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Explorations in Multiple-Input Shaker Shock Testing

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Abstract

Multiple shaker random vibration testing has gained favor in recent years due to the improved controllability and response accuracy allowed by utilizing multiple inputs at various locations and multiple-input/multiple-output control. As a typical vibration test workflow includes a range of inputs such as random vibration, sine sweeps, and shocks, it is important to extend multi-shaker test capabilities to include more than just random vibration. Several commercial control systems allow for multiple-input shock control however this test technique is still niche. A multiple shaker shock experiment was attempted at Sandia National Laboratories to begin exploring this problem and develop tools and techniques for future applications. Two shakers were arbitrarily placed on a plate test article and then used to replicate the response from a hammer hit. Issues in the results and questions about the effects of test setup on the results then motivated this work, where test design factors for multi-shaker shock tests are examined using simulations of a model of the plate test article. The effects of the number of shakers, their locations, and convolution errors are demonstrated and compared to a target shock response. Results of this study will provide useful guidance into how multi-shaker tests can be designed to improve accuracy of multi-input shocks.

Keywords: multiple-input, shock, vibration testing, MIMO

Introduction

Multiple-input/multiple-output (MIMO) vibration testing is a promising technique which can, in general, improve the accuracy of a laboratory vibration test. The use of multiple inputs provides control freedom to match the response of many locations and directions simultaneously. These MIMO techniques are not new but have not yet been widely adapted. At Sandia National Laboratories, most MIMO testing efforts in recent years have focused on stationary, random vibration problems [1, 2, 3]. Multiple successful MIMO random vibration tests have sparked some interest in MIMO shock testing with electrodynamic shakers. A MIMO shock capability is valuable for two reasons. First, some field shock environments may be multi-axial or best matched with multi-axial inputs. Second, if random vibration and shock can be completed by a single multi-input test setup, there could be significant schedule and cost benefits.

This work began with a request to perform a MIMO shock test on a unit after it was subjected to a multi-shaker random vibration test similar to [1, 3, 4, 5]. As this was the first MIMO shock test done with multiple shakers at Sandia, some basic development work was needed. A basic MIMO transient control approach was implemented and used in a simple test where two shakers were used to replicate response of a plate subjected to a hammer hit. Various issues arose during this test which either solved or investigated further with more tests and eventually a simulation-based study. So, this work is an initial effort in exploring MIMO shock testing and test design to scope out this class of MIMO transient control problems. The goals are to develop some basic MIMO transient control capability, demonstrate its use with models and simple experiments, and utilize MIMO transient control methods in a simulation-based test design process to answer some of the typical test design questions such as the required number and location of shakers to achieve a given target response.

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This paper is not intended to present novel methods, rather it is reflective of the issues and lessons-learned encountered when exploring a MIMO shock problem using simulations and experiments. The paper presents the basic theory of MIMO transient control of a linear system, wherein multiple inputs are estimated to best match the response at a set of gauge locations. Then, that transient control method is used in a series of simulations which explore different aspects of the MIMO transient problem and test design. Finite element model simulations show that much like MIMO random vibration problems, the results are affected by the choice of the number and location of inputs. Adding more shakers generally improves control, with the benefits mostly at the start of the transient event. Responses can be matched well later in time, after the forces have been applied, even with a small number of shakers.

A Motivating Experiment

This exploration into MIMO shock began with an experiment which exhibited various interesting phenomena. The experiment involved an aluminum plate with two shakers attached at the locations shown in Figure 1. Twelve accelerometers were mounted at the various locations on the plate, placed for independence of the modes below 2 kHz. A single input location was used to generate a set of transient target responses using a hammer hit. Because this plate test article is small and light the effect of attaching the shakers to the plate is non-trivial. To avoid the difference in system dynamics between shakers attached vs. not attached from corrupting results, the shakers were left attached when the hammer hit was performed. The accelerometer responses and measurements of the shaker-driven FRFs of the plate were used to estimate the shaker drive voltages needed to best match the target responses. As will be discussed in detail later, it was found that not zero-padding the responses in the input estimation step results in a convolution wrap-around error in both the estimate inputs, shown in Figure 2, and the resulting responses from the shaker excitation, shown in Figure 3. Using zero-padding in the input estimation step cleans up the early and late-time response and input error, as seen in Figure 4.

