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Combined Shaker-Acoustic Vibration Test Techniques

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Abstract

Ground-based vibration testing is critical to the assessment of aerospace components and systems subjected to field acoustic or aero-acoustic environments. Traditionally, ground-based testing of these environments is accomplished either with single-axis shaker testing or reverberant chamber acoustic testing. However, neither test method is perfect, with both having limitations due to mismatches in boundary conditions and load paths or the inability to achieve very high sound pressure levels. Here, a combined multi-shaker and acoustic vibration test technique is presented. This test technique utilizes the benefits of mechanical and acoustic inputs to achieve a desired response with the end goal of improved response accuracy and higher test levels. While this combined-inputs test is similar to other multiple-input tests, the combination of shaker and acoustic inputs presents unique problems, for example in the distribution of input energy between the different types of inputs. To remedy this issue, an algorithm is presented which allows for automatic biasing of input energy to shaker or acoustic inputs. This adaptive-gain algorithm is shown to be effective at balancing input energy between the shaker and acoustic inputs. These techniques are demonstrated using simulations and experiments. Results indicate that combining shaker and acoustic inputs can provide a high accuracy vibration test with reduced input requirements over shaker-only and acoustic-only tests, meaning this technique could enable higher test levels than traditional, single-input approaches.

Keywords: multiple-input, MIMO, acoustic testing, vibration testing

Introduction

Multi-shaker vibration testing has gained interest in recent years as several researchers have demonstrated that complicated environmental response can be accurately replicated in the laboratory using multiple shaker inputs and multiple-input/multiple-output (MIMO) control [1, 2, 3, 4, 5, 6]. In many of these works, shakers were used to replicate an acoustic or aero-acoustic response which would typically be tested in a laboratory setting using either reverberant or direct field acoustic tests. Often, the sound pressure levels (SPLs) needed to match a field environment may be in excess of the acoustic test equipment capabilities. This limitation motivated the current work, with the hypothesis being that multiple shakers could be combined with acoustics in a single MIMO test. Various questions arose from this combined-inputs test concept, including how to solve the input estimation (control) problem for combined inputs, and how to adjust the balance of shaker and acoustic inputs to maximize each input type. There are scenarios where combining shakers and acoustics together could be beneficial. For example, shakers could be used to supplement an acoustic test to increase the maximum test level. Alternatively, an acoustic input could be used to supplement a multi-shaker test to improve the response accuracy at frequencies where the shakers alone are not sufficient.

This paper explores the combined shaker-acoustic MIMO problem by first reviewing MIMO linear system theory, combined-inputs systems, and input estimation. Next, an algorithm is presented which allows for the different inputs to be scaled up or down by adjusting terms in a gain matrix applied to the inputs estimated from a MIMO input estimation solution. A specific use case of adjusting the acoustic inputs up or down based on how accurately shakers can replicate the response is used to devise an adaptive-gain algorithm which automatically chooses the acoustic input gain at each frequency line. This algorithm strategically notches the acoustic input, reducing the overall acoustic input requirements which could enable higher test levels to be achieved.

Next, a simulated dynamic system and field environment is contrived to provide a representative, sufficiently complicated example problem for demonstration of these techniques. Acoustic inputs are used in both the simulated field environment and in the simulated laboratory shaker-acoustic test so a method for generating acoustic loads on the dynamic system model is presented. The simulated system allows the features of a shaker-acoustic test to be examined. Additionally, the effects of using

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different gains on the shaker or acoustic inputs is demonstrated, achieving the desired notching behavior which results in reduced acoustic inputs with little sacrifice in test accuracy. Finally, an experiment is performed using a plate test article and loudspeaker and shaker inputs to demonstrate the same phenomena seen in the simulation results. Overall, this work shows that combined shaker-acoustic testing can be performed using typical MIMO control methods, and that the combined-inputs test has benefits over shaker-only or acoustic-only tests.

