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How Modal Analysis Can Bring Insight to Vibration Testing

PRESENTED BY

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Motivation for Understanding Modal Response for Vibration Testing

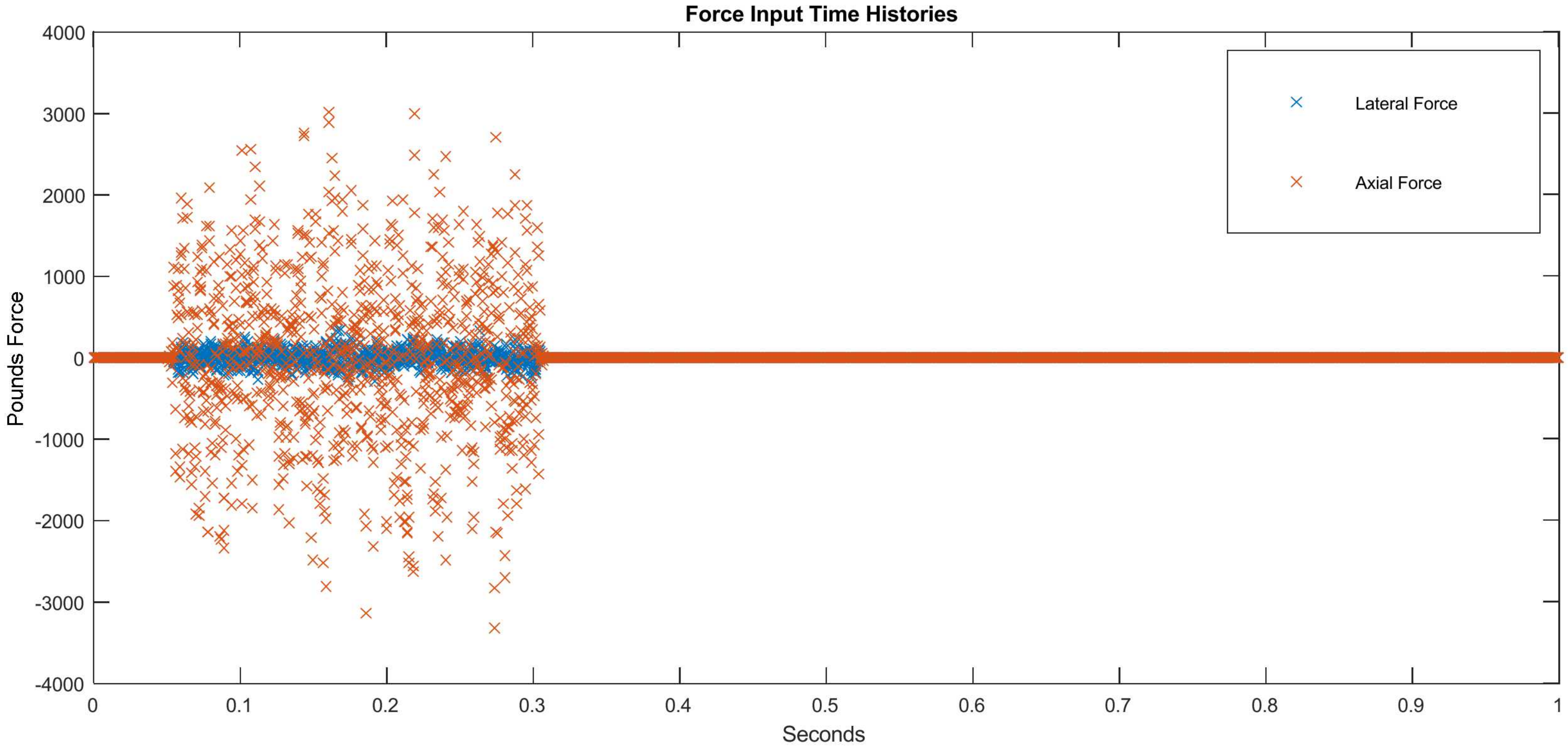
- Vibration qualification testing is designed to make sure the component (or payload) will survive the field environment.
- Large uncertainties exist because:
  1. The sensors from field test:
     - Were in the wrong location
     - Were too few
     - Ignored rotations
  2. Enveloping of measurements may add an unknown amount of conservatism
  3. Laboratory shaker environments, particularly on single axis shakers can significantly change the input from that experienced in the field because of constraints on 5 DoF. (This is true even if the shaker test "perfectly" controls to one DoF response. Response and/or force limiting are sometimes applied for high cost prototypes to mitigate part of this problem).
- Sometimes we unnecessarily break parts and force re-design due to these large uncertainties
- Although system level responses are quite complex, even in the modal domain, great insight can be obtained with a handful of modes at the component level. This understanding is useful at the specification and vibration testing level.
- Here we demonstrate analytically and with hardware that a few modal quantities can provide significant insight into both the system environment and the laboratory testing environment from the perspective of the component response.

Analytical Example

- We utilize a model of a 20 foot long rocket with a base mounted component.
- The nozzle of the rocket is forced with 1000 lbf rms random input in the axial and 100 lbf rms in the lateral direction up to about 1000 Hz.
- FE beam models are utilized in the 2 dimensional response.
- Three rigid body modes
- First bending mode is 21 Hz
- We animate the acceleration response to 2000 Hz.

Force input

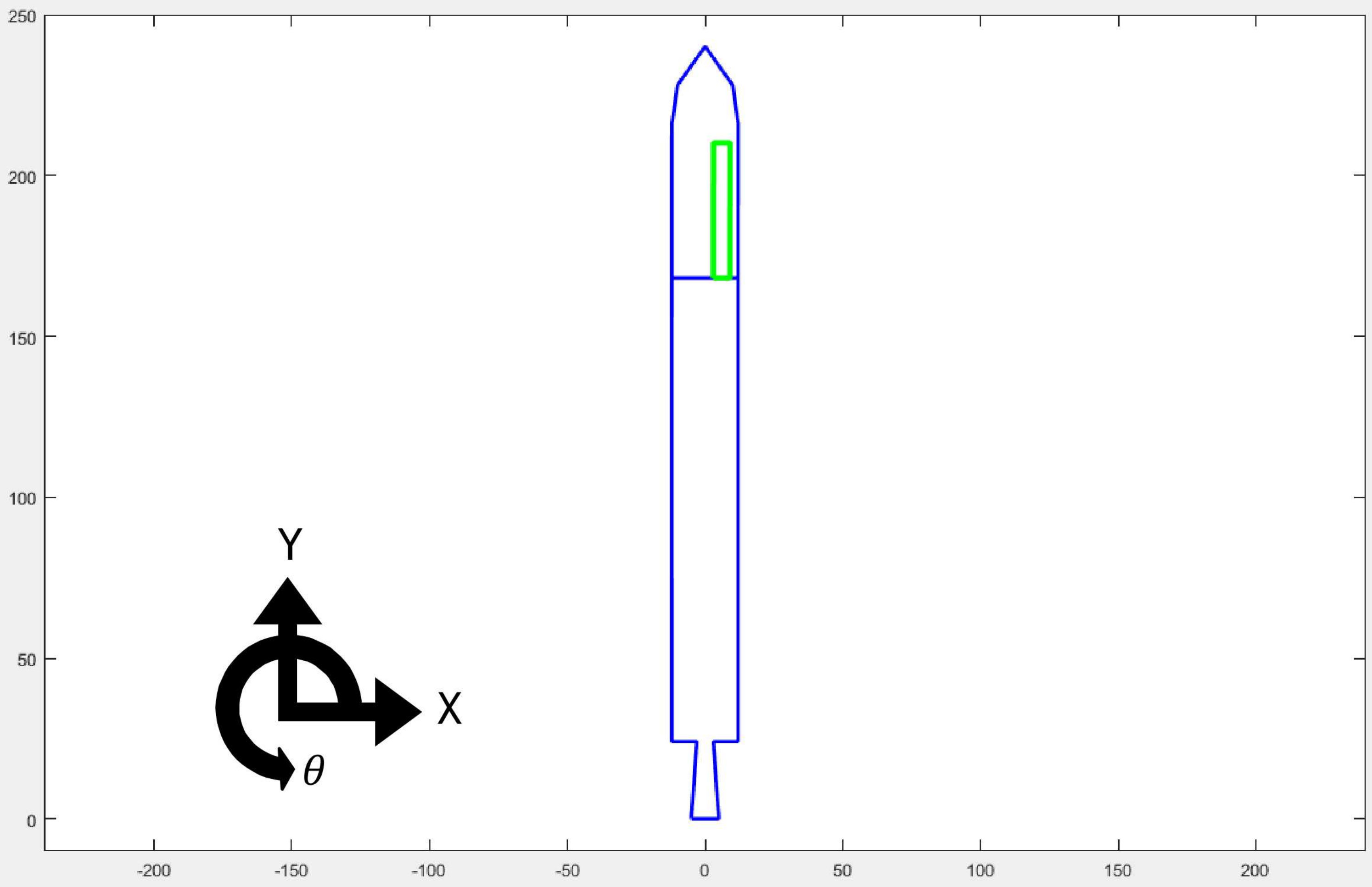
- Time Histories



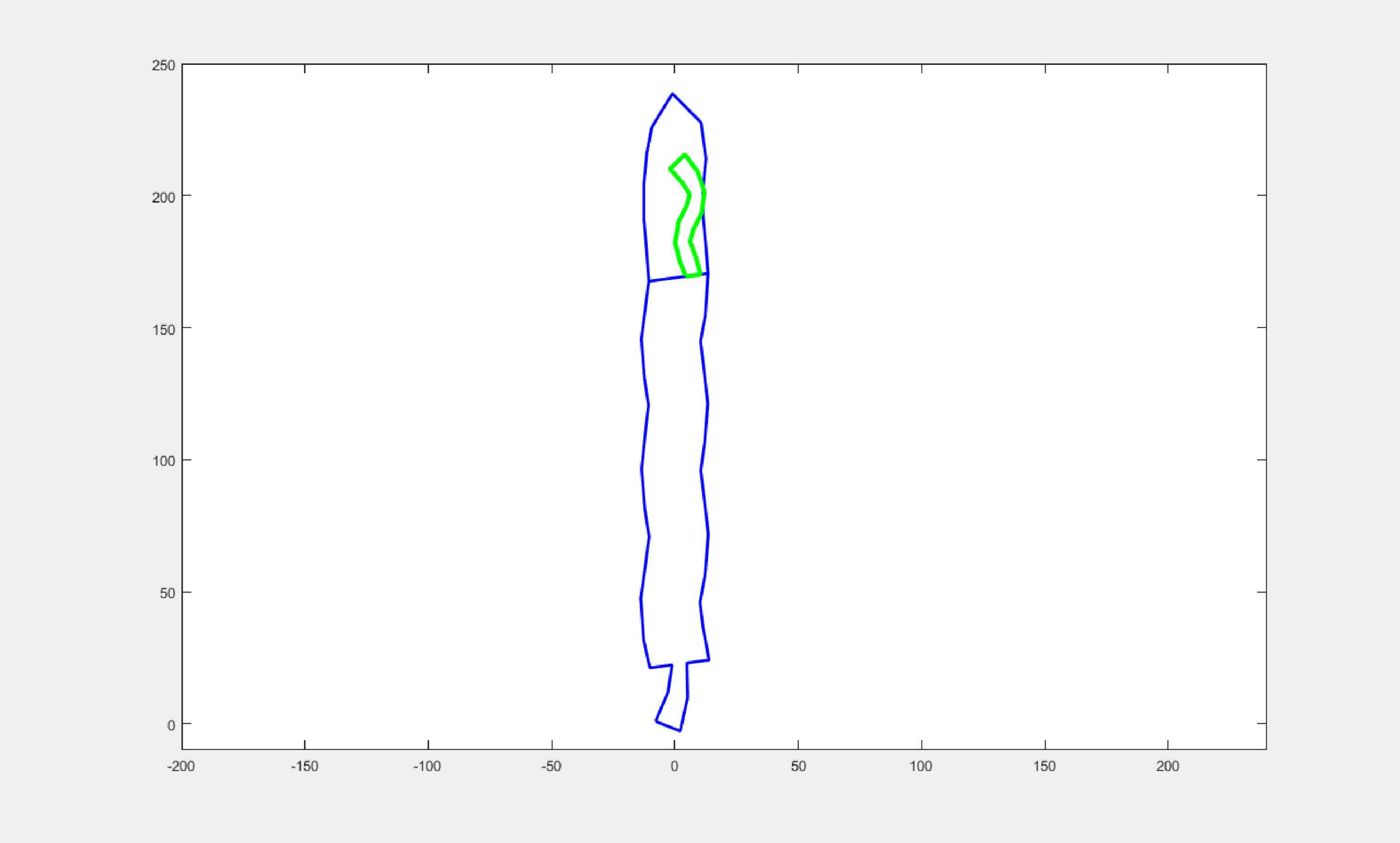
Analytical Example Response

- Question 1 – How many modes do you see active in the rocket system
- Question 2 - How many modes do you think can describe the motion of the component alone?
- Question 3 – How many modes do you think it takes to describe just the base motion of the component?

Random acceleration response to 2000 Hz due to nozzle force



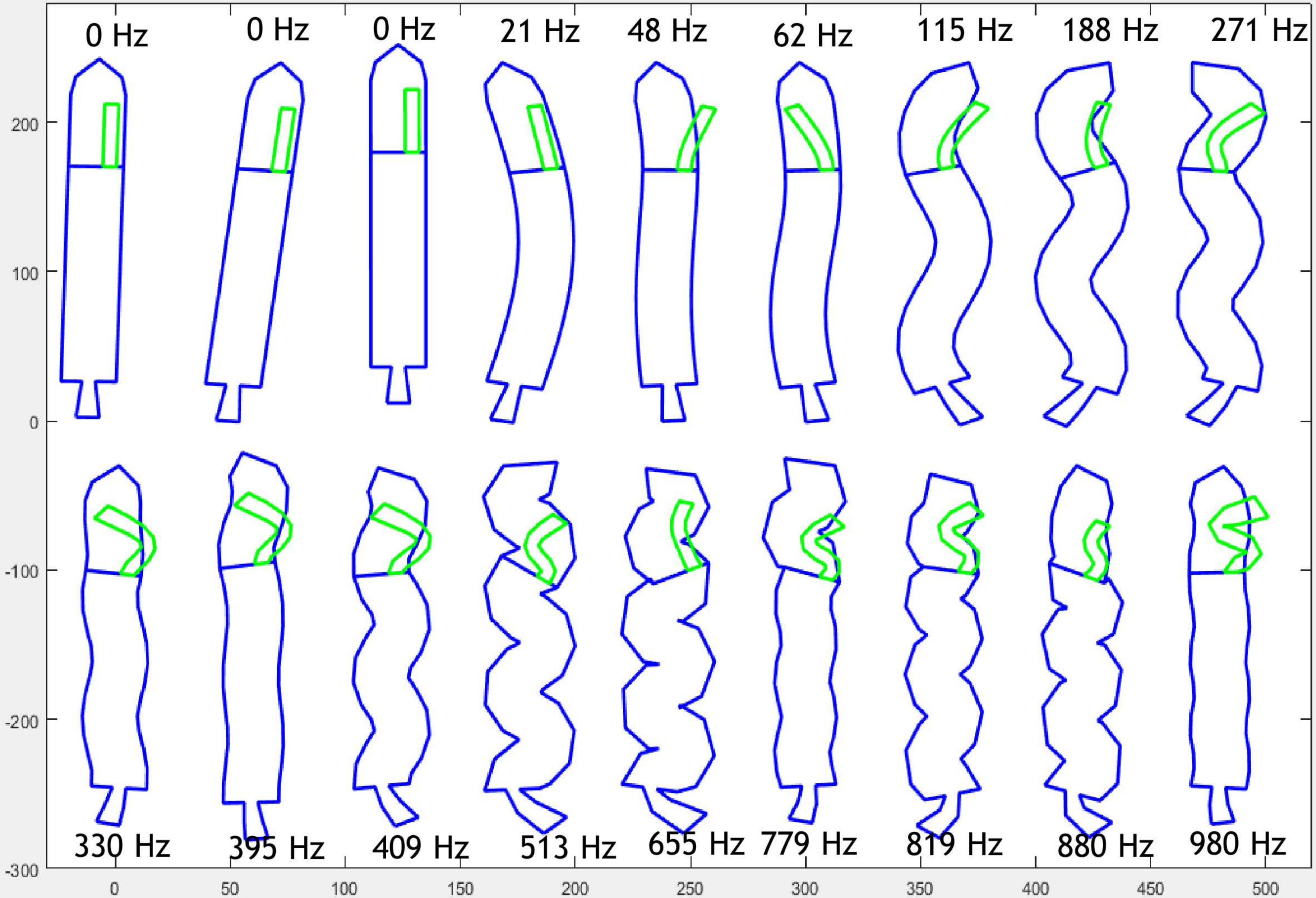
Approximately how many bending modes from this snapshot in time?



Data for Questions 1,2,3

- Question 1 – How many modes do you see active in the rocket system
- Question 2 - How many modes do you think can describe the motion of the component alone?
- Question 3 – How many modes do you think it takes to describe just the base motion of the component?
- There are 18 modes below 1000 Hz, 25 below 2000 Hz – too many to intuitively separate in the human mind of most engineers
- Base motion can be described by a lateral, vertical and pitch rigid body mode at that axial station
- The first 18 modes are plotted on the next slide – How many shapes are needed to describe the component motion?

