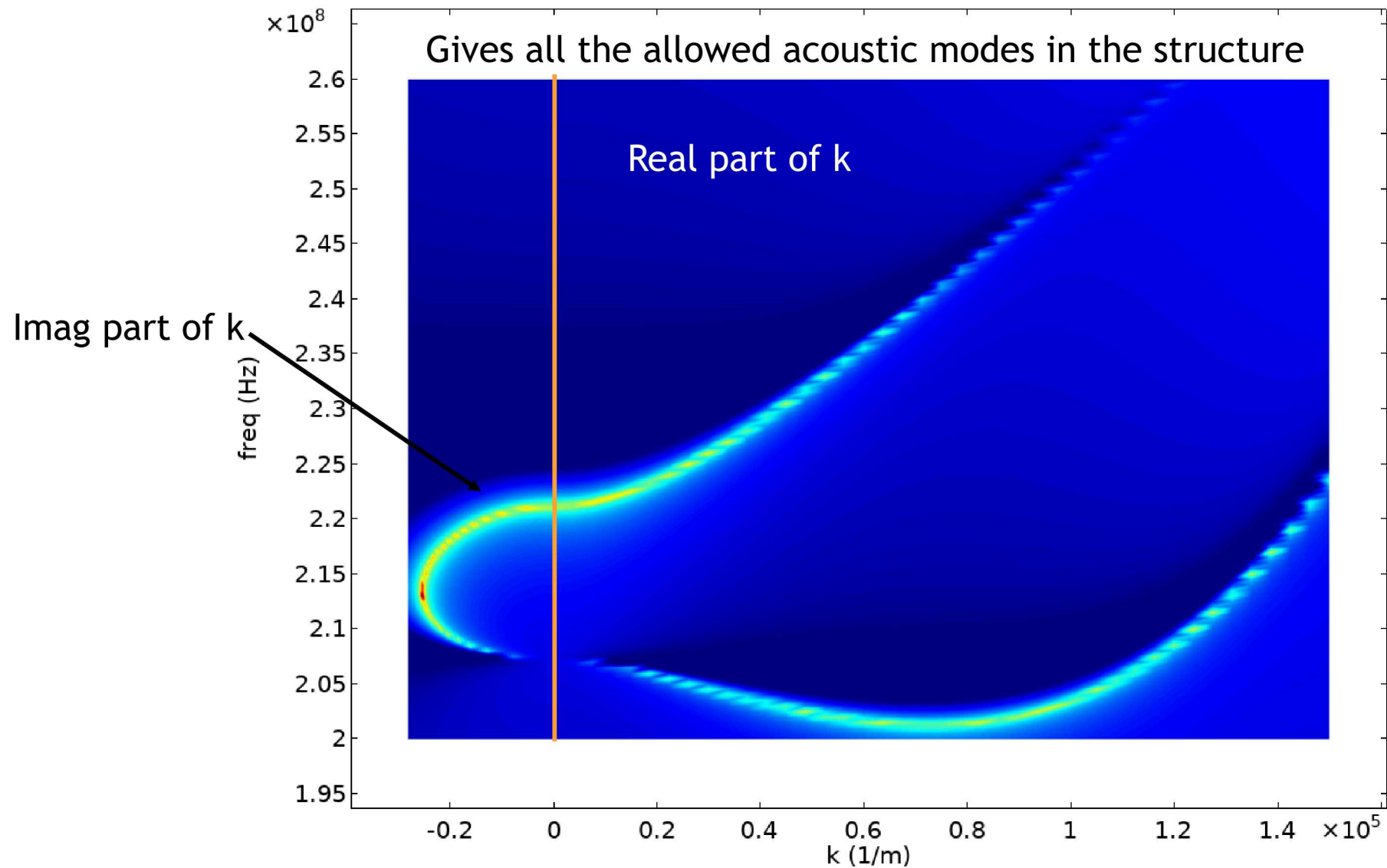
solid: Floquet conditions

$$u_{dst} = u_{src} e^{-ik(r_{dst} - r_{src})}$$

