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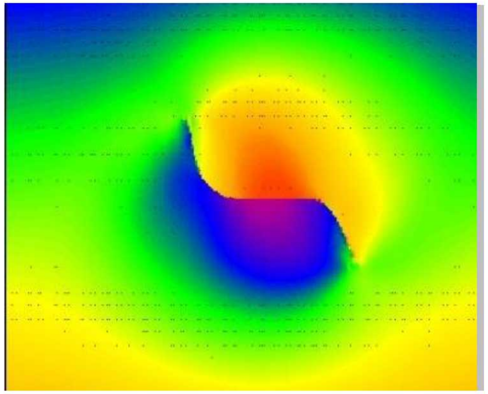

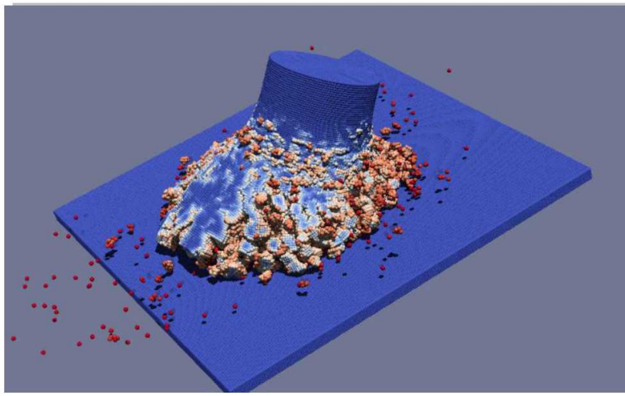

The Mechanics of Unstable Peridynamic Materials

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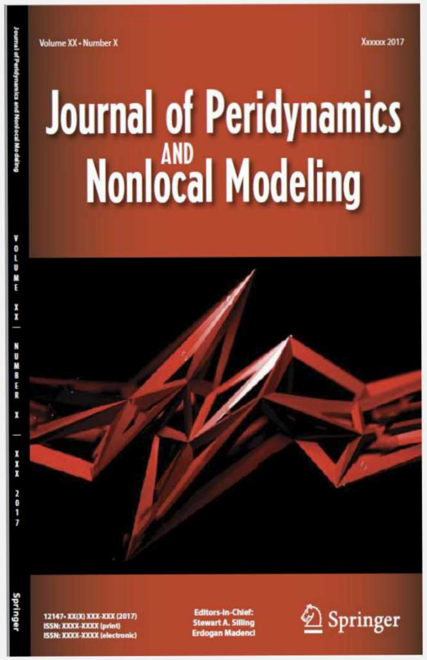
Outline

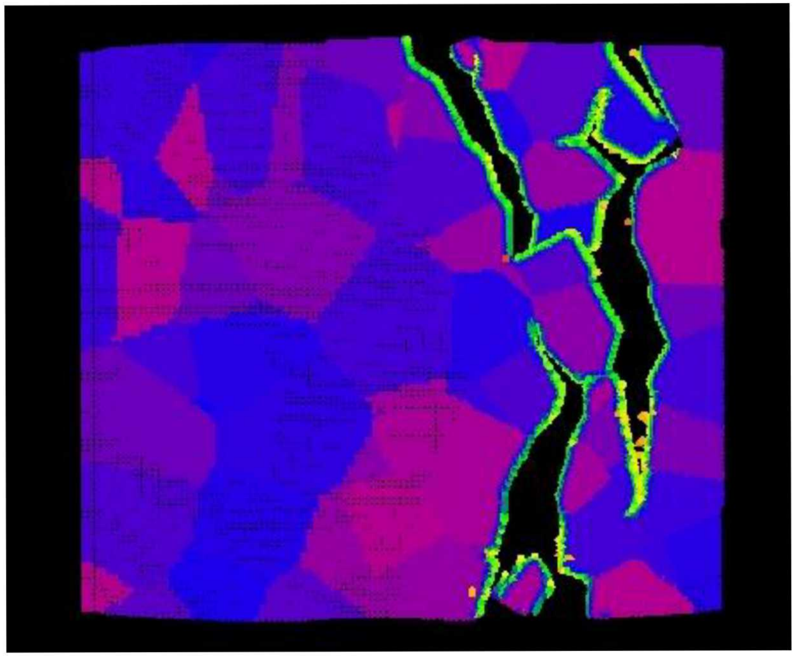
- Kinetics of material failure
- Peridynamic theory
- Non-propagating waveforms
- Growth of a small pulse
- Time to failure
- Rate effect

