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Using the Modal Craig-Bampton Procedure for the Test Planning of a Six-Degree-of-Freedom Shaker

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ABSTRACT

Recent research has shown the viability of using a six-degree-of-freedom (6-DOF) shaker to reproduce the system-level environments and stress fields exhibited by the component of interest. This paper presents a method to aid in the 6-DOF test planning process using the results derived from a free-free modal test of the component on a test fixture. This method incorporates the Modal Craig-Bampton (MCB) procedure that transforms the free-free modes to sets of fixed-base and rigid-body modes. To enable this modal transformation, the MCB procedure relies on an analytical model of the fixture and adequate instrumentation to capture the fixture's motion within the frequency band of interest. In its simplest form, the fixture may only consist of six rigid-body modes; however, it may also include fixture elastic modes. Subsequently, this transformation generates a transfer function between the component's response to the rigid-body modes of the fixture, thereby enabling a prediction of the 6-DOF shaker's ability to accurately reproduce the target response experienced by the component in a test environment. A numerical simulation predicts that driving the test component with a 6-DOF shaker accurately reproduces the response experienced during an acoustic test of a full system.

Keywords: Modal Analysis, Vibration Control, Six-Degree-of-Freedom Shaker, Modal Craig-Bampton, Test Planning

1 INTRODUCTION

The essence behind the Modal Craig-Bampton (MCB) procedure is to transform the modal coordinates derived from an experimental modal test to a linear combination of rigid-body and fixed-base modes. The primary purpose of this procedure is to ease the implementation of experimental dynamic substructuring that couples an experimental test article with an analytical model [1]. An alternative use for the MCB procedure includes deriving inputs for a vibration shaker that supplies base excitation to control the test article to a desired response [2, 3]. This abstract details a test-planning procedure that utilizes the MCB procedure with a modal test of the test article mounted on the test fixture to predict the 6-DOF shaker's ability to reproduce the component's response. The knowledge derived from this procedure can aid the test engineer in planning a vibration shaker test to better control the component to reproduce the desired response.

2 MODAL CRAIG-BAMPTION THEORY

The foundation of the MCB procedure is to derive a transformation matrix $\mathbf{T}$ that transforms the free-free modal coordinates $\mathbf{q}$ of the test article on the fixture to a linear combination of fixed-base modal coordinates $\mathbf{p}$ of the test article with the base motion constrained to zero and of rigid-body modal coordinates $\mathbf{s}$ that

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correspond to the rigid-body motion of the fixture:

$$\mathbf{q} = \begin{bmatrix} \mathbf{T}_p & \mathbf{T}_s \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} \tag{1}$$

Obtaining the transformation between $\mathbf{q}$ and $\mathbf{s}$ involves constraining the measured motion on the fixture to the motion derived from an analytical model of the stand-alone fixture. As the fixture is generally designed to be stiff compared to the test article, there is minimal elastic motion in the fixture itself, and the motion of the fixture can primarily be resolved using the six rigid-body modes derived using the mass properties (i.e., center of mass and moments of inertia) of the stand-alone fixture. This constraint leads to the transformation

$$\mathbf{q}_s = \underbrace{\mathbf{\Phi}_f^\dagger \mathbf{\Psi}_f}_{\mathbf{T}_s} \mathbf{s} \tag{2}$$

where $\mathbf{q}_s$ is the modal motion corresponding to the rigid-body motion, $\mathbf{\Phi}_f$ is the set of free-free experimental mode shapes corresponding to the measured fixture degrees of freedom, and $\mathbf{\Psi}_f$ is the set of rigid-body mode shapes derived from the mass properties of the fixture that also correspond to the measured fixture degrees of freedom. Also, the superscript $\dagger$ denotes the Moore-Penrose pseudoinverse.

Obtaining the transformation between $\mathbf{q}$ and $\mathbf{p}$ utilizes a slightly more complicated procedure that requires identifying a constraint matrix that enforces zero motion at the fixture [4]. The full derivation is omitted here for brevity, but can be found in [2]. The end result is the transformation

$$\mathbf{q}_p = \underbrace{\mathbf{L}\mathbf{\Gamma}}_{\mathbf{T}_p} \mathbf{p} \tag{3}$$