Rocket Modes up to 2000 Hz



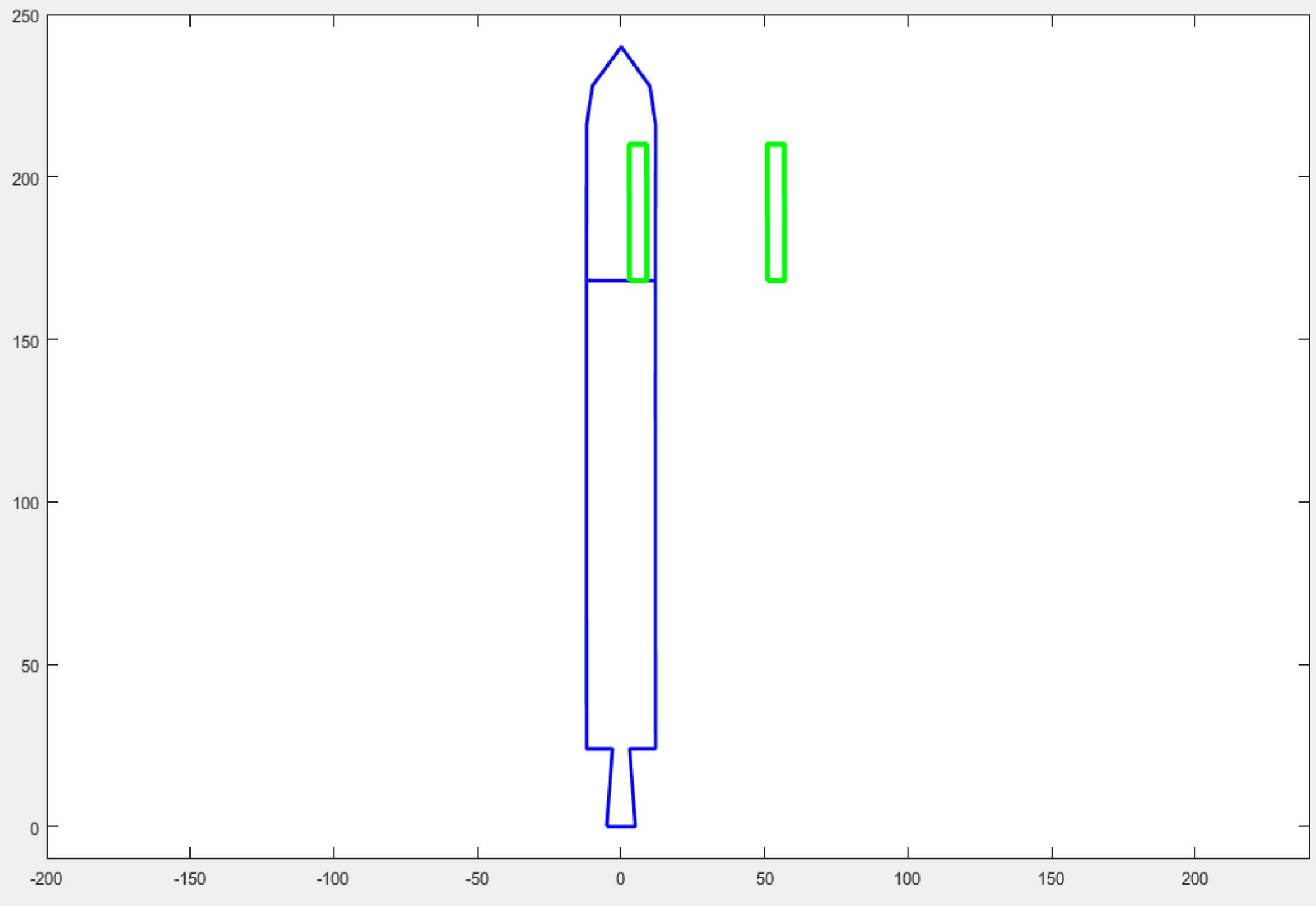
What have we learned so far

- Full system response is pretty complicated
- Component response looks like it be can approximately be reproduced by linear combination of a few modes (6,7,8?)
- The component elastic response that produces damaging strain can be reproduced by removing the rigid body modes from the few modes above, i.e. elastic response contained in even fewer modes (3,4,5?)

Approximate the component flight response with component mounted on a base fixture

- Aluminum fixture 12” wide, 2” thick, 24” deep
- Three rigid body modes
- Four elastic modes
  1. First bend 115 Hz
  2. 2nd bend 359 Hz
  3. 3rd bend 958 Hz
  4. Axial 1213 Hz

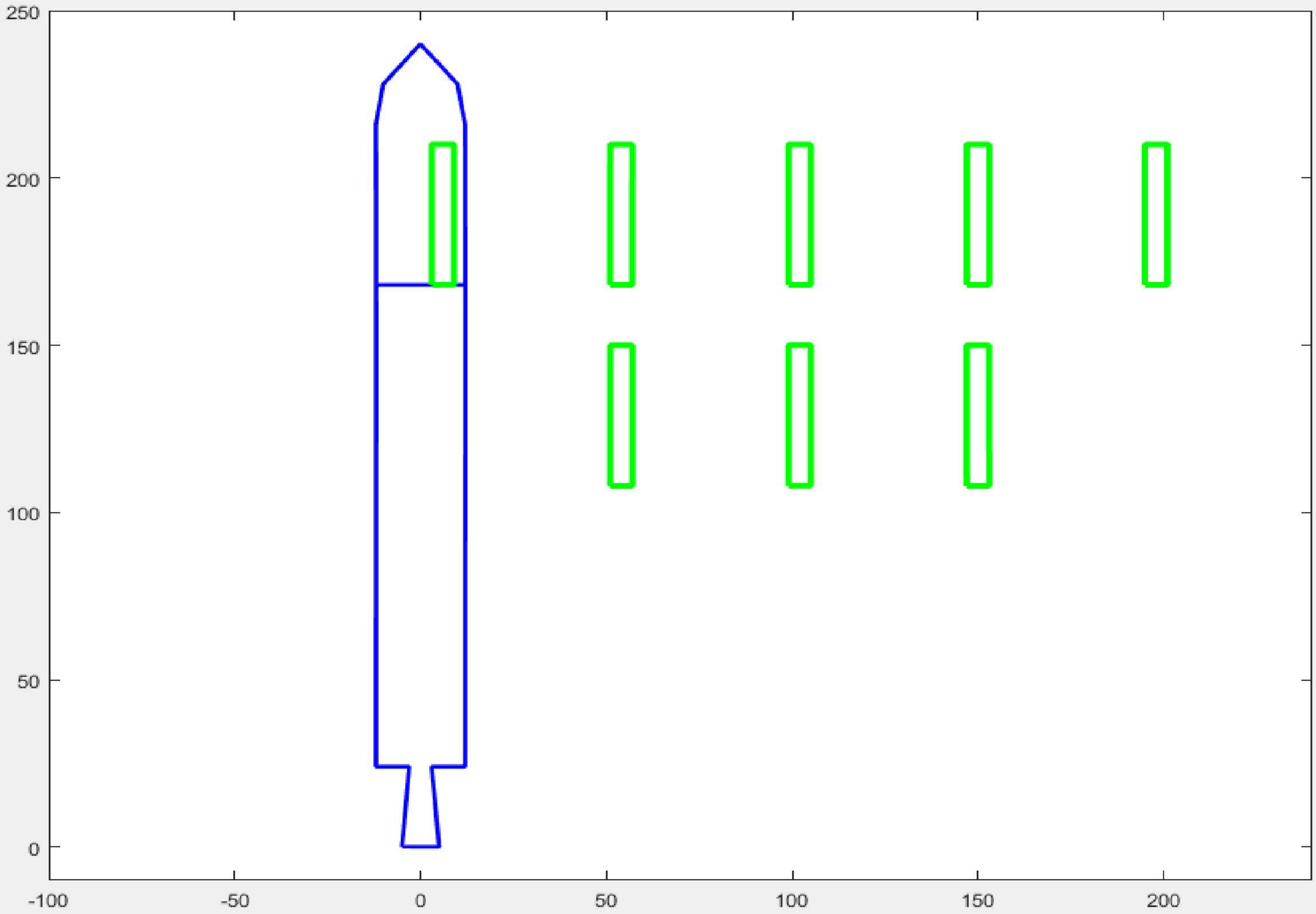
Rocket Field Response and free modes of beam on fixture fit



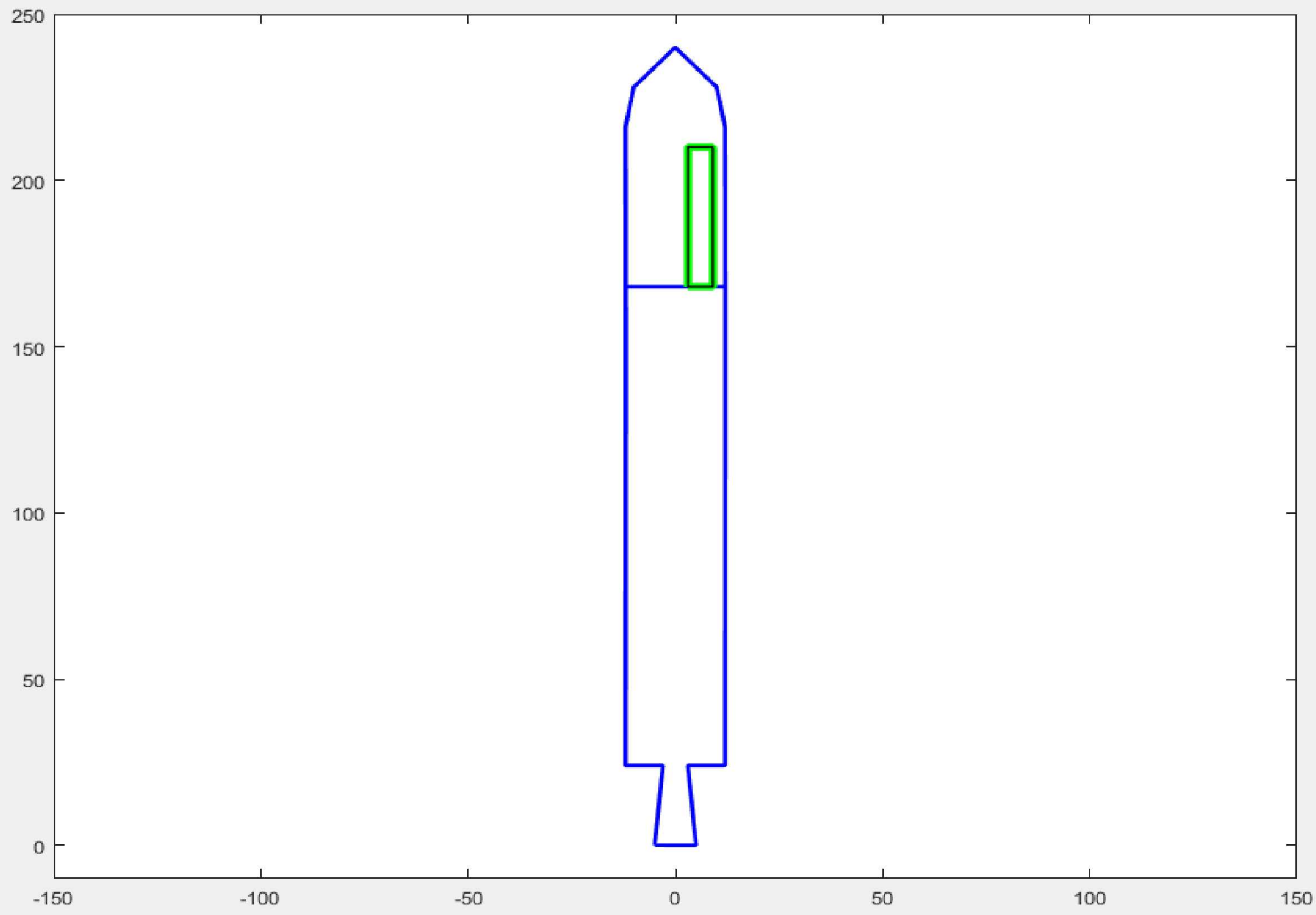
Let's convert to our 3 DoF Laboratory

- Convert 3 rigid body modes to the X, Y and theta responses of the shaker table
- Add to the rigid body modes four fixed base modes which will capture the elastic response
- Calculate the fixed base component frequencies
  - Wavespeed in aluminum 200,000 in/sec
  - L=42"
  - Axial frequency is wavespeed/(4*L)
  - EI=1e7 (psi) * .4*3^3/12 (in^4)
  - m (mass/length)=.1*.4*3/386 lbf-sec^2/in
  - Bending frequency is $\frac{\lambda_i^2}{2\Pi L^2}(\frac{EI}{m})^{.5}$
    - $\lambda_1 = 1.875$
    - $\lambda_2 = 4.694$
    - $\lambda_3 = 7.855$
- Now find linear combination of these 3 rigid body modes and 4 fixed base modes to attempt to fit component flight data
- Plot our approximate rigid body/fixed base modal response next to flight

Rocket Field Response and rigid body/fixed base modes isolated

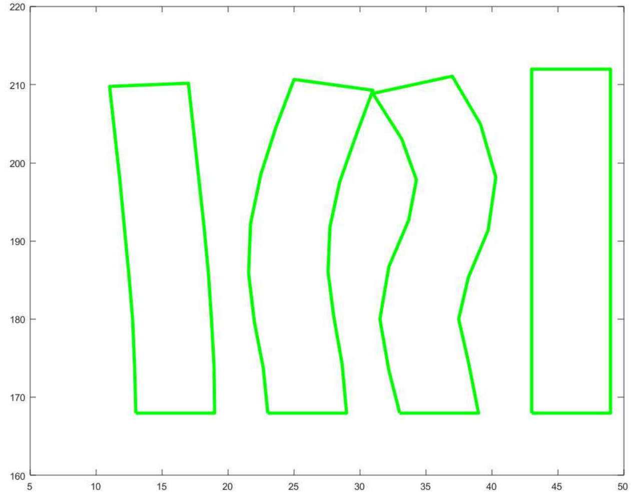


Rocket Field Response and rigid/fixed base modes fit (black line)



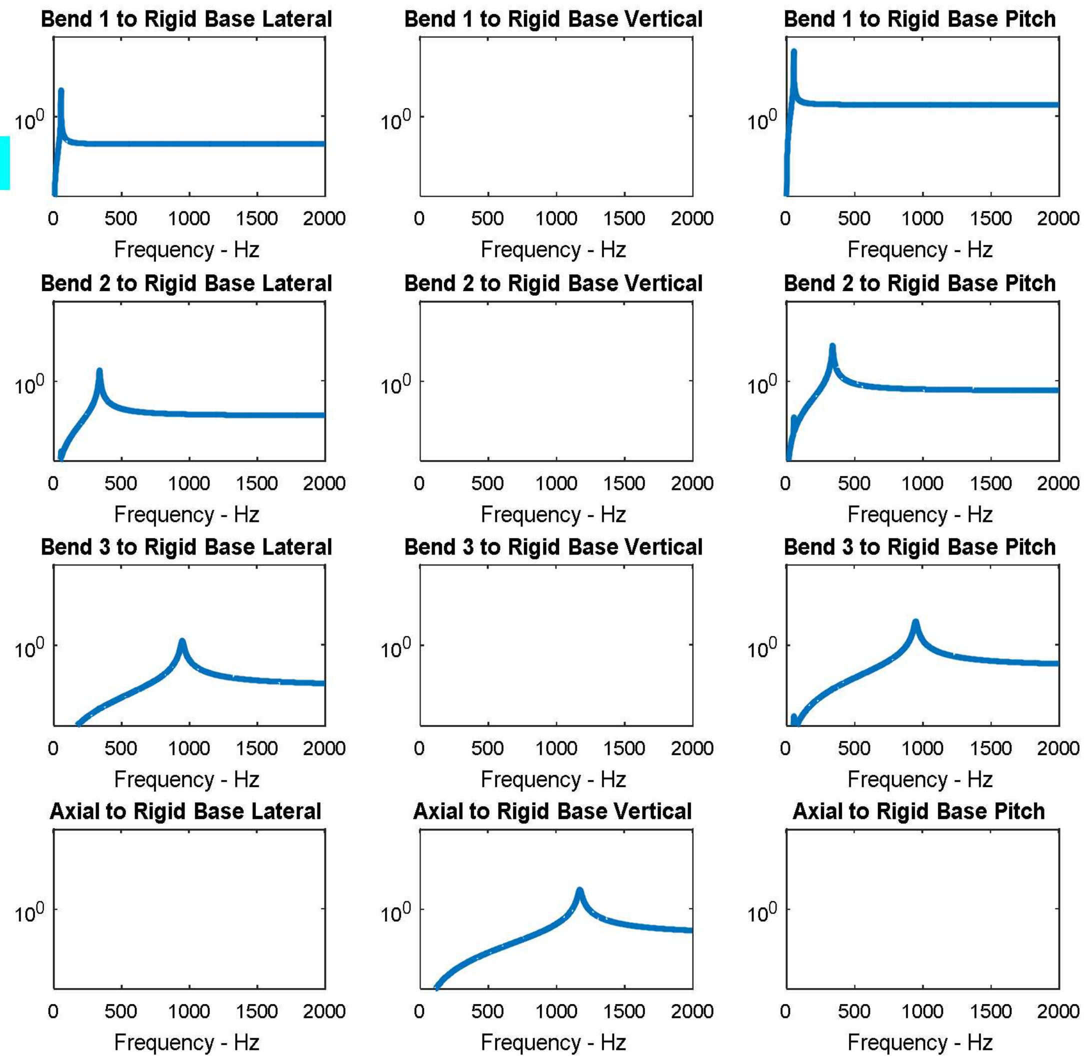
What have we learned

- We can get a pretty good simulation of component field response with just the rigid body modes and 4 fixed base modes that would be active on a 3 DoF shaker
- Insight into the component motion is quite strong
  - Damaging elastic strain response is captured with just 4 fixed base modes
  - 3 DoF table drive motion required to match field response is contained in the rigid body mode response referenced to the vibration table fixture x,y,theta coordinate system
- A significant portion of the response was driven by rigid body pitch (which is generally ignored in laboratory tests)



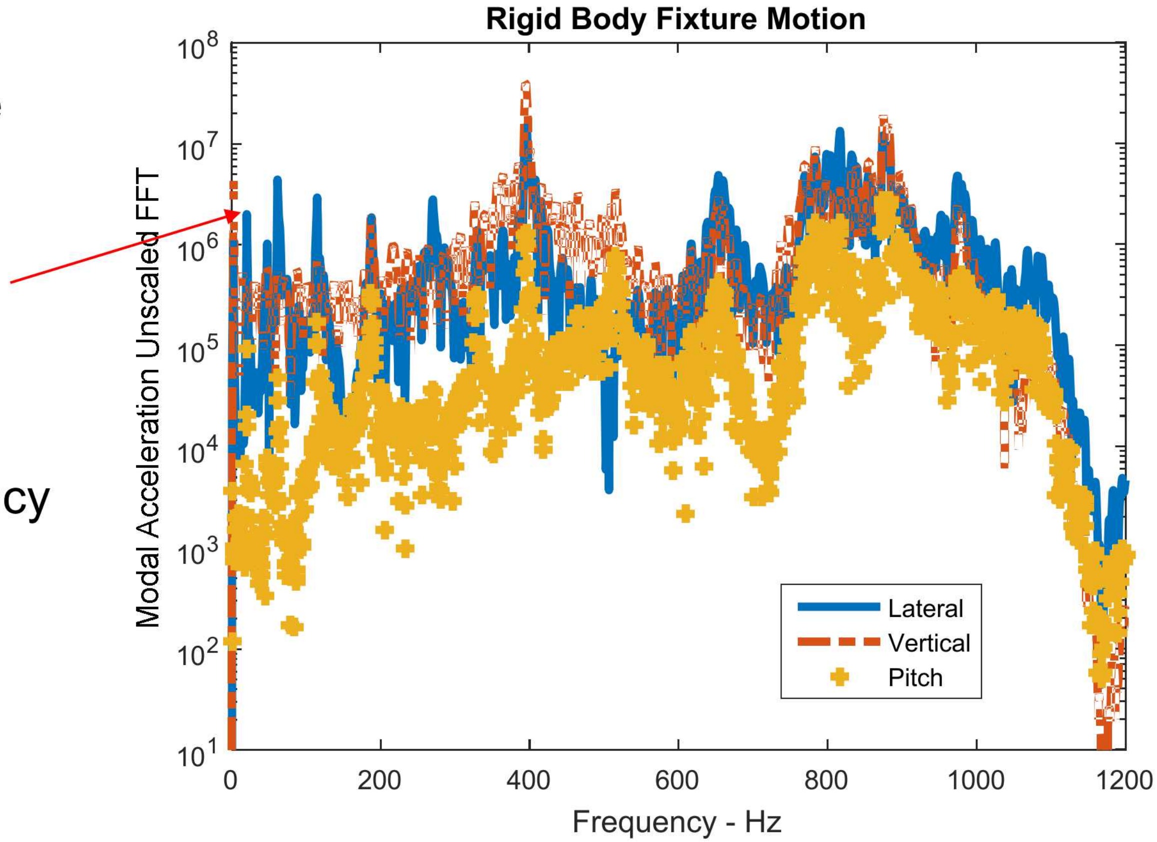
Now let's look at squiggly lines since we have a basic physical understanding of the 3 rigid body and 4 fixed base mode shape

