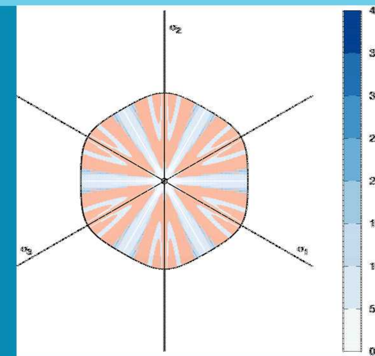
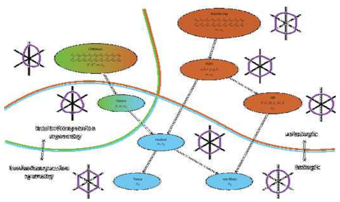
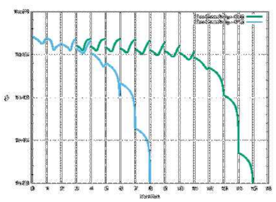
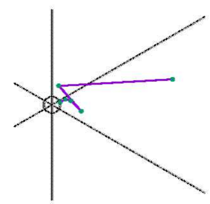
SAND2019-13748C

Implementation and Verification of Continuum Plasticity Models for Modeling and Simulation









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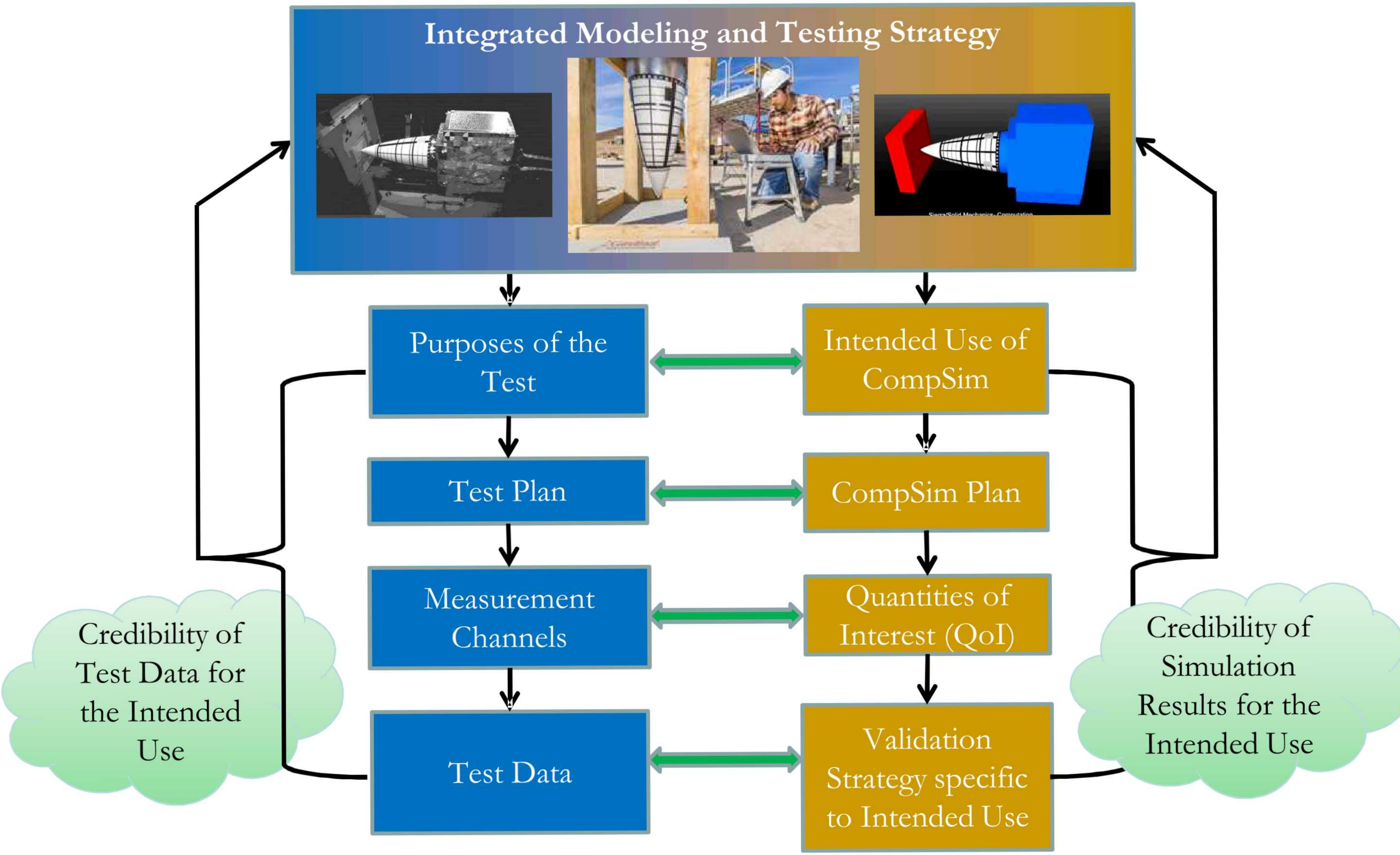
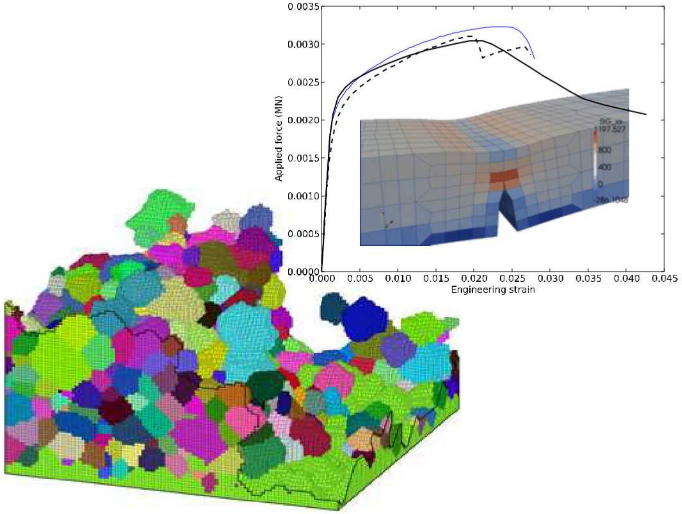
Integrated Modeling and Testing

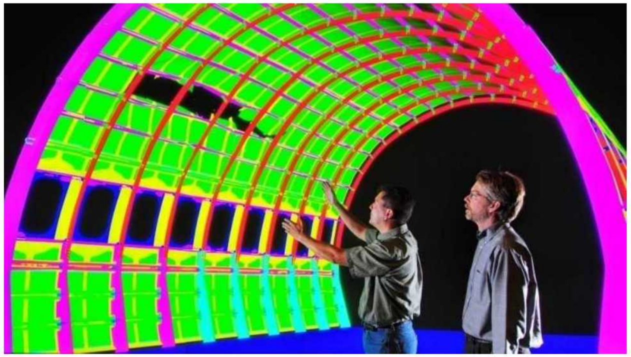


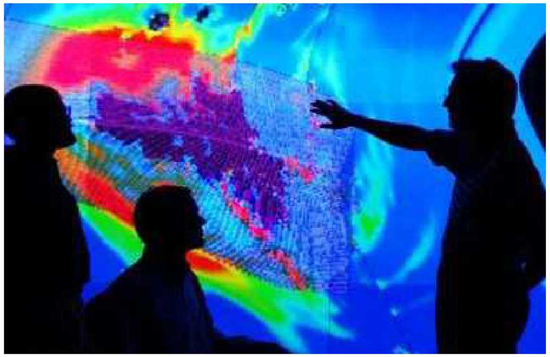
Modeling and Simulation

Enormous progress in computational mechanics over the past 3 decades.

- Computer architectures
- Geometric details
- Physics in computational models
- Scalable algorithms
- Multiphysics simulation codes



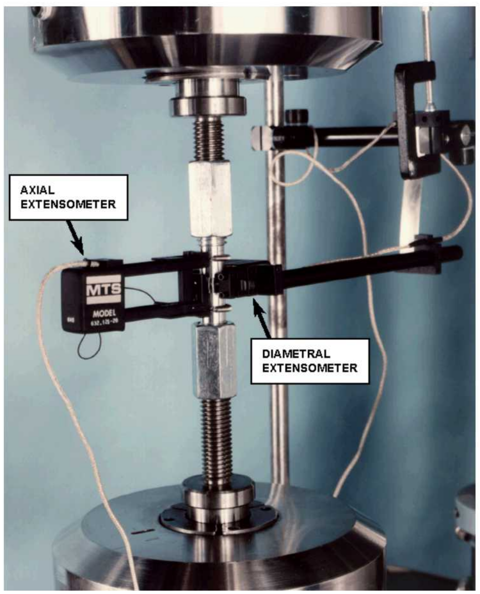




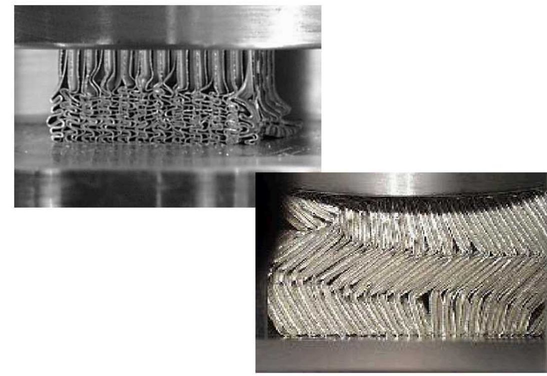
Solving previously intractable problems

Material Behavior

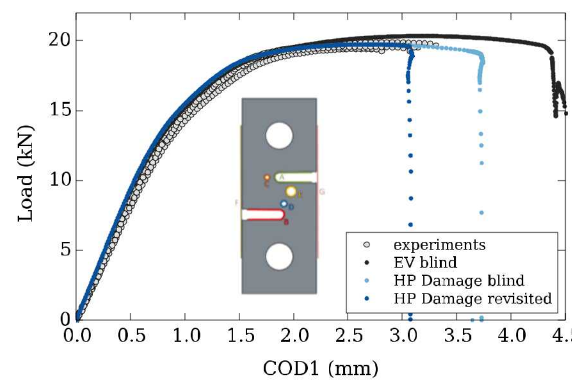
- Understanding and modeling material behavior is at the core of solid mechanics simulations



Courtesy of B. Antoun, Sandia National Laboratories



Courtesy of W. Y. Lu, Sandia National Laboratories



Karlson et al., 2016, *Int. Jrnl. Frac.*, 198: 179-195

Motivation

- The implementation of a model ***should*** *be an accurate representation* of its mathematical form
- This is assumed – implicitly – by any user of the model

Example: Johnson-Cook plasticity model *

$$\sigma = \lfloor A + B\epsilon^n \rfloor \lfloor 1 + C \ln \dot{\epsilon}^* \rfloor \lfloor 1 - T^{*\,m} \rfloor$$ flow stress

ϵ equivalent plastic strain

$\dot{\epsilon}^* = \dot{\epsilon}/\dot{\epsilon}_0$ dimensionless plastic strain rate

$$\sigma = \lfloor A + B\epsilon^n \rfloor \lfloor 1 + C \ln \dot{\epsilon}^* \rfloor$$

ignore temperature

2024-T351 Aluminum

$A = 265$ MPa
$B = 426$ MPa
$n = 0.34$
$C = 0.015$

* G. R. Johnson and W. H. Cook, "A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures," *Proc. 7th Int. Symp. On Ballistics,* pp. 541-547, 1983.