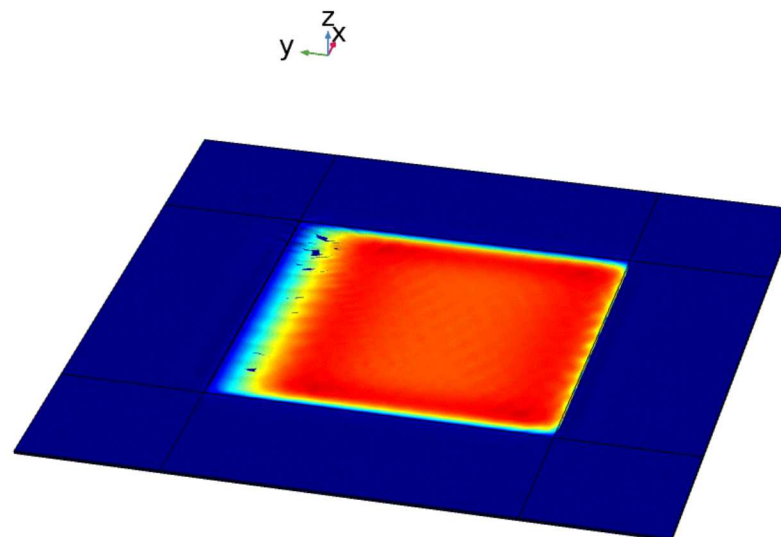
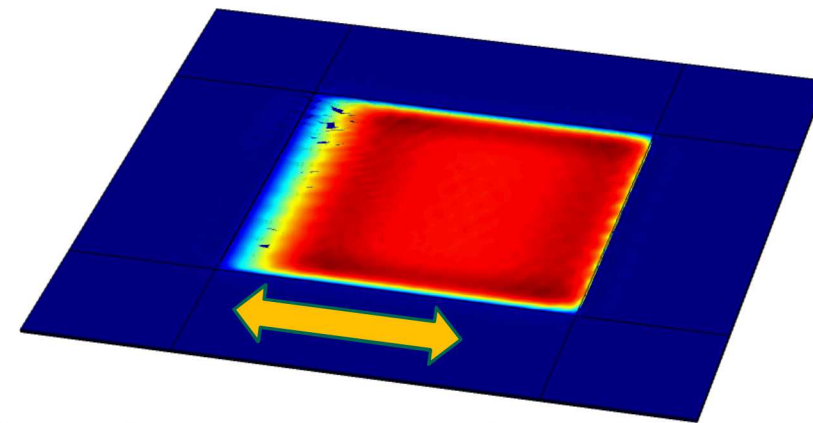
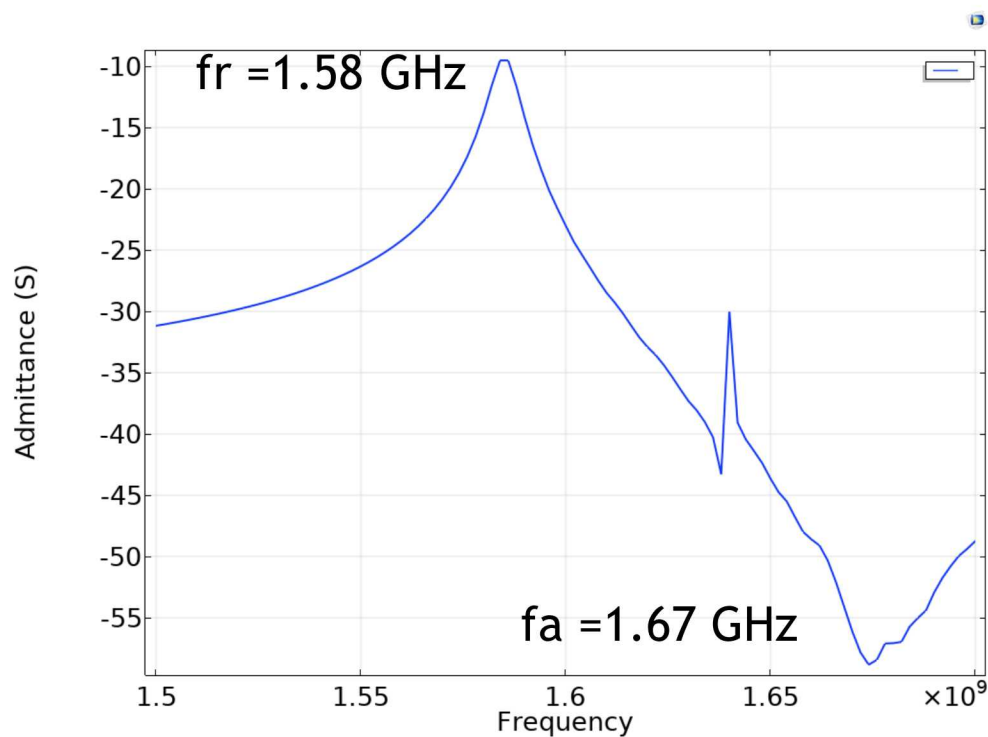
es: Floquet conditions