- From a FE model, a free modal model of the component on fixture, or an uncorrelated buzz test on a 6 DoF shaker we can extract the transmissibility matrix between rigid body inputs and fixed base mode outputs
- Scale is .001 to 1000
- Bend 1 – 54 Hz
- Bend 2 – 339 Hz
- Bend 3 – 938 Hz
- Axial – 1172 Hz
- Blank plots are many orders of magnitude down



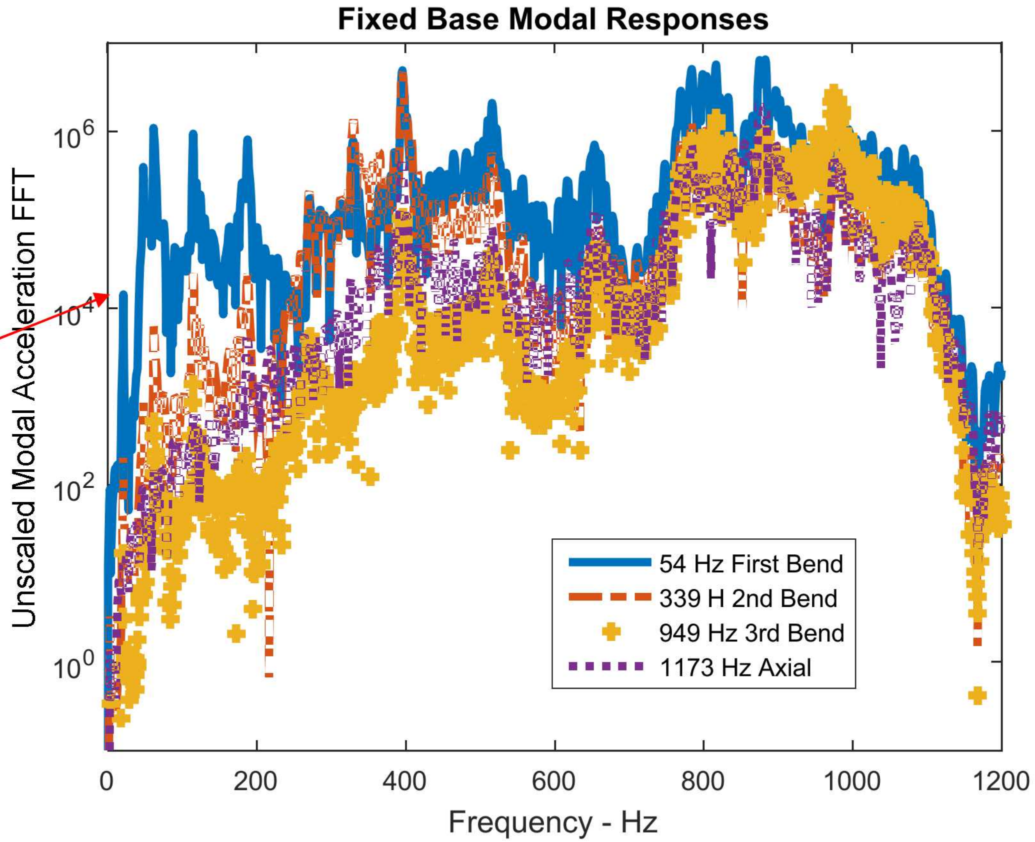
Modal Input for our 3DoF shaker - FFTs

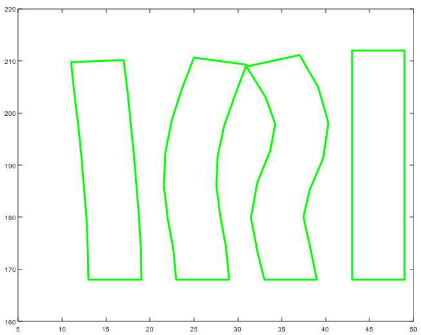
- Note that ALL input DoF have significant response throughout the 1000 Hz excitation band
- By driving the base input significantly, we can simulate field modal responses, e.g. 21 Hz is first rocket modal frequency
- No peaks appear at the fixed base frequencies



Modal Fixed Base Responses - FFTs

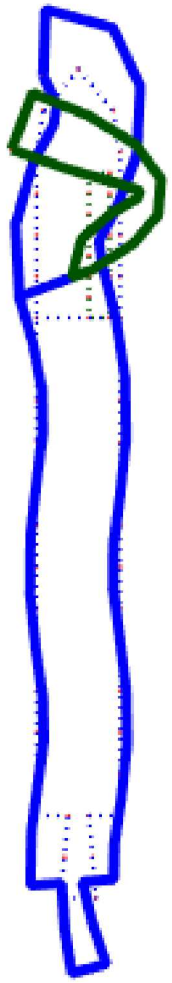
- Note that ALL modal DoF have significant response throughout the 1000 Hz excitation band
- These fixed base modal responses contain the strain in the component
- By driving the base input significantly, we can simulate field modal responses to forces, e.g. 21 Hz is first rocket modal frequency
- No peaks appear at the fixed base frequencies
- Check out the most notable frequency is about 395 Hz – Axial Mode of the Rocket





395 Hz Rocket Response

- Axial mode has base input pitch and lateral as well as axial for component

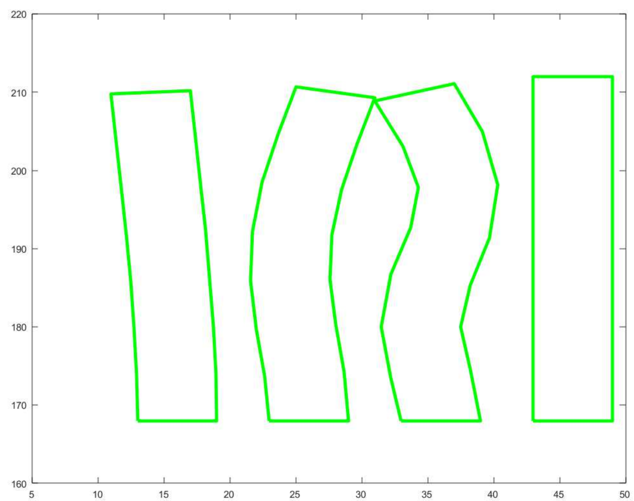


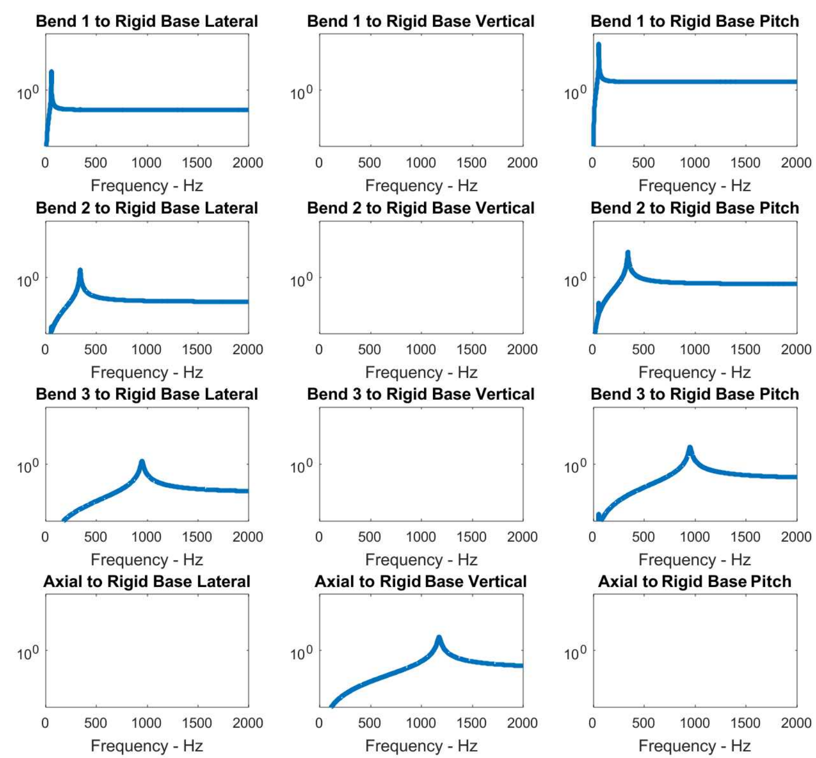
How well can you reproduce the component responses

If we know the fixed base modal response, we can back out the base input needed to best match it with our transmissibility matrix

- Once we know the fixed base responses we desire, we can back out the base input required to best fit the fixed base modal response
- Conversely, if the field response put in both *x* and θ to obtain a response and we only measured the *x* input, by inputting ONLY *x* we will get a very UNCERTAIN simulation

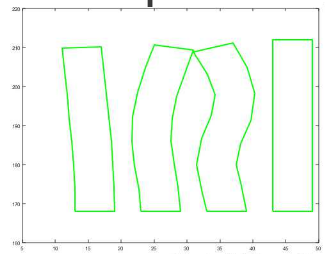
$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{bmatrix} \begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix}$$





Fixed base modal response FFT to perfectly controlled x input only (no theta or vertical input)

- Uncertain Simulation



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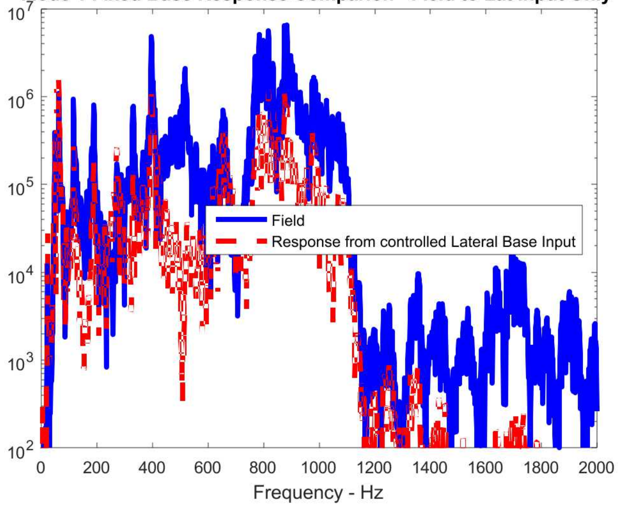
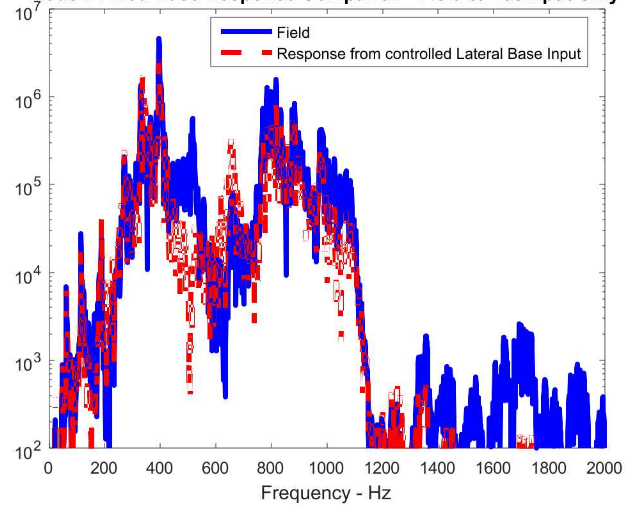
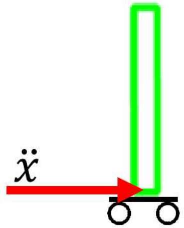
Mode 1 Fixed Base Response Comparion - Field to Lat Input Only

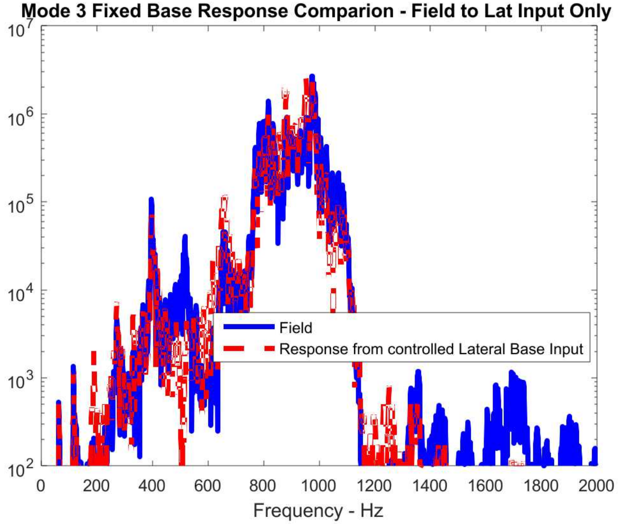


Mode 2 Fixed Base Response Comparion - Field to Lat Input Only

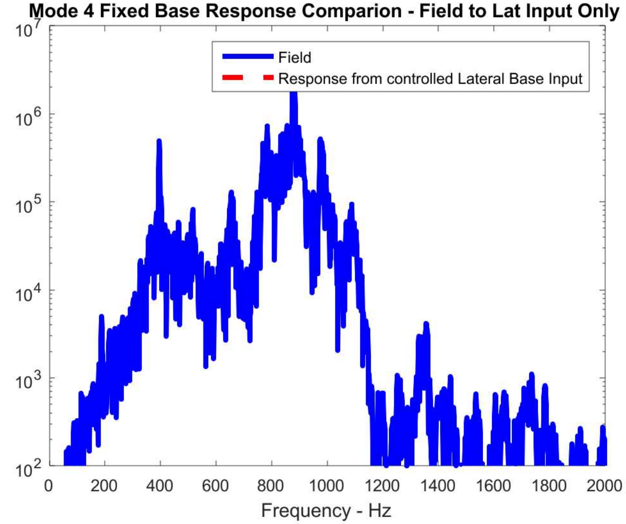




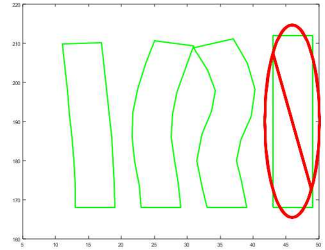
Mode 3 Fixed Base Response Comparion - Field to Lat Input Only



Mode 4 Fixed Base Response Comparion - Field to Lat Input Only



Fixed base modal response FFT to controlled x input least squares fit to fixed base modes 1-3



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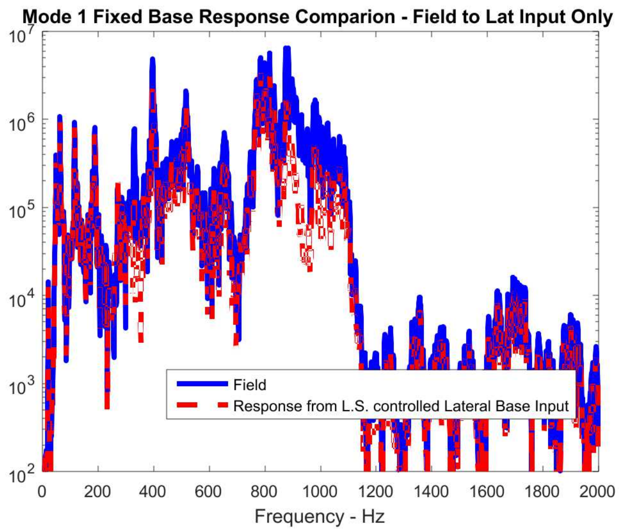
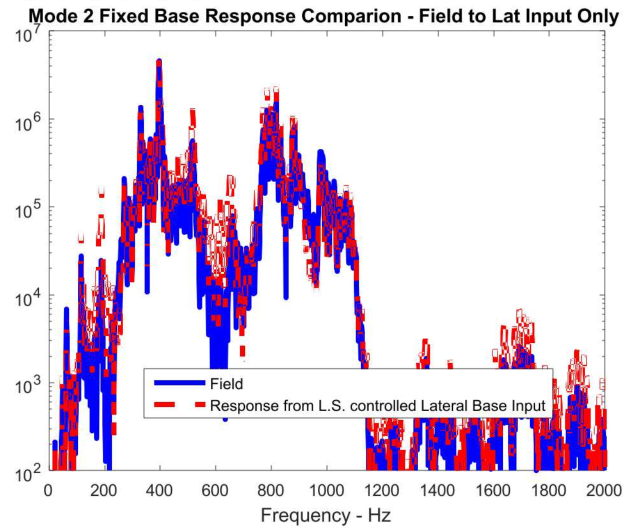
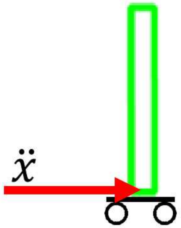
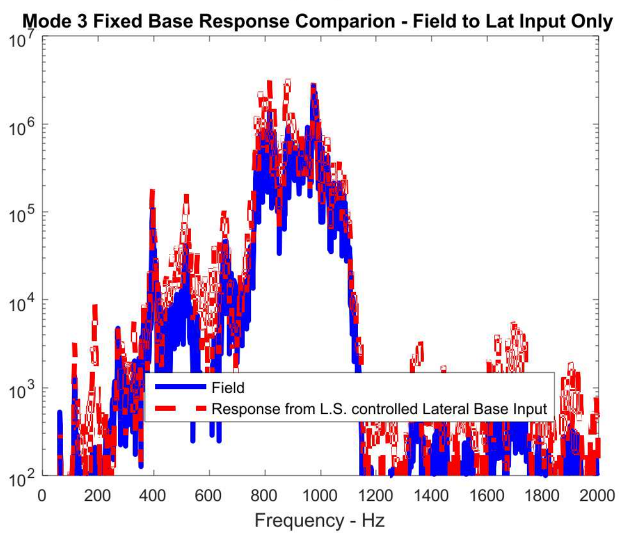
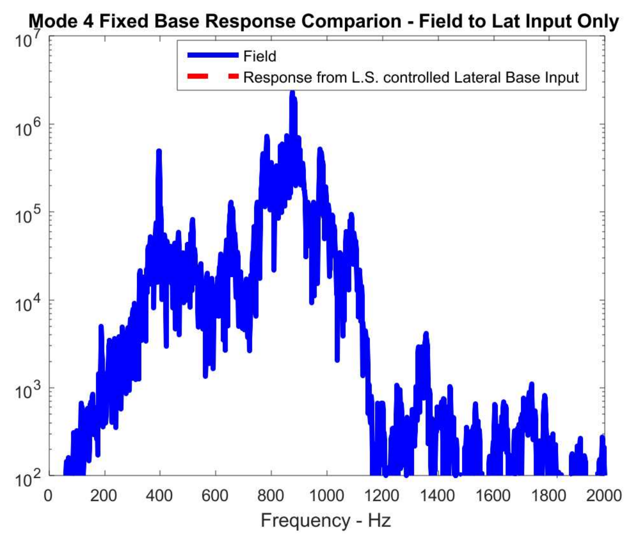
- Closer Simulation