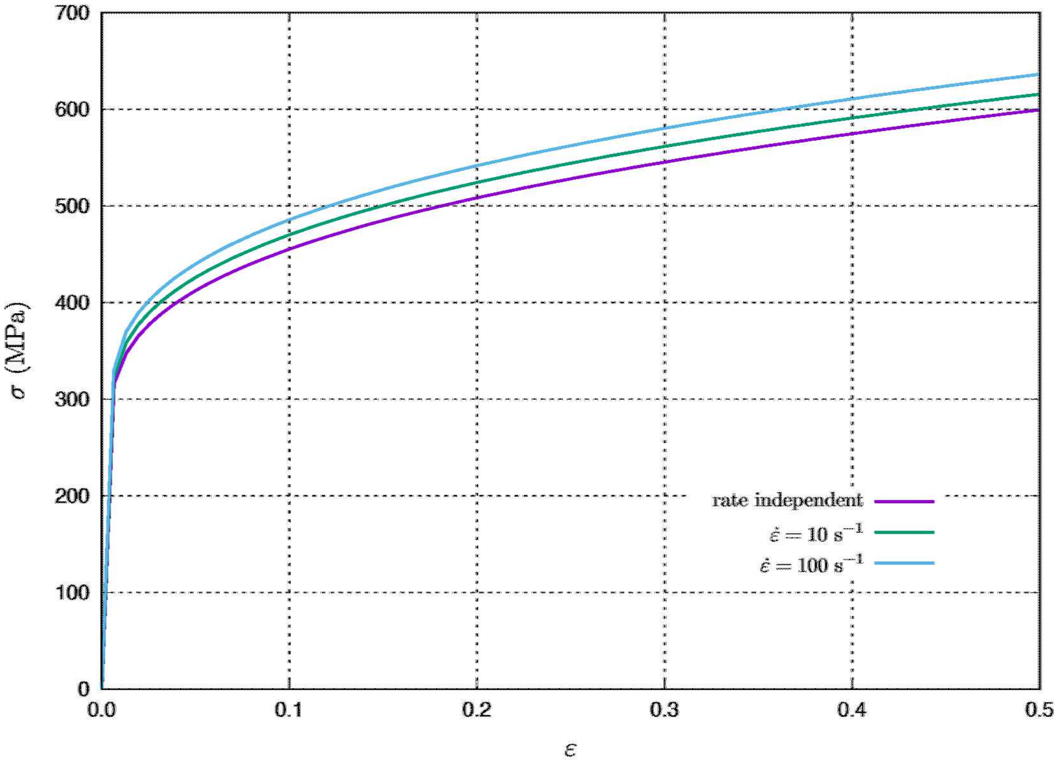
Motivation

Johnson-Cook plasticity model

$$\sigma = [A + B\epsilon^n]\,[1 + C \ln \dot{\epsilon}^*]$$

2024-T351 Aluminum

$$A = 265 \text{ MPa}$$
$$B = 426 \text{ MPa}$$
$$n = 0.34$$
$$C = 0.015$$



…is this correct?

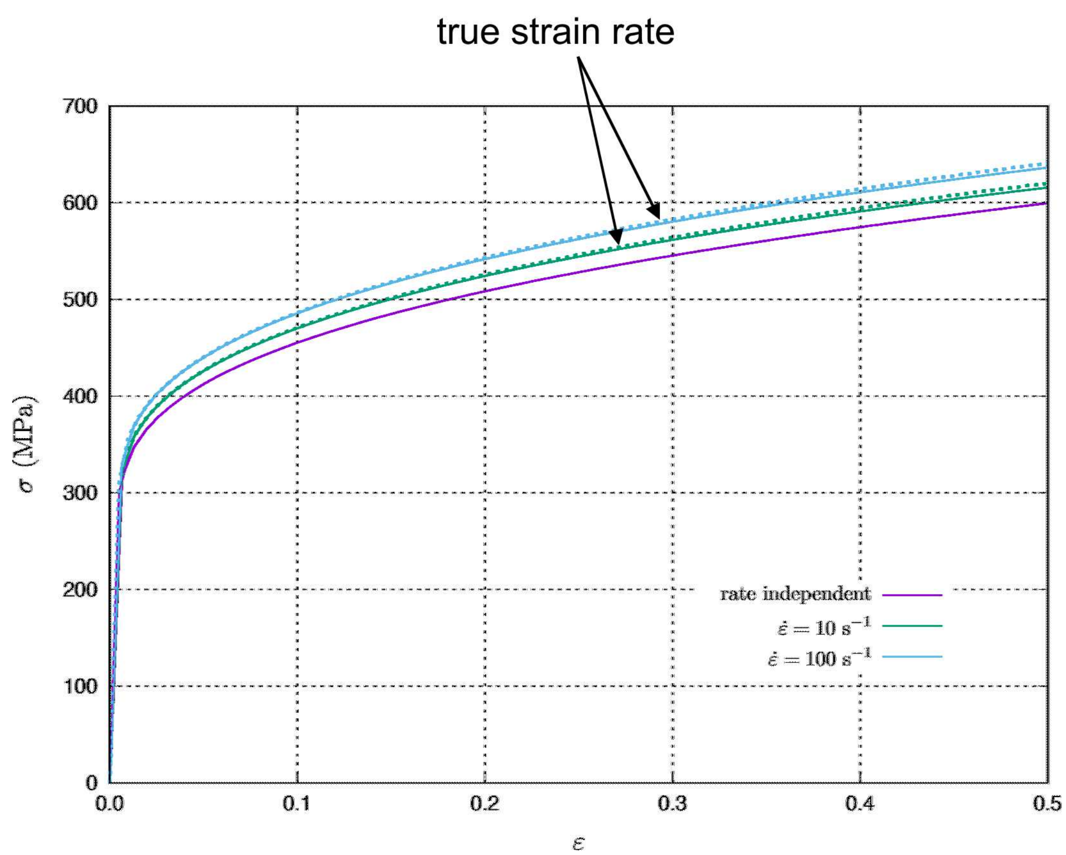
Motivation

Engineering strain rate

$$u_x(t) = L\,\dot{\varepsilon}\,t$$

True strain rate

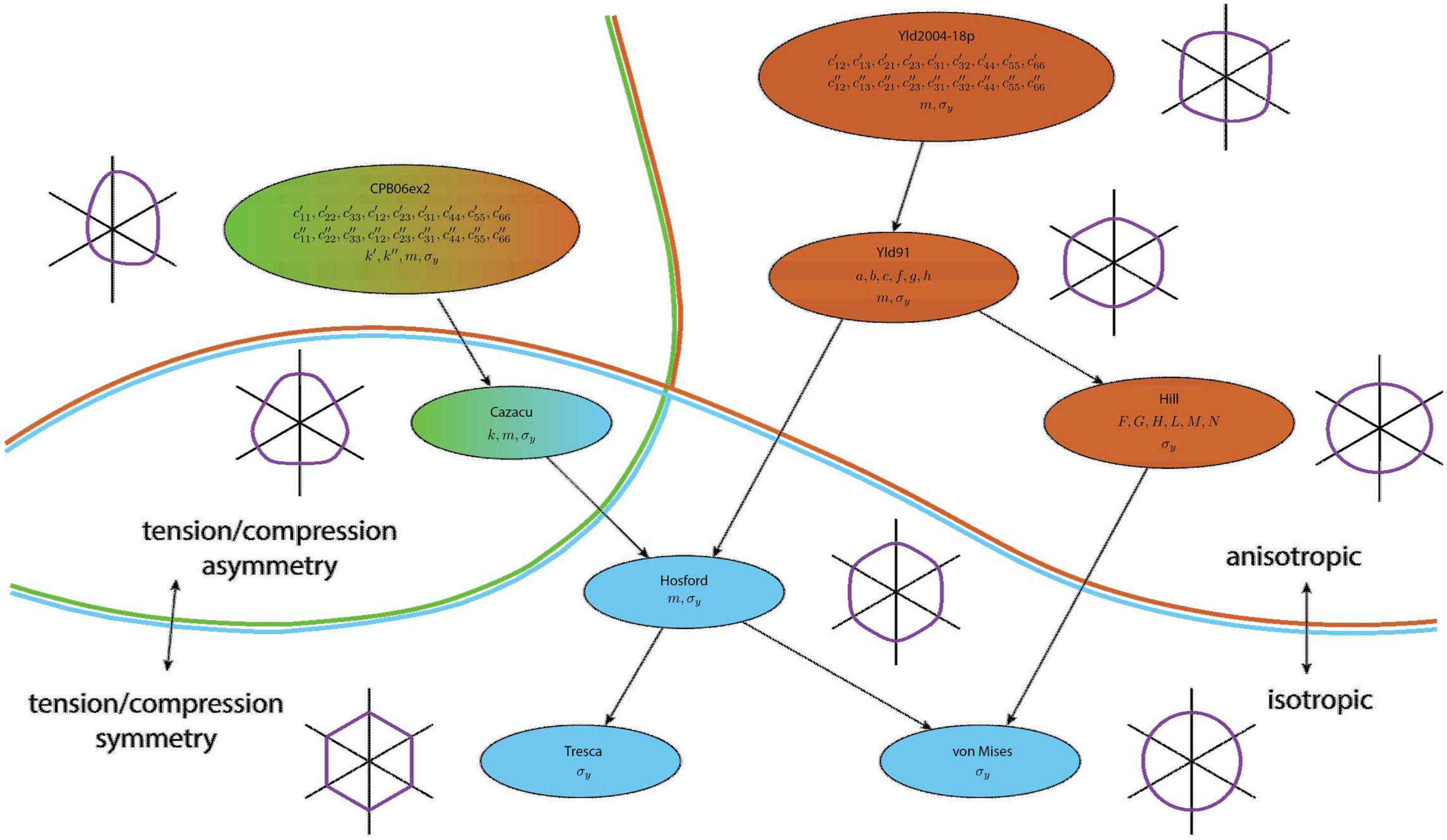
$$u_x(t) = \left[\exp(\dot{\varepsilon}\,t) - 1\right] L$$



…is this correct?

Motivation

A family of yield surfaces implemented in Sierra/SolidMechanics provides the basis for a flexible and **reliable** family of plasticity models



Outline

- Yield Surface Models
- Flow Stress Models
- Model Verification
- Performance
- Conclusions

Yield Surface Models

Yield Surfaces

Mathematical description

$$f(\boldsymbol{\sigma}, \bar{\varepsilon}^p) = \phi(\boldsymbol{\sigma}) - \bar{\sigma}(\bar{\varepsilon}^p) = 0$$

effective stress

shape of the yield surface

flow stress

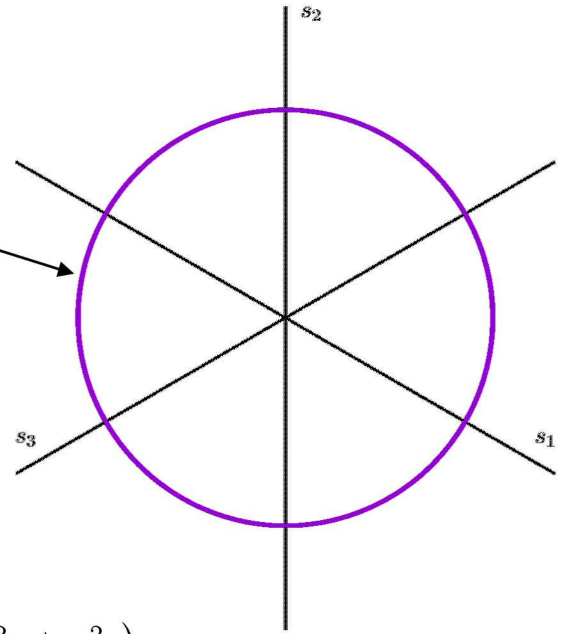
size of the yield surface

- Effective Stress Models
  - Isotropic
    - Von Mises
    - Hosford
      - Tresca
  - Anisotropic
    - Hill
    - Yld91
    - Yld2004-18p
  - Tension-Compression Asymmetry
    - Cazacu
    - CPB06

von Mises Model

Expressions for the effective stress

$$f(\boldsymbol{\sigma}, \bar{\varepsilon}^p) = \phi(\boldsymbol{\sigma}) - \bar{\sigma}(\bar{\varepsilon}^p) = 0$$



$$\phi(\boldsymbol{\sigma}) = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}}$$

$$\phi(\boldsymbol{\sigma}) = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

