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Expansion Methods Applied to Internal Acoustic Problems

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Abstract
Expansion techniques have been used for many years to predict the response of un-instrumented locations on structures. These methods use a projection or transformation matrix to estimate the response at un-instrumented locations based on a sparse set of measurements. The transformation to un-instrumented locations can be done using modal projections or transmissibilities. Here, both expansion methods are implemented to demonstrate that expansion can be used for acoustic problems, where a sparse set of pressure measurements, say from a set of microphones in a cavity or room, are used to expand and predict the response at any location in the domain. The modal projection method is applied to a small acoustic cavity, where the number of active modes is small, and the transmissibility method is applied to a large acoustic domain, where the number of active modes is very large. In each case, expansion is shown to work well, though each case has its benefits and drawbacks. The numerical studies shown here indicate that expansion could be accurate and therefore useful for a wide range of interior acoustic problems where only sparse measurements are available, but full-field information is desired, such as field reconstruction problems, or model validation problems.

Keywords: expansion, modal projection, interior acoustics, frequency response function, transmissibility

1 Introduction

Expansion is a process where sparse measurements are used to infer the response at other locations in the domain. Most typically, this is done with structures using accelerometer measurements at various locations on the structure and the System Equivalent Reduction Expansion Process (SEREP) to expands from those sparse accelerometer measurements to other locations of interest on the structure [1]. Often, a model of the structure is used to create the two mode shape matrices needed for SEREP. The first shape matrix is at the measured response locations, here called the a-set degrees of freedom (DOFs). The second shape matrix is at the expanded response locations. The expansion could be to all locations, the n-DOFs, or to a subset of other locations, the b- or c-DOFs. The shape matrixes need to have enough modes to span the space of the content in the measured responses and the number of a-set DOFs must be greater than the number of modes in the matrix. As such, if the response is broadband or involves many active modes, many a-set DOFs are required.

Just as expansion is useful for structural problems, expansion could be useful for many types of acoustic systems. For example, expansion could be used to take a small number of microphone measurements in the inside of an aircraft fuselage and provide the acoustic field at all locations within the fuselage. Expansion could also be used to determine the as-tested pressures on a test article in an acoustic environmental test. In that case, the acoustic domain would be the test chamber, the measured a-set DOFs would be a small number of microphones in the chamber, and the expanded, b-set DOFs would be the points on the surface of a test article. In that way, the tested pressure loads on the test article could be known in a full-field sense. These use cases motivated this work, which uses simulations of big and small systems to exercise expansion of acoustic domains and assess how typical structural expansion techniques may be used for a range of acoustic problems.

A span of acoustic domain sizes represents a span of modal density regimes. A small interior acoustic domain, such as an automobile cabin or the payload bay of a rocket, perhaps may only have tens of modes in a typical bandwidth of interest (e.g. below 2kHz). Conversely, a large interior acoustic domain, such as a large auditorium or reverberation test chamber, can have tens of thousands of modes in the same bandwidth. Thus, these represent very different types of dynamic systems which require different expansion approaches. For the small domain with a small number of active modes, SEREP could be very practical; the number of a-set DOFs (measurements) is tractable. However, the large domain, with a very large number of

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active modes cannot use SEREP expansion as the number of a-set DOFs would be impractical. Here, expansion of a large domain is accomplished instead with a transmissibility approach wherein the frequency response function (FRF) matrices are formed between the a-set DOF outputs and inputs as well as between the b-set DOF outputs and inputs. Then, the transmissibility matrix relating the b-set DOF outputs to the a-set DOF outputs is formed. In that way, the measured a-set DOF outputs can be used to expand to the b-set DOF outputs. With a transmissibility approach, there only needs to be at least as many a-set DOF as independent inputs, so it can work in very high modal density regimes.

