



Robust Inference for Physical Parameters with Bayesian Model Calibration

Introduction

- Bayesian model calibration (BMC) refers to coupling of limited experimental data with an expensive computer simulator^[1].

$$y(x_i) = \eta(x_i, \alpha, \gamma) + \delta(x_i) + \epsilon_i$$

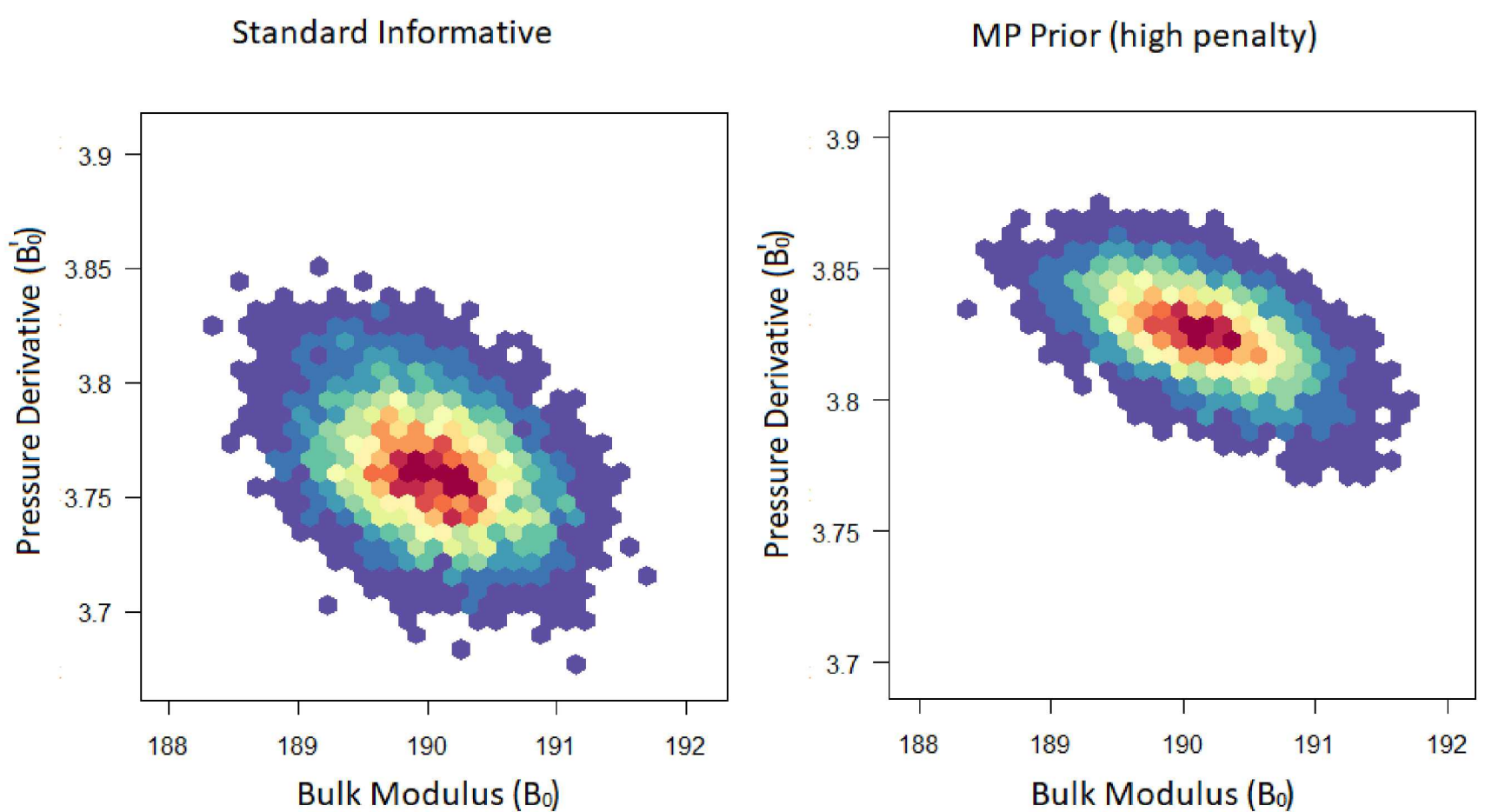
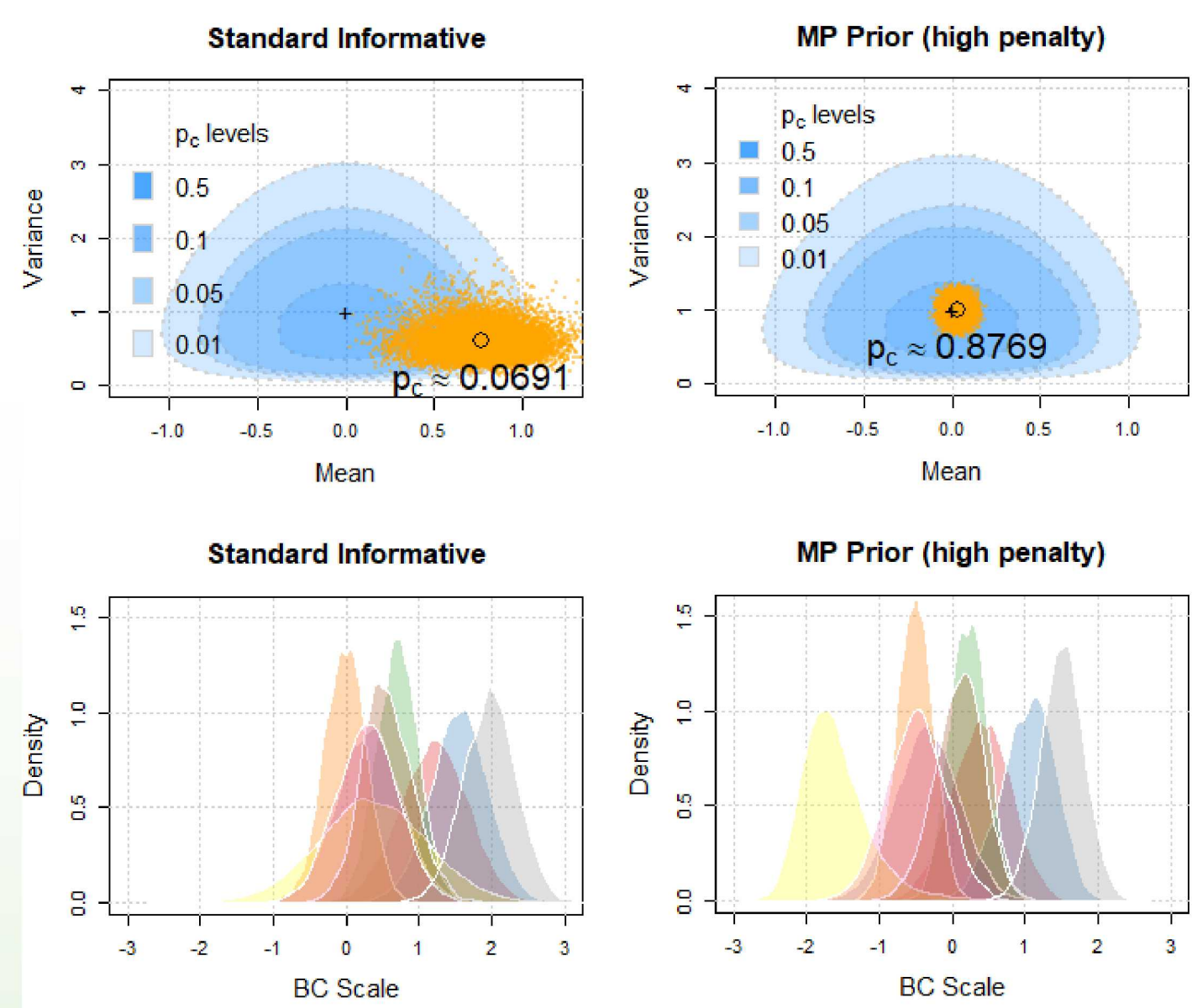
- Computer simulator takes a high dimensional set of inputs known as calibration parameters.
- Modeling the discrepancy function directly leads to overfitting and unidentifiability^[2].
- Goal: Use Bayesian inference to learn the posterior distribution of the physical parameters.

Regularization

- The addition of meaningful constraints in order to reduce overfitting.
- Example: Computer simulator takes 6 inputs, 3 of which vary across experiments.
- Some inputs represent measurement uncertainties, with known mean μ and variance \mathbf{V} .

- Overfitting can be identified with the probability of prior coherency and can be reduced or diagnosed via moment penalization^[3].

$$\pi_{\text{MP}}(\gamma) \propto \exp \left\{ -\lambda_1 (M - \mu)^2 - \lambda_2 (V - \nu)^2 \right\}$$



- Moment penalization leads to a shift in the posterior distribution of the physical parameters.
- More consistent with the physicists prior beliefs.

Modularization

- Sometimes physical parameters α and nuisance parameters γ are inherently jointly unidentifiable.

- Modularization improves identifiability by forfeiting the ability to learn about nuisance parameters, while rigorously accounting for the associated uncertainty^[4].

$$\pi_{\text{M}}(\alpha|y) = \int_{\mathbb{R}^q} \pi(\alpha|\gamma, y) \pi(\gamma) d\gamma$$

- The Modularization posterior is efficiently approximated numerically using the Sequential Gaussian Process (SGP)