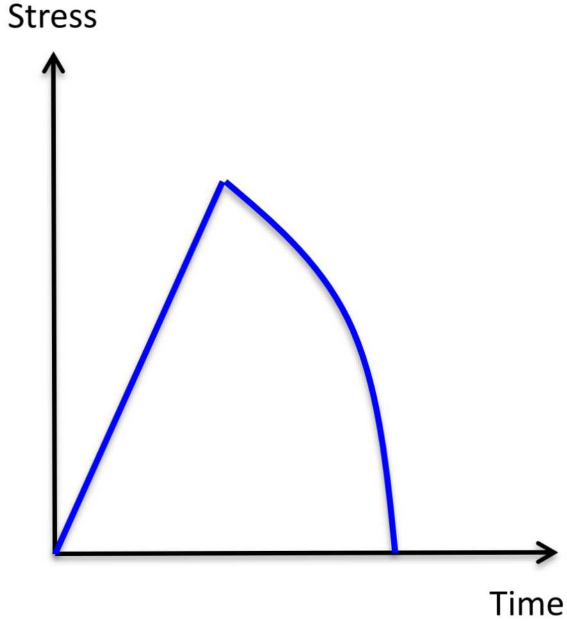


New Springer journal!
Scope is all things nonlocal

Failure kinetics: time it takes to fail

- If we pull slowly on a test specimen, it breaks when it reaches some strain.
- Once it reaches the failure strain, how much **time** does it take to fail?
- Failure involves small-scale processes that occur at a finite rate.
- How to model this in a homogenized material?
- **We will study this problem using an unstable peridynamic material model.**





Peridynamic mechanics

- General peridynamic equation of motion:

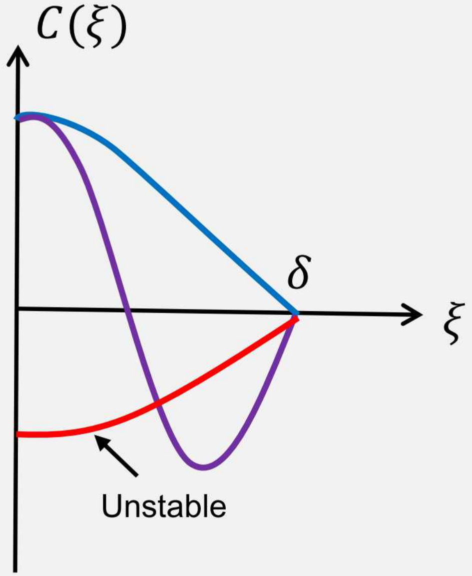
$$\rho(x)\ddot{u}(x,t) = \int_{-\delta}^{\delta} f(\eta,\xi)\, d\xi + b(x,t), \qquad \xi = q - x, \qquad \eta = u(q,t) - u(x,t)$$

where ρ=density, u=displacement, b=external force density, δ=horizon, ξ=bond, η=bond elongation.

- The horizon is a cutoff distance for nonlocal force interactions.
- Linear microelastic material:

$$f(\eta,\xi) = C(\xi)\eta, \qquad C(\xi) = \text{micromodulus.}$$

- The micromodulus contains the material properties.
- Assume an infinite homogeneous bar with zero external force.



Three choices of the micromodulus function

Dispersion and wave speeds

- Assume a wave of the form

$$u(x,t) = Ae^{i(kx-\omega t)}$$

where A=amplitude, k=wavenumber, ω=frequency.

- Equation of motion:

$$\rho\ddot{u}(x,t) = \int_{-\delta}^{\delta} C(\xi)\left[u(x+\xi,t) - u(x,t)\right]\, d\xi$$