In this paper, the theory of these two expansion approaches will be presented briefly. Then, SEREP will be used to expand acoustic responses inside a model of a cavity of a shell test article, which is an example of a small domain with low modal density. Next, transmissibilities will be used to expand acoustic responses in a model of a reverberation chamber, which is an example of a large domain with high modal density. Unfortunately, no test data was available at the time of this writing to demonstrate these techniques with actual measurements but results of these numerical example problems are promising and should motivate an experimental demonstration in the future.

2 Theory

SEREP expansion uses a modal projection to convert a set of measured DOFs, the a-set DOFs, to a set of expanded DOFs, the n-set DOFs. This is accomplished using a transformation matrix, $[T]$:

$$\{x_n(t)\} = [T]\{x_a(t)\}. \tag{1}$$

The transformation matrix is formed using a set of mode shape matrices, $[U_a], [U_n]$, at the a- and n-DOFs, respectively. A pseudo-inverse of the $[U_a]$ mode shape matrix projects the measured a-set DOFs into modal space, giving the modal responses. Those modal responses are then projected back to physical space at the n-set DOFs through the $[U_n]$ mode shape matrix. The SEREP transformation matrix is:

$$[T] = [U_n][U_a]^+, \tag{2}$$

where the superscript $[\cdot]^+$ indicates a pseudo-inverse of the shape matrix. While Equation 1 shows expansion of time responses, frequency domain quantities can also be expanded in the same way. For example, linear spectra at a-DOFs, $\{X_a\}$, can be expanded to n-DOFs, $\{X_n\}$, with an analogous equation:

$$\{X_n(\omega)\} = [T]\{X_a(\omega)\}. \tag{3}$$

Expansion can also be achieved using transmissibility functions, which are a ratio of two FRFs. The first FRF relates the measured responses at the a-DOFs to the input. This FRF is computed (if using the H1 FRF estimator) based on the CPSD of the outputs and inputs, $[S_{ai}]$, and the CPSD of the inputs, $[S_{ii}]$:

$$[H_{ai}] = [S_{ai}][S_{ii}^+]. \tag{4}$$

The FRFs at the n-DOFs can be similarly computed, giving $[H_{ni}]$. With the two FRFs relating outputs at a- and n-DOFs to some inputs i, the transmissibility matrix can be formed which relates the response at the n-DOFs to the response at the a-DOFs [2]:

$$[T_{na}] = [H_{ni}][H_{ai}]^+. \tag{5}$$

With the transmissibility matrix, $[T_{na}]$, the measured response at the a-DOFs can be used to estimate the response at the n-DOFs with:

$$\{X_n\} = [T_{na}]\{X_a\}. \tag{6}$$

Note that the number of a-DOFs required for SEREP must be equal to or greater than the number of modes in $[U_a]$ and $[U_n]$. Similarly, there is a minimum number of a-DOFs for the transmissibility expansion, where the number of a-DOFs must be equal to or greater than the number of inputs to the system. Also, with the transmissibility approach, the input locations and directions must be known and consistent. That is, the input locations used to determine the transmissibility matrix must be the same as the actual input locations in any measurement used to expand. If the input locations change, the results will not be valid. These techniques are general to any linear system and here are simply applied to acoustic systems to demonstrate their usefulness for these types of problems.

3 SEREP Expansion for Acoustic DOF in a Small Domain Model

Acoustic finite element models were created for the small shell cavity and the large reverberation chamber. The shell cavity model is based on a piece of test hardware shown in Figure 1. The interior air cavity is cylindrical with a 7 inch diameter and 24 inch length. The model was meshed using hexahedral elements with the element size of 0.5 inch chosen to give accurate results to at least 2kHz. A 1 ms haversine input is provided at the edge of one end of the cavity as an acoustic velocity on a set of element faces, shown in Figure 1. The pressure response due to this input is measured at the a-set DOF and then used to expand to the other DOF in the cavity. The first task was to determine the a-set DOF which can best measure the response due to a set of modes in the chosen bandwidth of 1kHz. This frequency range includes the first 20 modes, so at least 20 a-set

DOF are needed. Here 25 DOF were chosen from a set of 168 candidate locations using the effective independence (EFI) algorithm [3]. The candidate locations and the chosen a-set locations are shown in Figure 2.