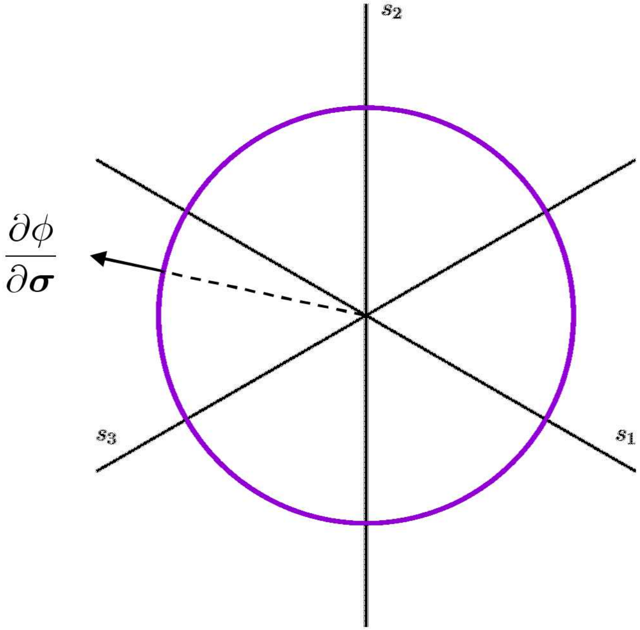
$$\phi(\boldsymbol{\sigma}) = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\sigma_{yy} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} + 3\left(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2\right)}$$

Associated flow

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\bar{\varepsilon}}^p \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \quad \rightarrow \quad \dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\bar{\varepsilon}}^p \frac{\partial \phi}{\partial \boldsymbol{\sigma}}\right)$$

Integration

$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\bar{\varepsilon}}^p \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right)$$



Backward Euler

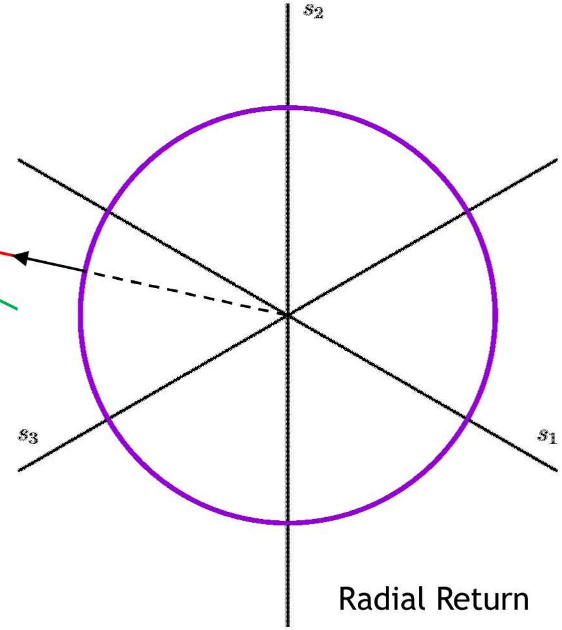
- Closest point projection

plastic corrector

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{\mathrm{tr}} - \mathbb{C} : \Delta\bar{\varepsilon}^p$$

$$\boldsymbol{\sigma}^{\mathrm{tr}} = \boldsymbol{\sigma}_n + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$

elastic predictor or
trial stress



Radial Return

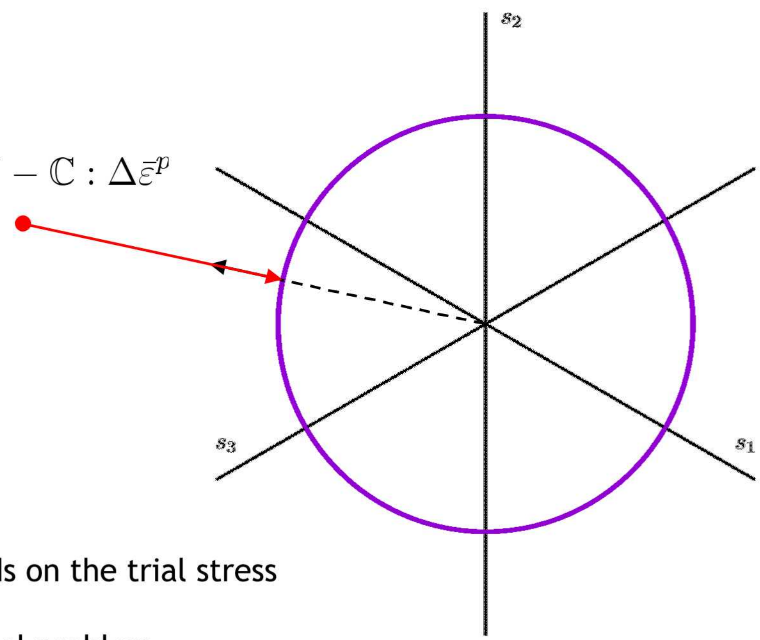
von Mises Model

Integration

$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\bar{\varepsilon}}^p \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right) \longrightarrow \Delta \boldsymbol{\sigma} = \int_{\Delta t} \dot{\boldsymbol{\sigma}} \, dt = \int_{\Delta t} \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\bar{\varepsilon}}^p \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right) dt$$

closest point projection

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{\mathrm{tr}} - \mathbb{C} : \Delta \bar{\varepsilon}^p$$



- only depends on the trial stress
- 1 dimensional problem

iterative solution

$$\bar{\varepsilon}^p_{(k+1)} = \bar{\varepsilon}^p_{(k)} + \Delta \bar{\varepsilon}^p_{(k)}$$

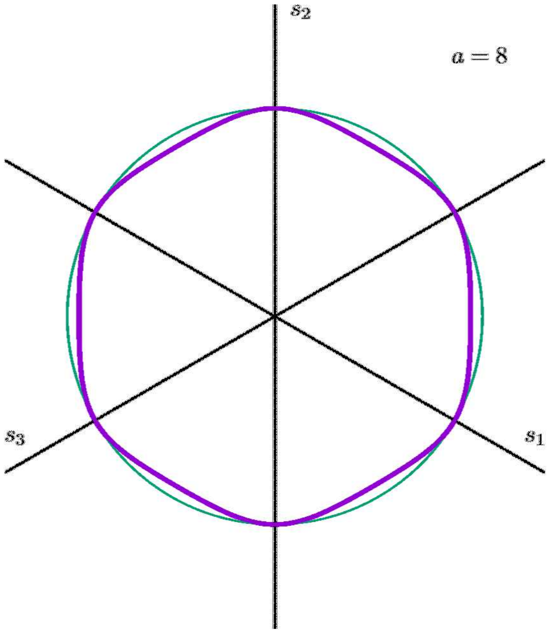
$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} - 3\mu \Delta \bar{\varepsilon}^p_{(k)} \frac{\mathbf{s}_{(k)}}{\phi(\boldsymbol{\sigma}^{(k)})}$$

$$\Delta \bar{\varepsilon}^p_{(k)} = \frac{\phi(\boldsymbol{\sigma}^{(k)}) - \bar{\sigma}(\bar{\varepsilon}^p_{(k)})}{3\mu + H'_{(k)}}$$

Hosford Model *

Effective stress

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{2} \left[|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a \right] \right\}^{1/a}$$



the exponent affects:

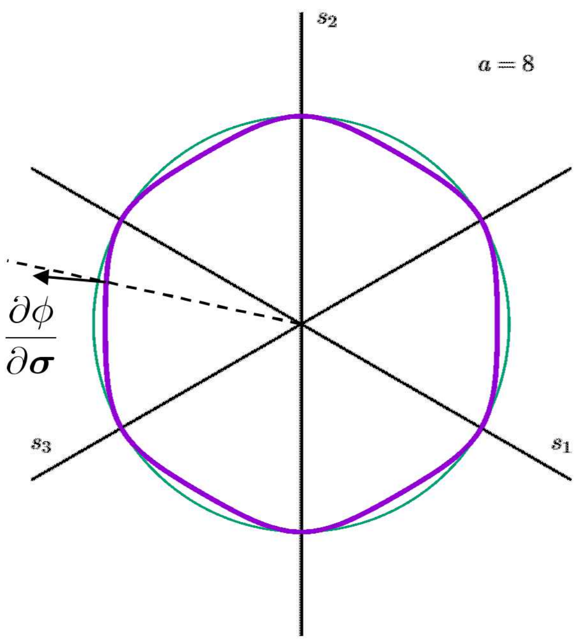
- where the material yields
- the plastic flow direction
- the curvature of the yield surface

* W. F. Hosford, *J. Appl. Mech.*, **39** (1972) 607-609

Hosford Model

Integration

$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\bar{\varepsilon}}^p \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right)$$



Backward Euler

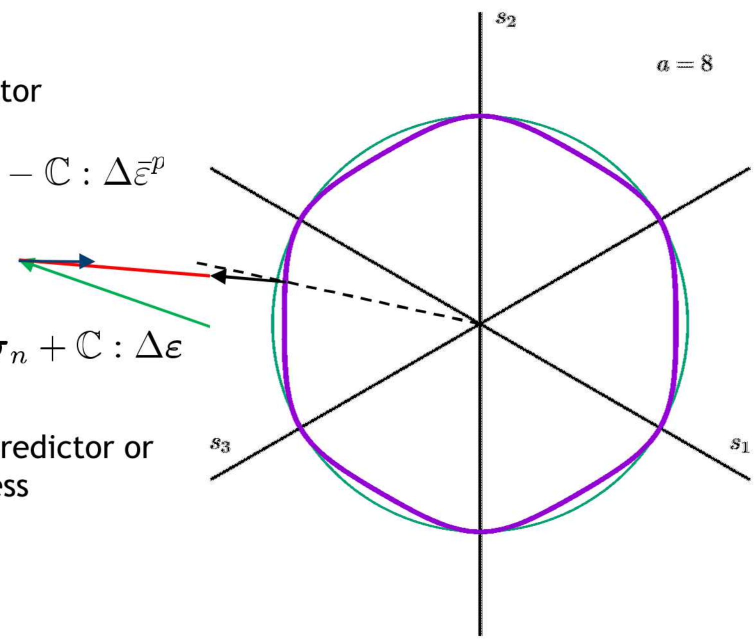
- Closest point projection

plastic corrector

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{\text{tr}} - \mathbb{C} : \Delta\bar{\varepsilon}^p$$

$$\boldsymbol{\sigma}^{\text{tr}} = \boldsymbol{\sigma}_n + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$

elastic predictor or
trial stress



Return Mapping Algorithm

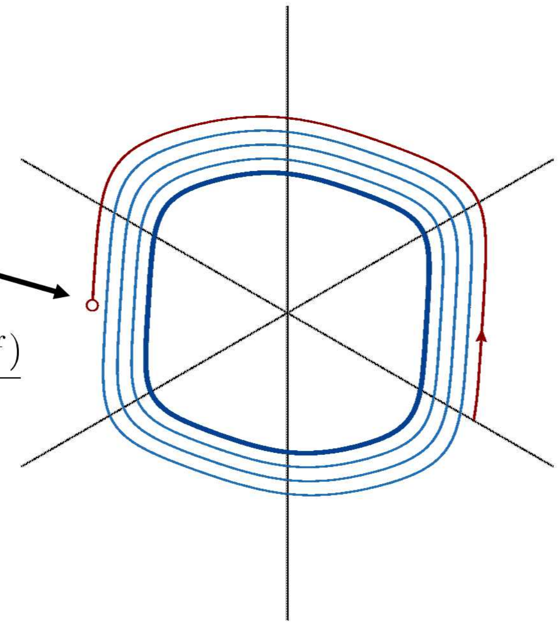
Return mapping algorithm (RMA) uses an augmented Newton-Raphson with a line search *

- How do we test it?