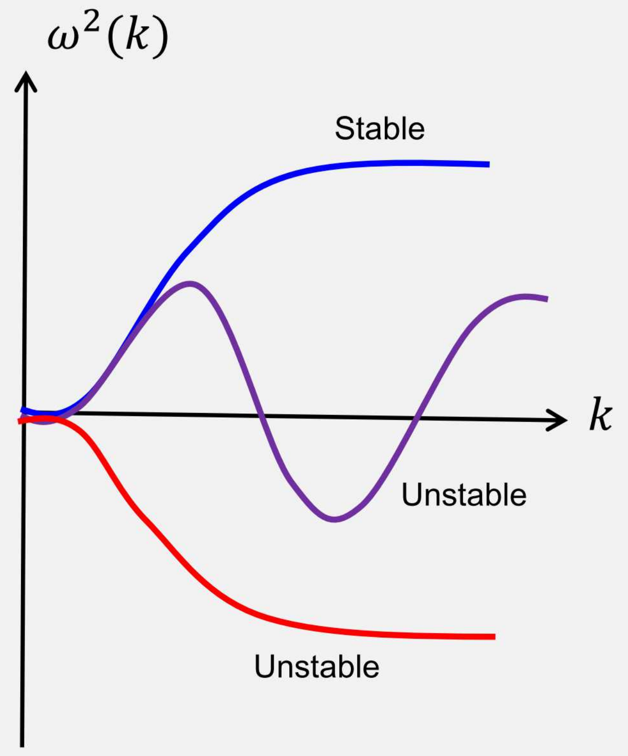
- leads to

$$-\rho\omega^2 = \int_{-\delta}^{\delta} C(\xi)\left[e^{ik\xi} - 1\right]\, d\xi$$

- Therefore the dispersion relation is

$$\omega(k) = \sqrt{\frac{P - \bar{C}(k)}{\rho}}, \qquad P = \bar{C}(0)$$

where $\bar{C}(k)$ is the Fourier transform of $C(\xi)$.



Dispersion curves for 3 materials

Unstable waveforms grow exponentially

- Initial data in the infinite bar:

$$u(x,0) = A\cos kx, \qquad \dot{u}(x,0) = 0 \qquad \forall x.$$

- If $\omega^2(k) > 0$:

$$u(x,t) = \frac{A}{2}\left[\cos(kx - \omega(k)t) + \cos(kx + \omega(k)t)\right].$$

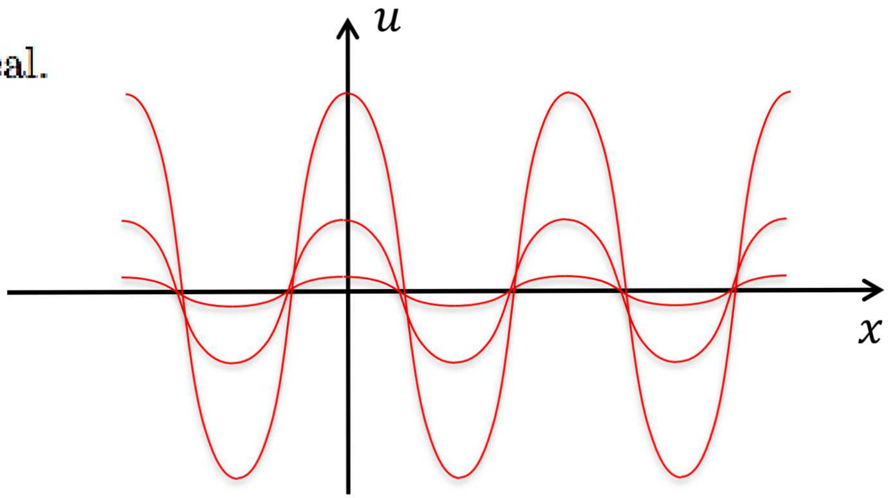
- If $\omega^2(k) < 0$:

$$u(x,t) = A\cos(kx)\cosh(\lambda(k)t)$$

where

$$\lambda(k) = \sqrt{-\omega^2} \qquad \text{real.}$$

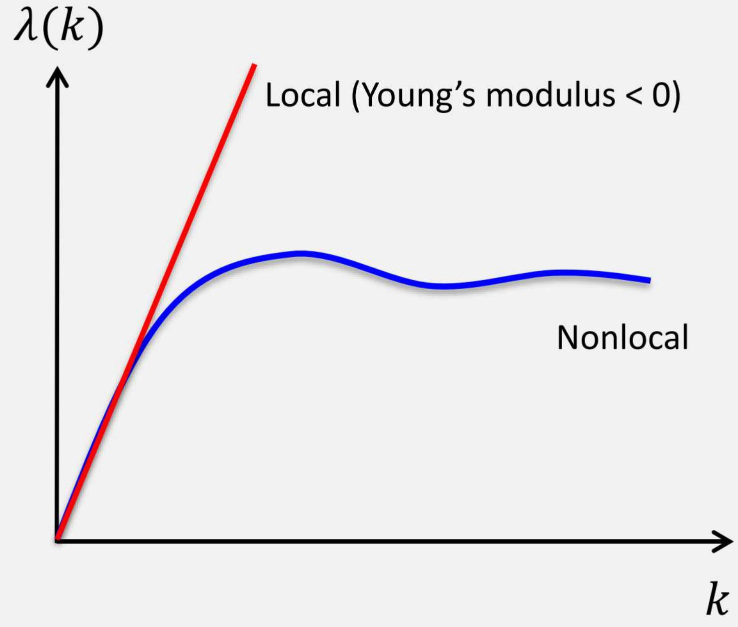
- λ is called the *blow-up rate*.