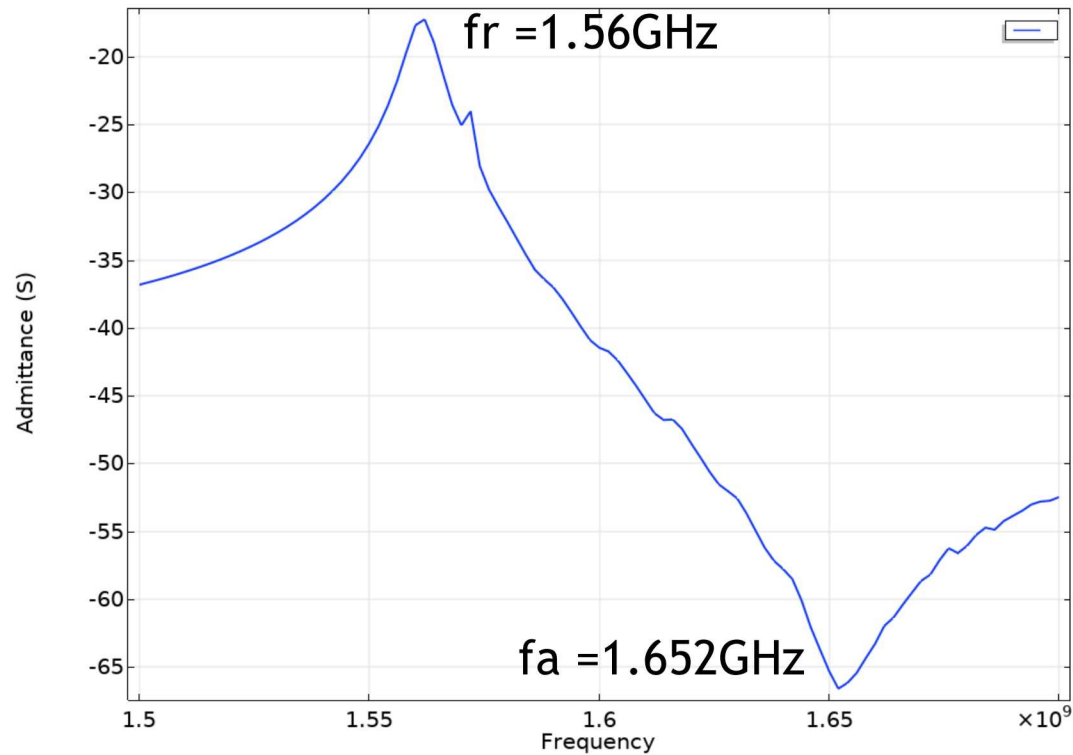
BAW Dispersion, cont'd



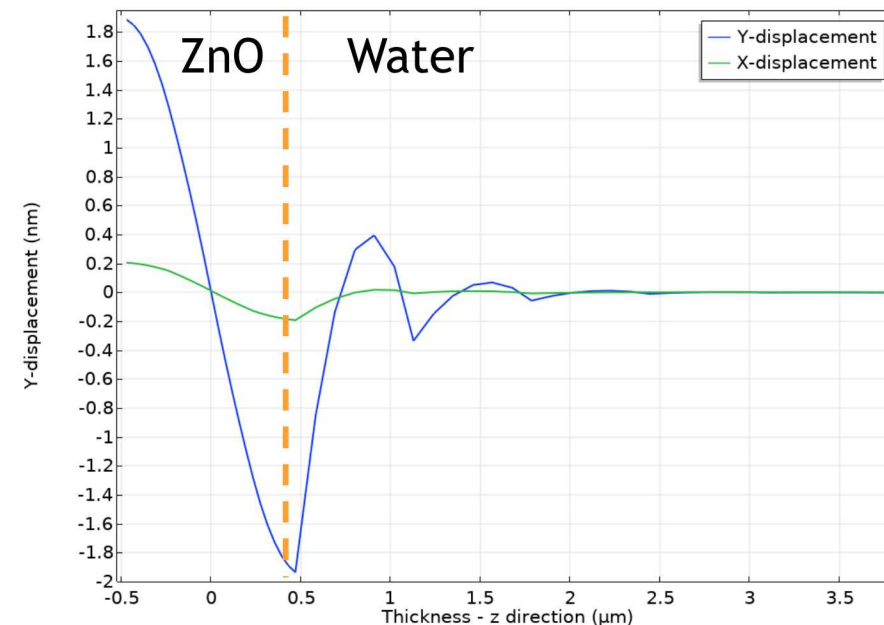
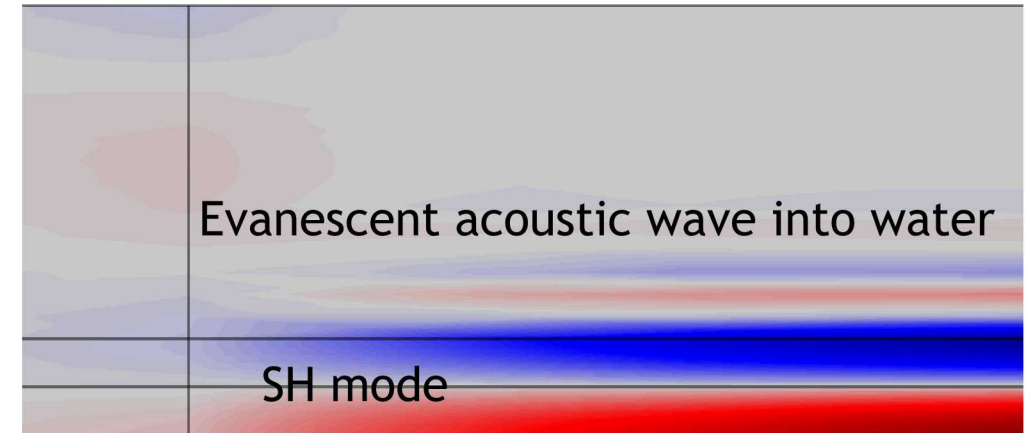
ZnO Shear BAW in Air, 28° c-axis tilt