RMA only depends on trial stress state (we don't care how we got it…)

For every trial stress state, record the number of iterations required for convergence

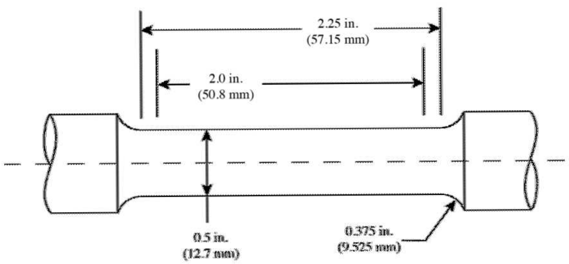
$$R^{\mathrm{tr}} = \frac{\phi(\boldsymbol{\sigma}^{\mathrm{tr}})}{\bar{\sigma}}$$

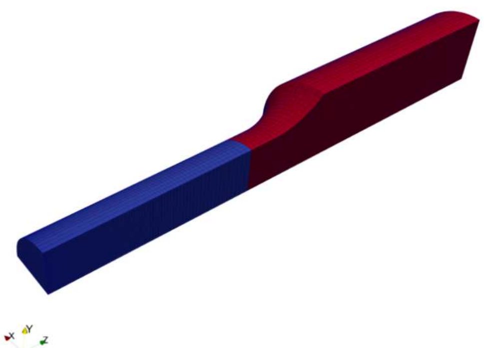


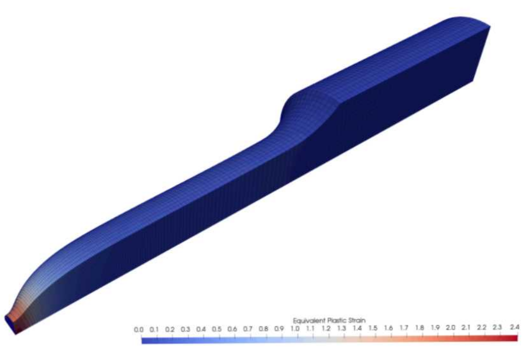
* A. Perez-Foguet and F. Armero, *Int. J. Num. Meth. Eng.*, **52** (2002) 331-374, W. Scherzinger, *Comp. Meth. Appl. Mech. Eng.*, **317** (2017), 526-553.

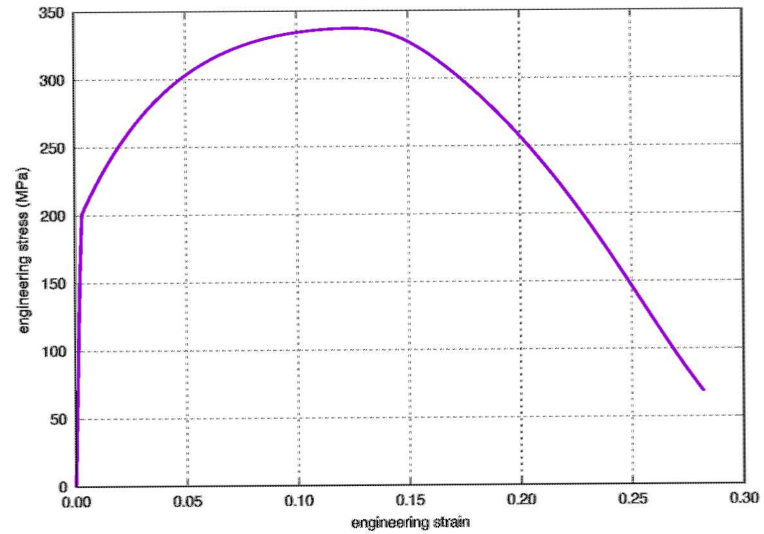
Return Mapping Algorithm – Testing

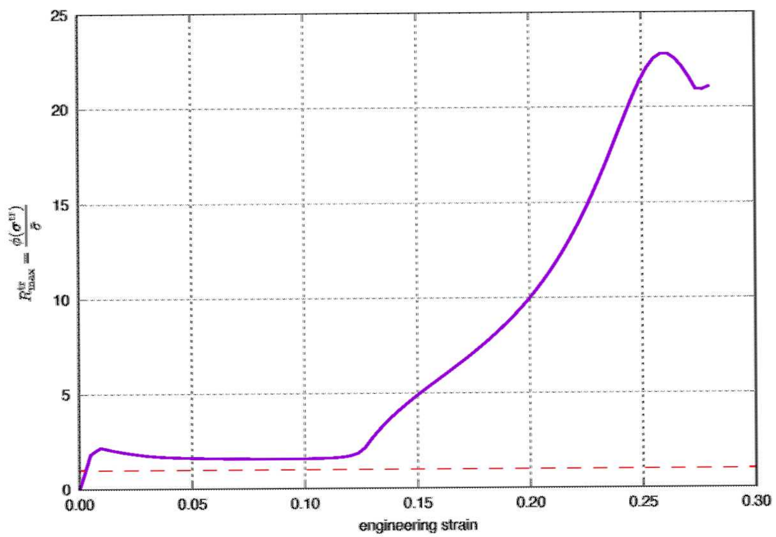
ASTM E8 specimen







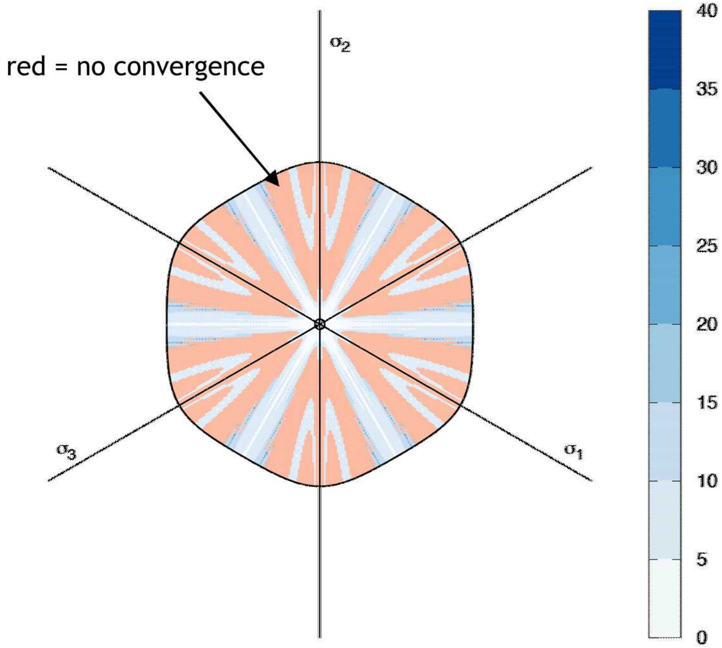




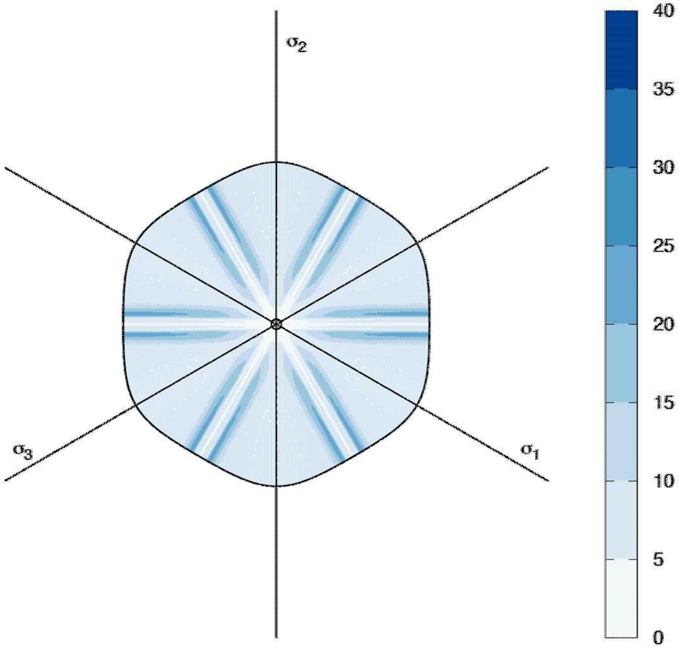
Return Mapping Algorithm – Testing

Compare Newton-Raphson with and without line search

$$R^{\text{tr}}_{\max} = 30$$



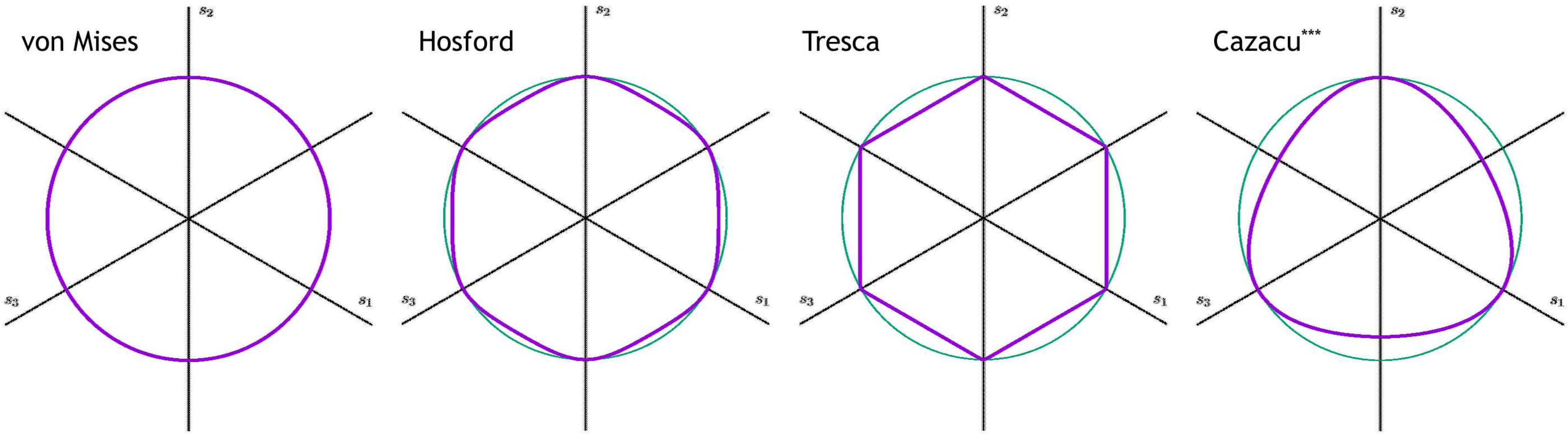
Newton-Raphson



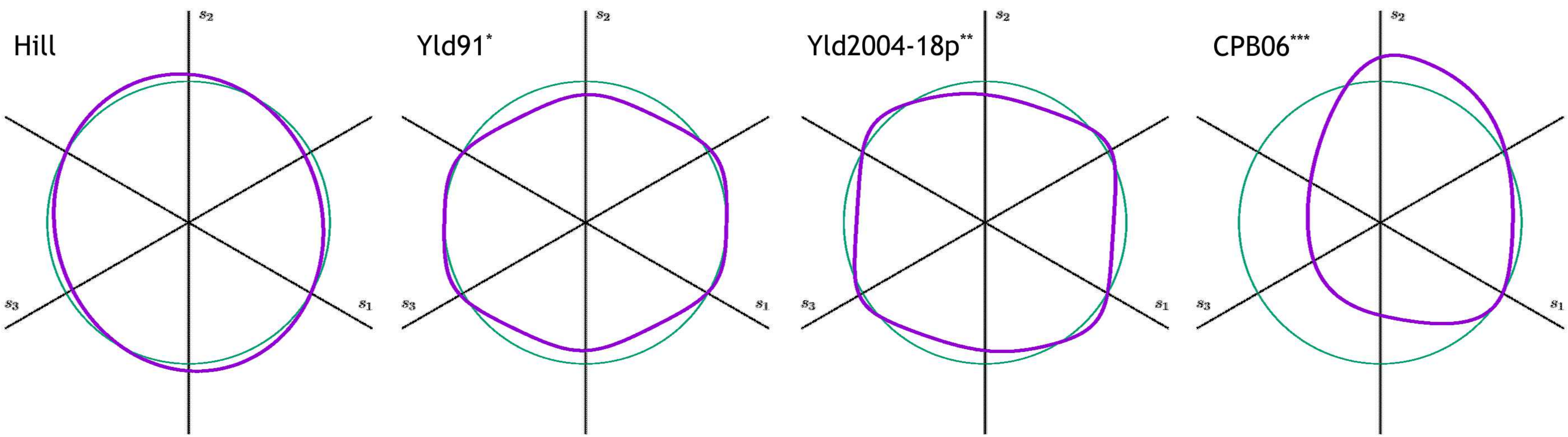
Line Search

Yield Surface Models

isotropic



orthotropic

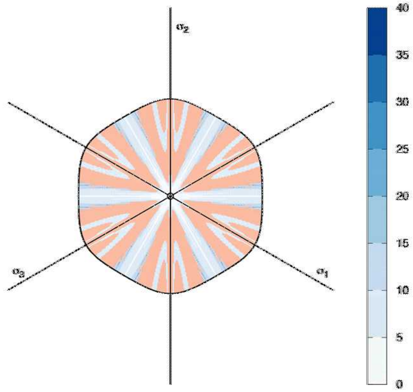


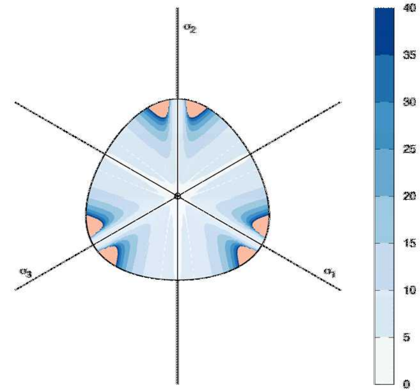
* F. Barlat et. al., *Int. J. Plast.*, **7** (1991) 693-712, ** F. Barlat et. al., *Int. J. Plast.*, **19** (2005) 1009-1039, *** B. Plunkett et. al., *Int. J. Plast.*, **24** (2008) 847-866