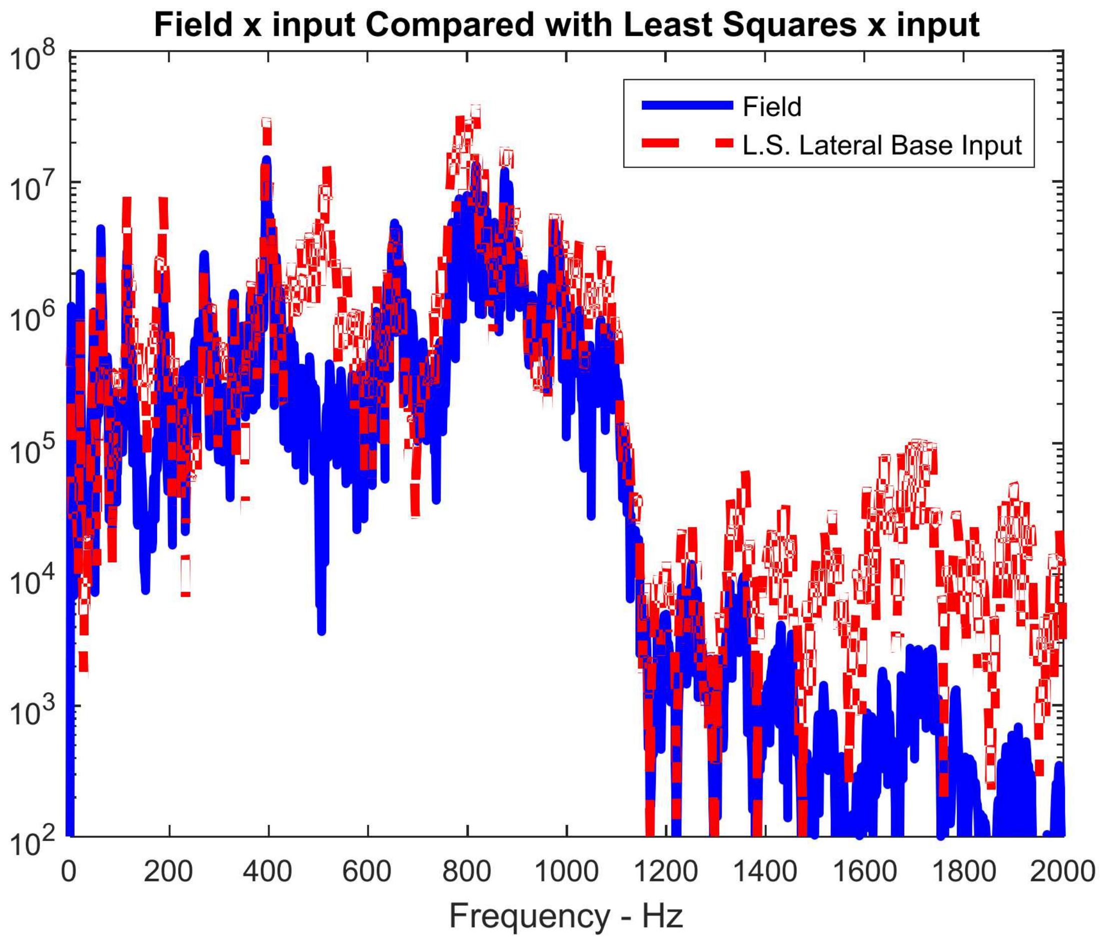


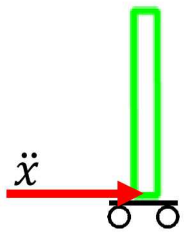




Compare field x FFT input to least squares x FFT input

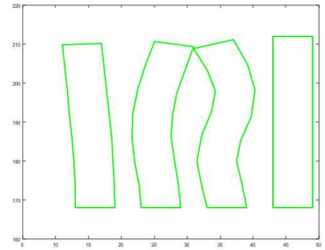
- Least squares x input has to compensate for lack of theta input





Fixed base modal response FFT to Least Squares 45 degree input only (part lateral part axial)

- 45 degree input both lateral and axial



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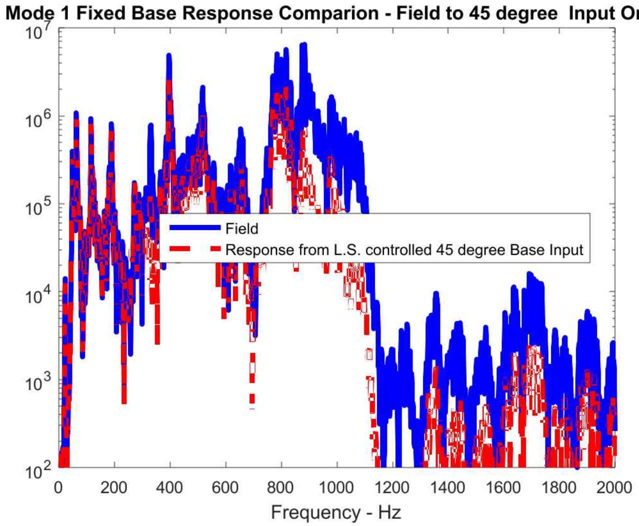
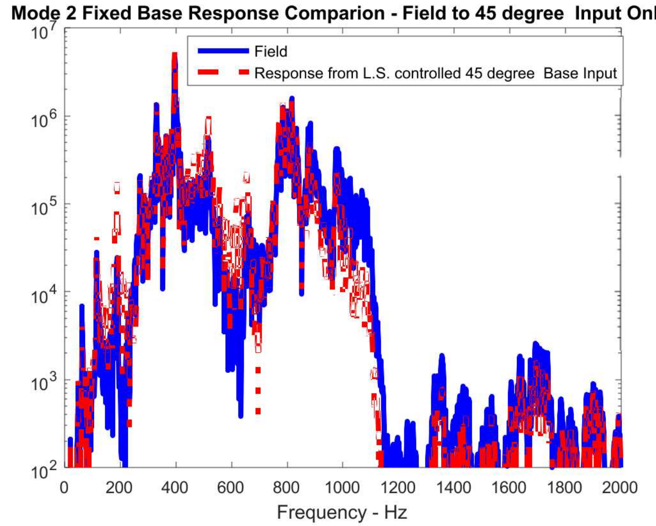
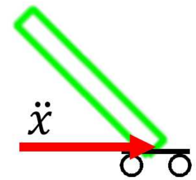
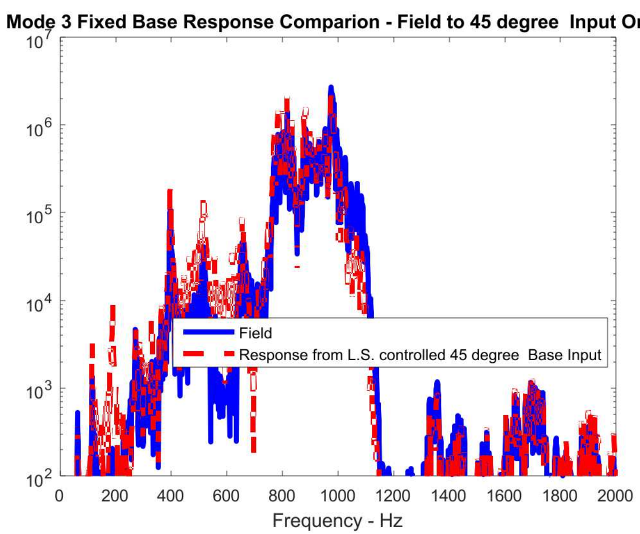
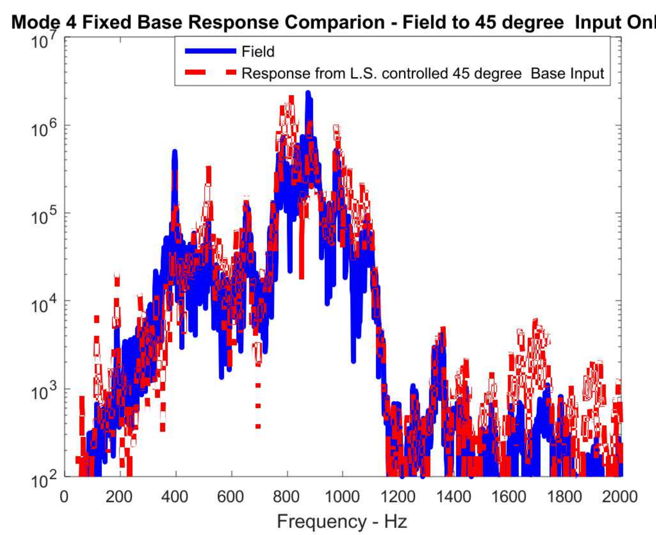




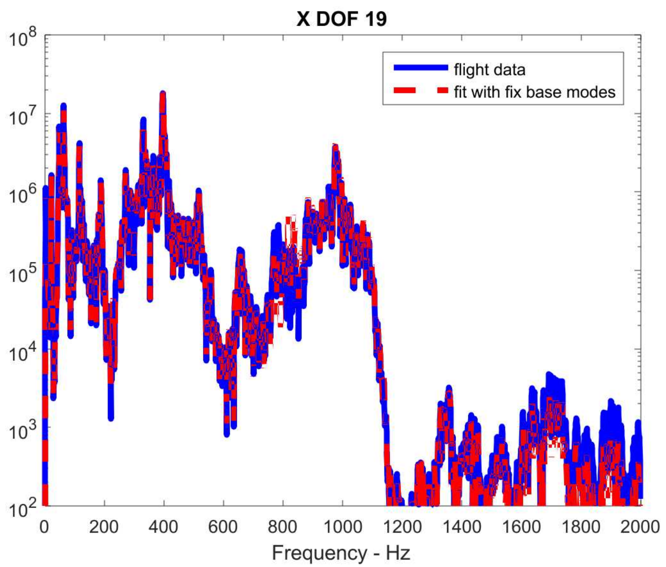


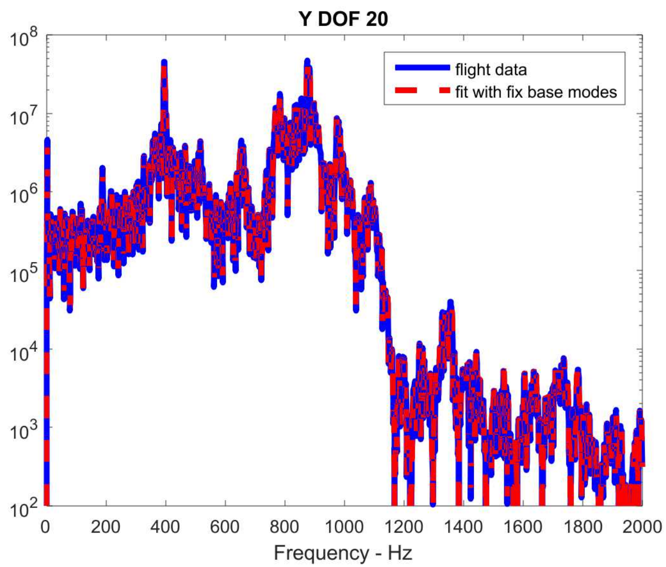


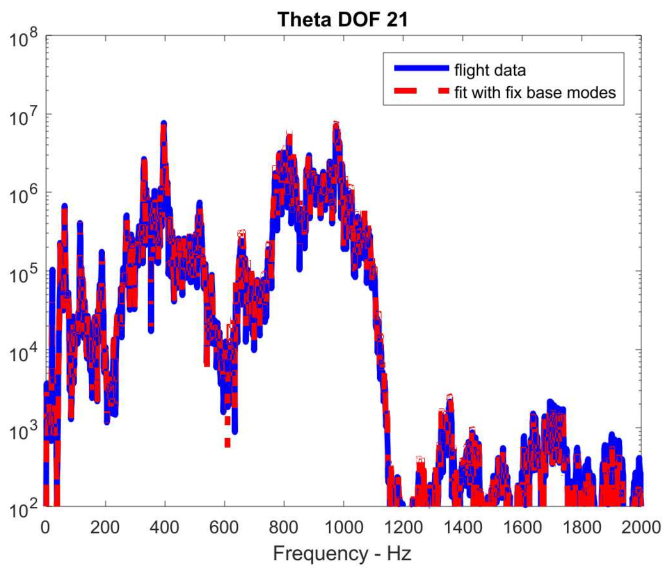


Lab 3 dof match to component flight response

Component beam centerline acceleration response 6” from top of component







Traditional Single DOF Test Specification and Methods

Traditional 1DOF shaker test specifications

- Manually enter test specification breakpoints so, limited number of points
- Broad plateau's to allow for test article modes to shift due to unit to unit variability.
- Basis is typically a few or 1 component input locations in the assembly. Will use 1 in these examples

Types of test specifications discussed here

- Base input
- Response limited base input
- Least squares base input to match responses

Source Data

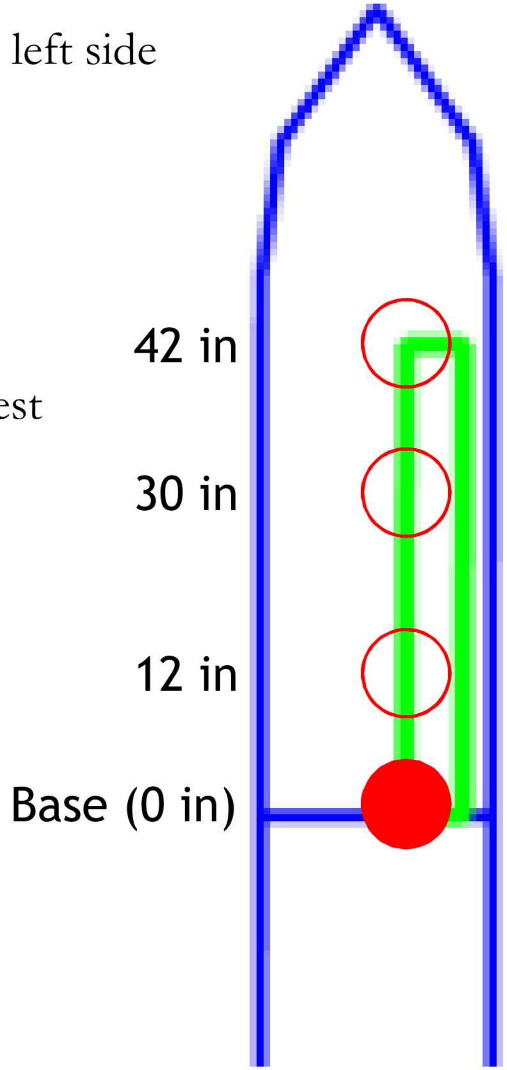
Acceleration response time history data along the left side of the component

Acceleration auto-spectral densities calculated

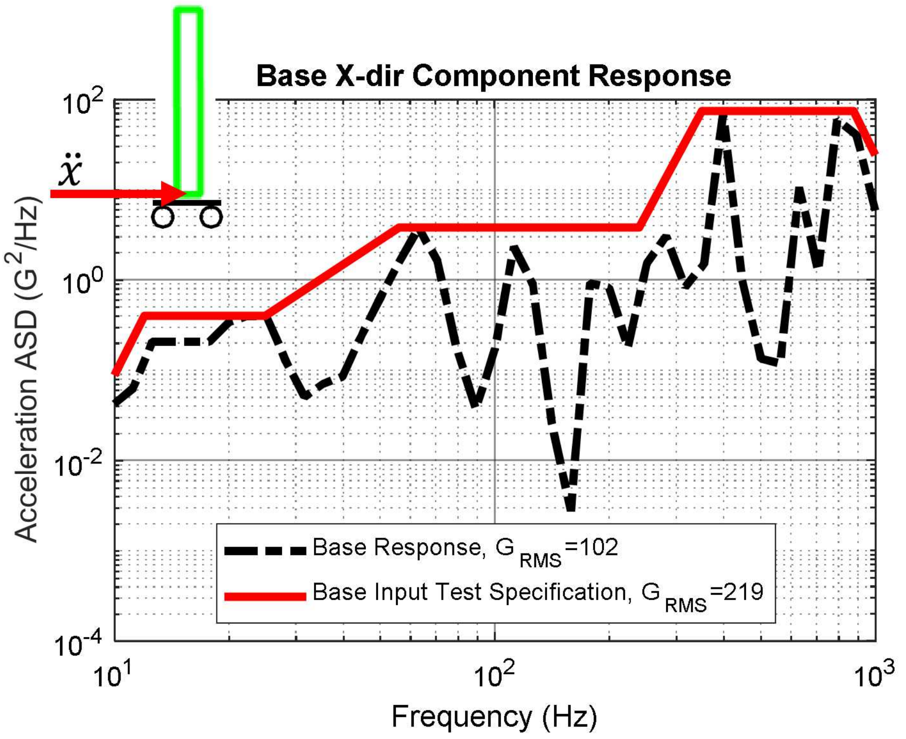
Considered from 10 to 1000 Hz

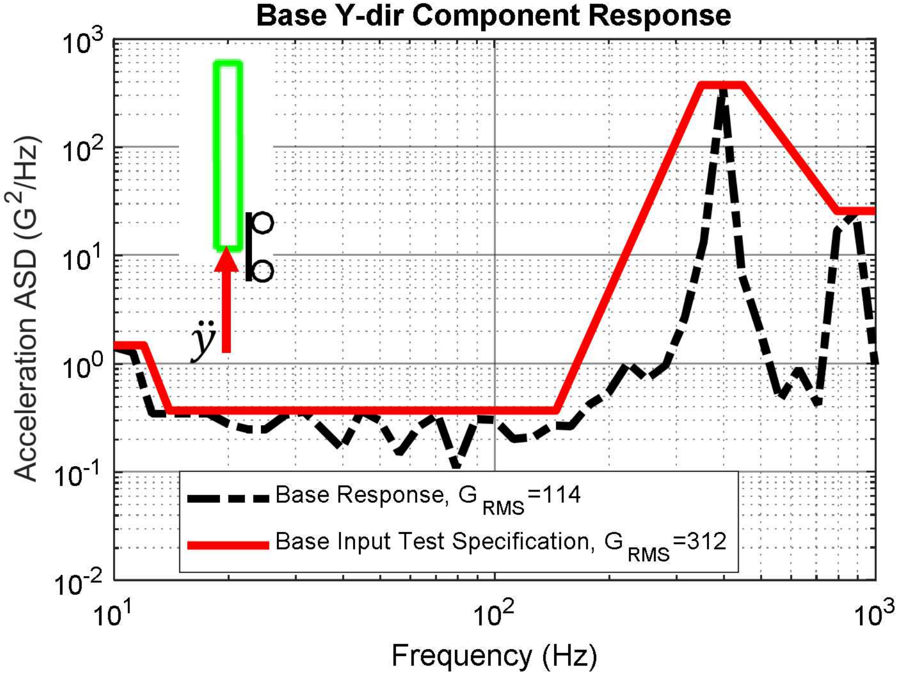
- The data drops off above 1000 Hz

Base location is taken as the source data for the test specification



Traditional Single Axis Test Specifications





Components acceleration time Histories provided at 0, 12, 30, & 42" up the side of the component

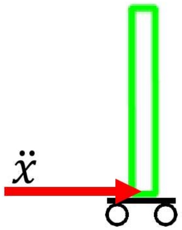
1/6th octave ASDs generated from the data