With results from this preliminary experiment, several questions arose which warranted more thorough investigation. First was the convolution issue. Next was the addressing the inaccuracy in the replicated response from the shakers. The basic form of the response looks good, but there is some inaccuracy, primarily early in time near the initial shock input. It was supposed that this issue was due to an inability to control the plate, which could be improved by adding more inputs or changing where they are located. The simulation studies in the next sections examine these issues in more detail.

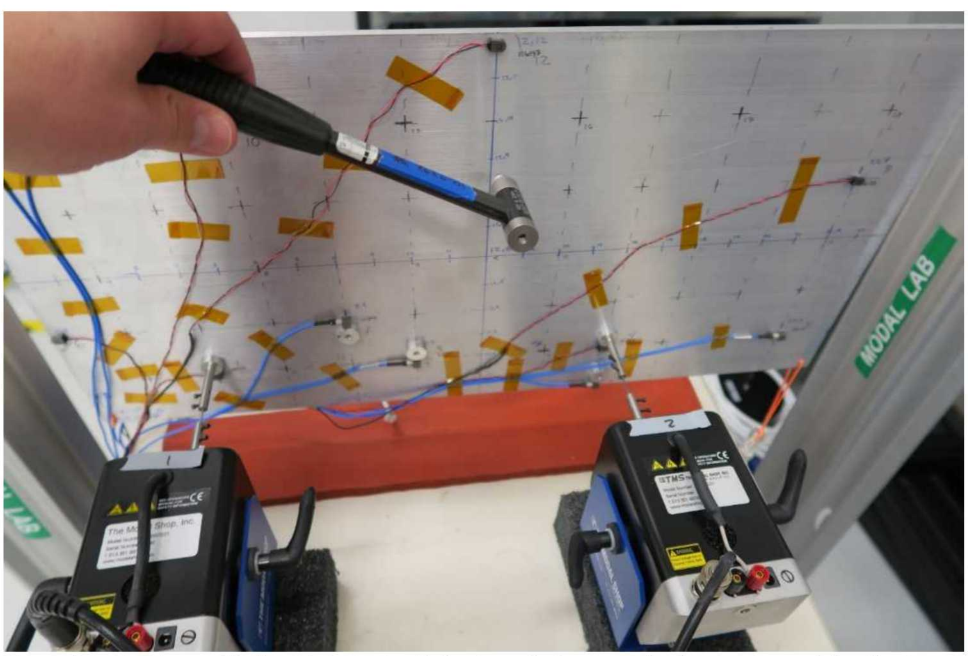

Figure 1: Experiment setup with the plate test article, two shakers, and a hammer used for the target response excitation

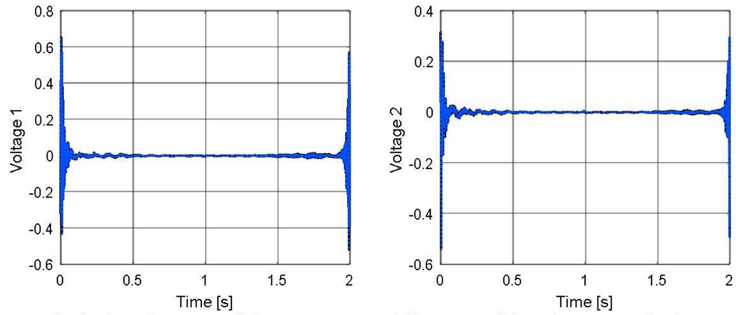

Figure 2: Estimated shaker inputs without zero-padding resulting in convolution wrap-around error

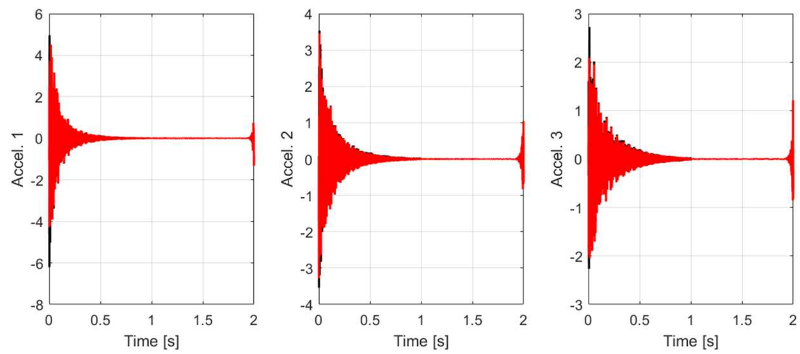

Figure 3: Responses for outputs 1-3 from inputs estimated without zero-padding resulting in convolution wrap-around error