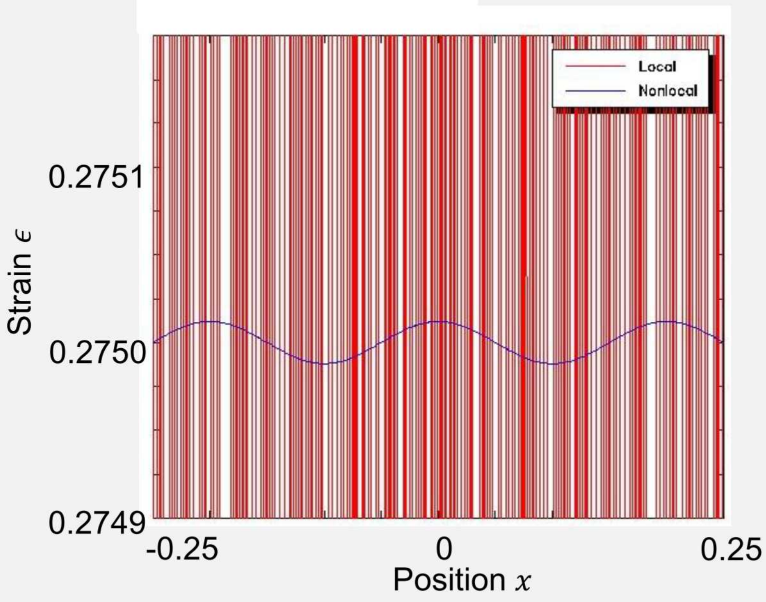
Unstable mechanics is fundamentally different in local & peridynamic theories

- Blow-up rate is bounded in peridynamics for "most" materials for all k.
- Unbounded in the local theory for large k.
  - Hadamard instability (loss of ellipticity, "imaginary wave velocity").
  - Initial value problems generally don't have sensible solutions.

Blow-up rate

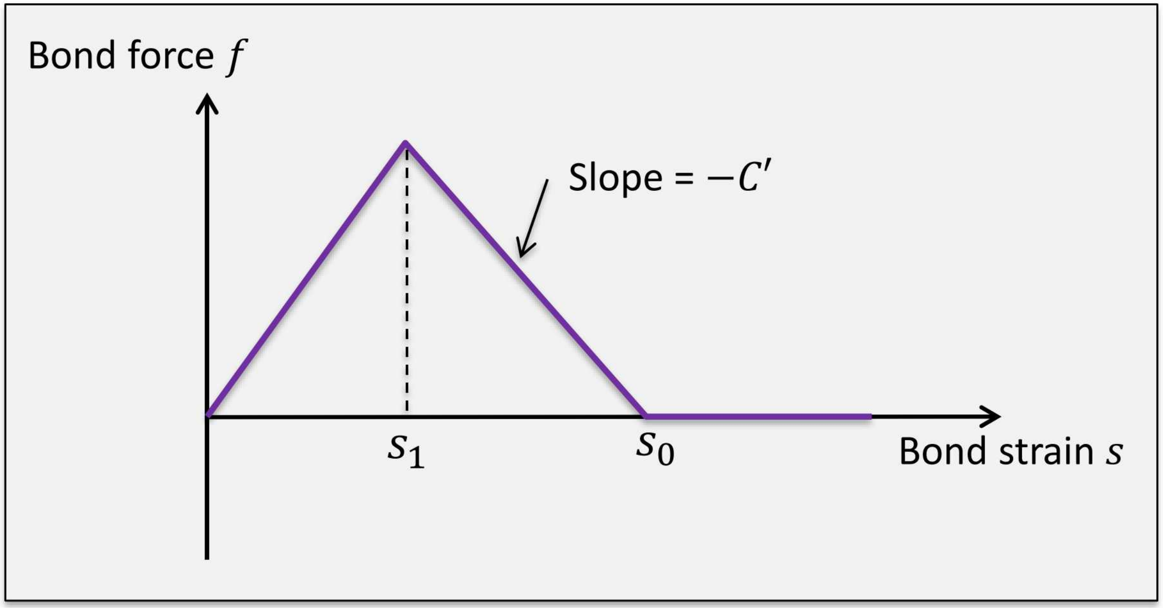


What you get numerically in the local theory with Young's modulus < 0



Failure kinetics: a peridynamic material that changes from stable to unstable

- As the microelastic material is stretched, the bond force passes through a peak into an unstable region.
- Lipton showed problems are well posed. They may spontaneously produce Griffith cracks.