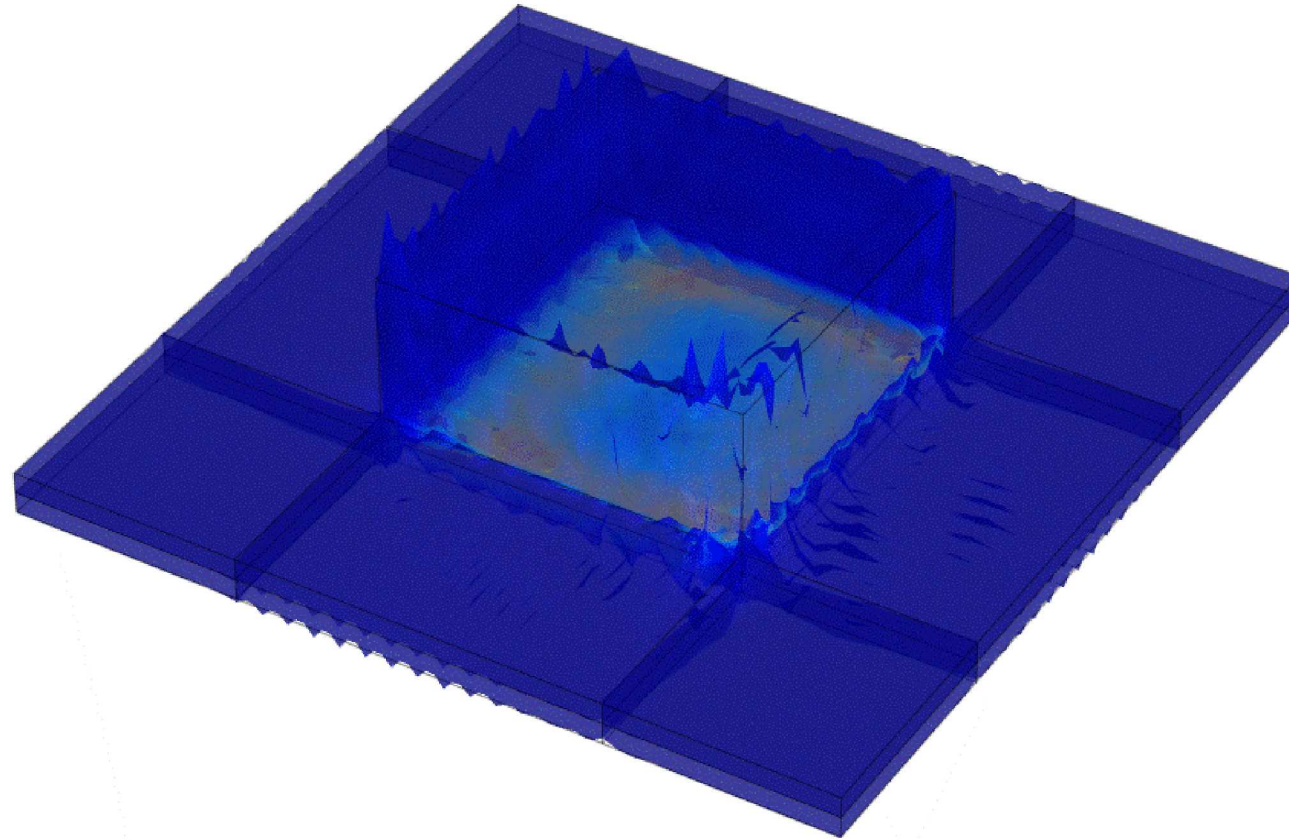
ZnO Shear BAW with Fluid Loading, 28° c-axis tilt