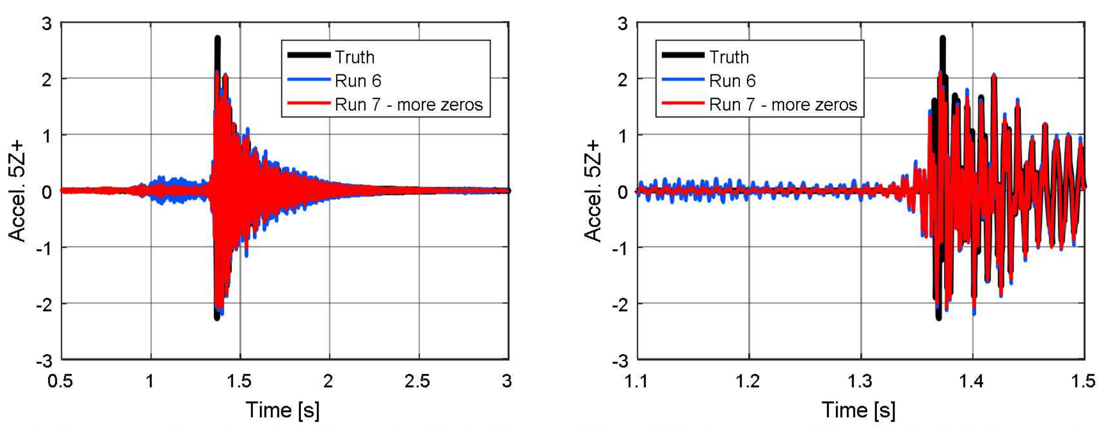

Figure 4: Response from inputs estimated with (red) and without (blue) zero-padding. Left: Full time scale. Right: Zoomed to start of the record.

Theory

For a linear system with a frequency response function, $[H_{yx}]$, the outputs, $\{X_y\}$, due to inputs $\{X_x\}$ is given by:

$$\{X_y\} = [H_{yx}]\{X_x\}, \tag{1}$$

where $[H_{yx}]$ is size M outputs by N inputs and the outputs and inputs are vectors of linear spectra. Equation 1 is written for a single frequency line, and in practice is formed and evaluated on a frequency line by frequency line basis. The time domain form of Equation 1 is:

$$\{y\} = [h_{yx}]\otimes\{x\}, \tag{2}$$

where now the system information is contained in the M by N impulse response matrix $[h_{yx}]$ and the outputs and inputs are vectors of M by 1 and N by 1 time responses, respectively, and $\otimes$ is the convolution operator. So, the response of a system due to multiple transient inputs can be obtained by evaluating Equation 2. Alternatively, the transient response can be obtained by converting the input time responses to the frequency domain with the Fourier Transform, evaluating Equation 1, and then inverse Fourier Transforming the output linear spectra into time responses. This time to frequency and back to time approach is the method used throughout this work.

Naturally, the outputs are affected by the system and the inputs, and as such the number and location of inputs affects the content in $[H_{yx}]$ and therefore the response achieved in $\{X_y\}$. While the inputs required to get a test response, $\{X_{y1}\}$, to best match some desired response, $\{X_{y0}\}$, can be estimated with a least-squares, pseudo-inverse solution:

$$\{X_{x1}\} = [H_{yx}]^{+}\{X_{y0}\}, \tag{3}$$

the form (i.e. the best number and location of inputs) of $[H_{yx}]$ cannot be obtained from a simple direct solution. Instead, the typical practice is to locate as many shakers as is practical at locations that are practical or perhaps picked with intuition. A search problem, where multiple sets of shaker locations are evaluated and compared, is also an option although the cost of evaluating that search space can be large due to the large number of combinations of candidate locations. Here, the input locations for the initial cases will not be chosen to be optimal, but rather to be different and represent different use cases to demonstrate the effects of location on response results.