Performance of Yield Surface Models – Newton-Raphson

Isotropic

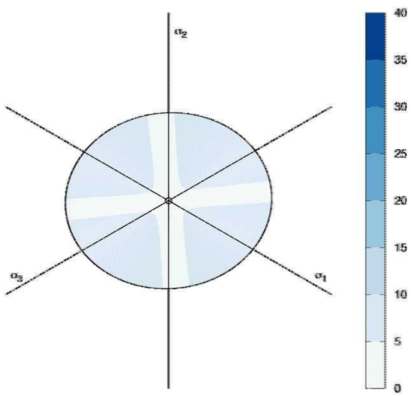
Hosford



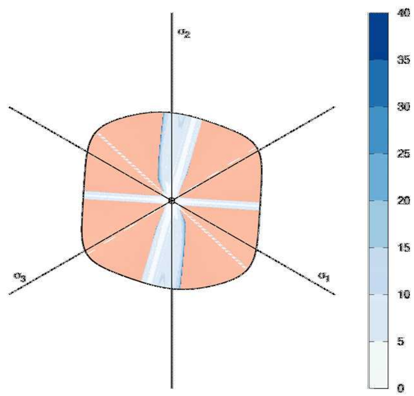


Cazacu, et. al.**

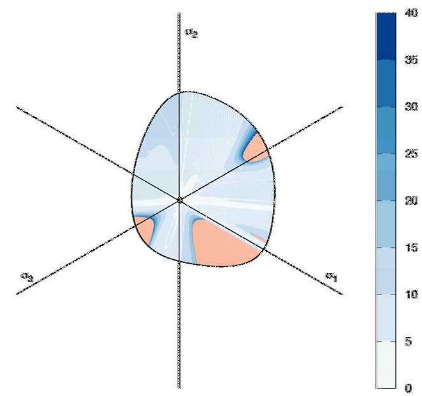
Anisotropic



Hill



Yld2004-18p*



CPB06ex2**

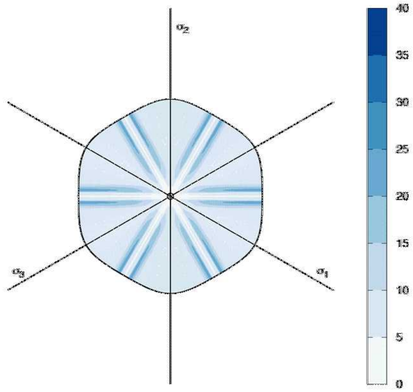
* F. Barlat et. al., *Int. J. Plast.*, **19** (2005) 1009-1039

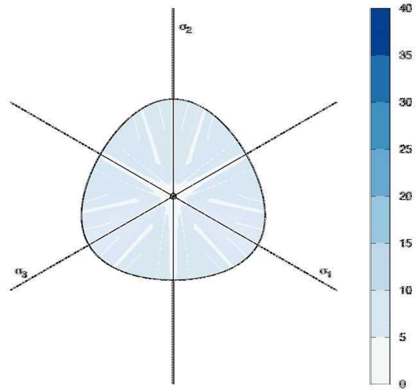
** B. Plunkett et. al., *Int. J. Plast.*, **24** (2008) 847-866

Performance of Yield Surface Models – Line Search

Isotropic

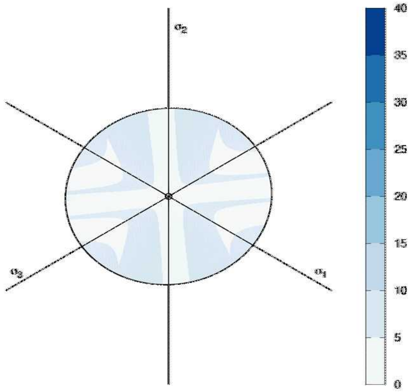
Hosford

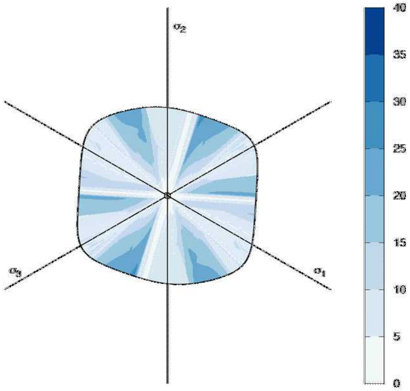


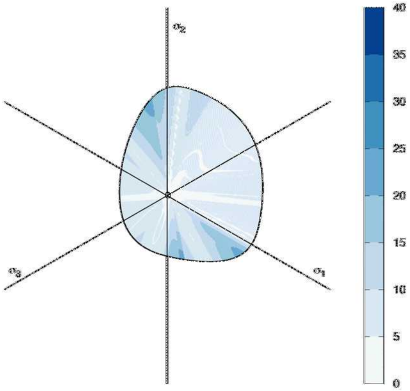


Cazacu**

Anisotropic







Hill

Yld2004-18p*

CPB06ex2**

* F. Barlat et. al., *Int. J. Plast.*, **19** (2005) 1009-1039

** B. Plunkett et. al., *Int. J. Plast.*, **24** (2008) 847-866

Flow Stress Models

Flow Stress Models

$$f(\boldsymbol{\sigma}, \bar{\varepsilon}^p) = \phi(\boldsymbol{\sigma}) - \bar{\sigma}(\bar{\varepsilon}^p) = 0$$

Rate independent hardening models

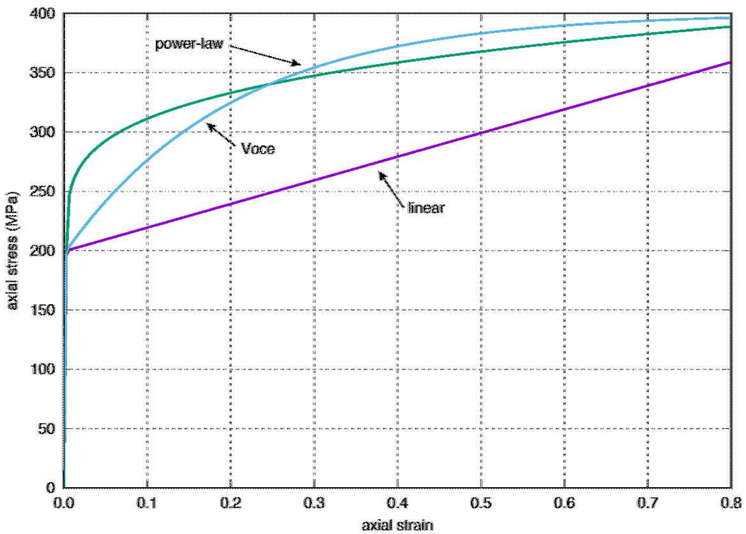
linear $\bar{\sigma} = \sigma_y + H\bar{\varepsilon}^p$

power-law $\bar{\sigma} = \sigma_y + A\left(\bar{\varepsilon}^p\right)^n$

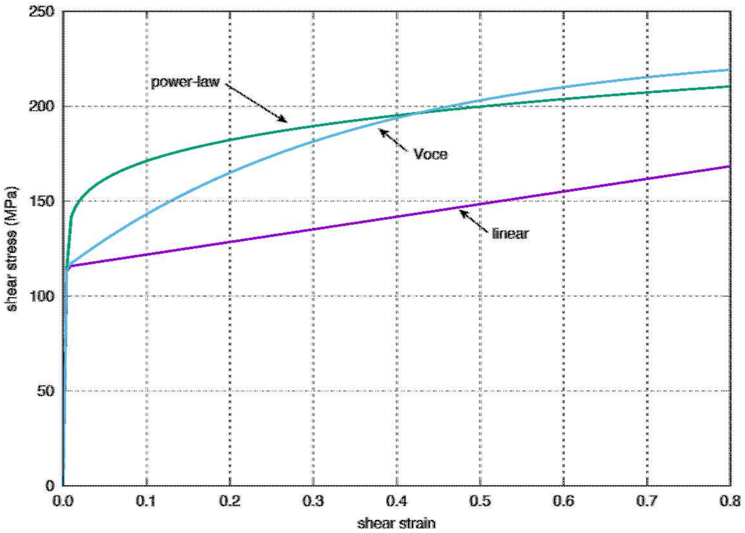
Voce $\bar{\sigma} = \sigma_y + A\left(1 - \exp(-b\bar{\varepsilon}^p)\right)$

user defined $\bar{\sigma} = \sigma_y + h(\bar{\varepsilon}^p)$

axial response



shear response



* von Mises model

Flow Stress Models – Rate Dependence

$$f(\boldsymbol{\sigma}, \bar{\varepsilon}^p) = \phi(\boldsymbol{\sigma}) - \bar{\sigma}(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p) = 0$$

$$\downarrow$$

$$\bar{\sigma}(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p) = H(\bar{\varepsilon}^p) g(\dot{\bar{\varepsilon}}^p)$$

Two choices for rate multiplier:

Johnson-Cook

$$g(\dot{\bar{\varepsilon}}^p) = \begin{cases} 1 + C \ln\left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0}\right) & \text{if} \quad \dot{\bar{\varepsilon}}^p > \dot{\varepsilon}_0 \\ 1 & \text{if} \quad \dot{\bar{\varepsilon}}^p \le \dot{\varepsilon}_0 \end{cases}$$

power-law breakdown

$$g(\dot{\bar{\varepsilon}}^p) = 1 + \sinh^{-1}\left[\left(\frac{\dot{\bar{\varepsilon}}^p}{g}\right)^{1/m}\right]$$

Model Verification

Verification

"The process of verification assesses the fidelity of the computational model to the mathematical model." *