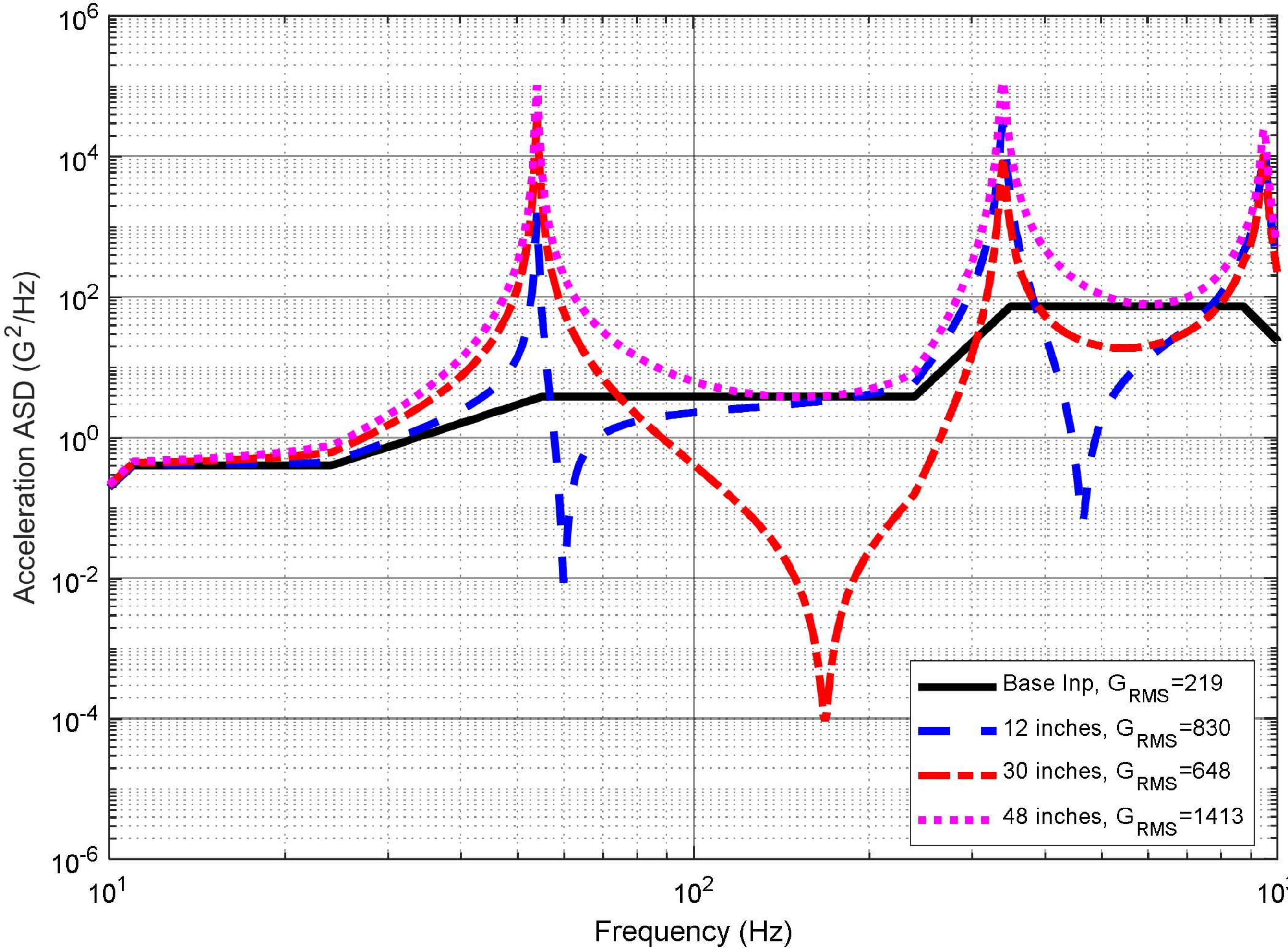
Use base (0") in the X- and Y-directions, in turn. Ignore rotations

Draw straight-line test specification over 1/6th octave data

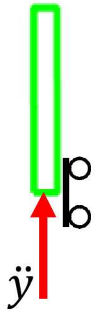
Apply to base of component as the input, in turn

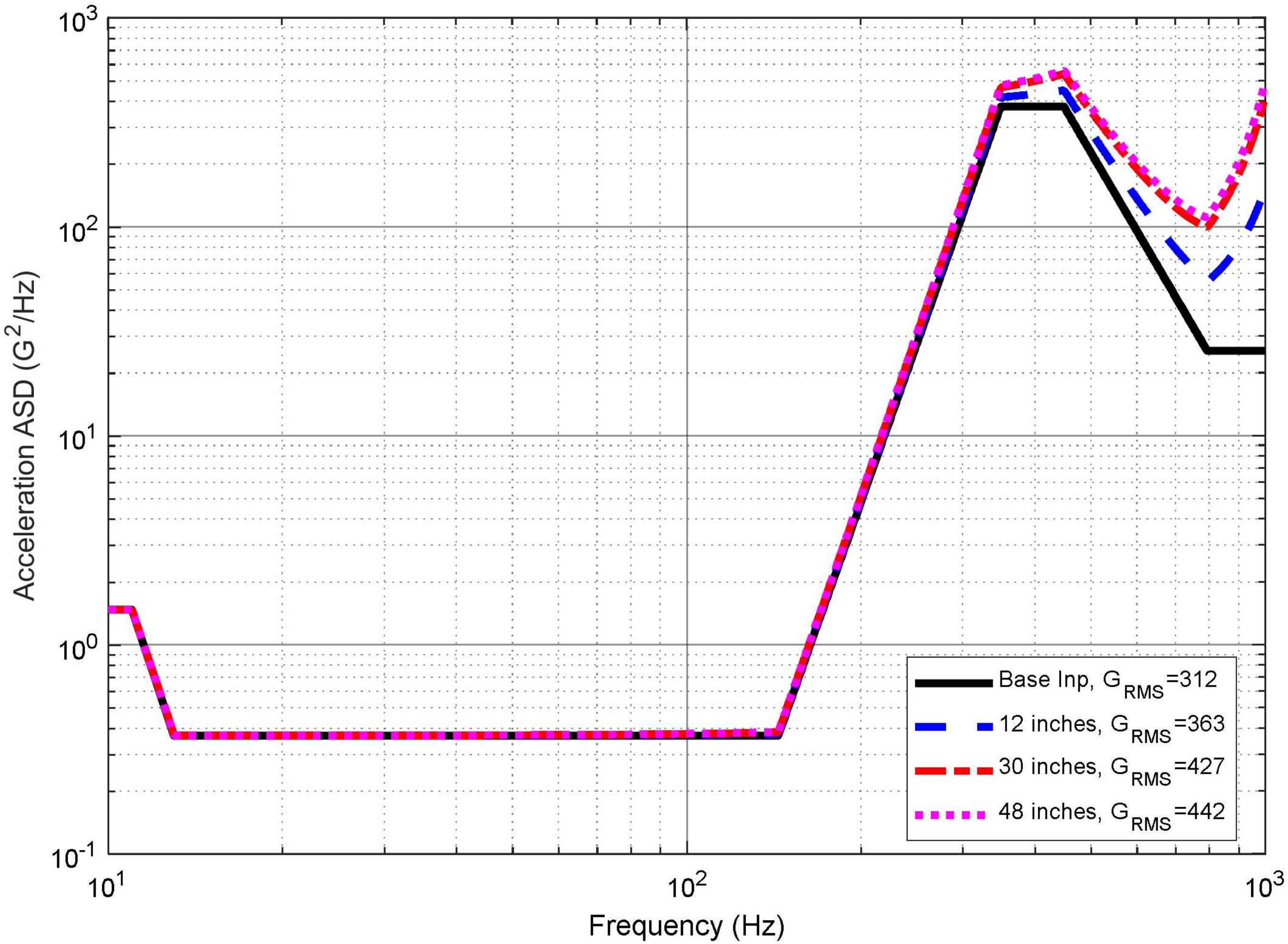
Responses to X-Direction Input



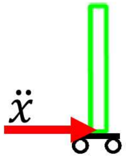


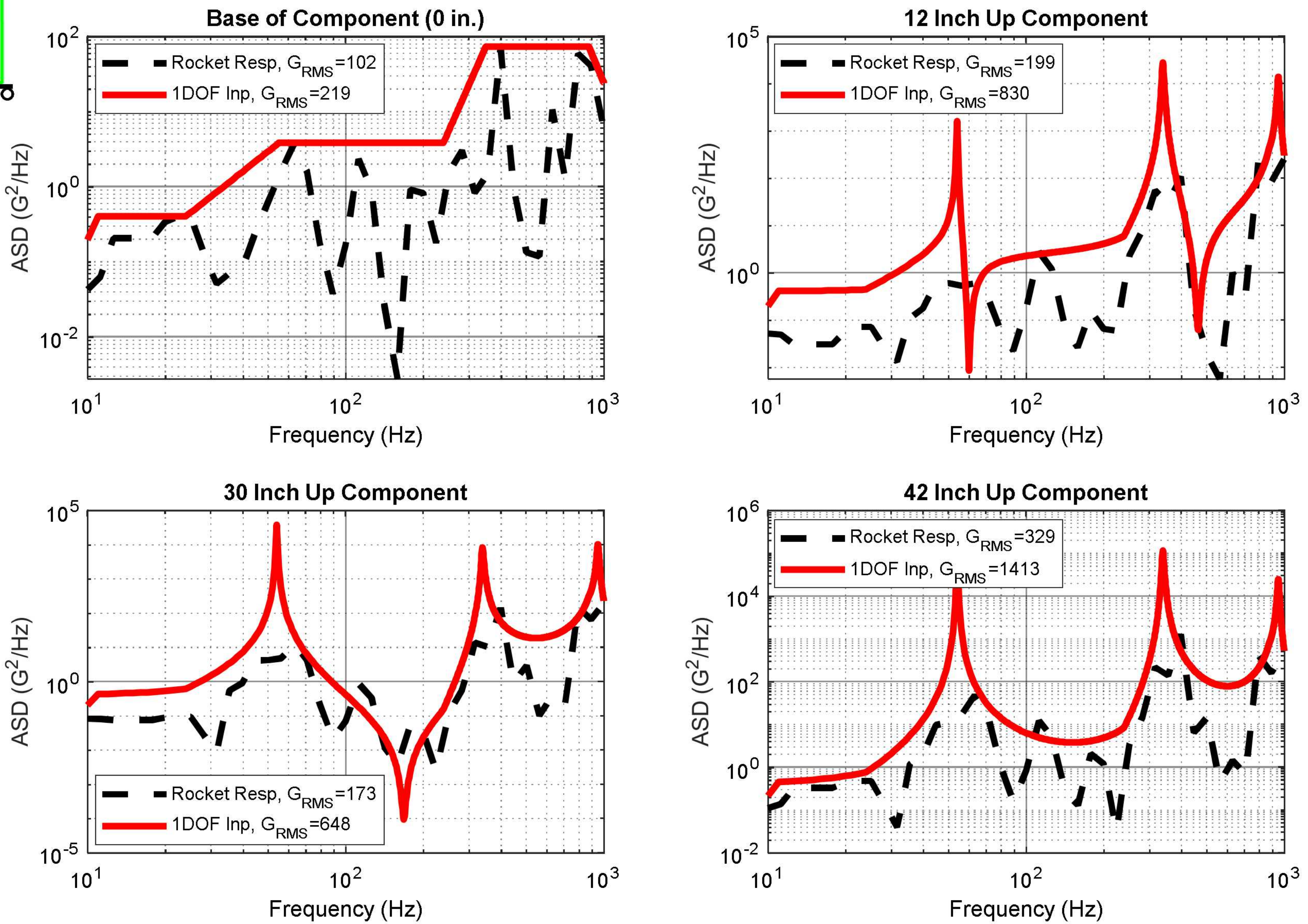
Responses to Y-Direction Input



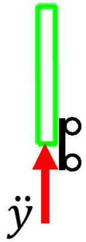


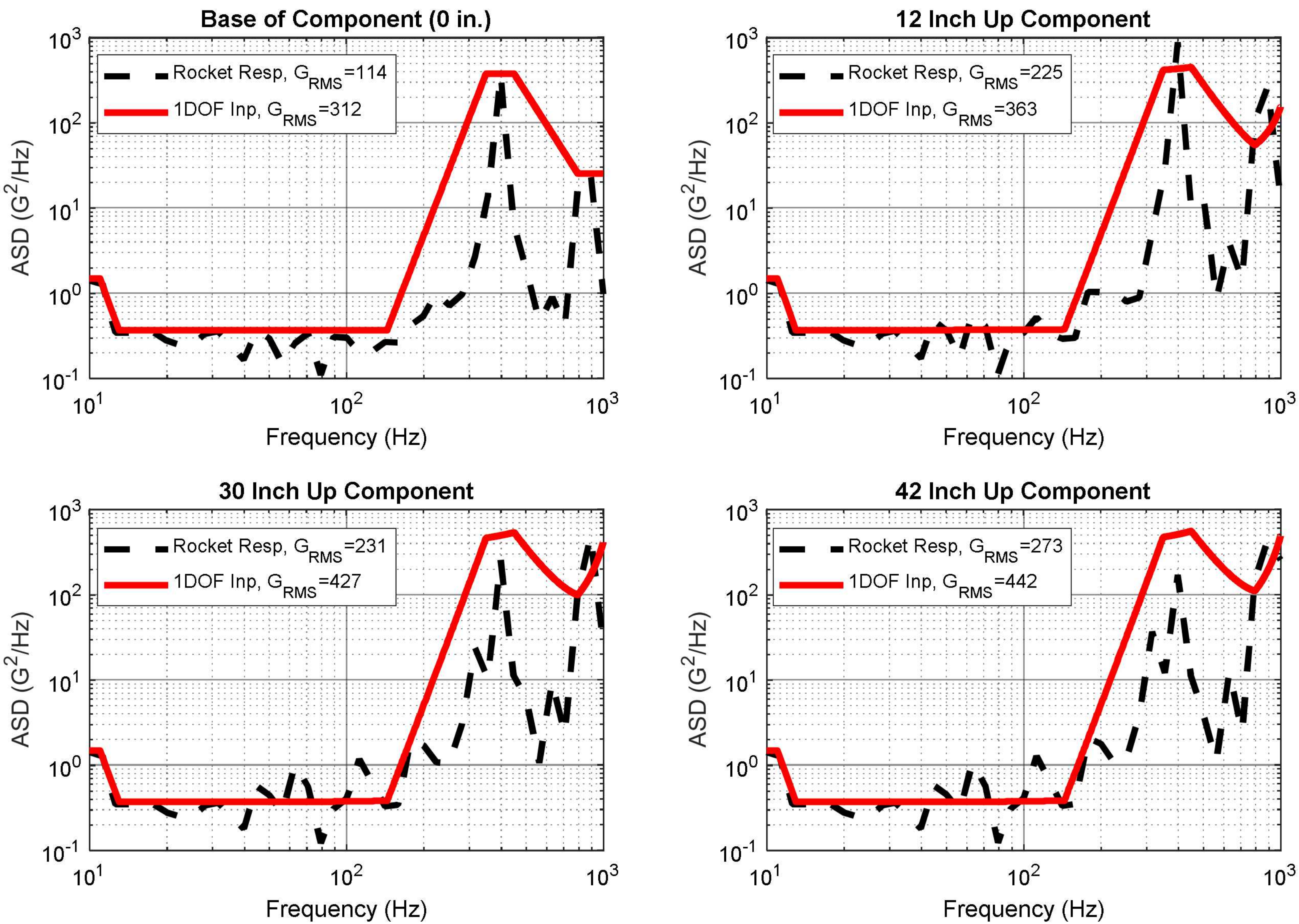
X-Direction 1DOF Input Test Response Compared to Rocket Response





Y-Direction 1DOF Input Test Response Compared to Rocket Response



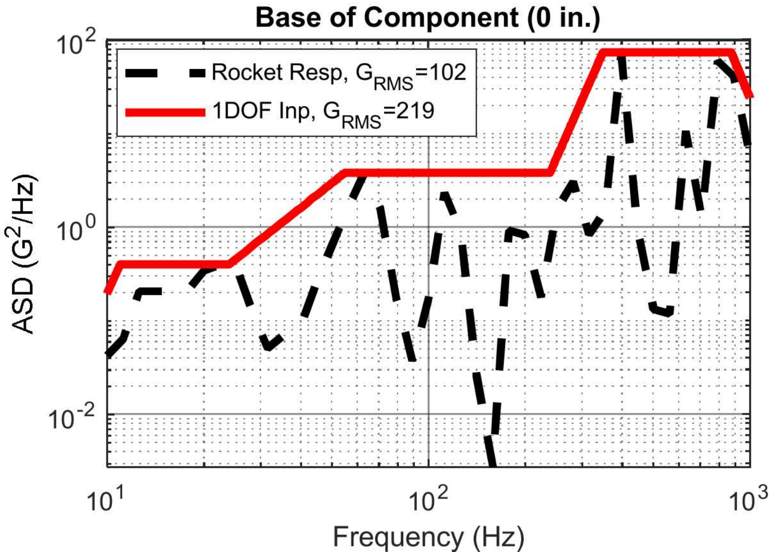


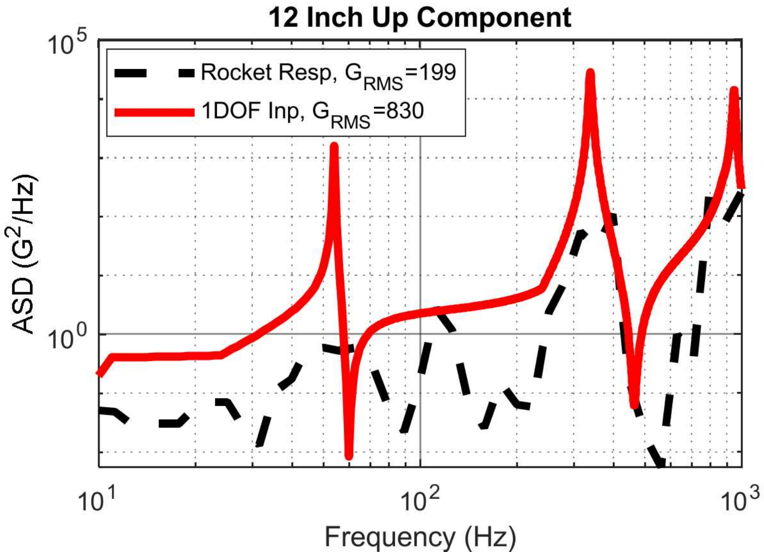
Take Away from Single DOF Testing

Over excitation test article responses

- Test specification is generally a conservative, coarse straight line envelope of the reference data
  - Filling in valleys in the data
  - Wider peaks than in the data
- Possibly much higher responses than the difference between the test specification and the reference data at that location from the system
- Might be more response than the design can and should be subjected to

By itself, doesn't give an indication of how representatively the damage mechanism from the comparable load in the assembly is being engaged





Response Limited Single DOF Testing

When test article is too responsive which causes failure, response limiting may be applied to reduce the energy in the test article.