Example Dynamic System

Exacting results of a MIMO problem are system-specific, but there are trends and features which are common to many MIMO problems. Here, a plate system is used to demonstrate the effects of test design factors on the inputs and responses for a MIMO shock problem. The plate is modeled with finite elements having aluminum material properties and dimensions of 12 by 24 by ¼ inch. It has 12 accelerometer locations which were chosen to have good independence to discriminate the modes out to 2 kHz. An image of the hardware version of the plate model, as well as the finite element model mesh with accelerometer locations in red, is shown in Figure 5. The modes of the plate model were used to synthesize FRFs from a set of candidate input locations, shown in blue in Figure 6. A simulated field shock environment is created using 1 ms haversine forces applied at two locations shown in the left plate of Figure 6. The response from this field environment is then used to estimate the required forces at two different locations for the simulated laboratory test, also shown in the right plate of Figure 6. Different field and lab input locations is meant to represent the often unknown nature of field loads. The transient response is achieved using the linear spectrum of the haversine input forces applied to the FRF matrix to get the linear spectrum of the responses, which is then inverse Fourier Transformed to get the time response at the 12 accelerometer locations.

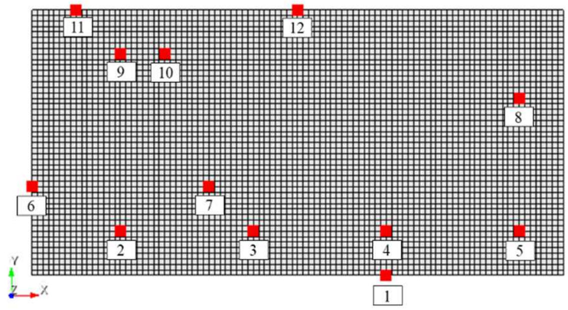

Figure 5: Plate FE model showing 12 response locations

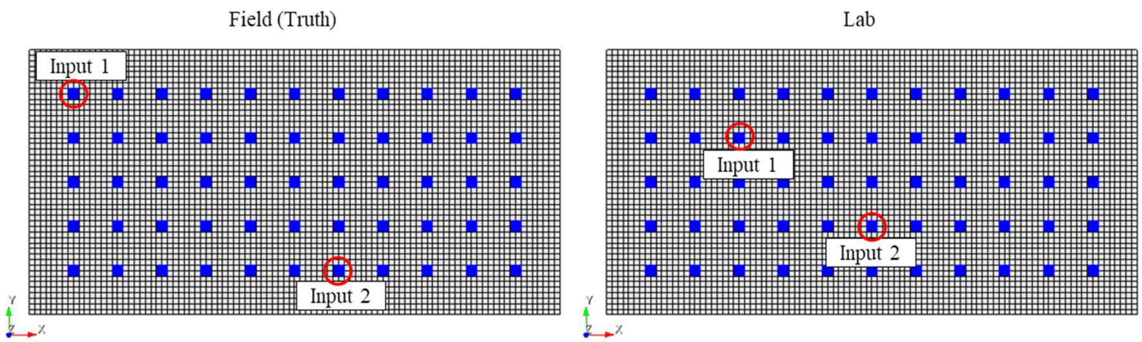

Figure 6: Candidate input locations with two field test (left) and two laboratory test (right) locations

Test Design Effects

The plate model with field and laboratory inputs can be used to evaluate various MIMO shock test configurations and considerations. The first is to simply examine how a MIMO transient input estimation problem works (or doesn't work). Next, the effects of the number and location of inputs is examined. While the response at all 12 accelerometers could be examined, for brevity, just the response at accelerometer 8 will be shown. Each response figure shows the response comparison between the field and laboratory tests over the 0.5 second frame, along with zoomed-in plots of the start, middle, and end of the frame.