Four approaches

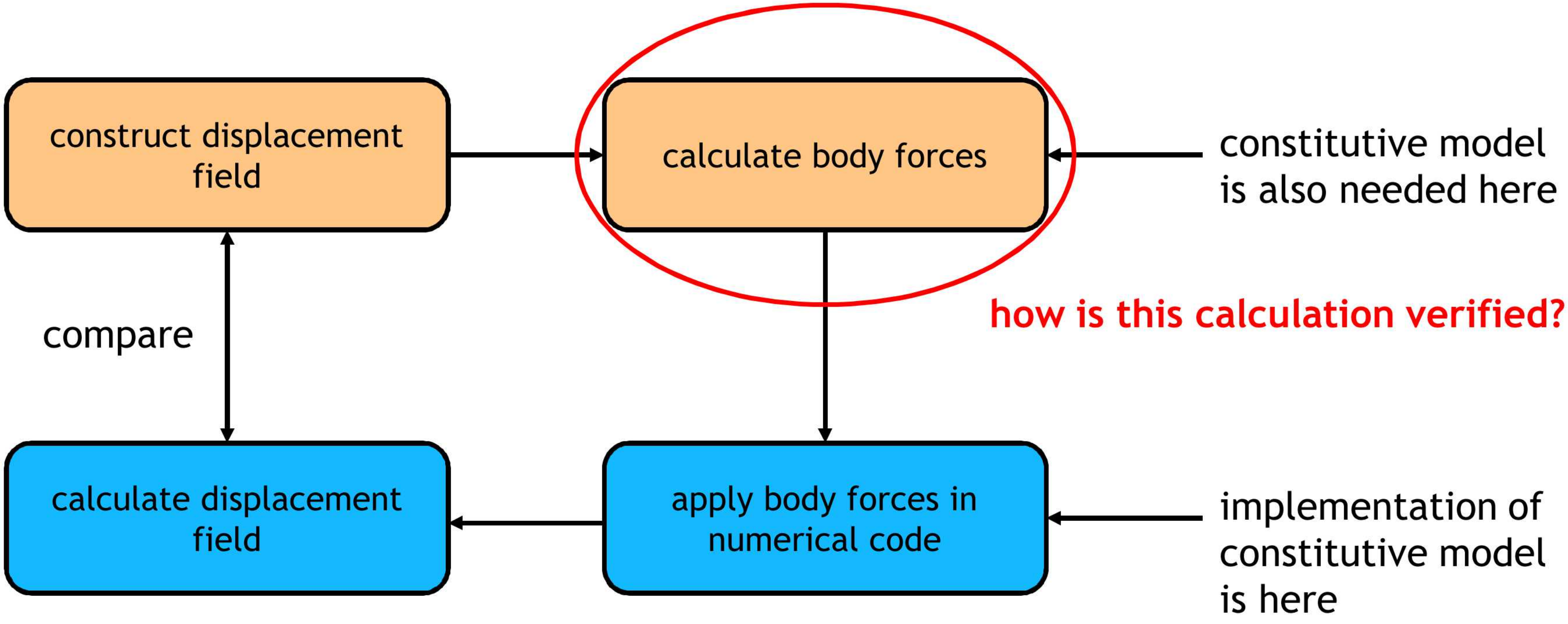
- *Analytical Solutions* – difficult to find
- *Method of Manufactured Solutions* – forcing function depends on material model
- *Numerical Benchmark Solutions* – semi-analytical, code-to-code
- *Consistency Tests* – "complementary to the other types of algorithm tests"

"With the ever-increasing complexity in CSM [computational solid mechanics] models, ***especially constitutive models***, the task of verification becomes more difficult because of a lack of relevant analytical solutions." *

* Guide for Verification and Validation in Computational Solid Mechanics - ASME V&V 10-2006 (reaffirmed 2016)

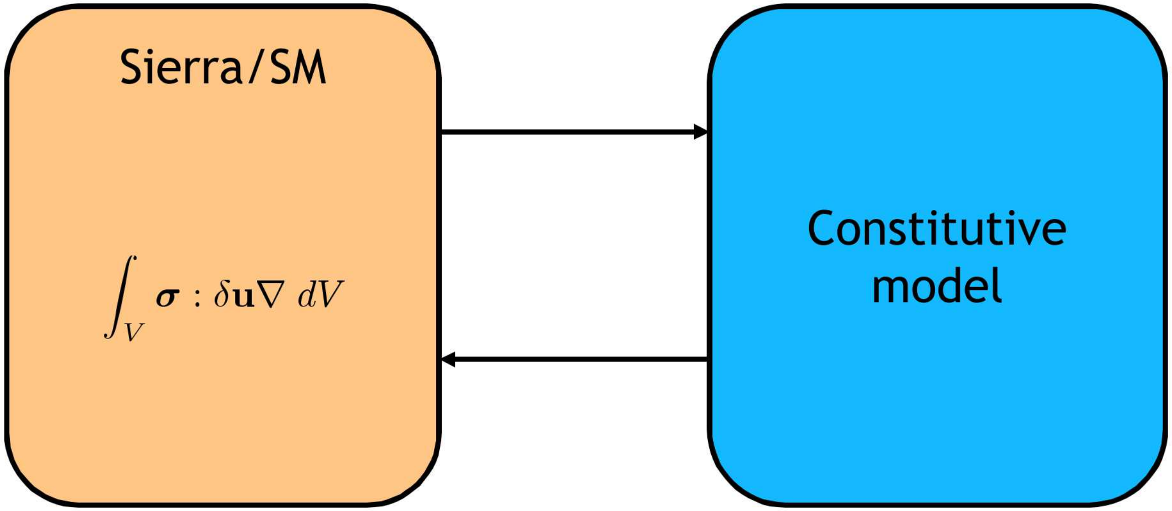
Verification

- Method of Manufactured Solutions (MMS)
  - Standard and effective method for verification of solid mechanics codes
  - Difficult to use for nonlinear, path dependent material models



- Material Point Driver (MPD)
  - Code that exercises **only** the material model

$$\sigma = f(\varepsilon) \qquad \dot{\sigma} = f(\dot{\varepsilon})$$



Verification

- Use a Sierra/SolidMechanics as MPD

  - Requires knowledge of
    - Constitutive model behavior
    - Finite deformation kinematics
    - Implementation in the code

  - Tasks
    - Find a solution you can quantify
    - Carefully construct boundary/initial conditions
    - **Document and peer review**

Verification – Analytical Solutions

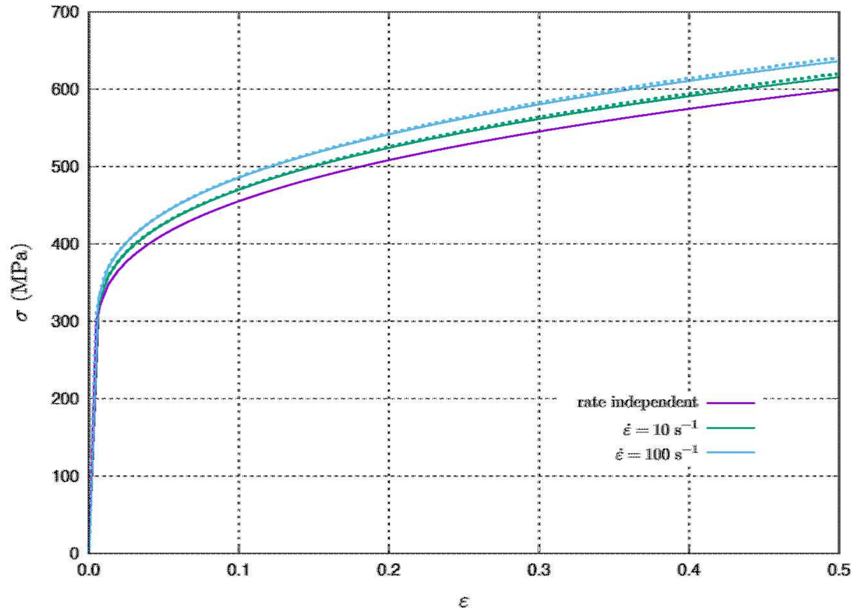
Derive stress/strain paths to get the "correct" result

Strain paths

- Uniaxial strain
- Simple shear
- ***Pure shear***

Stress paths

- ***Uniaxial stress***
- ***Pure shear***
- Biaxial stress

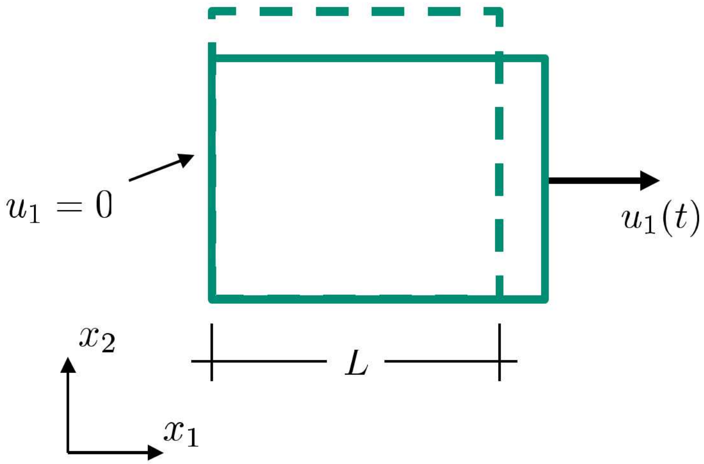


$$\bar{\sigma} = \left[\sigma_y + A\left(1 - \exp(-b\bar{\varepsilon}^p)\right)\right] g(\dot{\bar{\varepsilon}}^p)$$

$$g(\dot{\bar{\varepsilon}}^p) = \begin{cases} 1 + C \ln\left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0}\right) & \text{if} \quad \dot{\bar{\varepsilon}}^p > \dot{\varepsilon}_0 \\ 1 & \text{if} \quad \dot{\bar{\varepsilon}}^p \leq \dot{\varepsilon}_0 \end{cases}$$

Uniaxial Stress

$$\sigma_{11} \neq 0 \quad ; \quad \sigma_{ij} = 0 \text{ otherwise}$$



What is the displacement history?

Use the deformation gradient

$$\mathbf{u}(t) = (\mathbf{F}(t) - \mathbf{I}) \cdot \mathbf{X}$$

For uniaxial deformation this becomes

$$u_1(t) = \left[\exp\left(\varepsilon_{11}(t)\right) - 1\right] L$$

What is the strain history?

Uniaxial Stress

Additive decomposition of the strain

$$\varepsilon_{11} = \varepsilon_{11}^e + \varepsilon_{11}^p$$

Elastic strain

$$\sigma_{11} = E\varepsilon_{11}^e \quad \rightarrow \quad \varepsilon_{11}^e = \frac{\sigma_{11}}{E}$$

Plastic strain

$$\dot{\varepsilon}_{11}^p = \dot{\bar{\varepsilon}}^p \frac{\partial \phi}{\partial \sigma_{11}} \quad \rightarrow \quad \varepsilon_{11}^p = \bar{\varepsilon}^p \frac{\partial \phi}{\partial \sigma_{11}}$$

$$u_1(t) = \left[\exp\left(\frac{\bar{\sigma}\left(\bar{\varepsilon}^p(t)\right)}{E} + \bar{\varepsilon}^p(t) \frac{\partial \phi}{\partial \sigma_{11}} \right) - 1 \right] L$$

Displacement history is a function of the equivalent plastic strain

Works for isotropic models, but more involved for orthotropic models…

Given strain rate $\bar{\varepsilon}^p(t) = \int_0^t \dot{\bar{\varepsilon}}^p(t) dt$

Initial stress state for rate dependent models

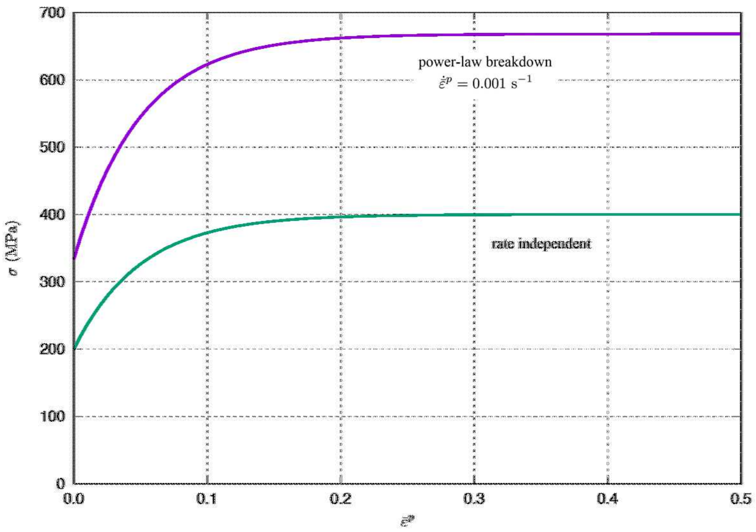
$$\sigma_0 = \sigma_y g(\dot{\bar{\varepsilon}}_0^p)$$