Cross-section through the thickness

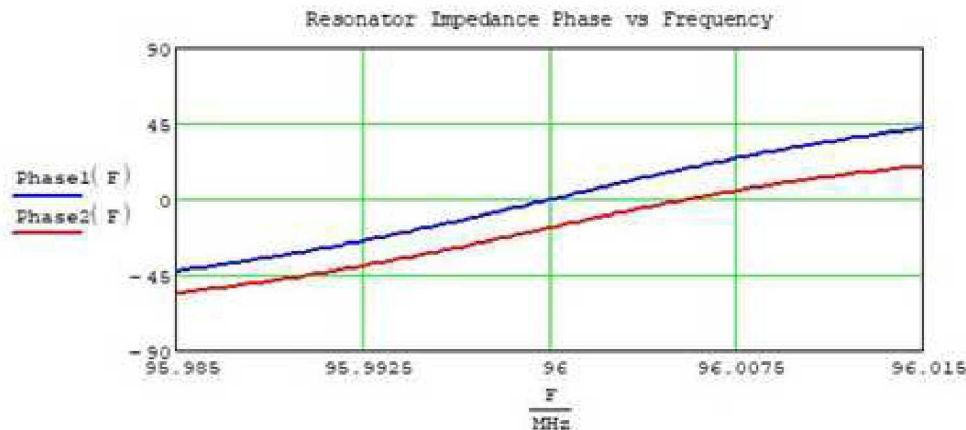
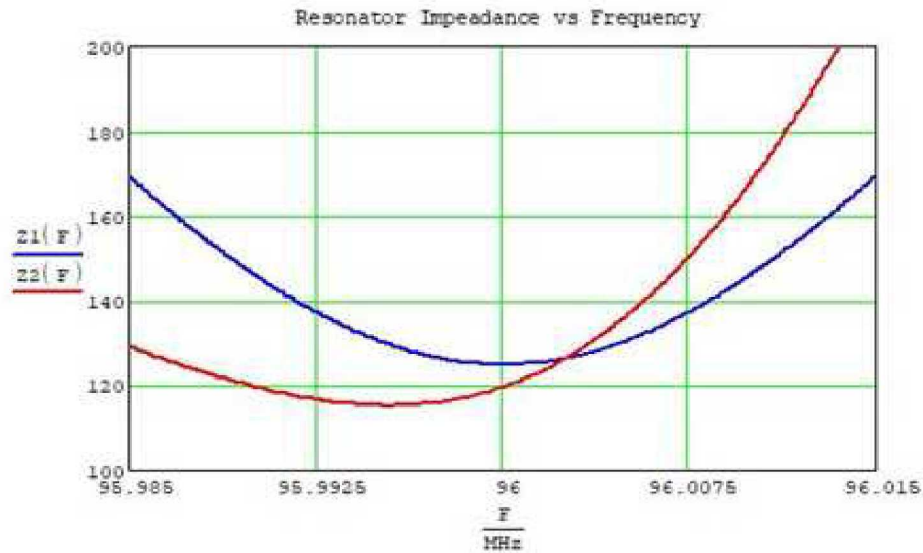


ZnO Shear BAW with Fluid Loading



Specialized Monitoring Circuits for One-Port Resonators

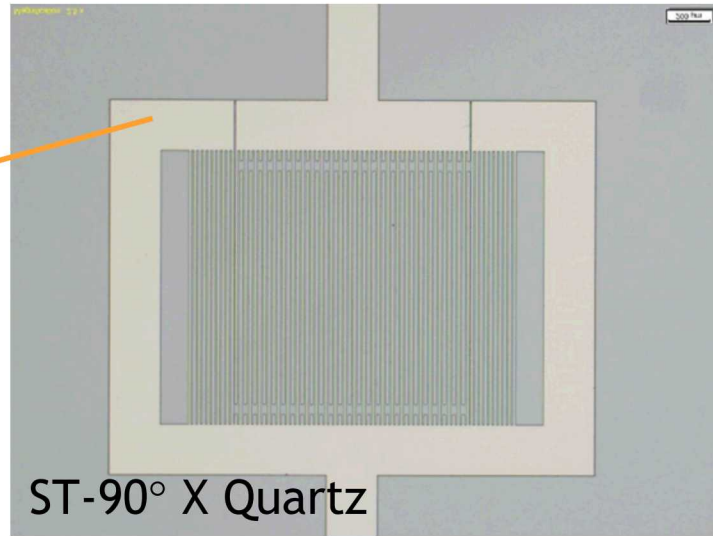
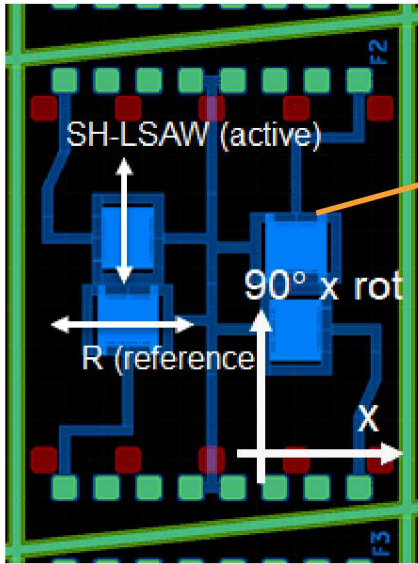
Impact of shunt capacitance C_o on the resonator. Red: Non-zero $C_o = 4\text{pF}$ and Blue: $C_o = 0$.