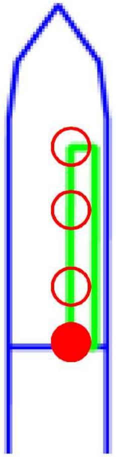
- We use it when, we know that it is responding too much
- When test article has failed previous testing

What causes the situation wherein response limited might be used?

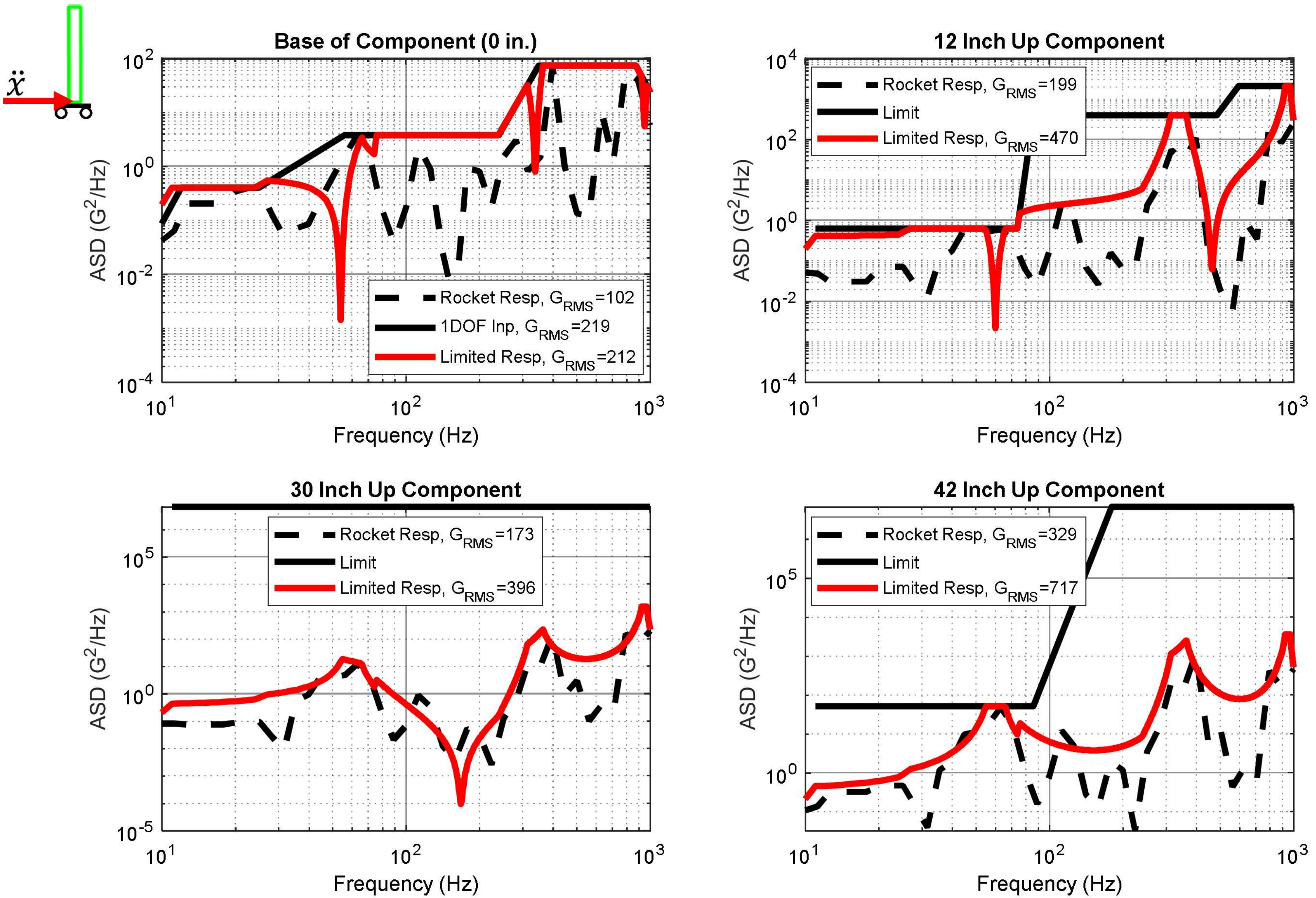
- Shaker essentially provides infinite impedance
  - Especially in the off axis test directions where zero motion is enforced with infinite force.
- Boundary condition causes test article to have modes much different than in the assembly
- Load application is not representative of what happens in the assembly (E.G. no rotational loading. Only one axis at a time)

How does it work

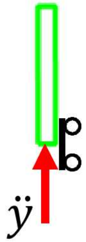
- Compare the test response to measured or analysis based response in the next assembly for the same loading condition
- Response profile is determined as do not exceed responses.
- Shaker controls system only engages at frequency values where the test inputs cause the response to exceed the established limit
- If a response limit is exceeded, the control system reduces the input until the response matches the limit.

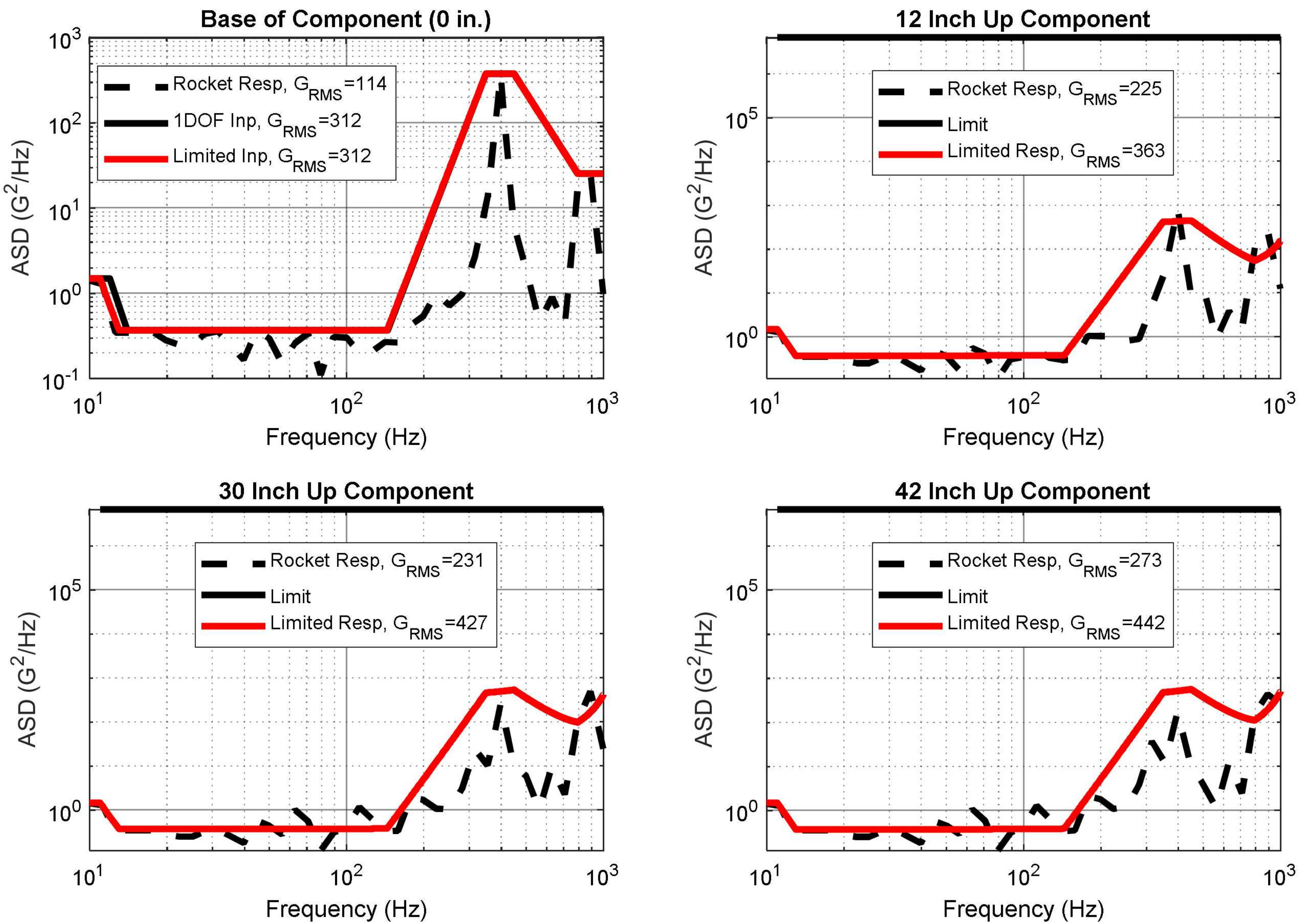


X-Dir Notched Inputs with Response Limits



Y-Dir Notched Inputs with Response Limits





Take Away from Response Limited Single DOF Testing

Can prevent over testing condition

Only works if you can measure the response at the location of interest in the test

Need to know the appropriate levels to limit the response to

By itself, doesn't give an indication of how representatively the damage mechanism from the comparable load in the assembly is being engaged

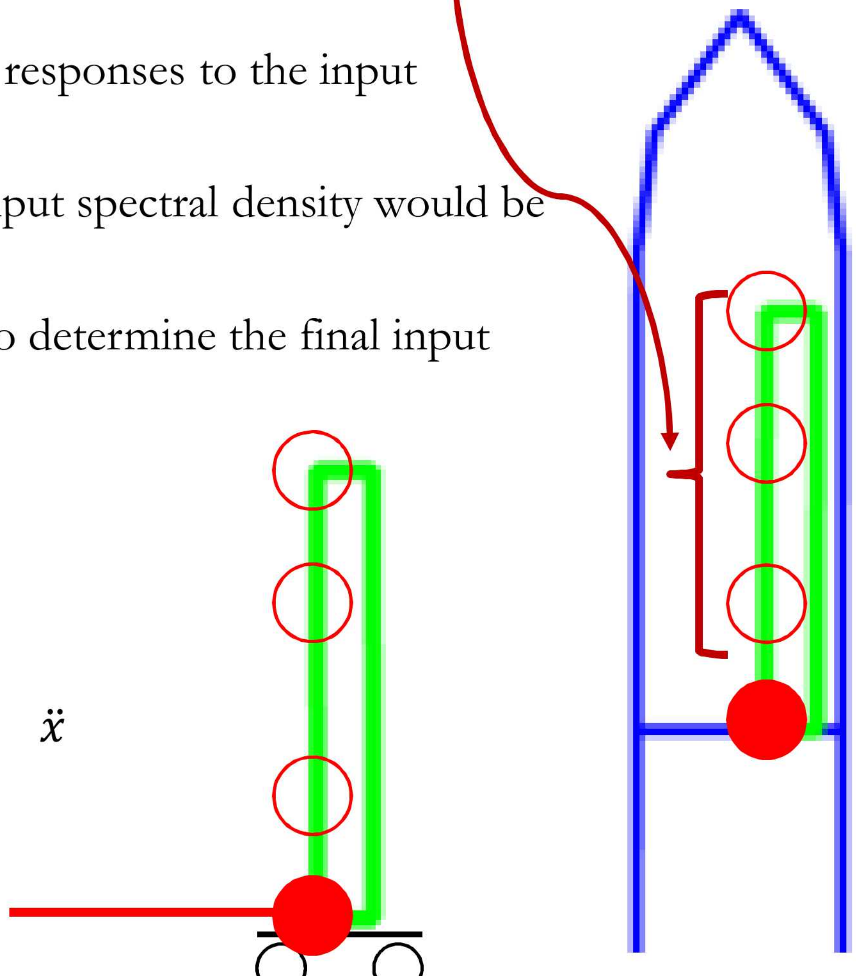
Least Squares Input Spectrum from the measured response locations

Use the responses at several locations as the basis for developing the input

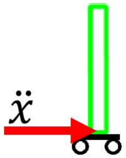
Measure the spectral density ratios from the responses to the input location in the test configuration

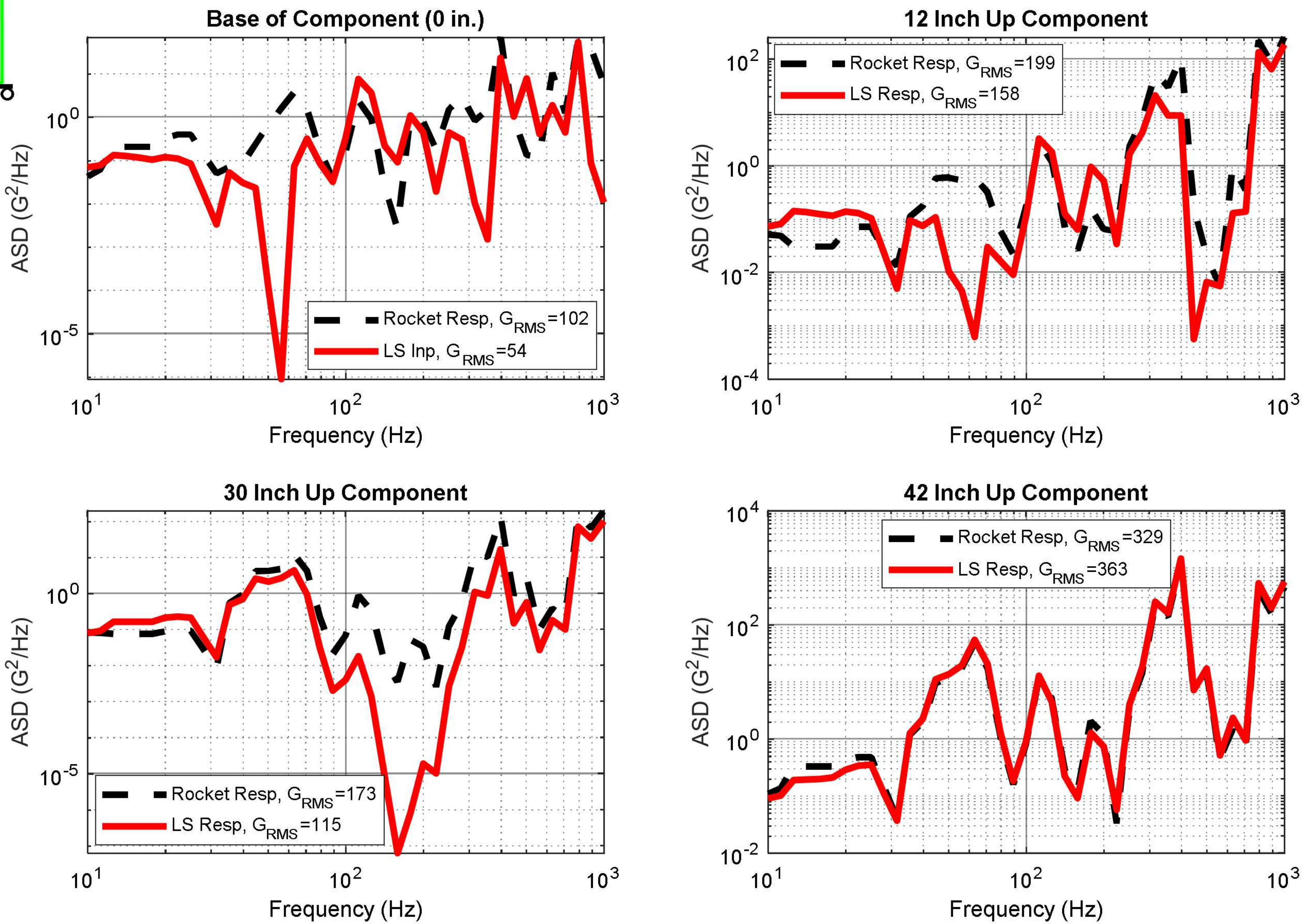
Use the ratios to determine what the ideal input spectral density would be to obtain each response