Theory of MIMO and Combined-Inputs Problems

Shaker-acoustic testing can be considered a type of general MIMO problem where multiple inputs, in this case several shaker and acoustic inputs, are applied to a dynamic system and generate a response which is measured at several output locations. It should be noted that this work considers linear systems in stationary random vibration environments only. The responses, $\{X_y\}$, of a linear system defined by a frequency response function (FRF) matrix, $[H_{yx}]$, due to inputs, $\{X_x\}$, is given by

$$\{X_y\} = [H_{yx}]\{X_x\} \tag{1}$$

in linear space and

$$[S_{yy}] = [H_{yx}][S_{xx}][H_{yx}]^H \tag{2}$$

in power space, where $\{X\}$ are vectors of linear spectra, $[S]$ are cross-power spectral density (CPSD) matrices and $[\cdot]^H$ is the Hermitian, or conjugate transpose, of the matrix. In a laboratory MIMO vibration test the objective is to use a control system to provide an excitation which causes the response of the laboratory test article to match some target response. The various control systems in the market typically have proprietary control algorithms, but in general this can be considered a MIMO input estimation problem such as

$$[S_{xx}] = [H_{yx}]^+[S_{yy}][H_{yx}]^{+H}, \tag{3}$$

where $[S_{yy}]$ is the target response and $[S_{xx}]$ is the laboratory input needed to best match that target response using a least-squares solution via the pseudo-inverse of the FRF matrix, denoted as $[H_{yx}]^+$.

A combined-inputs problem can be simply viewed as an FRF matrix composed of one or more FRF matrices (or vectors). Take the example below where one set of inputs, A, are combined with a second set of inputs, B, to give the combined inputs FRF matrix $[H_{AB}]$:

$$[H_{AB}] = [[H_A]\ \ [H_B]]\,, \tag{4}$$

noting that this is the two FRF matrices stacked side-by-side and not multiplied together. If this combined-inputs system is used in a MIMO test, the MIMO input estimation solution will yield an input CPSD matrix which has components of the A inputs, $[S_{xx,AA}]$, components of the B inputs, $[S_{xx,BB}]$, and also components which relate A and B, $[S_{xx,AB}] = [S_{xx,BA}]^H$:

$$[S_{xx,AB}] = \begin{bmatrix} [S_{xx,AA}] & [S_{xx,AB}] \\ [S_{xx,BA}] & [S_{xx,BB}] \end{bmatrix}. \tag{5}$$

Changing the Gain to Different Inputs

A relevant consideration to any MIMO problem, and in particular to the combined-shaker-acoustic problem, is how to adjust the input levels that result from the input estimation step. That is, it is possible the solution of the input estimation problem would require too much of one input than another or be above the equipment capabilities. In that case, the test engineer would want to adjust the levels, or gains, on the inputs to balance the problem to something more desirable. This work presents one possible approach to adjusting gains in a MIMO problem. This approach involves estimating inputs, adjusting them with some gains and then adjusting them a second time to get the response levels correct. The problem begins with a target response CPSD, $[S_{yy0}]$. Laboratory inputs are initially estimated using Equation 3 to give $[S_{xx1,1}]$, where the subscript "1" indicates the first step in this process. Those initial inputs are modified by pre- and post-multiplying them by a diagonal matrix of gain values, $[g]$, as

$$[S_{xx1,2}] = [g][S_{xx1,1}][g]^H, \tag{6}$$

$$[g] = \begin{bmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{bmatrix}, \tag{7}$$

which gives the adjusted inputs $[S_{xx1,2}]$. Note that the gain values in $[g]$ can be constant or functions of frequency. Because the input levels are changed by the application of the gain matrix the responses will not match the target responses. The inputs now need to be re-adjusted by a single value (scaling all the inputs together the same). This can be done by calculating a scale term, σ_g:

$$\sigma_G = \{G_{yy1,2}\}^+\{G_{yy0}\}, \tag{8}$$

where $\{G_{yy1,2}\}$ are the response auto-power spectral densities (APSDs) predicted using the inputs $[S_{xx1,2}]$ and $\{G_{yy0}\}$ are the target response APSDs (the diagonals of $[S_{xx0}]$). This scale term can then be applied to $[S_{xx1,2}]$ to get the third and final set of laboratory inputs, $[S_{xx1}]$ with:

$$[S_{xx1}] = \sigma_g[S_{xx1,2}]. \tag{9}$$

These laboratory inputs can then be applied to the system to get the laboratory response, $[S_{yy1}]$, which should be a good match to the target response, $[S_{yy0}]$, and have the desired balance of input power needed by the test engineer for that particular test.

Having some control over the input levels may be very important for shaker-acoustic testing because it is possible the least-squares solution obtained from the input estimation process may be very biased toward shaker inputs or acoustic inputs. Ideally, there should be a balance of input power from both types of sources which would allow each to be maximized and result in the highest possible test response levels. It is typical that shakers can cause the response to match very well near the peaks in the response but less well between peaks where there are multiple mode contributions. So, this gain approach can be used to modify the balance of shaker and acoustic inputs to increase the acoustic input gain at frequencies where the shakers alone are not accurately matching the response. Here, an adaptive-gain algorithm was developed to accomplish this task. The algorithm begins by solving the input estimation problem for shakers alone and predicting the response from those shaker inputs. The shaker-only response is compared with the target response to give a dB error in the sum of the response APSDs at each frequency line:

$$dB_{error} = 10\log_{10}\left(\frac{\Sigma\{G_{yy1}\}}{\Sigma\{G_{yy0}\}}\right). \tag{10}$$

Next, the test engineer specifies a threshold dB_{error} value which controls when the acoustic gain will be increased. For example, 1.5 dB was used in this work as the threshold value so when the error in the sum of the APSDs is greater than 1.5 dB, the gain on the acoustic inputs will be increased. This is done by normalizing the dB_{error} value:

$$norm_{error} = \frac{dB_{error}}{threshold}. \tag{11}$$

Next, the acoustic input gain is adjusted from a nominal value of g_{w0} to a new value of g_w via:

$$g_w = g_{w0} * norm_{error}. \tag{12}$$

This gain value is then used in the gain matrix, $[g]$, to adjust the balance of shaker and acoustic input levels via the process described above. With this algorithm, the balance of shaker and acoustic inputs can be automatically adjusted at each frequency line depending on how the inputs can solve the problem. The effect is to notch out the acoustic inputs when the shakers are effective and then turn them back up when the shakers are not effective, which reduces the overall acoustic input requirements. This means that the overall test level could be increased, assuming the acoustic input is the limiting factor, which is plausible in many situations.

Modeling Acoustic Loads on a Structural Dynamics Model

The inputs applied to the dynamic system in the equations above were in the form of a CPSD matrix. The CPSD matrix defines the levels of each input via the diagonal terms which are the APSDs of each input, and the interrelationships between each input and all the other inputs which are the off-diagonal terms. Acoustic loads on a dynamic system can be modeled as a CPSD matrix of forces which are applied to many locations on a discretized surface of a dynamic system. A typical example is to apply forces, which are related to the sound pressure, to the nodes on the outer surface of a finite element model (FEM). In that case, the acoustic load CPSD matrix would have a row/column for each node on the surface of the FEM. The relationship of the applied pressures at one node to the other nodes is defined by the off-diagonal terms in the CPSD matrix. If that relationship is known or assumed, then an acoustic loads CPSD matrix can be generated and used with Equation 2 to get response predictions for the dynamic system in an acoustic environment.