Verification - Results

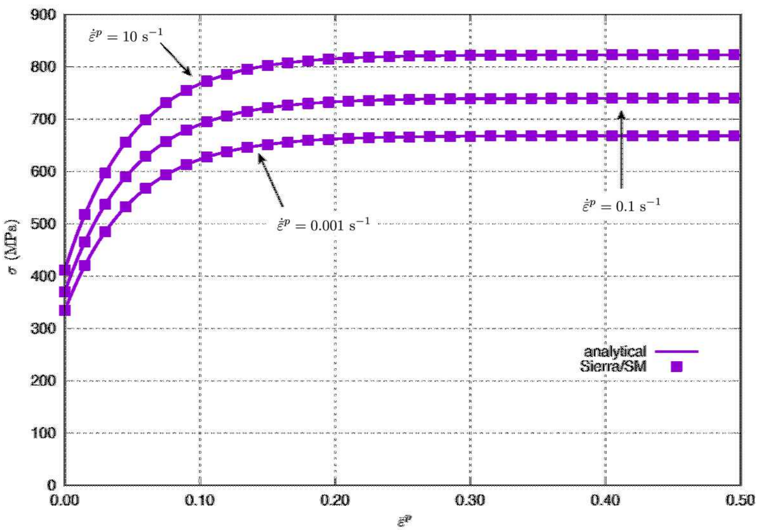
$$\bar{\sigma} = \left[\sigma_y + A\left(1 - \exp(-b\bar{\varepsilon}^p)\right)\right] g(\dot{\bar{\varepsilon}}^p)$$

$$g(\dot{\bar{\varepsilon}}^p) = 1 + \sinh^{-1}\left[\left(\frac{\dot{\bar{\varepsilon}}^p}{g}\right)^{1/m}\right]$$

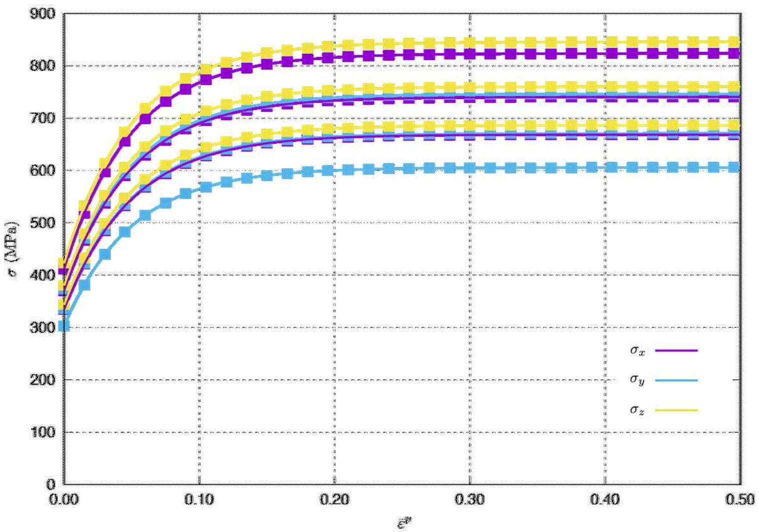
$$\sigma_y = 200 \text{ MPa}$$
$$A = 200 \text{ MPa}$$
$$b = 20$$
$$g = 0.210 \text{ s}^{-1}$$
$$m = 16.4$$



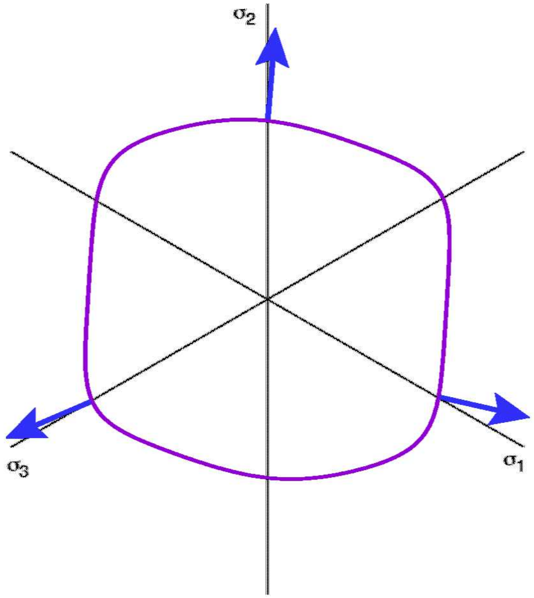
Hosford

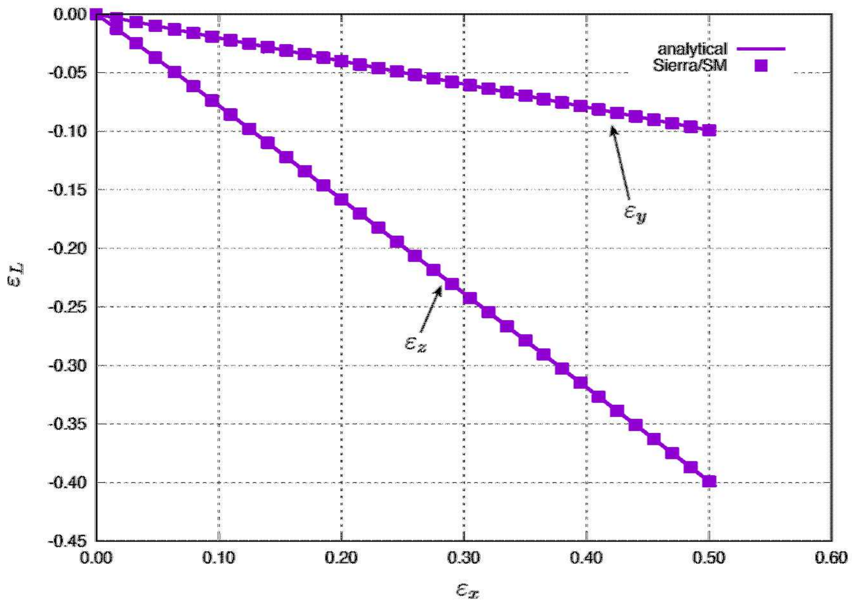


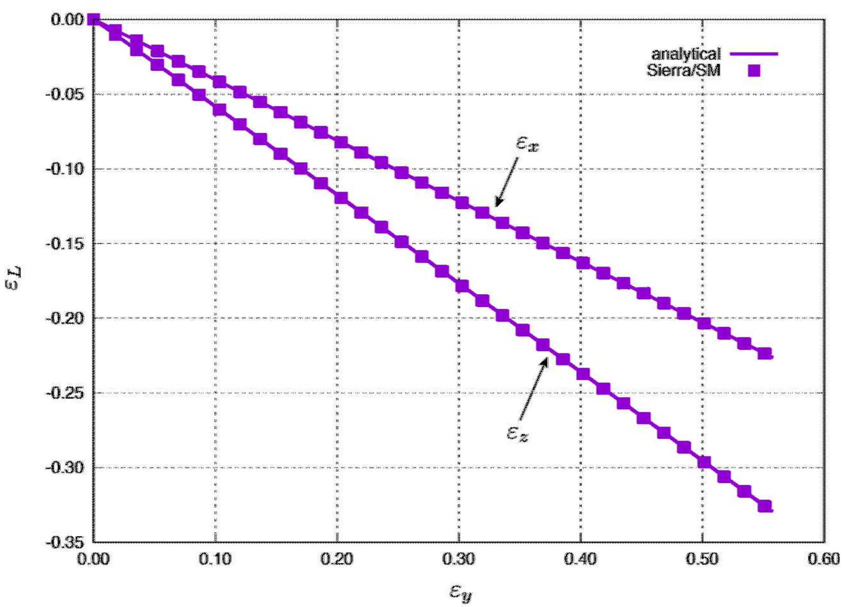
Barlat

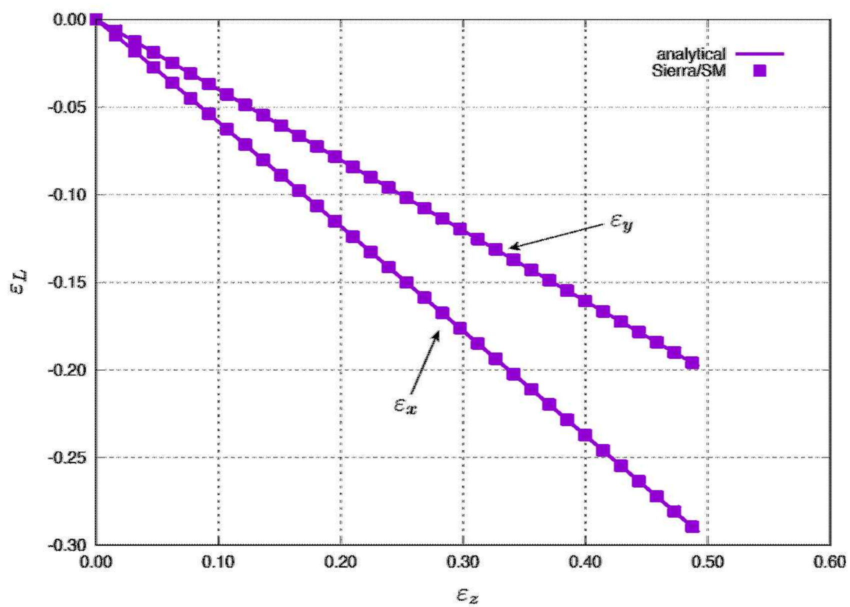


Verification - Results



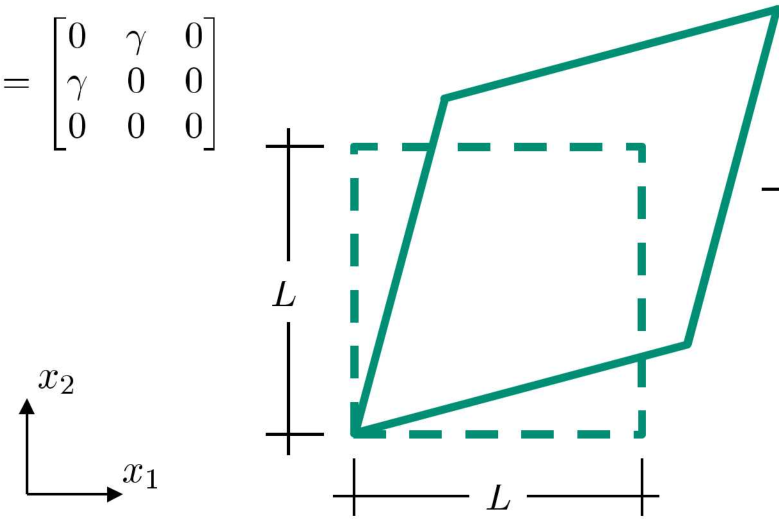


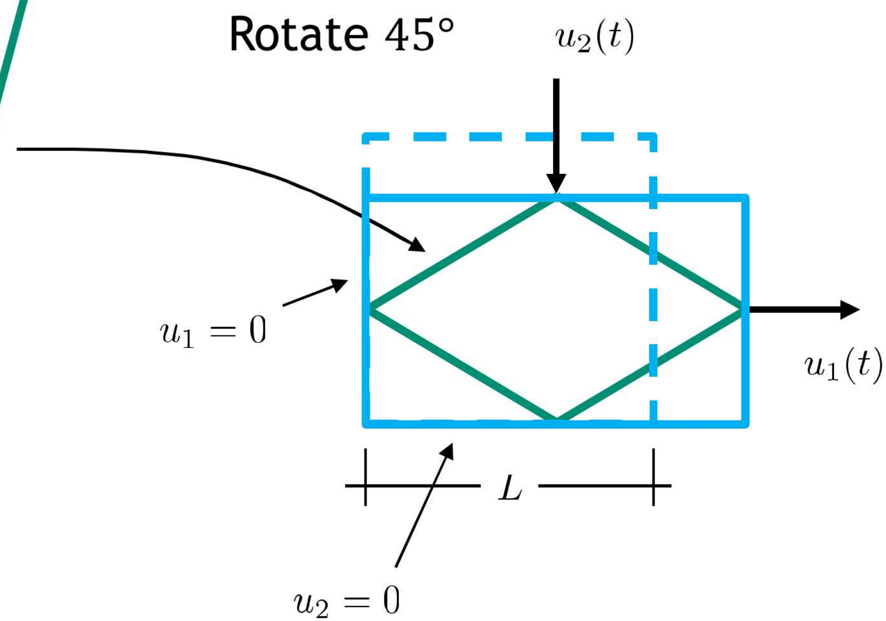




Pure Shear

$$[\varepsilon_{ij}] = \begin{bmatrix} 0 & \gamma & 0 \\ \gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$





What is the displacement history?