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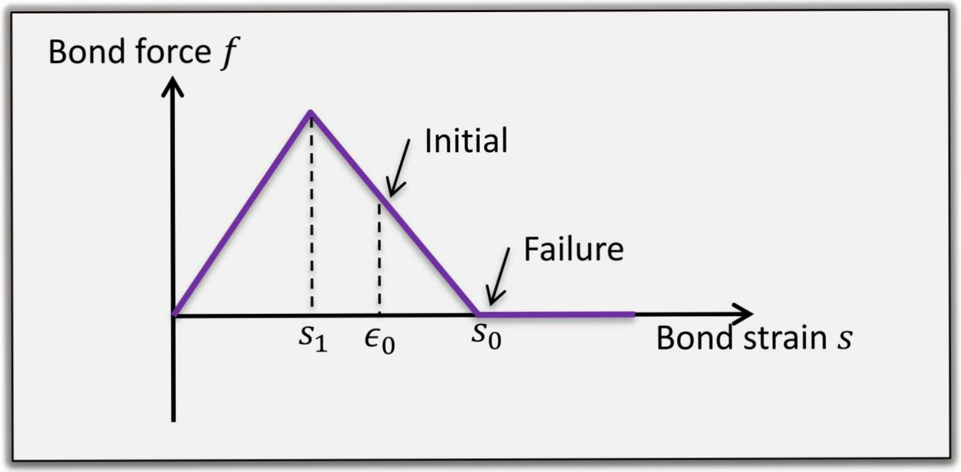
Time to failure with initial data inside the unstable region



- Material model for bond force as shown.
- Initial data in the infinite bar:

$$u(x,0) = \epsilon_0 x + \frac{\tilde{\epsilon}_0}{k}\cos kx, \qquad \dot{u}(x,0) = 0 \qquad \forall x$$

where

$$\epsilon_0 = \frac{s_0 + s_1}{2}$$

and $\tilde{\epsilon}_0$ is a small strain ($\approx 10^{-6}\, s_0$).

- Use the blow-up rate $\lambda(k)$ to find how much time it takes for any bond to fail (bond strain $\geq s_0$).
- Result:

$$t_{fail} = \frac{1}{\lambda(k)} \cosh^{-1}\left(\frac{s_0 - \epsilon_0}{\tilde{\epsilon}_0}\right).$$

Initial data with a spectrum of wavenumbers

- Recall for a single wavenumber k in the unstable region:

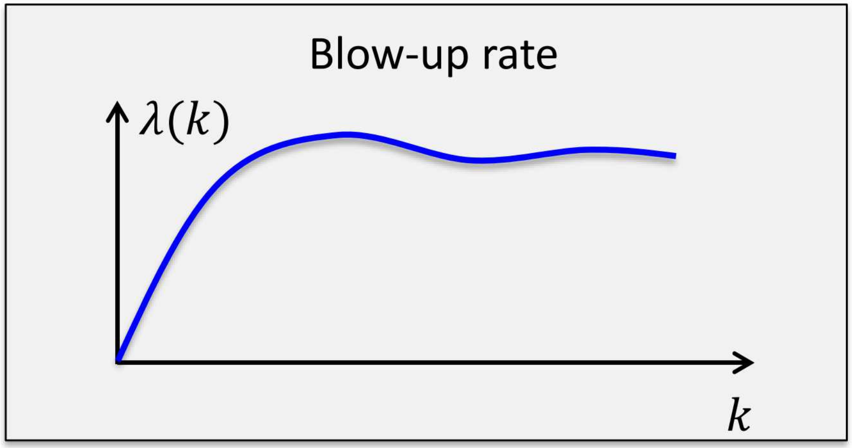
$$u(x,t) = A\cos(kx)\cosh(\lambda(k)t).$$

- For general initial data (even function of x for simplicity):

$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{u}_0(k)\cos(kx)\cosh(\lambda(k)t)\,dk$$

  where $\bar{u}_0$ is the Fourier transform of the initial displacement.

- Usually large k wins the race among Fourier components over long time periods.