With the components of $\mathbf{T}$ defined, applying this transformation in the frequency domain results in a coupling between $\mathbf{p}$ and $\mathbf{s}$. Rearranging the resulting equations to solve for $\mathbf{p}$ with respect to the prescribed shaker inputs $\mathbf{s}$ results in

$$\mathbf{p} = \underbrace{-\left\lceil \frac{1}{\omega_{\text{fix},j}^2 - \omega^2 + i2\zeta_{\text{fix},j}\omega_{\text{fix},j}\omega} \right\rfloor \left[\mathbf{K}_{ps} + i\omega \mathbf{C}_{ps} - \omega^2 \mathbf{M}_{ps} \right]}_{\mathbf{H}_{ps}} \mathbf{s} \tag{4}$$

where $\lceil \cdots \rfloor$ denotes a diagonal matrix, $\mathbf{H}_{ps}$ is the transfer function relating the elastic modal motion of the test article to the shaker inputs, and ω_{fix} is the fixed-base natural frequencies that are estimated with the MCB procedure. This transfer function provides a measure of how each of the elastic modes respond to the various shaker inputs. Converting back to physical coordinates results in the transfer function $\mathbf{H}_{xs}$ that relates the physical response of the component $\mathbf{x}_c$ and $\mathbf{s}$:

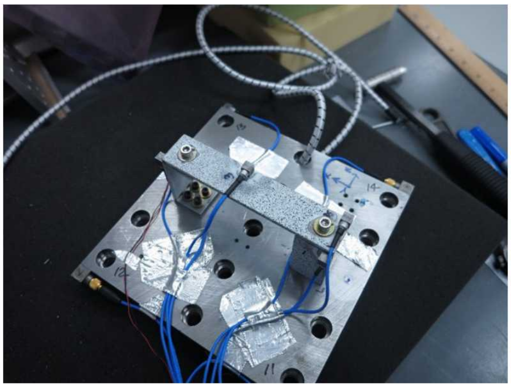
$$\mathbf{x}_c = \underbrace{\mathbf{\Phi}_c(\mathbf{T}_p \mathbf{H}_{ps} + \mathbf{T}_s)}_{\mathbf{H}_{xs}} \mathbf{s} \tag{5}$$

This equation facilitates an estimation of the required input PSD $\hat{\mathbf{S}}_{ss}$ given a target response $\mathbf{S}_{xx}$ located at the control points on the component:

$$\hat{\mathbf{S}}_{ss} = \mathbf{H}_{xs}^\dagger \mathbf{S}_{xx} \left(\mathbf{H}_{xs}^\dagger\right)^H \tag{6}$$

3 MODAL TESTING

A modal test of the component on the fixture generates all of the terms that comprise $\mathbf{H}_{xs}$. Figure 1 shows a picture of the test setup where the fixture was suspended by a bungee cord to approximate a free-free boundary condition. The test also included 4 triaxial accelerometers located at various locations on the component and 4 additional triaxial accelerometers located at the corners of the fixture. Characterizing the system up to 2000 Hz required hammer impacts at multiple locations; Fig. 1 shows the modal parameters extracted from the tests.



Mode	f_n [Hz]	ζ [%]
1	389.8	0.19
2	1044	0.49
3	1140	0.83
4	1648	0.97
5	1827	0.33
6	2013	0.16

Figure 1: Test setup of the RC mounted on the fixture (left) and a list of the modal parameters (right)

4 SHAKER PERFORMANCE PREDICTIONS

The performance predictions of the 6-DOF shaker utilized the component's response originating from an acoustic test of the Modal Analysis Test Vehicle (MATV) conducted at the Atomic Weapons Establishment (AWE). This hardware consisted of a conical shell encapsulating the component mounted to a flat plate. A description of the hardware used in the acoustic test can be found in [5].

Figure 2 shows the autospectral densities (ASDs) for each of the twelve acceleration responses on the component for both the truth data derived from the MATV acoustic test and the estimated response on the 6-DOF shaker. These results show that the 6-DOF shaker can accurately reproduce the response at the control DOFs at nearly all frequency lines.

This procedure can also derive performance estimates for single-axis shakers by constraining all DOFs but the active DOF to zero. Figure 3 shows the ASDs for each acceleration response for both the truth data and the optimal shaker input when only considering shaker motion in the x-direction. Limiting the excitation to only the x-direction reproduces only the low-frequency responses in the x-direction; however, there are significant discrepancies in all other directions. In this case, an advantage of the 6-DOF shaker is the ability to reproduce the responses in all directions simultaneously with a single test.

5 CONCLUSIONS

This abstract outlined a method for planning vibration tests on 6-DOF shakers using the results from a modal test of the component mounted on the fixture. Combining these results with the Modal Craig-Bampton procedure enabled an estimation on a shaker's ability to excite the component to reproduce a desired response. The performance predictions show the increased ability of a 6-DOF shaker to reproduce the required responses when compared to classical shaker testing in a single direction.