Convolution Error

The first issue when performing or simulating the MIMO transient problem was a convolution wrap-around error. This occurred when simply using a simulated field response which starts near zero time and nearly rings down at the end of the frame. Since the transient response is a convolution of the system impulse response with a set of loads, there is a wrap-around effect which must be accounted for. The error occurs when estimating inputs using Equation 3 if the target responses do not have enough zeros at the start or end. The effect, shown in Figure 7, is a ring-up which occurs near the end of the frame due to the convolution wrapping around. This is fixed by simply zero padding the target response time histories and similarly interpolating the system FRF matrix to match. The addition of zeros at the front and back of the signal can remove this convolution wrap-around error, as seen in Figure 8. In this example problem, the sample rate was 32,768 Hz, the target response signal had 16,384 points (0.5 seconds long), and 16,384 zeros were added to both the front and back of the target response signal. Zero-padding the target response signal means the FRF matrix frequency resolution must be finer to be compatible with the target response linear spectrum frequency resolution. This was done by interpolating that FRF matrix. The effects of using fewer zeros at the start or end was not explored in detail, but it is likely that fewer zeros could be used and obtain similar results. Here, the target responses were windowed with a Tukey window with 32 or 128 points in the taper region at the start and end, respectively, to ensure a smooth transition to zero before appending zeros. By windowing and zero-padding the target responses and interpolating the FRF matrix to accommodate the longer block size, the effects of convolution wrap-around on the estimated inputs and resulting responses can be mitigated.

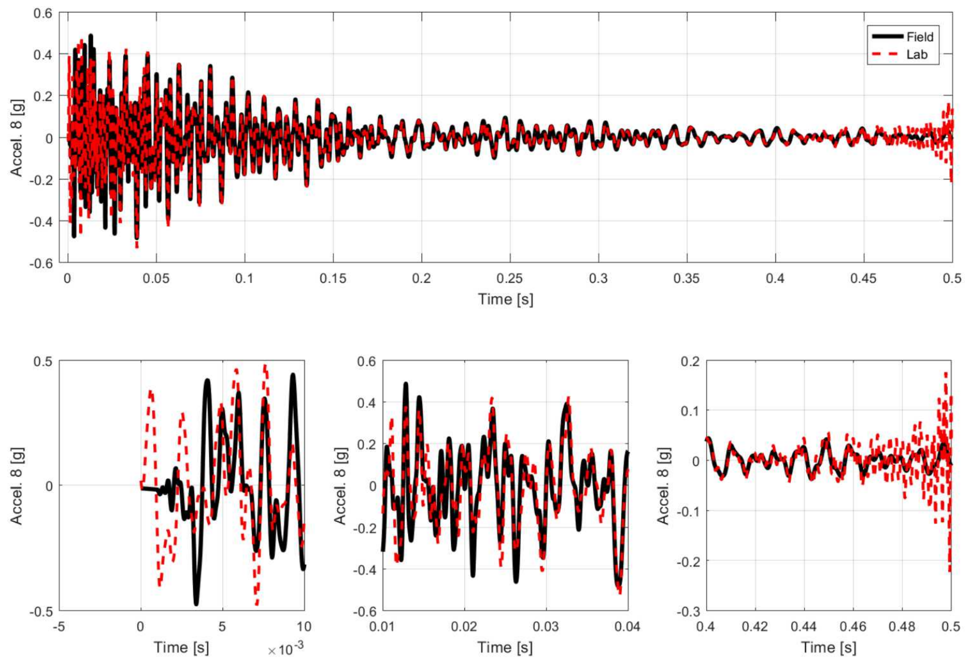

Figure 7: Response at accel. 8 when using no zero padding to estimate laboratory inputs