Time to failure with a Gaussian initial perturbation in strain

- Initial data in the infinite bar:

$$u(x,0) = \epsilon_0 x + \frac{\sqrt{\pi}\ell\tilde{\epsilon}_0}{2}\mathrm{erf}\left(\frac{x}{\ell}\right), \qquad \dot{u}(x,0) = 0 \qquad \forall x$$

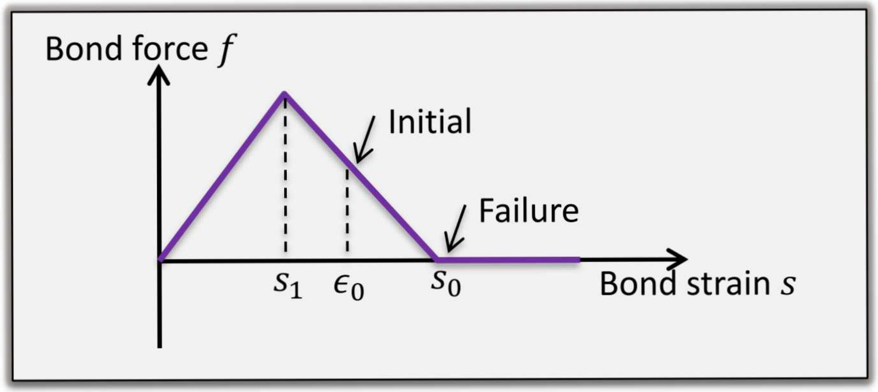
where and $\tilde{\epsilon}_0$ is a small strain erf is the error function and $\ell \ll \delta$ is the half-width of the Gaussian.

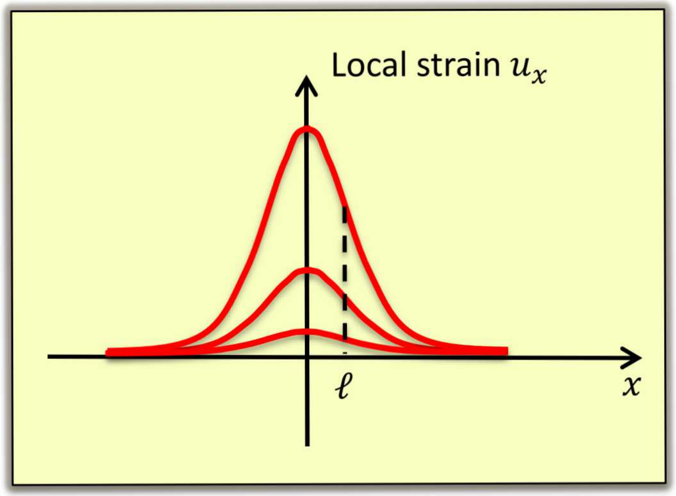
- This differs from the previous example in that there are many Fourier components.

- To solve, use the Fourier transform equation of motion and initial data.

- Result:

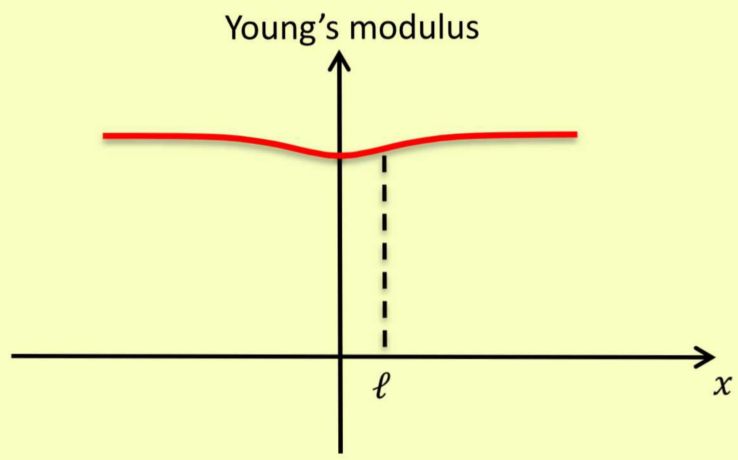
$$t_{fail} \approx \frac{\delta}{C}\sqrt{\frac{s_0 - s_1}{6s_1}}\cosh^{-1}\left(\frac{s_0 - \epsilon_0}{\tilde{\epsilon}_0}\right).$$