Key Points:

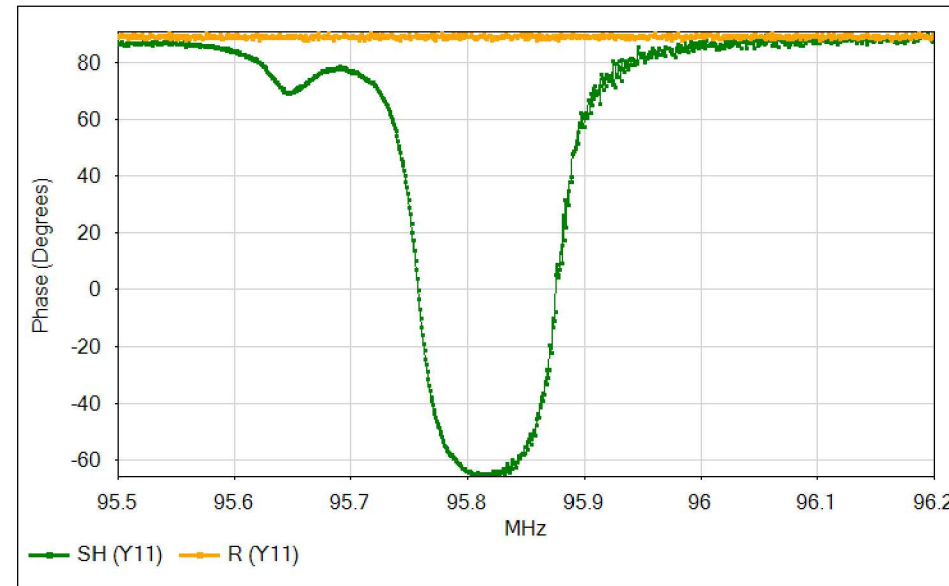
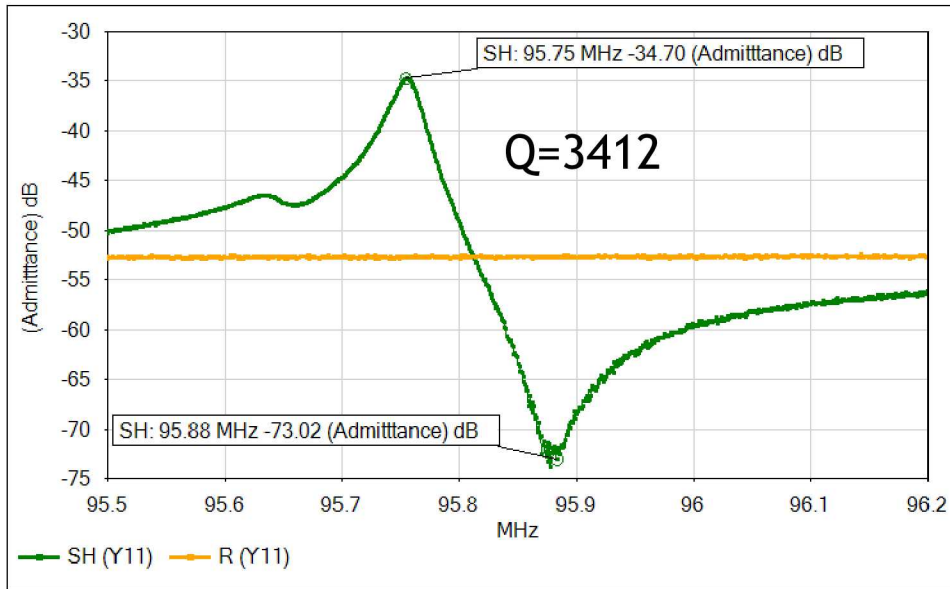
- Non-zero shunt capacitance alters the impedance by shifting the location of the minimum and the zero phase crossing while causing significant distortion.
- The distortion for non-zero shunt capacitance becomes increasingly worse as the resonator loss increases (i.e. low Q).
- To obtain the true F_s and resonator loss it is highly desirable remove the shunt capacitance.
- When $C_o = 0$, the series resonance occurs exactly where the phase crosses zero at the minimum impedance or series resonance.