For a pure diffuse acoustic field, the relationship (called the spatial correlation) between one point in space and another point in space is defined by a sinc function [7]:

$$R_{ij} = \frac{\sin(kr_{ij})}{kr_{ij}} \ , \tag{13}$$

where the spatial correlation between a point i and a point j is R_{ij} , k is the acoustic wavenumber, and r_{ij} is the distance between those points. If one ignores the effects of scattering and assumes the applied acoustic field on the surface of the dynamic system is purely diffuse then this spatial correlation can be used to populate the acoustic pressure CPSD matrix as [8]:

$$[S_{PP}] = \begin{bmatrix} R_{11}P_{11} & \cdots & R_{1N}P_{1N} \\ \vdots & \ddots & \vdots \\ R_{N1}P_{N1} & \cdots & R_{NN}P_{NN} \end{bmatrix} , \tag{14}$$

where $P_{ij} = \sqrt{P_{ii}P_{jj}}$ and P_{ii} is the pressure APSD at the i-th location on the surface. Conversion of this pressure CPSD matrix to a matrix of forces which can be applied to nodes of a FEM can be done by multiplying each pressure by a diagonal matrix with the area associated with each node, $[A]$:

$$[S_{FF}] = [A^2][S_{PP}] \ . \tag{15}$$

For other types of acoustic loads the same approach could be taken so long as the spatial correlation is known.

Simulation Demonstration

A contrived system, field environment, and combined shaker-acoustic test was developed to simulate the effects of using shaker and acoustic inputs and to demonstrate the effectiveness of the adaptive-gain algorithm. This contrived situation was designed to have a simple but representative dynamic system and simple but representative vibration and acoustic loads in the field environment. An example of a solar panel on a satellite was chosen as the panel is a simple geometry (can be modeled as a plate) and is subject to a vibration input at the connection and an acoustic input from the launch environment. No attempt was made to make this match any particular real system or environment, but rather to represent a sufficiently complicated, plausible system and environment. The panel was modeled as a 12 x 24 x ¼ inch aluminum plate using a finite element model. Frequencies and mode shapes from this model were used to synthesize FRFs for the plate. This example system and some example mode shapes and FRFs can be seen in Figure 1.

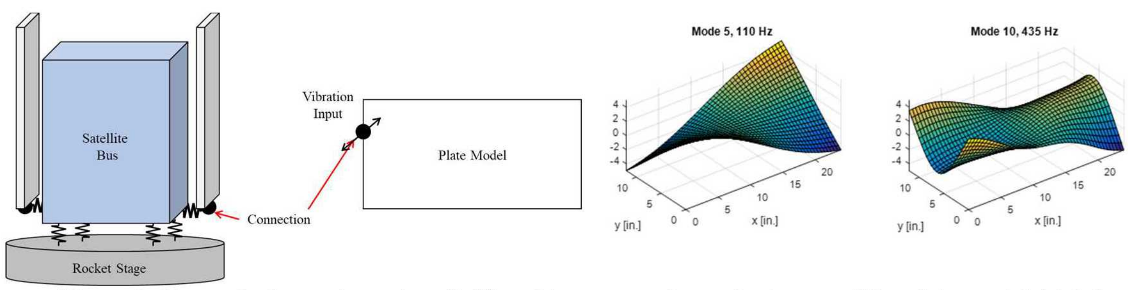

Figure 1: Example dynamic system (left) and two example mode shapes of the plate model (right)

The vibration and acoustic field inputs were taken from SMC-S-016 [9]. The vibration load was specified as a transverse acceleration with a PSD from the Unit Vibration Test section of SMC-S-016 and was applied at the connection node on the left edge of the plate. A concentrated mass was used at this connection node to simulate the boundary condition effects of the connection of the panel to the bus. The acoustic sound pressure level was taken from the Unit Acoustic Test section and was applied at the nodes in the center of two inch patches on the plate surface, as shown in Figure 2. To make the acoustic load somewhat more complicated than a uniform diffuse field (which is the assumed form of the acoustic load to be solved for with input estimation), a gradient of 6 dB was applied with the left side of the plate at a higher level than the right side of the plate. This is purely a contrivance used to ensure that the input estimation problem would not just solve for exactly the original field and makes this example more realistic because the laboratory conditions are almost always different than the field conditions. In addition to a different acoustic load shape, the laboratory test used shaker locations away from the field vibration input at the connection point and free-free boundary conditions with no concentrated mass at the connection node. The vibration and acoustic load applied to the plate model generate a response which is captured at ten gauge locations distributed on the plate surface as shown in Figure 2. These ten responses are the target responses which will be used in a combined input estimation problem. For simplicity, a bandwidth of 800 to 1200 Hz will be examined.