Use the least square of the multiple inputs to determine the final input

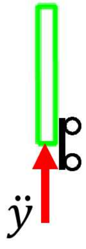


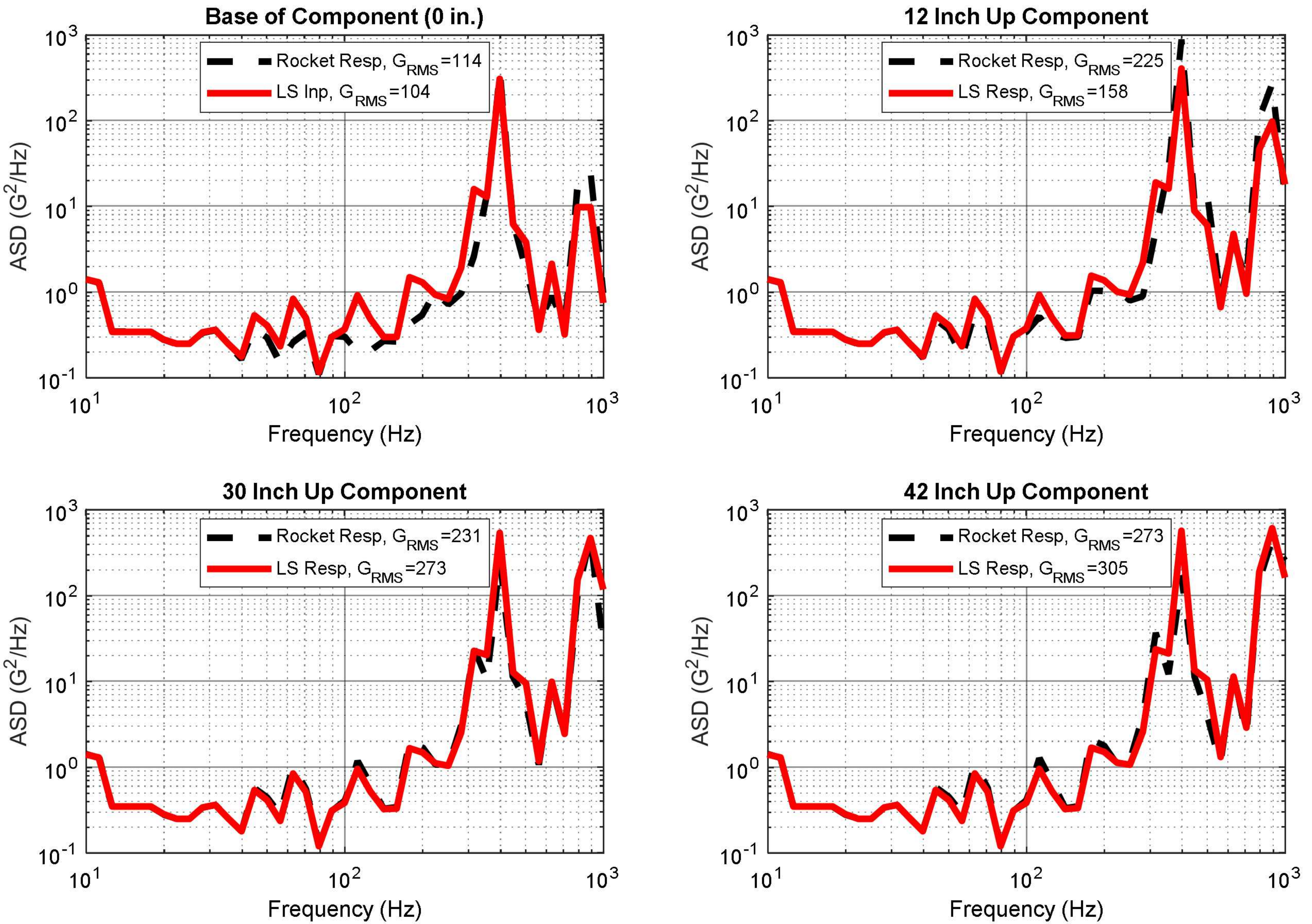
X-Dir Least Squares Input



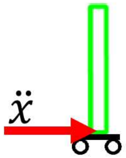


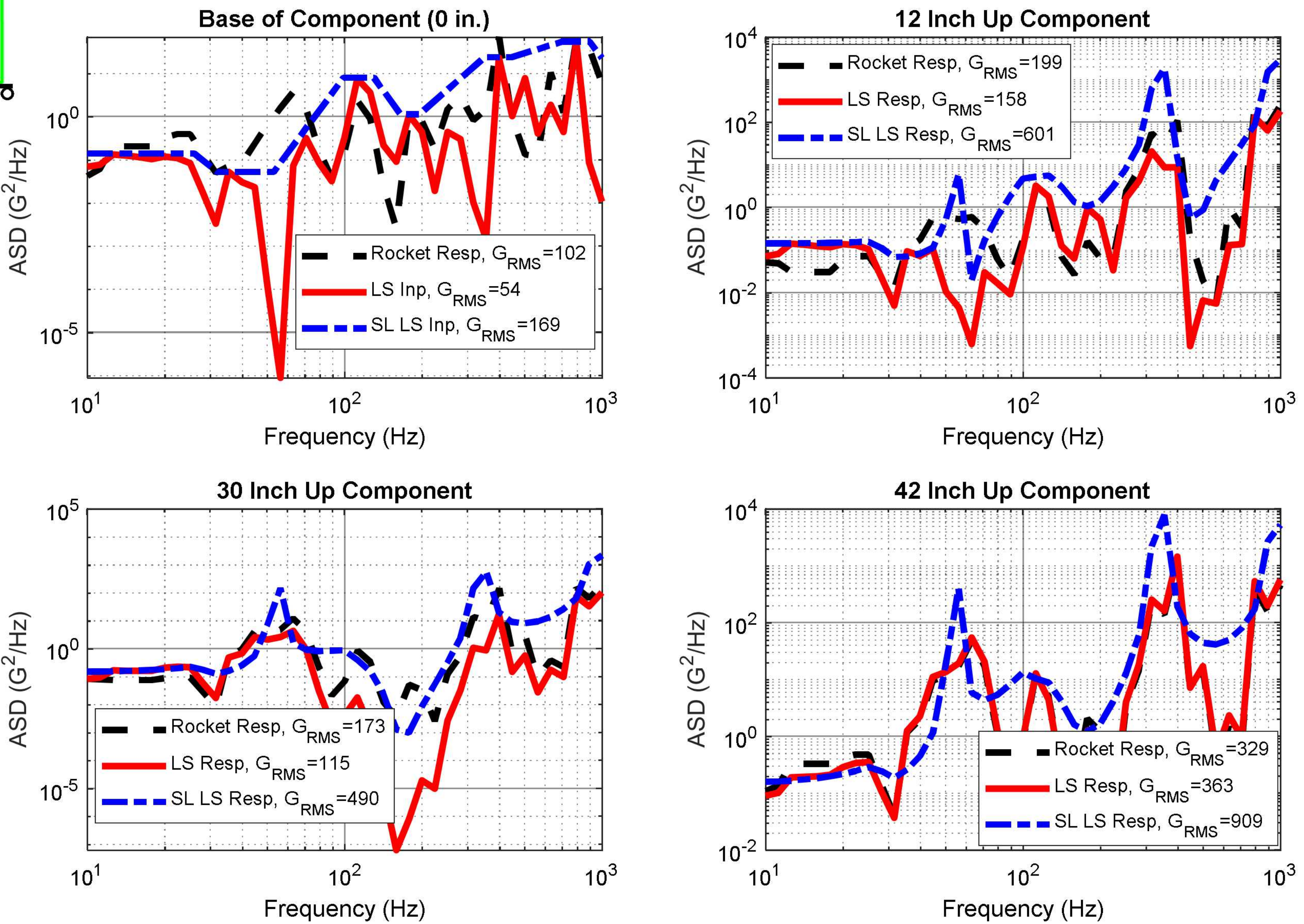
Y-Dir Least Squares Input



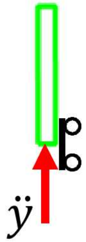


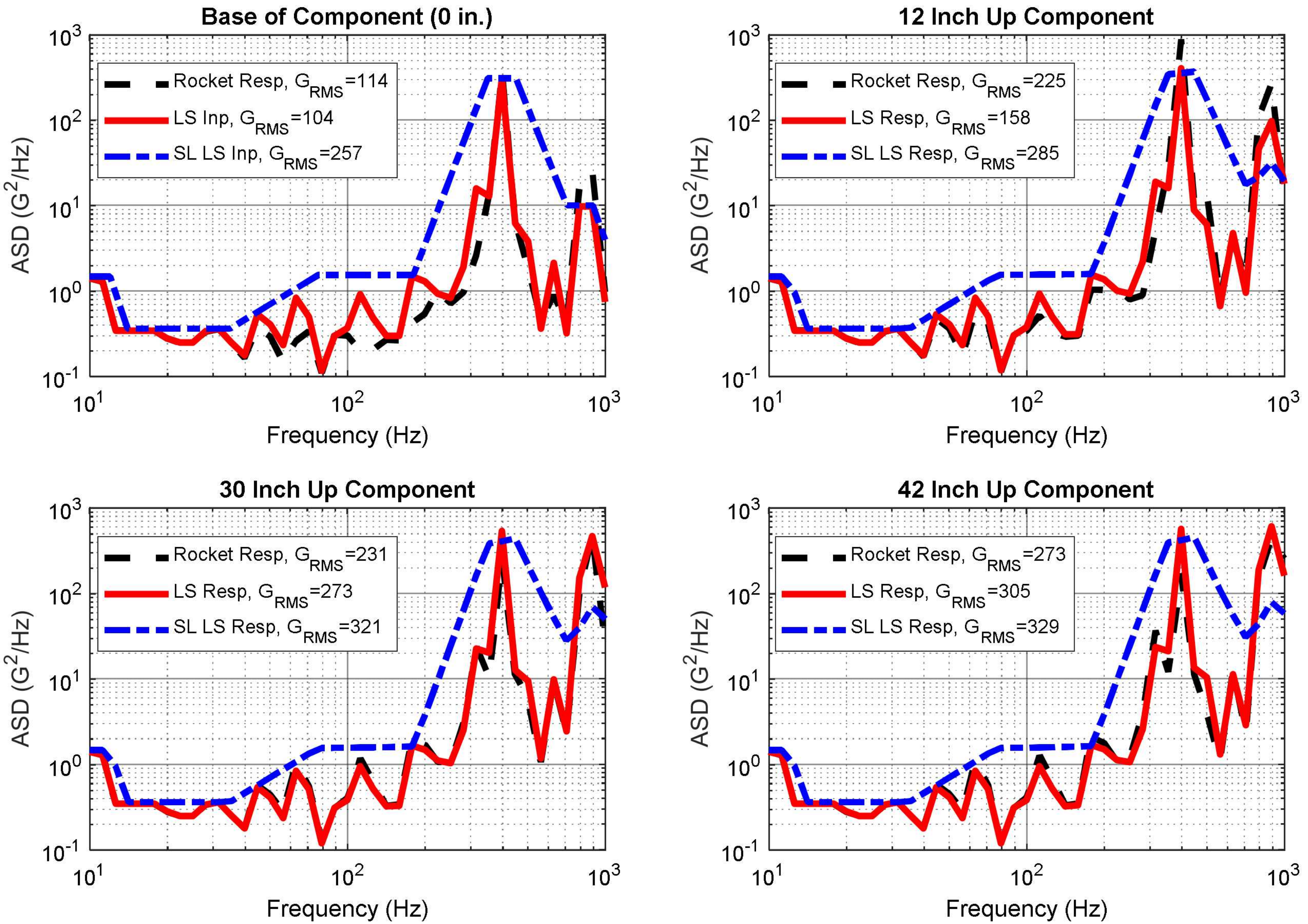
X-Dir Least Squares Straight Line Test Specification Responses





Y-Dir Least Squares Straight Line Test Specification Responses





Take Away from Input Determined from Least Squares Fit to Match Test Article Responses

Can help match responses better

Input likely will not match reference data at the input location as well

Developing straight line specification can take away some of the ability to match responses

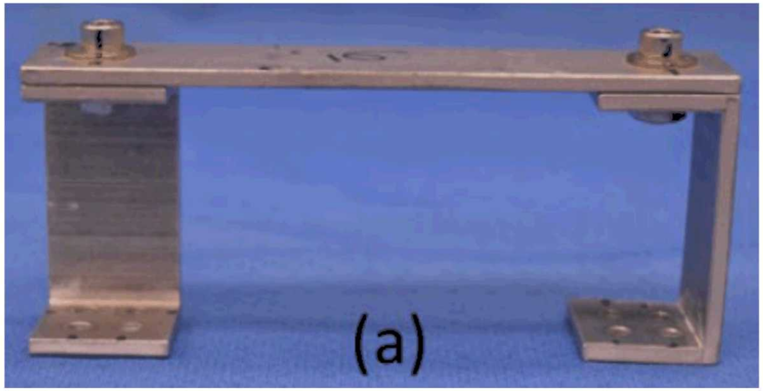
Should be conservative at the response locations

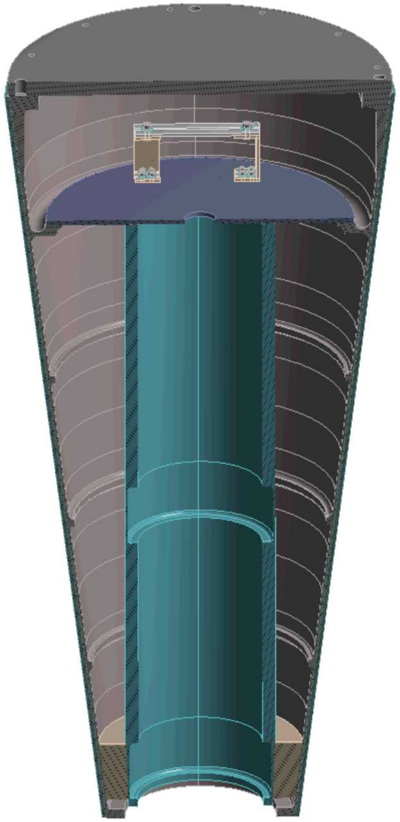
Can't get all of the responses exactly right. Limited to one target response.

By itself, doesn't give an indication of how representatively the damage mechanism from the comparable load in the assembly is being engaged

Proof of Concept Field Hardware

- System was Modal Analysis Test Vehicle (MATV)
- Hardware was developed by the Atomic Weapons Establishment, AWE, UK

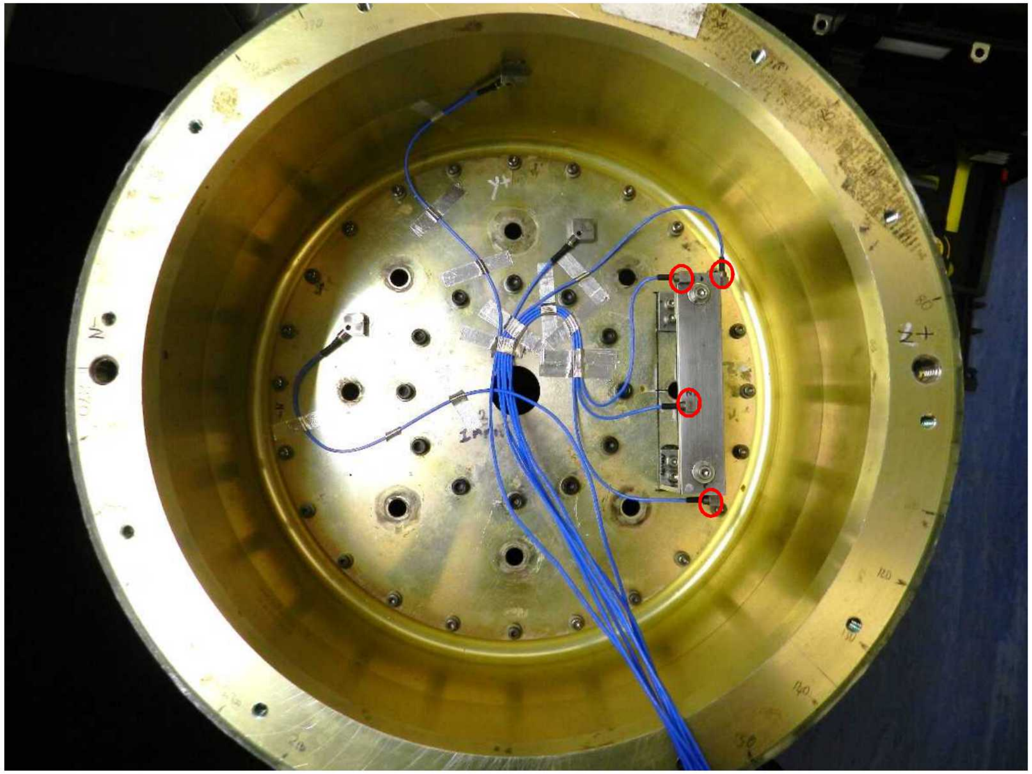




- Component is the removable component (RC), a round robin test article developed for the dynamic environments community ESTECH/SAVE/IMAC

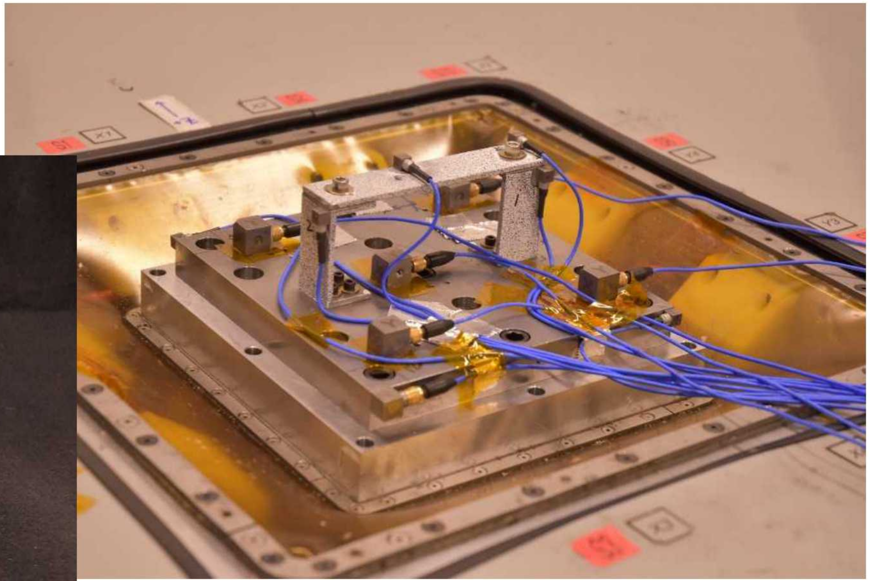
Acoustic Field Test and Instruments

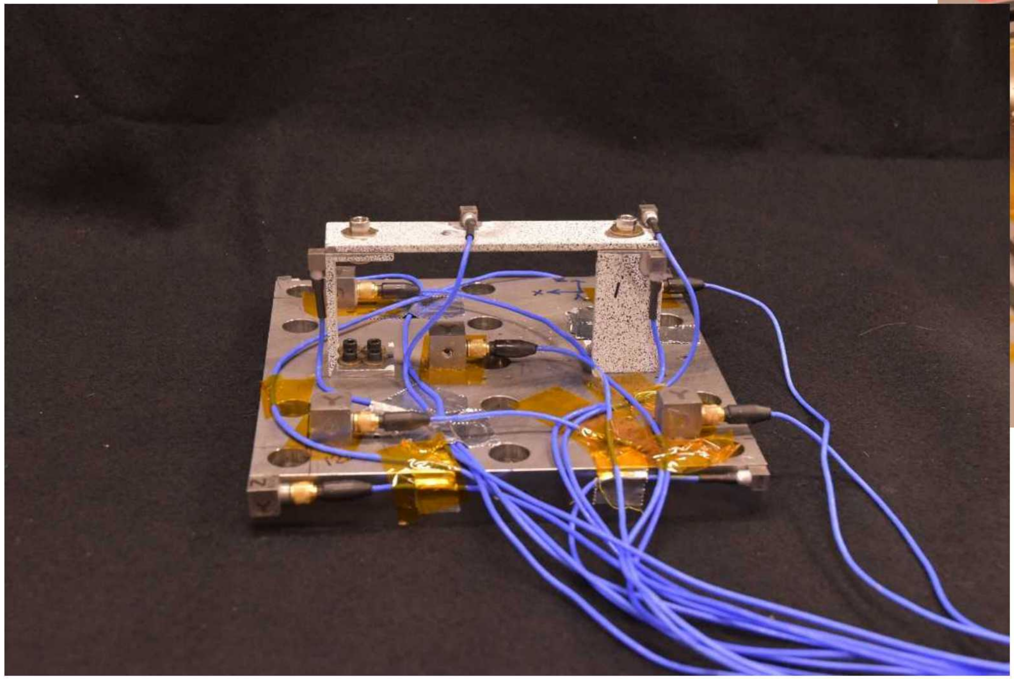
- The acoustic test was performed to 147 dB
- Data were gathered on 4 triax accelerometers on the RC



Laboratory Hardware

- Another RC was mounted on a steel plate and instrumented with 4 triax accelerometers in the same locations as the field test as well as 4 triax accelerometers on the corners of the plate.