Specialized Monitoring Circuits for One-Port Resonators, cont'd



- Device has one active resonator and a dummy or reference that has approximately the same shunt capacitance
- The resonators can be SAW, BAW, or MEMs devices

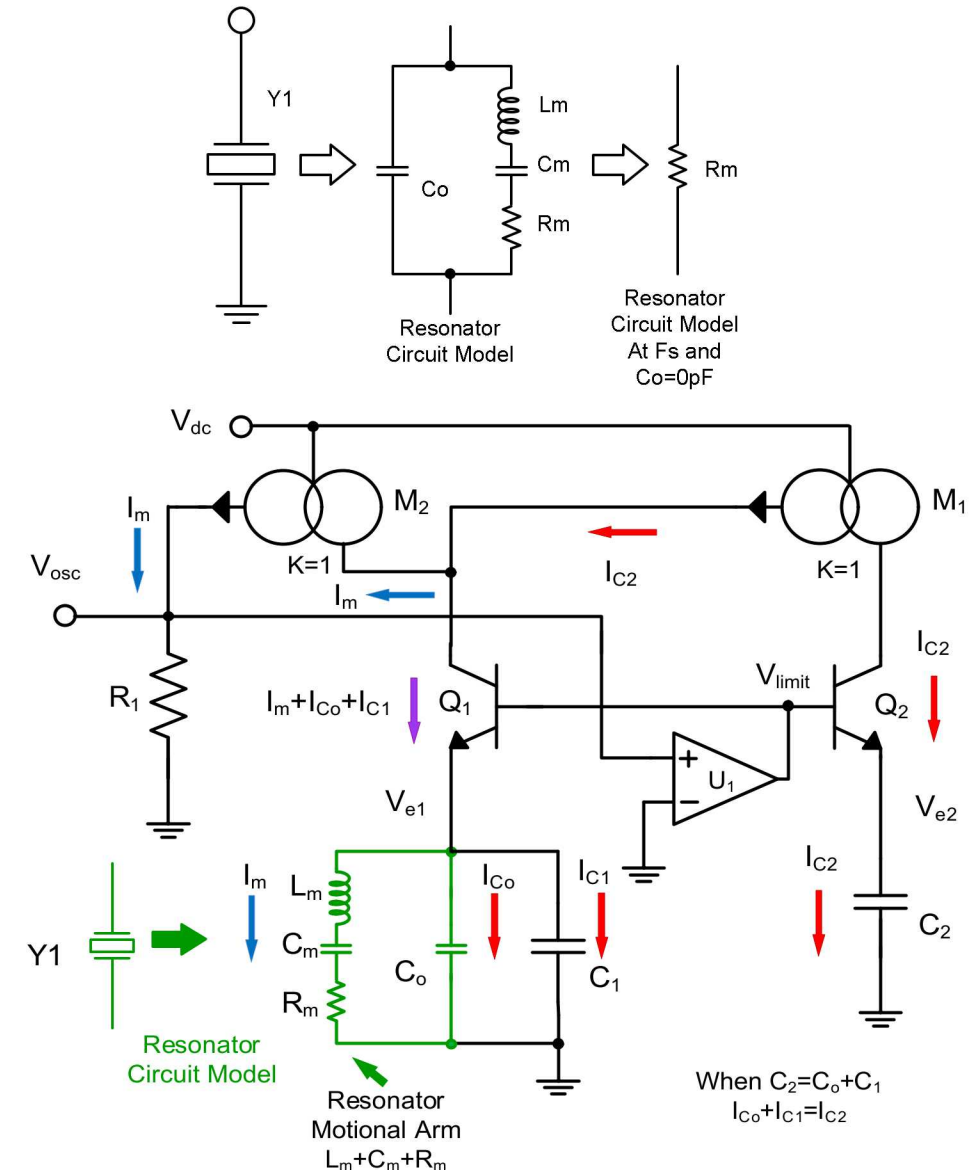
$$Q_s = \frac{F_s}{2} \frac{d\angle Z_{11}}{df} = \frac{2\pi F_s L_m}{R_m}$$