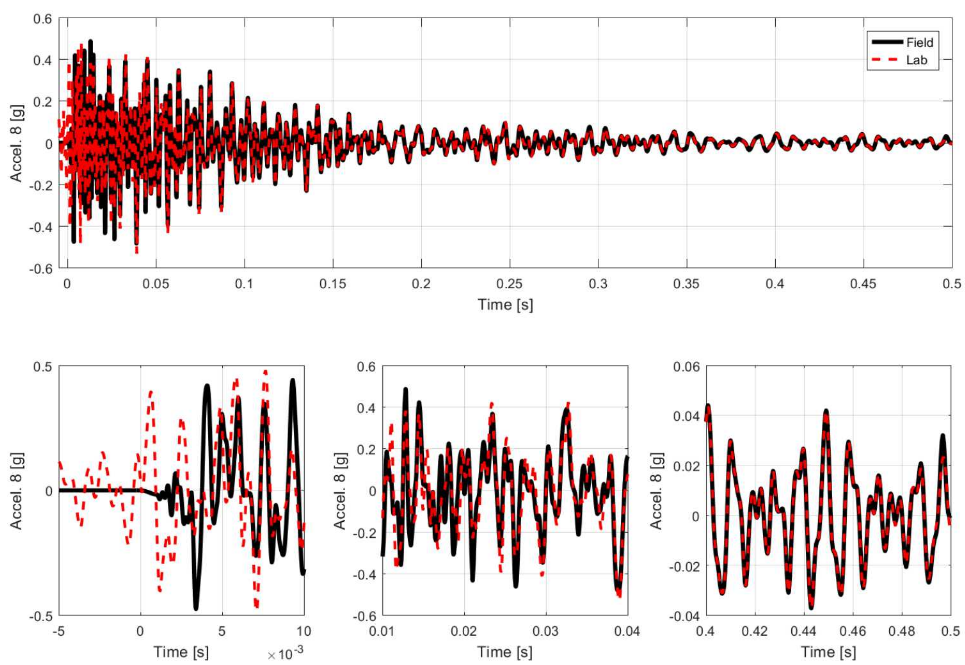

Figure 8: Response at accel. 8 when using zero padding to estimate laboratory inputs

Effects of the Number of Inputs

Now that the problem can be properly solved from a mathematical standpoint, the next consideration is how the problem can be better solved by changing the experiment. The main consideration is the number of inputs, or shakers, required to match the target response. Intuitively, adding more shakers should improve the response accuracy as there is more control freedom. While the accuracy may improve, it is not necessarily true than simply adding more shakers will result in a perfect match to the target response. If the laboratory inputs are different than the field inputs, then the response may not match exactly.

To explore how the number of shakers affects results, simulations were performed with two, four, and six inputs on the plate at the locations shown in Figure 9. These locations were chosen arbitrarily, with the only constraint being to avoid the two field input locations, as that would bias the results. As can be seen in Figure 10, the addition of more inputs does improve the response accuracy. The improvement is primarily seen early in the time, near the onset of the shock event. There is some small improvement in the middle of the response, and little improvement later in time where even just two inputs can match the response well. However, even with six inputs, the response is not matched perfectly early in time in this case. The main takeaway here is that more inputs can improve results, although simply adding more inputs will not guarantee perfect replication of the response time histories.