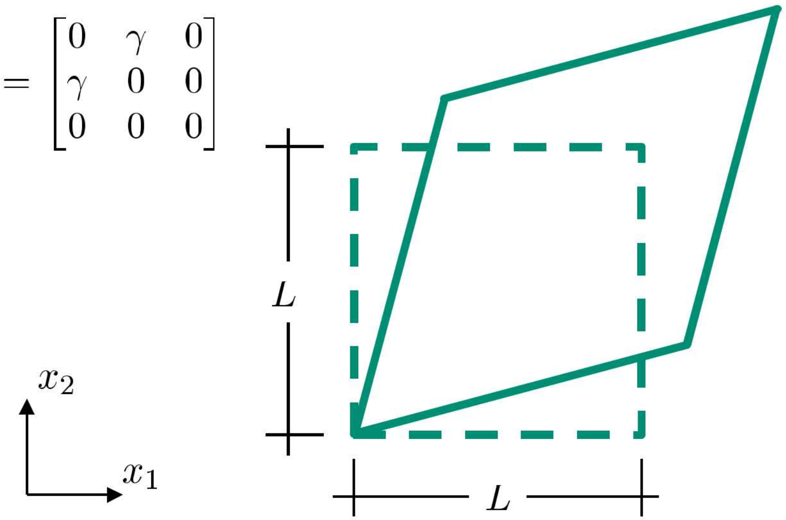
$$[\varepsilon^*_{ij}] = \begin{bmatrix} \gamma & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\gamma(t) = \ln \lambda(t)$$

$$\lambda(t) = \exp(\gamma(t))$$

Pure Shear

$$[\varepsilon_{ij}] = \begin{bmatrix} 0 & \gamma & 0 \\ \gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



von Mises

$$\gamma(t) = \frac{1}{\sqrt{3}}\left(\frac{\bar{\sigma}(\bar{\varepsilon}^p(t))}{2\mu} + \frac{3}{2}\bar{\varepsilon}^p(t)\right)$$

$$\lambda(t) = \exp(\gamma(t))$$

What is the displacement history?

$$u_1(x_1, x_2; t) = \frac{1}{2}\Big[\left(\lambda(t) + \lambda(t)^{-1} - 2\right) x_1 + \left(\lambda(t) - \lambda(t)^{-1}\right) x_2\Big]$$

$$u_2(x_1, x_2; t) = \frac{1}{2}\Big[\left(\lambda(t) - \lambda(t)^{-1}\right) x_1 + \left(\lambda(t) + \lambda(t)^{-1} - 2\right) x_2\Big]$$

Verification - Results

$$\bar{\sigma} = \left[\sigma_y + A\left(1 - \exp(-b\bar{\varepsilon}^p)\right)\right] g(\dot{\bar{\varepsilon}}^p)$$

$$g(\dot{\bar{\varepsilon}}^p) = 1 + \sinh^{-1}\left[\left(\frac{\dot{\bar{\varepsilon}}^p}{g}\right)^{1/m}\right]$$

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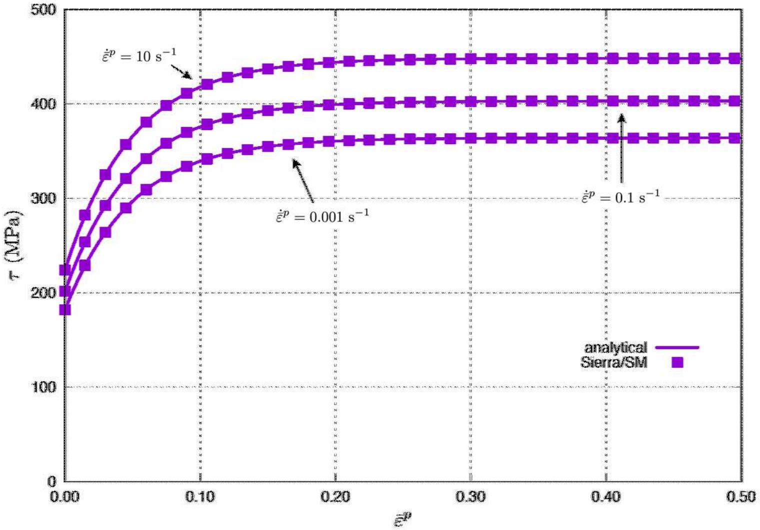
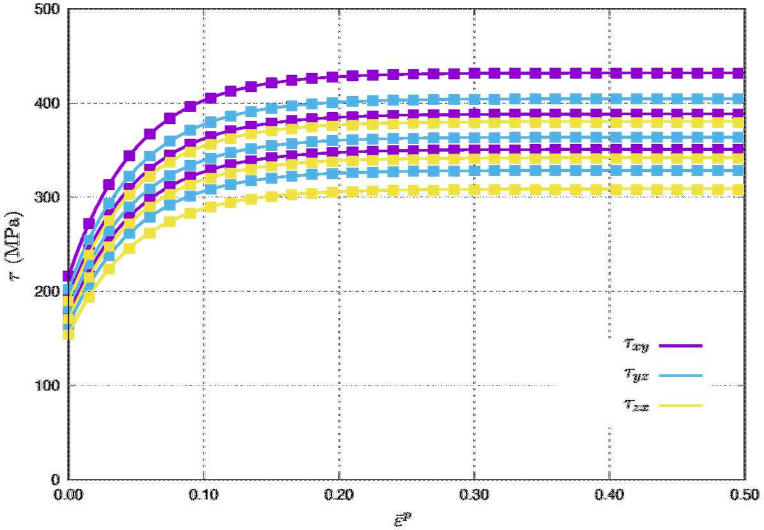


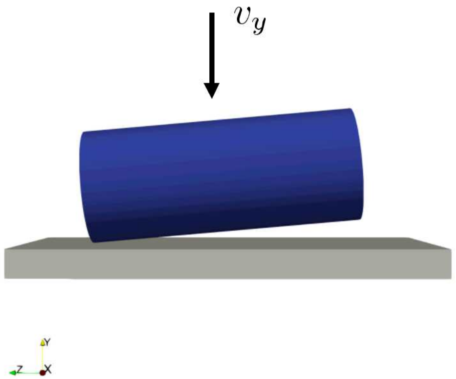
Barlat

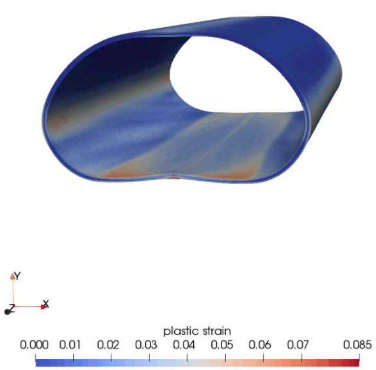


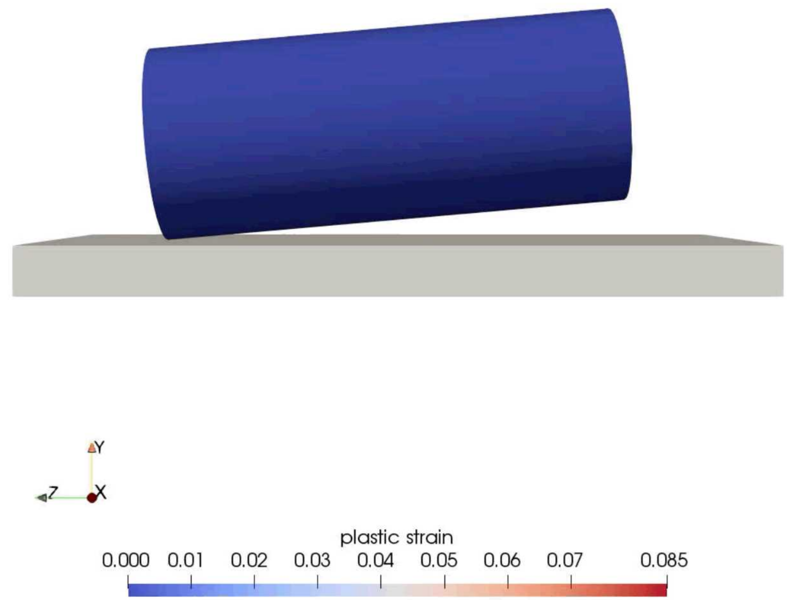
Performance

Model Performance

Impact of a can on a rigid plate



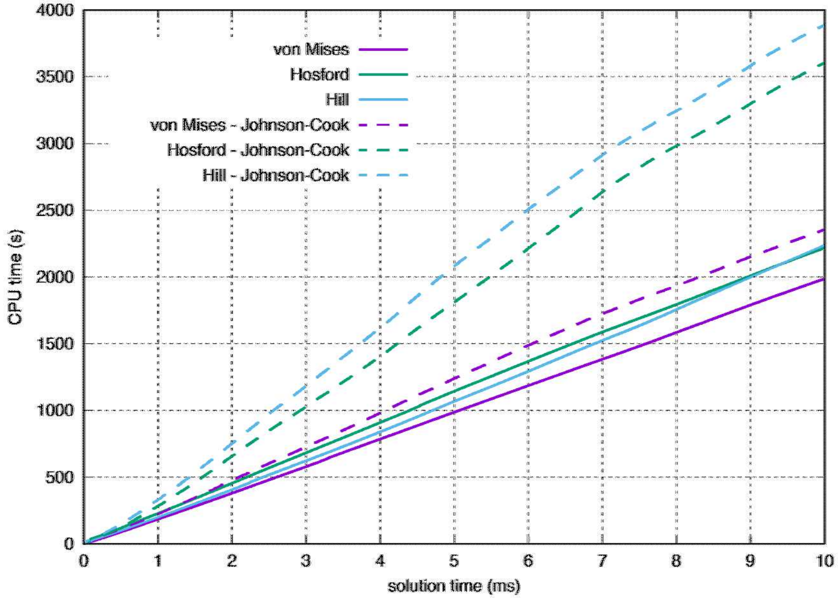


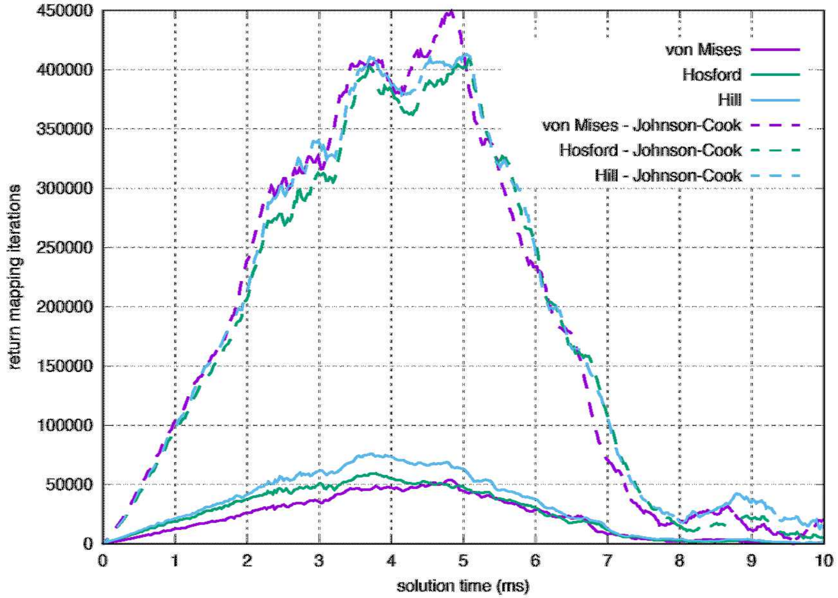


Model Performance

Performance of can model
- ~250,000 elements
- ~46,000 time steps
- 16 processors

Rate dependence slows down analysis
- Know what the problem is
- Developing approach to fix it





Conclusions

Conclusions

- Constitutive models that are used in modeling and simulation to support decision making require extensive verification and testing
- Verification is difficult
  - Show that a model is not verified
  - Test the algorithm -> test the implementation
- Test to fail
  - Avoid positive reinforcement
- Get it right, then make it fast
- Generate a lot of results
- *Documentation and peer review*