Specialized Monitoring Circuits for One-Port Resonators, cont'd

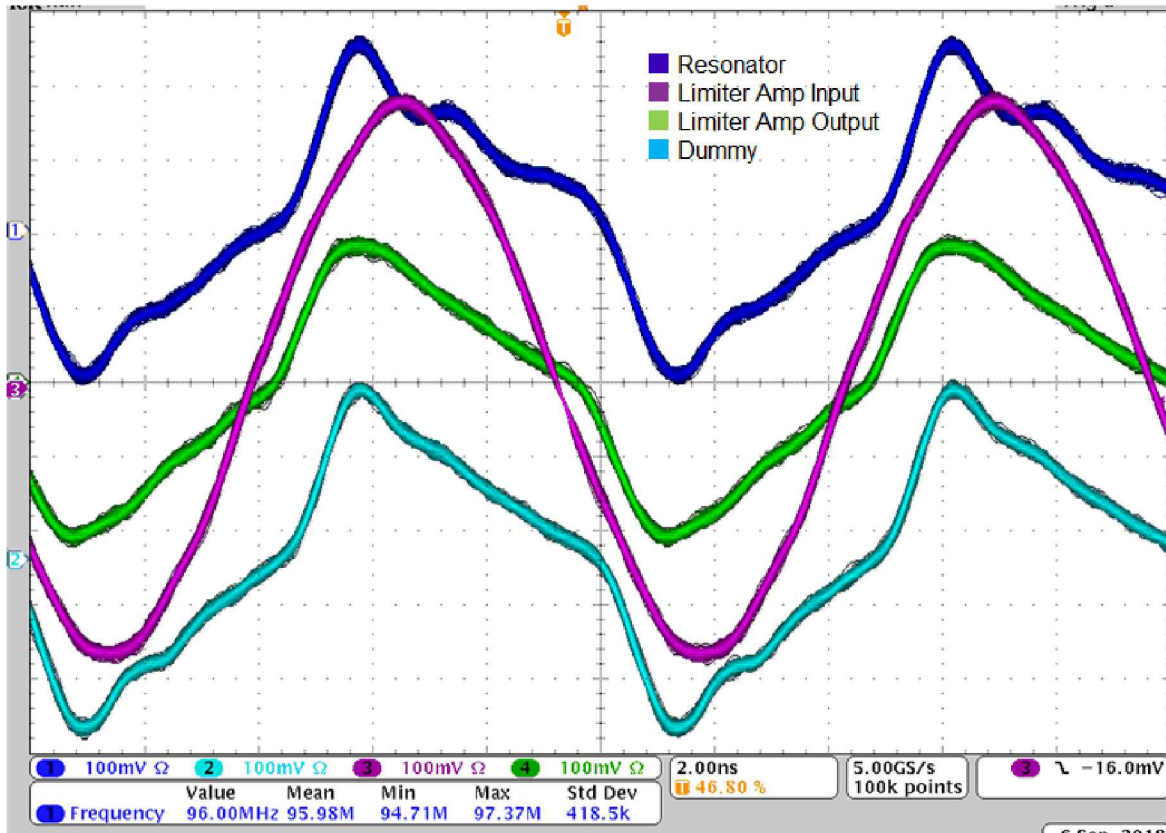
Concept:

- The oscillator effectively removes shunt capacitance across the motional arm of the resonator therefore oscillating very close to the series resonance frequency (F_s).
- The circuit outputs a dc voltage proportional to the amplitude of oscillation, which is also proportional to resonator loss, R_m .
- The circuit uses a limiter amplifier U1 and emitter followers, Q_1 and Q_2 as active ports to convert impedances of the resonator and shunt capacitances $C_o + C_1$ (emitter of Q_1) and a dummy capacitance, C_2 (emitter of Q_2) into currents. The currents are summed via current mirrors $M1$ and $M2$ and converted to a voltage at R_1 . This equivalent circuit functions as an oscillator where the frequency only depends on the motional arm of the resonator circuit. The circuit will oscillate when the loop gain is greater than one and the oscillation frequency occurs where the loop function is real, or 2π radians. This condition only occurs when $XC_m + XL_m = 0$ or when: $F_{osc} = 1 / (2\pi\sqrt{L_m C_m})$.



Specialized Monitoring Circuits for One-Port Resonators, cont'd

Demonstration at 96 MHz



- The oscillator used ST-90° rotated X-cut Quartz resonators, where the active port had $F_s = 96$ MHz and the dummy is a Rayleigh SAW that is out of band compared to the SH mode due to the orthogonal propagation direction.
- To confirm the circuit is removing the shunt capacitance and tracking the true series resonance, capacitance was added to the active and dummy oscillator ports in sequence. Adding a 2.5 pF capacitor to the active or dummy resonator port caused an equal but opposite frequency shift +30 ppm.
- The oscillator output was 96 MHz, where it can operate with a **series resistance from 10 Ω to 2k Ω without impedance matching.**



Thank you!