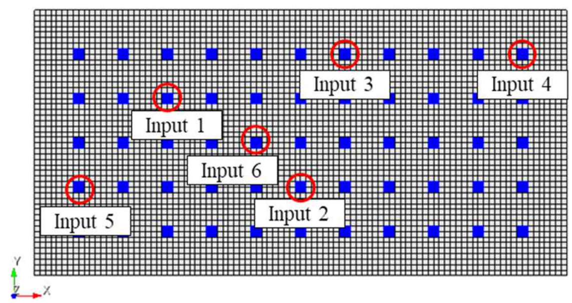

Figure 9: Six input locations

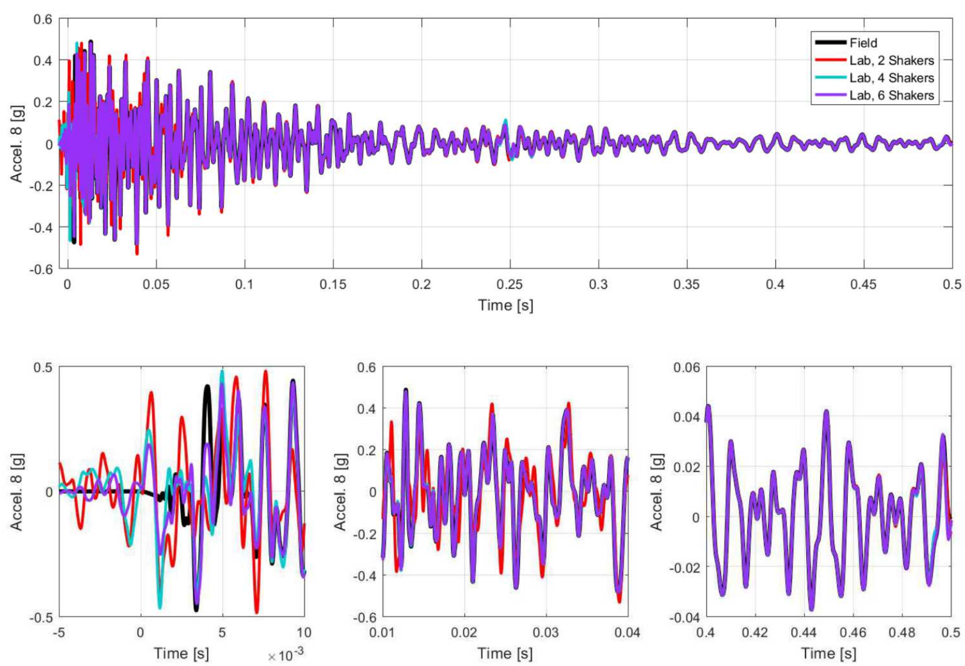

Figure 10: Response at accel. 8 when using 2, 4, or 6 shakers

Effects of the Location of Inputs

Just as the number of inputs affects controllability and response accuracy, so should the location of inputs. Of course, if the laboratory inputs match the location of the field inputs, then just two inputs would be required and the response would match exactly. However, because the field inputs are often unknown in location and form, it cannot be expected that the laboratory input locations can be made to match the field locations. So, some method is required to determine a best set of input locations given constraints of the number of inputs (limited by available equipment) and the possible input locations (limited by logistics). Assessing the quality of a MIMO test remains difficult as there is too much response information to condense down into a single, scalar value to use as a pass/fail metric. However, if trying to compare results of various test configurations, such as the effects of input locations, some metric is required. Here, the time response assurance criterion (TRAC) is used [6]. The TRAC is a normalized dot product of two time history vectors and results in a scalar value between zero and one, with one indicating the vectors are very similar in shape and zero indicating the vectors are not similar. The TRAC is identical to a modal assurance criterion (MAC) but applied to time histories instead of mode shapes [7].

The original set of candidate locations and the symmetry of the modes of this free-free plate mean that many of the candidate input locations would excite the modes to exactly the same level as a symmetry input counterpart. To avoid considering several redundant input locations, a subset of 18 candidate locations were chosen for the location search problem, shown in Figure 11. In total, there are 153 combinations of 2 input locations from these 18 candidate locations. Each of these 153 input location sets were simulated. The response accuracy was assessed by comparing the TRAC values for 4 output locations, at output 1, 7, 8, and 11. The minimum TRAC value for any of those four outputs was used to assess how well that set of inputs matched the target response. As seen in Figure 12, there is a distribution of TRAC values, with some sets of locations resulting in a very close match to the target response, indicated by TRAC values approaching one, and some sets resulting in a poor match to the target response, indicated by TRAC values near zero. This general behavior, with several sets being fairly good and a few being quite bad is similar to what is seen in location assessments for MIMO random vibration problems [2, 3, 8]. To understand the difference between a good set of input locations and a bad set of input locations, the results of input locations with the best and worst minimum TRAC values are shown in Figure 13. While the best locations provide a response which matches very well in the middle and end of the record, and fairly well at the start of the record, the worst locations provide a response which is a poor match over the entire record. So, the best locations provide a larger controllable space while the worst locations provide a small controllable space. The best locations are shown in cyan and the worst locations are shown in red in Figure 11. The worst locations are both on the centerline, which means they have no ability to affect certain modes, limiting the controllability.

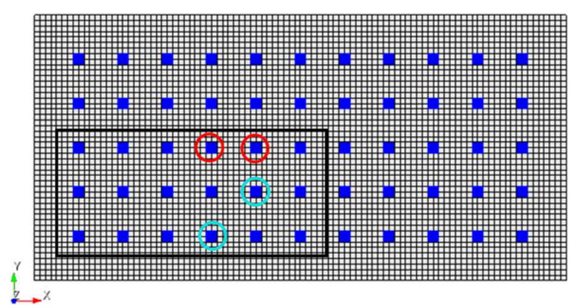

Figure 11: Candidate locations for the location search (black box region) and the best locations (cyan) and worst locations (red) identified