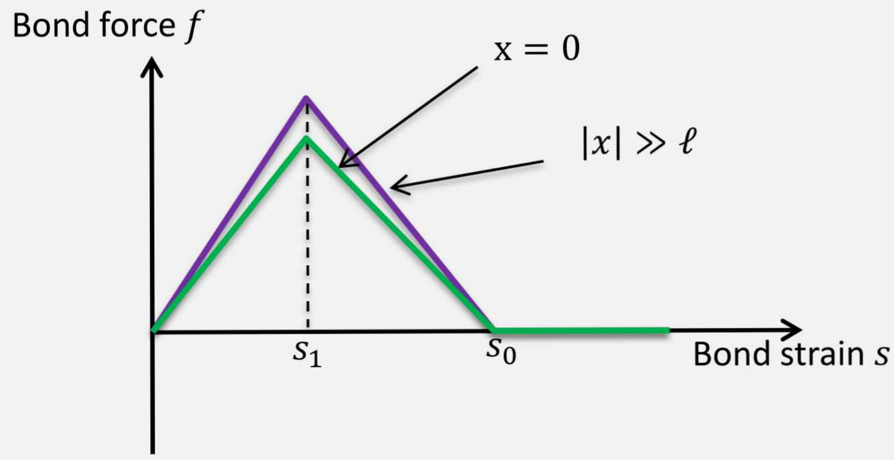


Stretching of a bar with a small defect

- Bar is slightly more compliant in a small region near $x = 0$.
- Bar is initially stretching at a constant rate r_0.
- Effect of the defect is amplified as the strain passes the peak in the material model.



Bar with a small heterogeneity

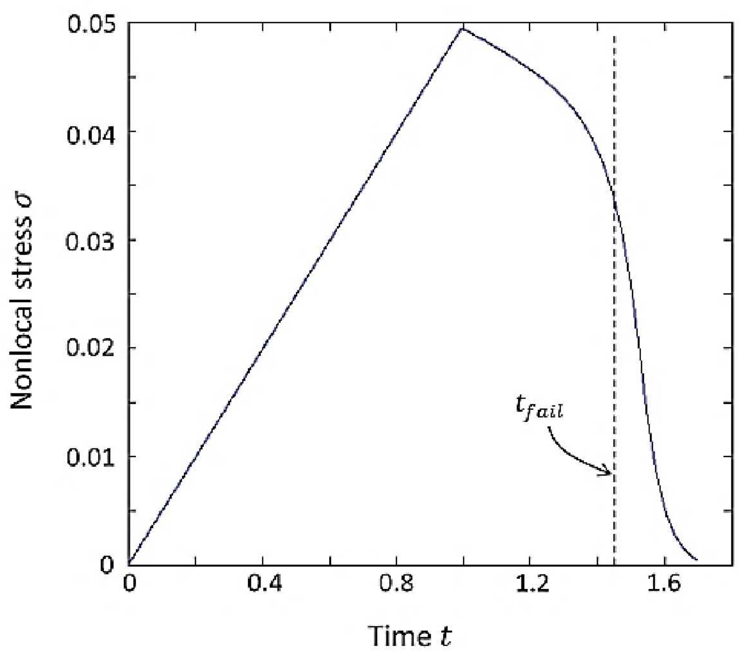


Position-dependent material model

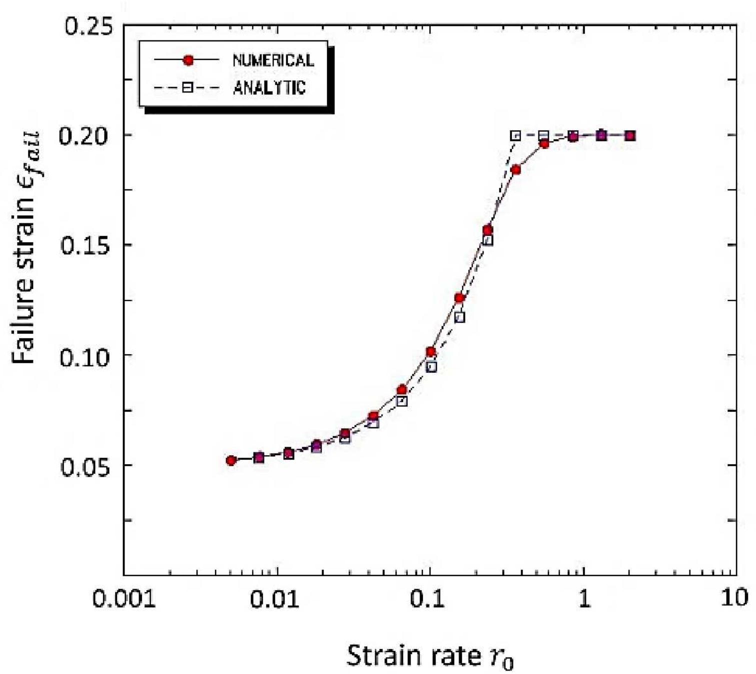
Defect nucleates failure

- Finiteness of the failure time after the peak results in a strain rate effect on the remote failure strain.
- Approximate analysis and Emu code give nearly the same results.