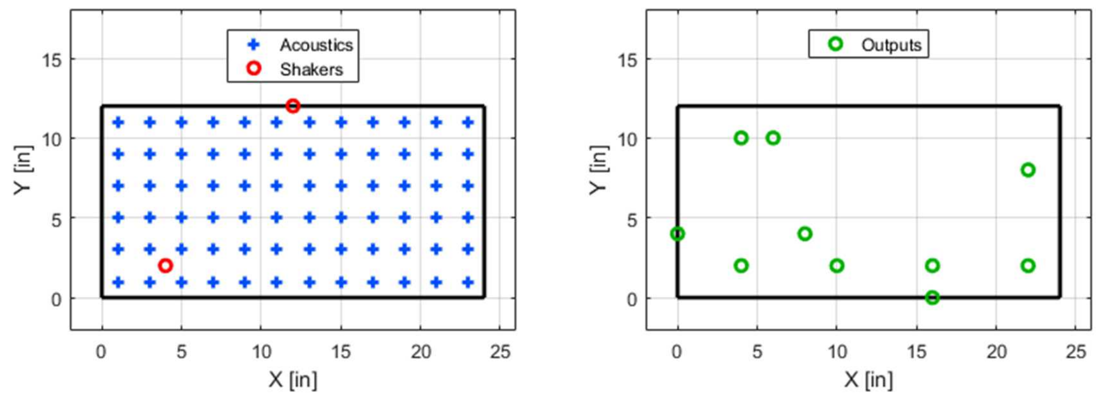

Figure 2: Locations of acoustic and shaker inputs (left) and outputs (right)

The first combined-inputs test case considers using two shakers and one uniform diffuse acoustic load to match the target response. The two shakers are located at the red dots and the acoustic forces are applied at the blue crosses in Figure 2. A combined shaker-acoustic configuration is compared with acoustic-only and shaker-only configurations. As seen in the response plot in Figure 3, the sum of APSDs is well matched overall by all three methods in this case, however there is some improvement when using the combined shaker-acoustic approach. The root mean square (RMS) shaker input levels when using shakers-only is slightly reduced when adding in the acoustic input, as seen in Figure 4. The acoustic input level is significantly reduced compared with the acoustic-only input case. This represents a case where an acoustic test could be supplemented with shakers to reduce the acoustic input requirements. That is, for a given acoustic input capability, shakers could be added to increase the overall test response levels.

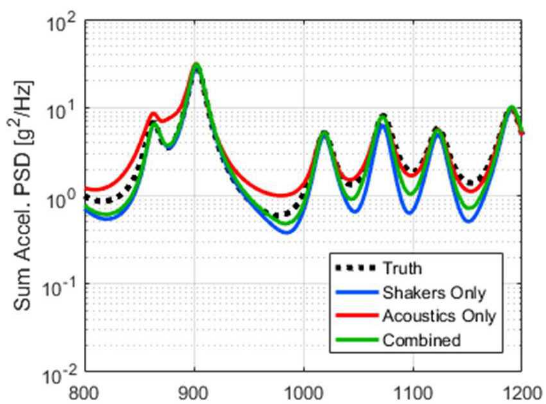

Figure 3: Response shown as the sum of APSDs comparing the truth (field) with shaker only, acoustic only, and combined shaker-acoustic inputs