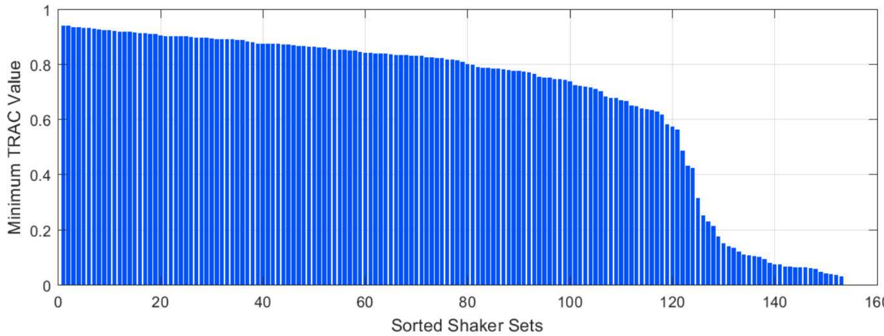

Figure 12: Minimum TRAC values for each of the 153 input location sets

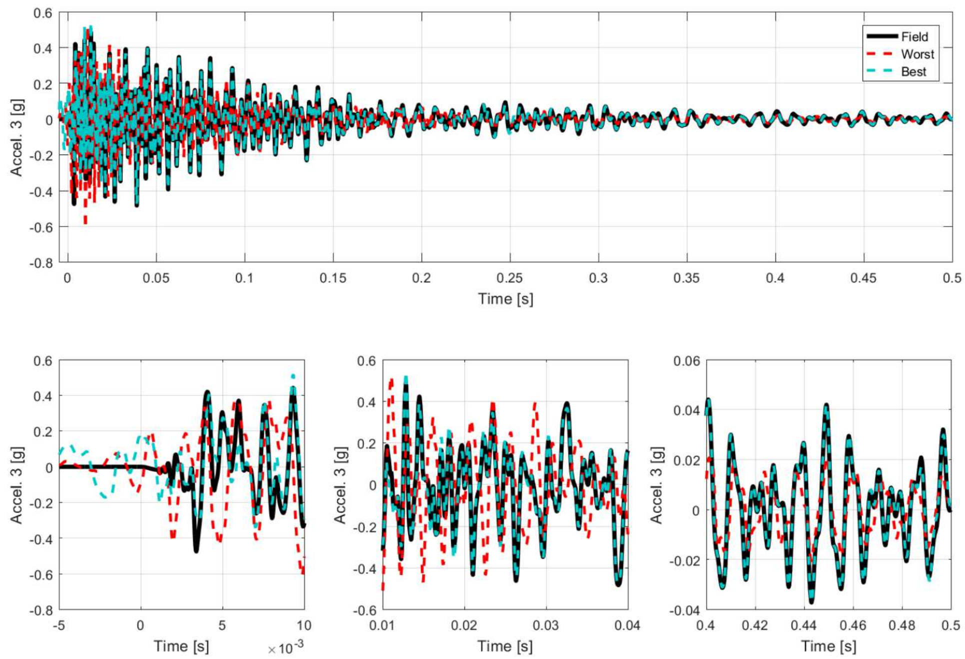

Figure 13: Response at accel. 8 when using 2 inputs at the best vs. worst locations

Discussion and Conclusions

Multi-input shock is still a niche capability. One step toward increasing its usage is developing an understanding of how this class of problems behaves, and how test design factors affect results. This work aimed to answer some of those basic questions, which arose during a multi-input shock test, using models and simulation. A plate model was used as an example dynamic system, and a simulated field shock environment was simulated with two haversine force inputs to the plate. A laboratory multi-input shock test was simulated by solving for forces at various locations on the plate which aim to best match the field response. It was found that simple evaluation of the linear system input estimation equation resulted in convolution errors in the estimated inputs, and therefore also the resulting responses. This manifested as errors at the start and end of the frame, with the end of the frame showing large errors. Simply zero-padding the target response time histories fixes this convolution error. The same behavior was encountered with experimental data of a plate test article with two shakers used to replicate a measured hammer hit. In terms of test design, it was observed that using more shakers can improve the match to the response, and the match is improved predominantly early in time; a small number of shakers was effective at matching the response later in time. Shaker locations were determined using an exhaustive search method to determine the best location of two inputs and a TRAC as the objective function. Results of this search indicate that there are many sets of locations which are result in relatively accurate responses, but here are some locations which result in poor replication of the target response. Importantly, the search was able to identify two shaker locations which allowed for very accurate replication of the target response. This exercise of models for multi-input shock test design indicate how models and simulations can be used to design an effective multi-input shock test and can explore the space of possible results including the effects of the number and location of shakers on the expected results.

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