Stress at $x = 0$

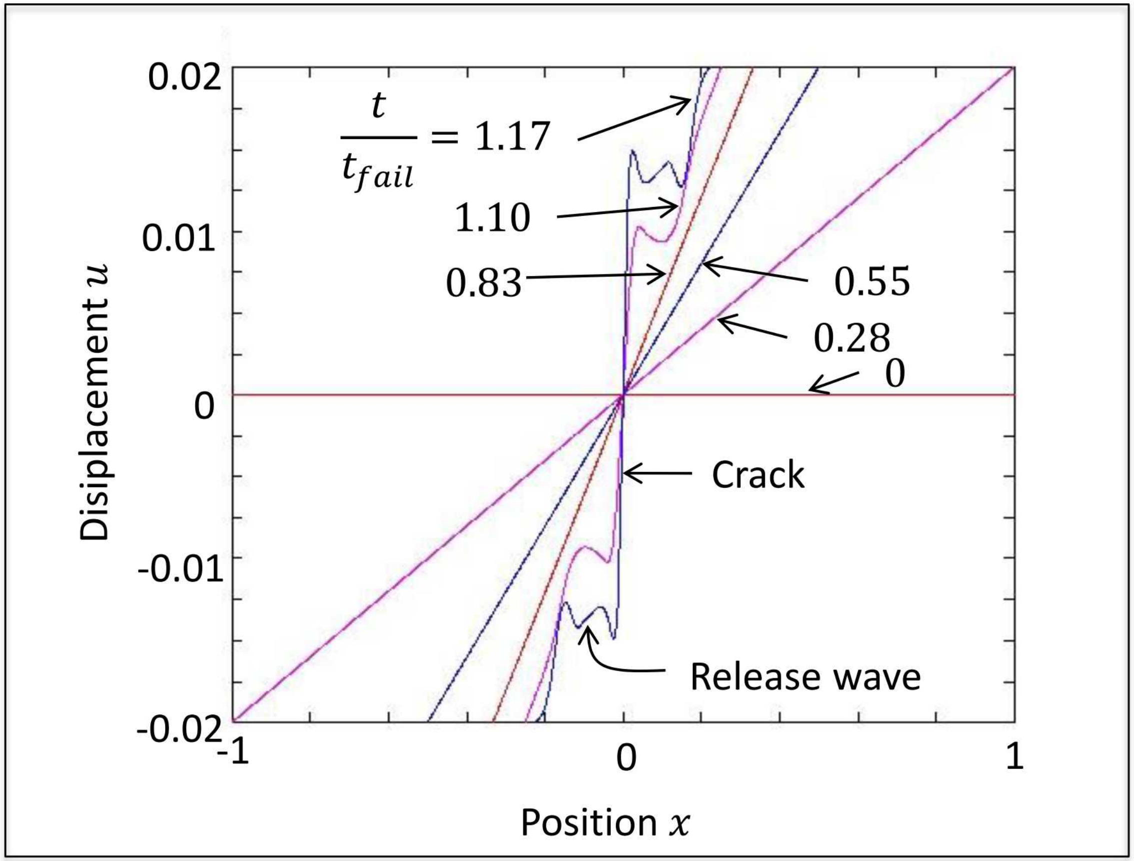


Remote strain at time of failure



Defect nucleates failure: Numerical simulation

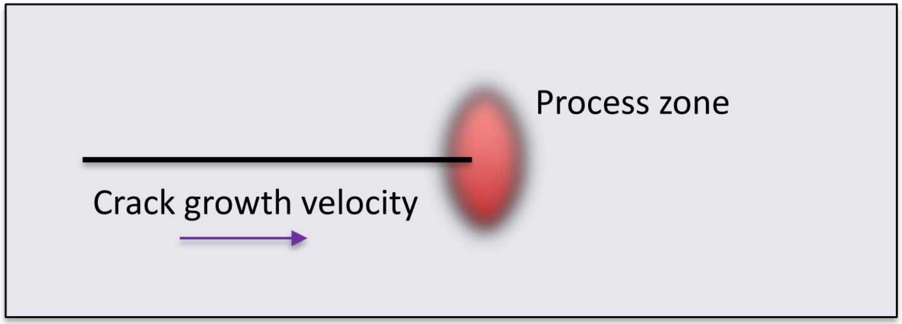
- A crack opens up at $x = 0$ after the bar enters the unstable region.



Discussion

- We get a kinetic effect on material failure “for free” with a nonconvex peridynamic material model.
- Conjectures:
    - This stuff happens near a growing crack tip.
    - Size of the (unstable) process zone influences the blow-up rate.

- Need more research on numerics further for unstable peridynamic materials.
- Need more general theories of convexity and material stability with peridynamics.



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