SEREP expansion of this acoustic domain is demonstrated two ways. First, the pressure modes of the cavity are expanded. In this case, the cavity modes from the finite element model at the a-set DOF were used to expand to all the DOF in the model. To make this expansion of "perfect" model data more realistic, random noise was added to the a-set "measured" shapes. As shown in Figure 3, the expansion of the acoustic pressure mode shapes of the cavity worked properly with this set of modes and a-set DOF. The expanded mode shapes are nearly identical to the actual mode shapes at all the DOF in the cavity.

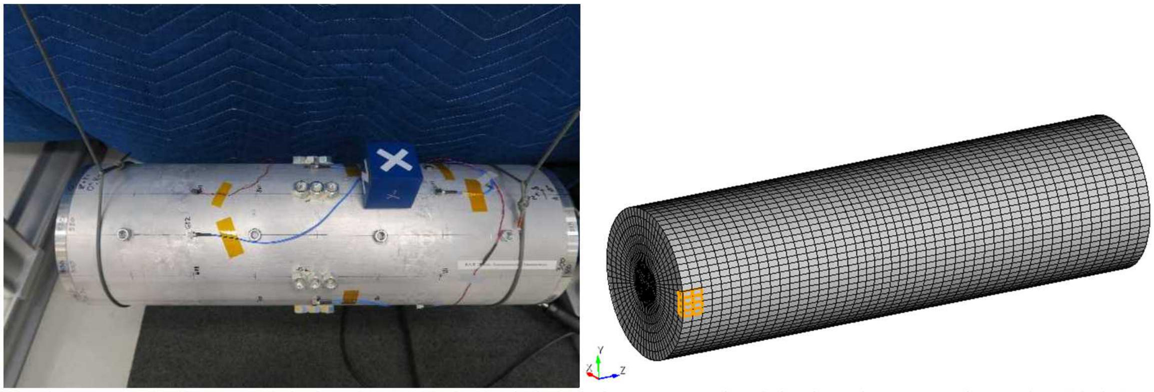

Figure 1: Shell test article hardware (left) and finite element mesh of the interior acoustic cavity (right)

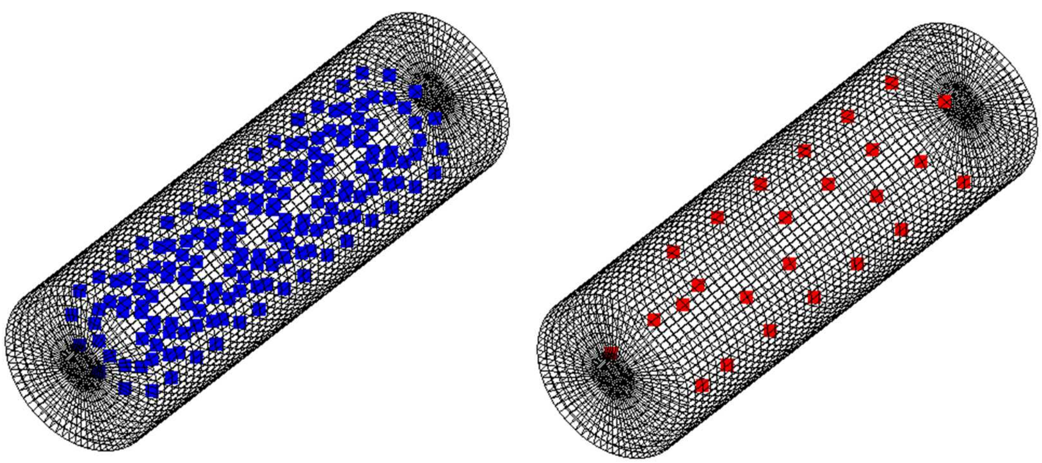

Figure 2: 168 Candidate measurement DOF (left) and 25 a-set DOF selected with EFI (right)