Transform to Rigid Body Modes and Fixed Base Modes

Modal Craig-Bampton procedure transforms free-free modes to a set of fixed-base (p) modes + rigid-body (s) modes

Free-free modal params.

$$\mathbf{x} = \mathbf{\Phi q}$$

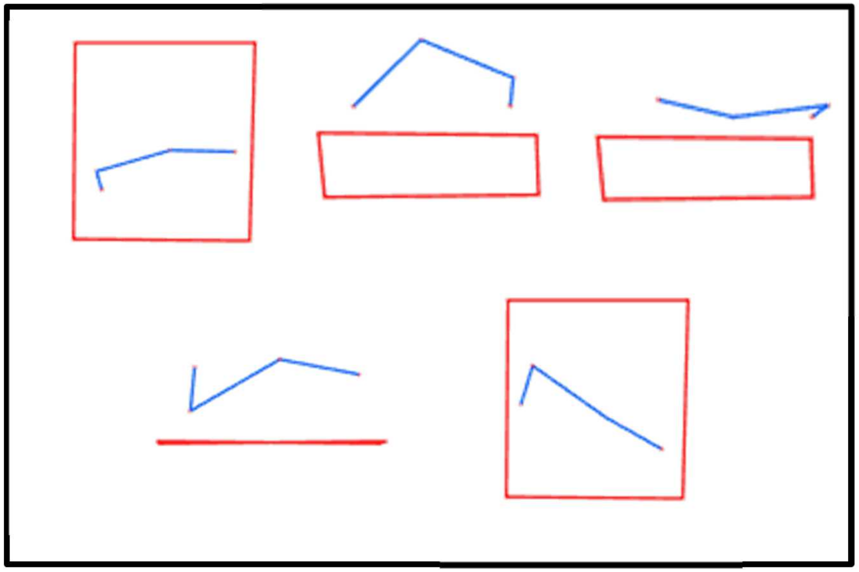
$$(\omega_{\text{free},r}^2 - \omega^2 + i2\zeta_{\text{free},r}\,\omega_{\text{free},r}\omega)q_r = 0$$

→

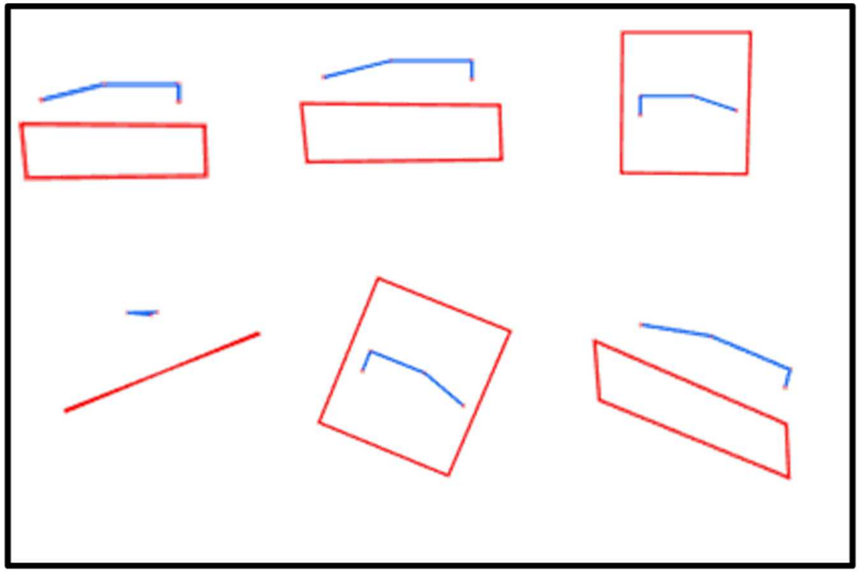
Transformation

$$\mathbf{q} = [\mathbf{T}_p \quad \mathbf{T}_s]\begin{Bmatrix}\mathbf{p}\\ \mathbf{s}\end{Bmatrix}$$

Fixed-base shapes:



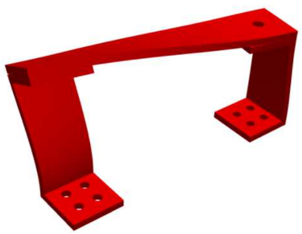
Rigid-body shapes:



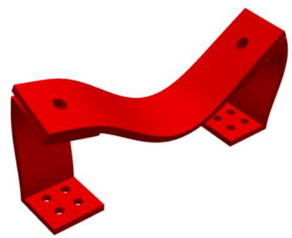
Mayes, Randall L., "A Modal Craig Bampton Substructure for Experiments, Analysis, Control and Specifications", *Proceedings of the 33rd International Modal Analysis Conference*, Orlando, FL, February 2015, paper number 353.

Better Visualization of RC Fixed Base Modes from FE Model

Here are the five fixed base mode shapes active up to 2000 Hz

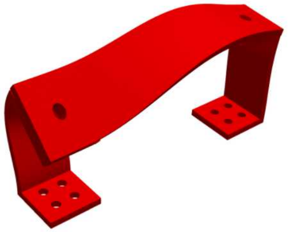


383 Hz

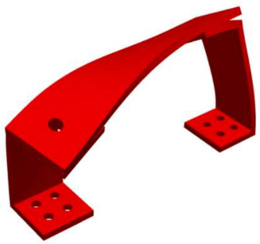


1026 Hz

1125 Hz



1651 Hz

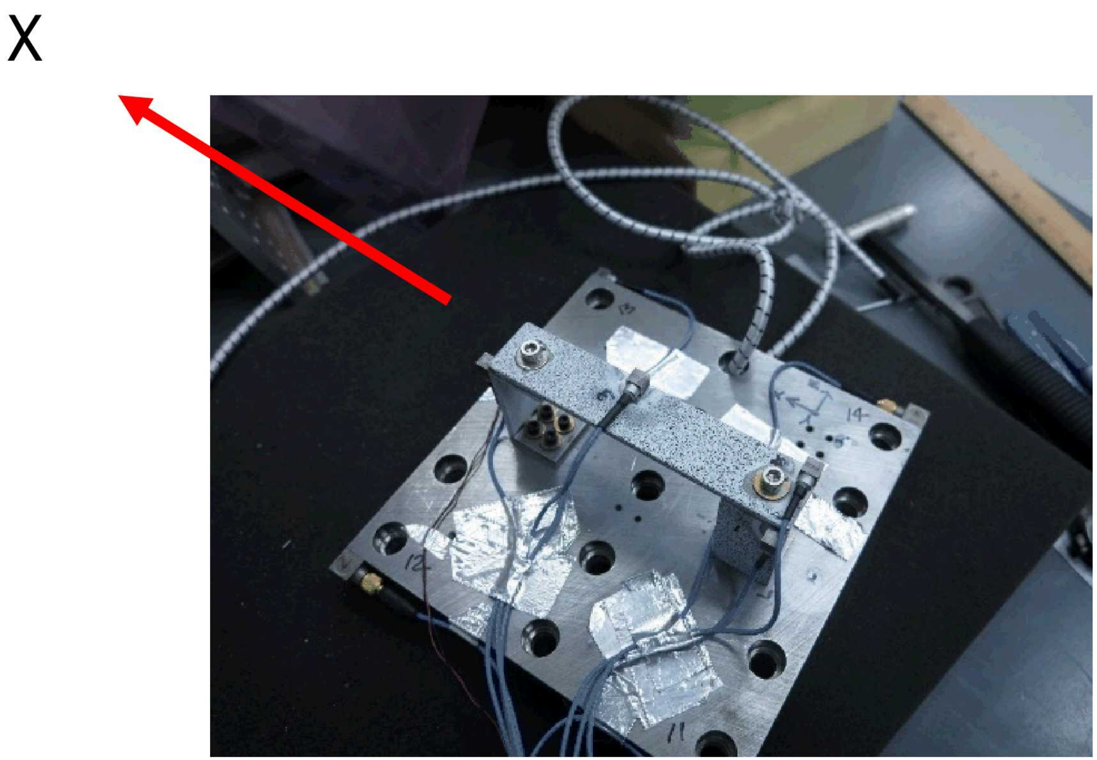


1883 Hz

Results from 1 DOF X axis input extended to physical ASDs

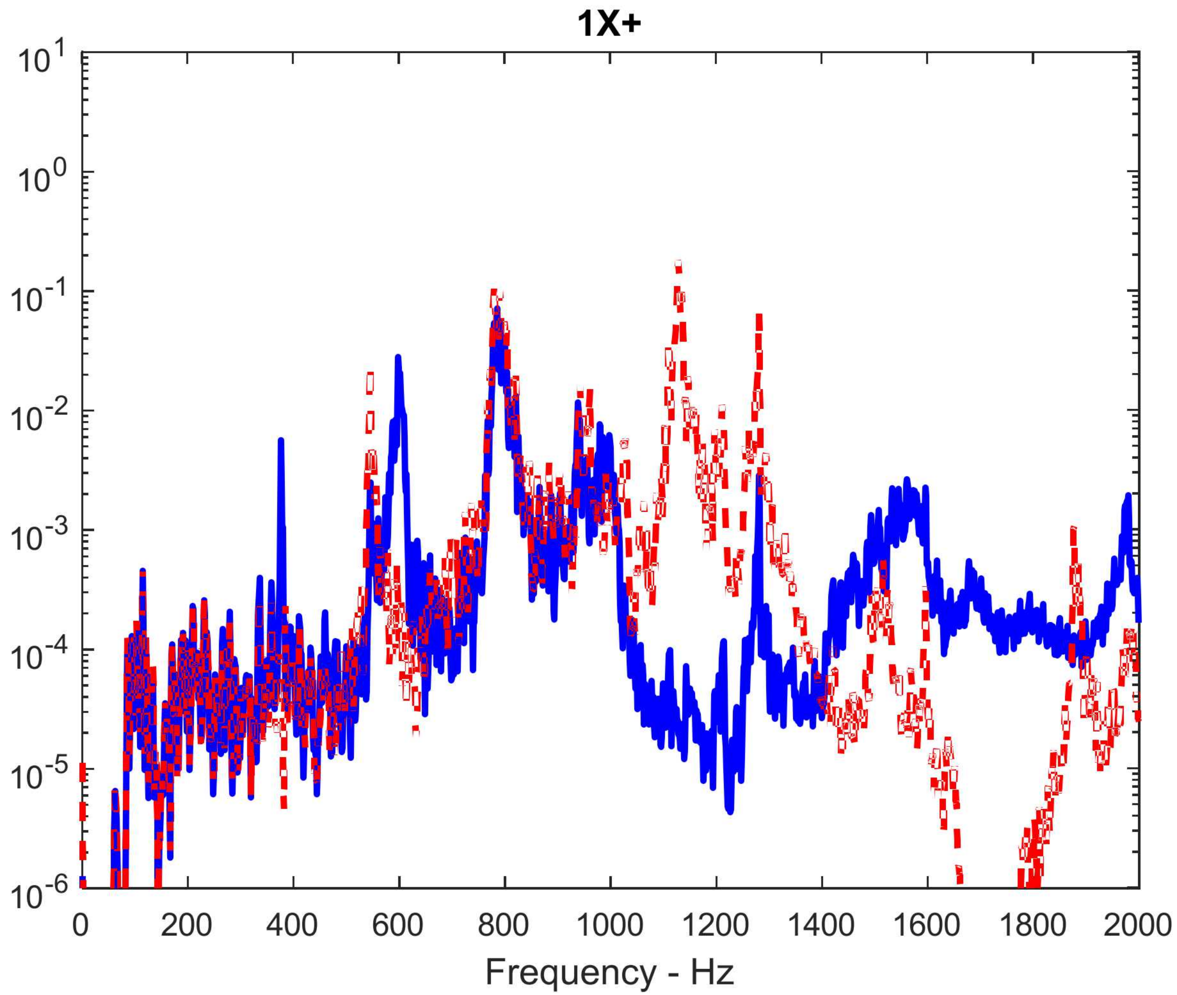
Transmissibilities were calculated with buzz test from the RC accelerometers to the rigid base DOF inputs (X,Y,Z,RX,RY,RZ)

Acceleration from MATV in X direction about 3 inches away from RC was used as input in X direction only



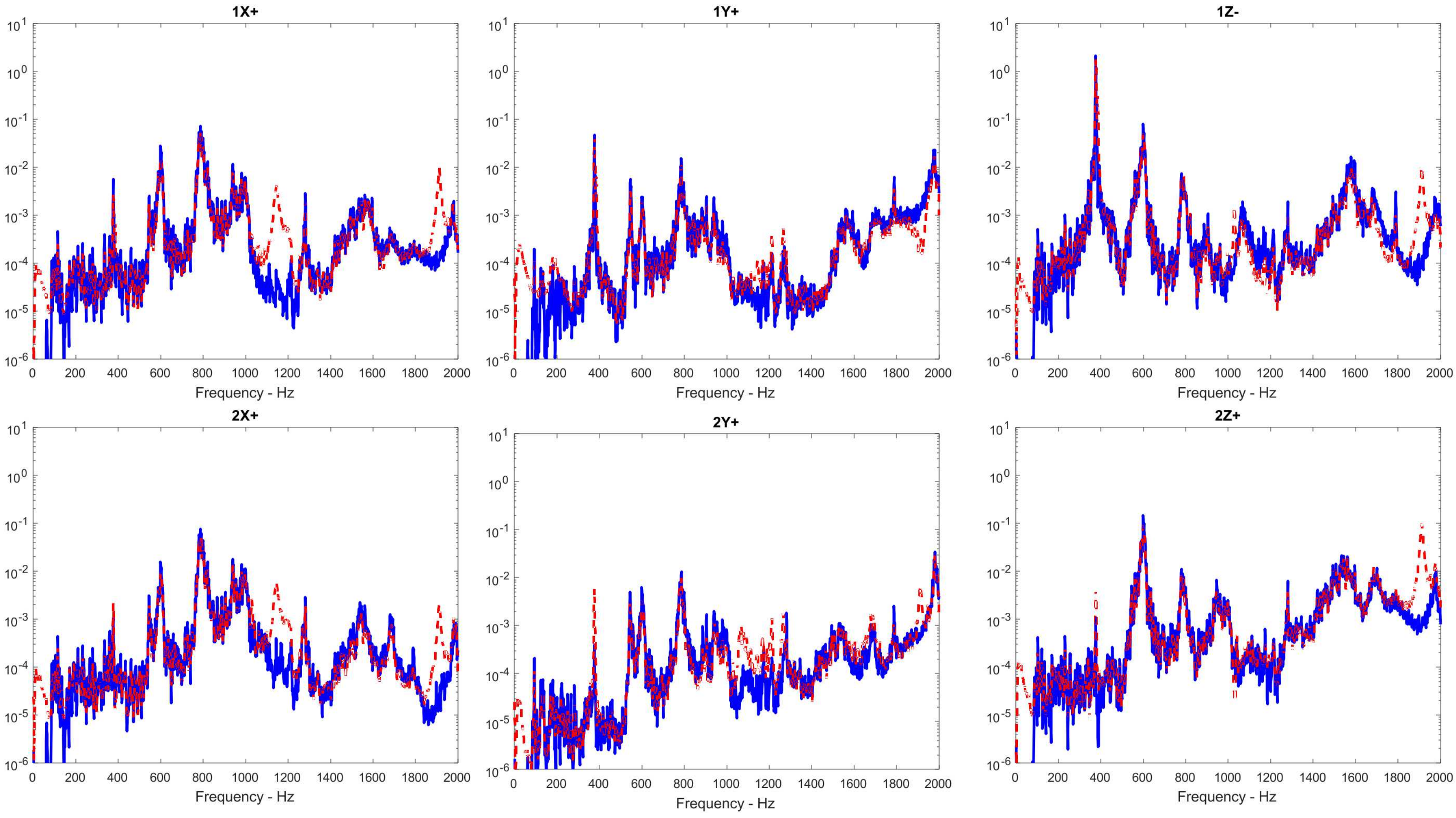
Results from 1 DOF X input analytically extended – ASD for 1X

ASDs from Acoustic Test-blue; 1 DOF shaker-red



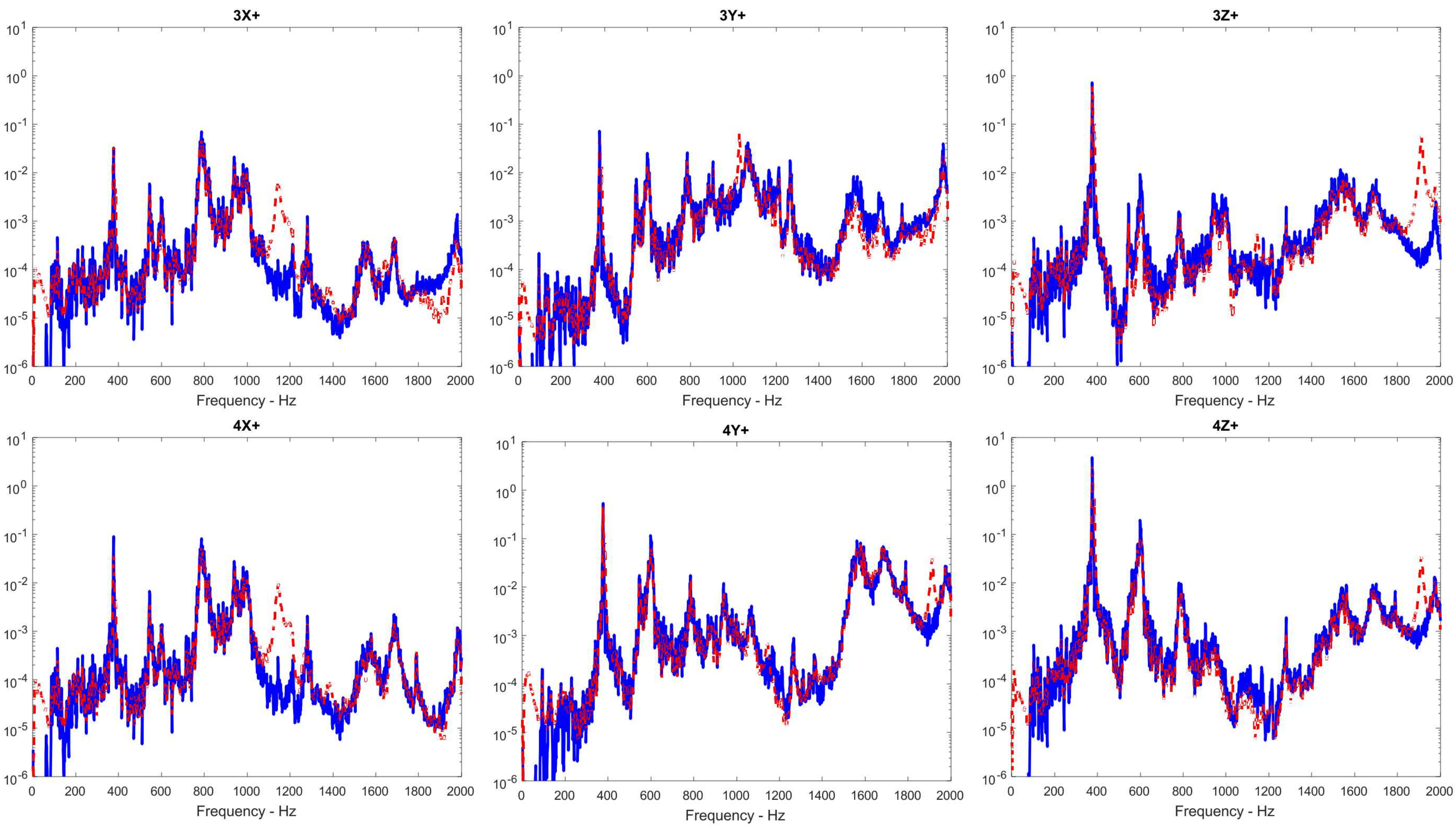
Results from 6 DOF test to control (50-2000 Hz) to 12 x 12 acoustic test cross spectral matrix (Paripovic/Nelson/Schultz)

- 6 ASDs from Acoustic Test – blue; 6 DOF shaker - red



Results from 6 DOF test to control to 12 x 12 acoustic test cross spectral matrix – page 2

- 6 ASDs from Acoustic Test – blue; 6 DOF shaker - red



Wrap-up for base mounted component modal response

Even though system response may be quite complex, a few fixed base component modes with rigid body modes may represent the component field response.

Physical insight is available with a small number of component fixed base mode shapes.

Strain (and its damage) is captured in fixed base mode shapes and their modal DOF.

Rigid body accelerations are defined by the base laboratory input.

MDOF and SDOF laboratory base inputs can be tailored to best match field response on component.

Matching fixed base modal responses with tailored base input provides better simulation on entire component than response limiting which can only address a few locations.

Whatever specification is used, knowing the fixed base modal field response and the fixed base modal specification response allows one to quantify the conservatism.