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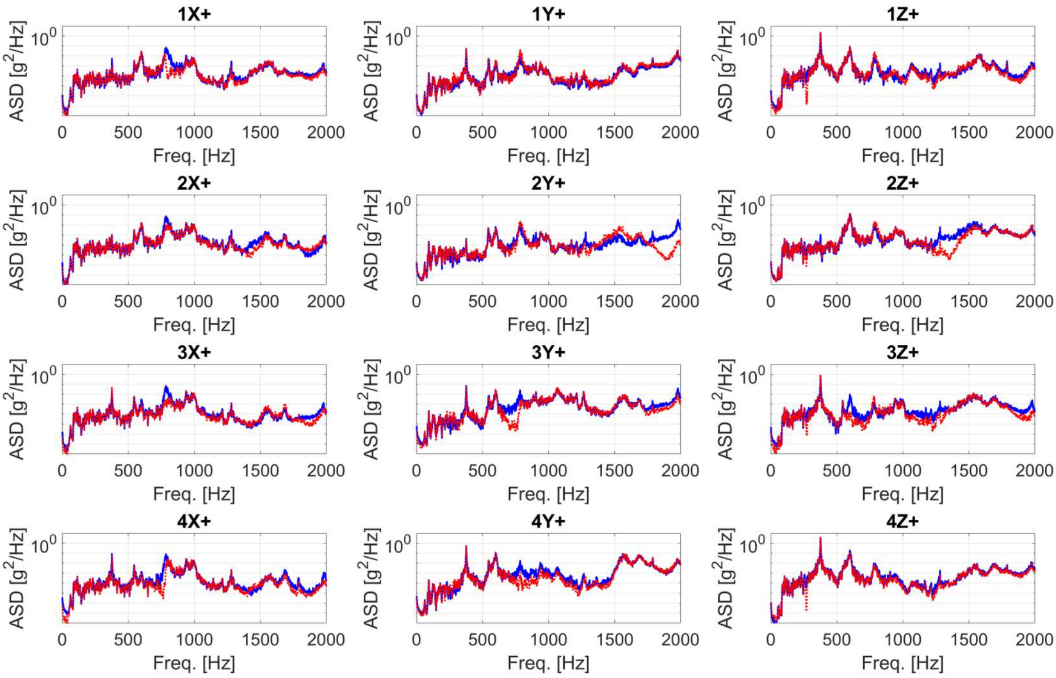

Figure 2: ASDs for each of the twelve acceleration responses on the RC. These plots include the truth data derived from an acoustic test (blue) and the estimated response of the MCB model to a controlled base input response on the 6-DOF shaker (red).

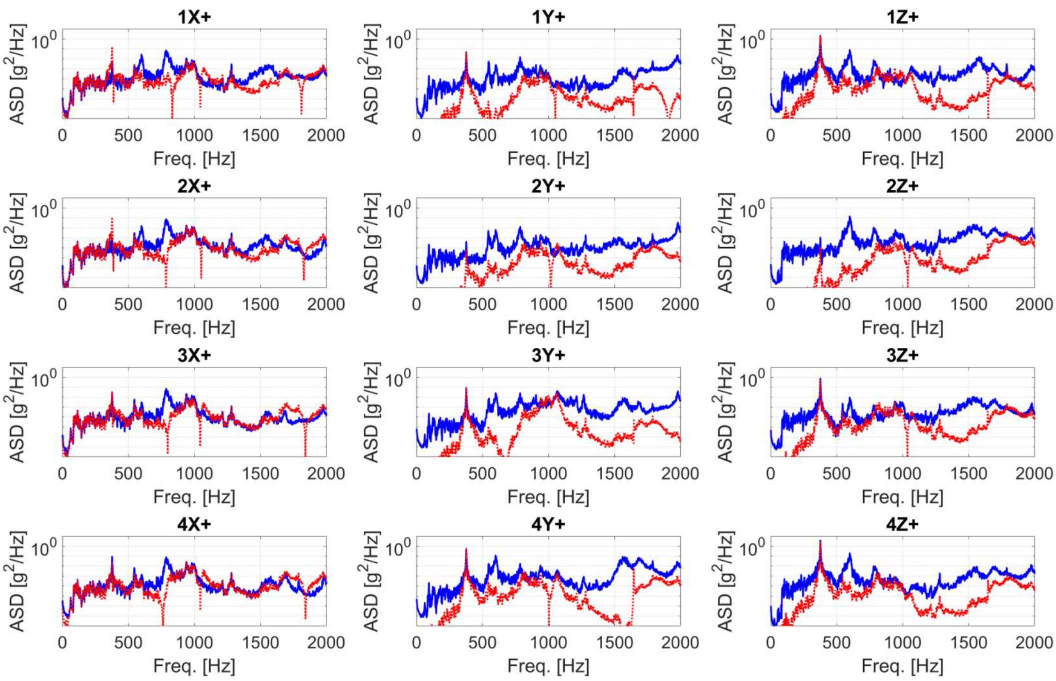

Figure 3: ASDs for each of the twelve acceleration responses on the RC. These plots include the truth data derived from an acoustic test (blue) and the estimated response of the MCB model to a controlled base input with the shaker motion constrained to the x-direction (red).

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