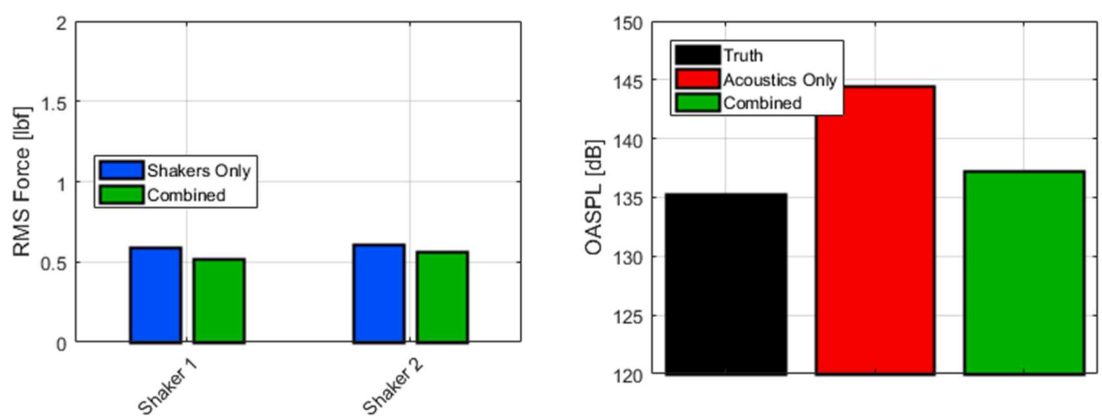

Figure 4: RMS shaker force (left) and overall sound pressure level (right) for the single-input and combined-input cases

The next combined-inputs test case uses the same two shakers and single diffuse acoustic field inputs but introduces a constant or frequency-variable gain. In this example, the shaker gains are both set to two and the acoustic gain is set to one. This biases the simulated test to be shaker-dominant. The adaptive-gain algorithm is then used to adjust the acoustic gain based on the accuracy of the shaker-only solution with a normalized error threshold of 1.5 dB. Each of the three cases produce approximately the same response, with the results using the gain being slightly more accurate at frequencies between peaks, as seen in Figure 5. The constant and adaptive gain responses are nearly identical (curves overlay). The RMS shaker force is nearly the same in all cases as well as shown in Figure 6. However, the acoustic input, shown as the overall sound pressure level, is reduced by using gain and especially by using the adaptive gain. As seen in Figure 7, the sound pressure level is notched over several frequency ranges in the bandwidth shown. This indicates that in these frequency ranges the shakers are effective at accurately matching the target response and so no acoustic input is needed. By notching out the acoustic input at those frequencies, the overall acoustic requirement is significantly reduced, by around 6 dB in this example.

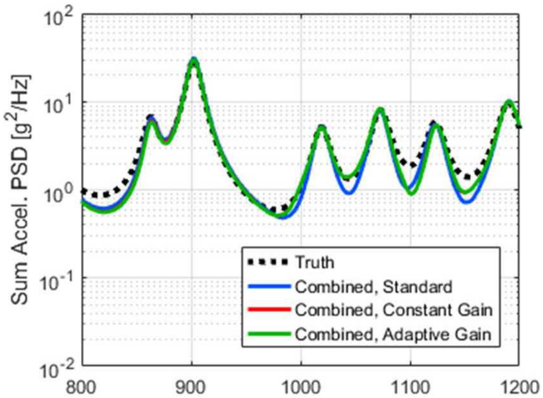

Figure 5: Response shown as the sum of APSDs comparing the truth (field) response with response from combined shaker-acoustic configurations using no gain, a constant gain, and a frequency-variable (adaptive) gain

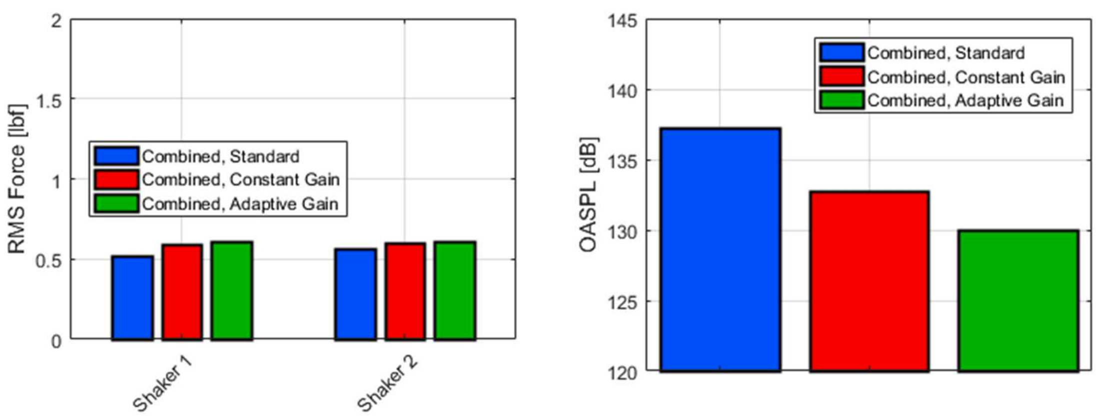

Figure 6: RMS shaker force (left) and overall sound pressure level (right) using no gain, a constant gain, and a frequency-variable (adaptive) gain

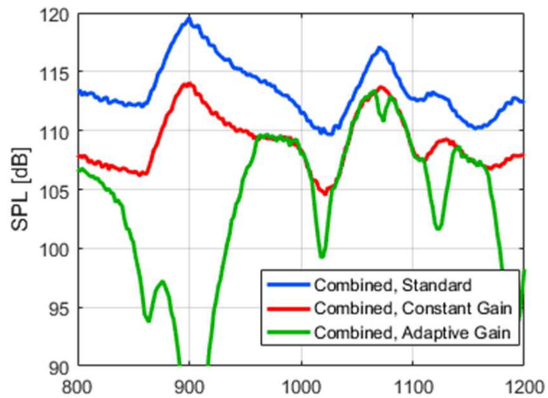

Figure 7: Acoustic sound pressure level using no gain, a constant gain, and a frequency-variable (adaptive) gain

Experimental Demonstration

To demonstrate that these test techniques can be implemented in the laboratory, an experiment was performed using an aluminum plate test article, several small loudspeakers, and three small shakers. The intent was not to try and replicate the simulation cases exactly. Instead, the plate was subjected to an acoustic test using loudspeakers stacked in the arrangement shown in the left image of Figure 8. In total, 24 loudspeakers were used and driven with 3 independent input signals in groups of 8 speakers each. A customizable open-loop controller was used in these tests which enabled the gain algorithms to be implemented. Responses from this acoustic-only test are the target responses which were then replicated using a combination of three shakers and loudspeakers as shown in the right image of Figure 8. To ensure the acoustic field was different for each case (field vs. laboratory tests), the loudspeakers were moved to different locations and driven with a single input signal.

Figure 8: Experiment setups for the acoustic-only "field" test (left) and the combined shaker-acoustic laboratory test (right)

As shown in Figure 9, combining the acoustic input with the shakers provides a more accurate response, in general, than the shaker-only test. In this case, the shaker input requirements were reduced by about 20 percent or more for each of the 3 shakers by adding in the acoustic input as shown in Figure 9. Next, the gain methods were demonstrated experimentally, again using no gain, a constant gain, and a frequency-variable gain. As shown in Figure 10, the response is well matched using any of the three methods. The difference is in the acoustic input requirements. As shown in Figure 10, the acoustic input can easily be modified with the use of the gain method described here. By modifying the balance of inputs between the shakers and speakers, the test can still be run accurately with greatly reduced overall acoustic input requirements. These experiment results show that the combined shaker-acoustic test can be run, can improve performance, and show that the gain algorithms are effective at modifying the input levels as-needed based on test requirements.

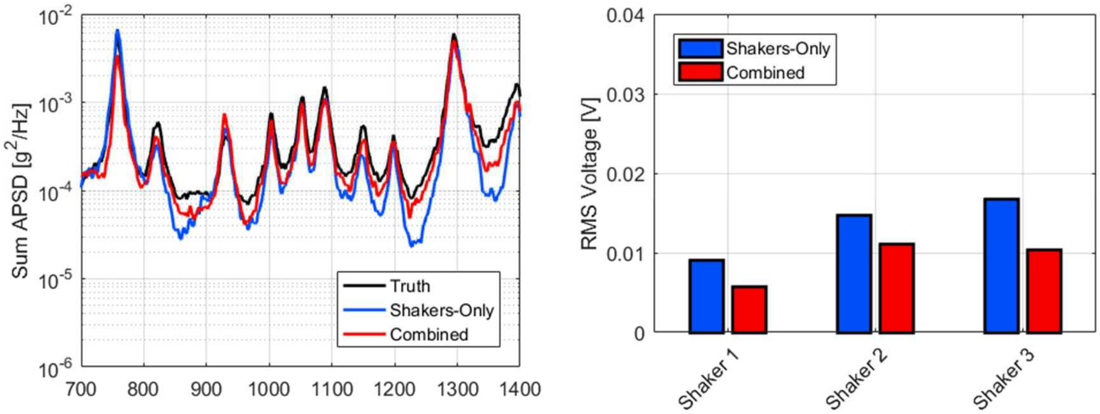

Figure 9: Response (left) and RMS shaker drive voltage (right) for the experiment using shakers-only and a combination of shakers and an acoustic input

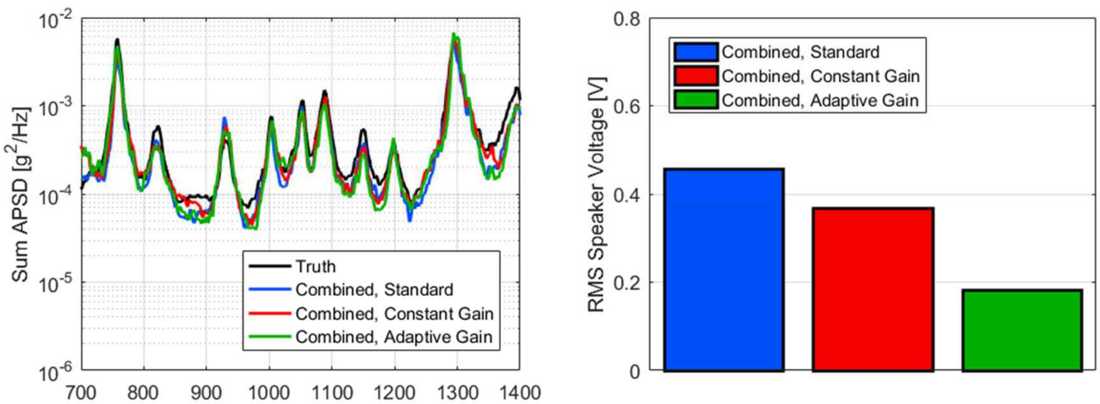

Figure 10: Response (left) and RMS speaker drive voltage (right) for the experiment using no gain, a constant gain, and a frequency-variable (adaptive) gain

Conclusions

Through simulation and experimental demonstrations, this work showed how a combination of shaker and acoustic inputs can be used to successfully perform a MIMO vibration test in the laboratory, accurately replicating a complicated target response. The combined shaker-acoustic test may be useful in various cases such as when a typical acoustic test is level-limited or when a shaker test is not accurately matching responses. While a combined shaker-acoustic test is not much of a stretch beyond normal MIMO problems, enabling user control over the balance of inputs is an important and practically necessary addition. Here, a general input gain modification method was developed and demonstrated. An extension specific to shaker-acoustic testing was also presented. The adaptive-gain algorithm determines how much acoustic gain to apply depending on how well the shakers match the response at each frequency line. This effectively notches out the acoustic inputs where the shakers are sufficient and then increases the acoustic inputs as-needed. In this way, the overall acoustic input can be substantially reduced, saving damage to acoustic test equipment or enabling higher level tests to be run.

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