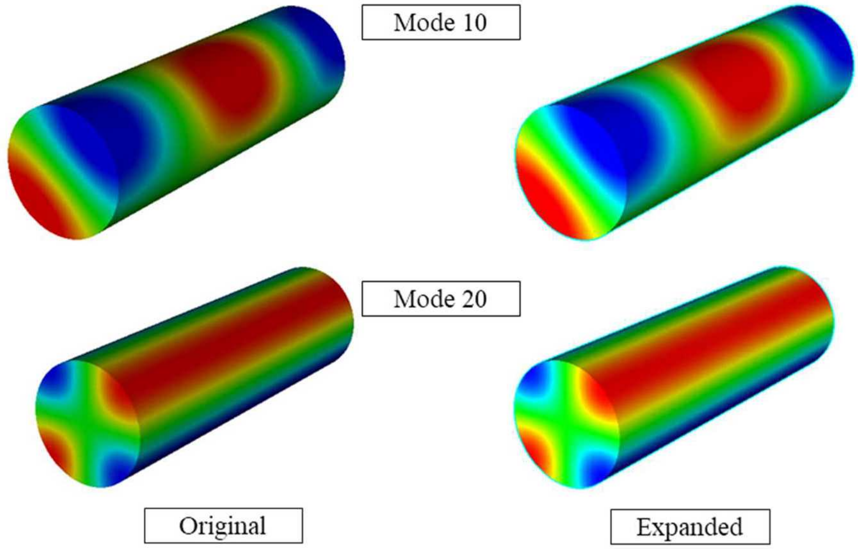

Figure 3: Example mode shapes of the shell cavity. Left: Actual mode shapes. Right: Expanded mode shapes.

Next, expansion of transient pressure response is demonstrated by simulating the transient response of the cavity due to the 1 ms haversine input at the corner of the cylinder. Again, some random noise was added to the measured a-DOF response to simulate the effects of having imperfect response data. While the response of all the DOF could be used in the expanded b-set DOF as in the mode shape expansion case, just two nodes specified in Figure 4 were used in the b-set to simplify visualization. The expansion results are shown in terms of the time response, in Figure 5, and the power spectral density (PSD) of that time response, in Figure 6. Using either metric, the expanded response of the two b-set DOF in this acoustic domain match very well with the actual response, indicating that SEREP is a useful method for acoustic expansion of not only mode shapes, but transient responses as well. A typical artifact of mode-based expansion methods is mode truncation errors that are caused by not including enough modes in the mode shape matrices used to make the SEREP transformation matrix. The effects of mode truncation are demonstrated in Figure 7 where just three modes were used to populate the transformation matrix. This is exactly the expected behavior, with the expanded response matching well at low frequencies and not as well at higher frequencies.

The results from SEREP expansion of the acoustic response in terms of modes and time responses show that SEREP is effective for internal acoustic problems as well as structures. While this isn't surprising, it is useful to see the technique demonstrated with this atypical application in mind. Just like with structural system expansion, acoustic system expansion is sensitive to problem setup considerations such as mode truncation, as demonstrated here, and also factors such as a-set DOF and mode selection. For systems with a small number of active modes, such as some automobile cabins or small rocket payload sections, this expansion technique could be effective in generating full-field responses from a small number of measurements.

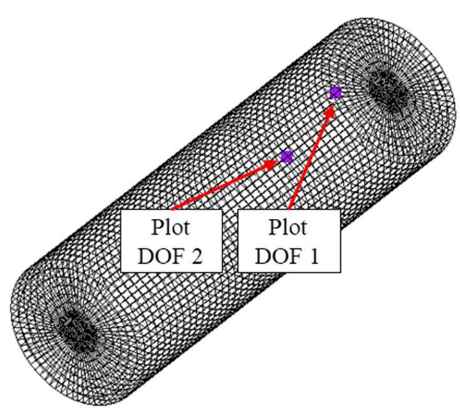

Figure 4: Locations of two expanded b-set DOF in the interior of the cavity

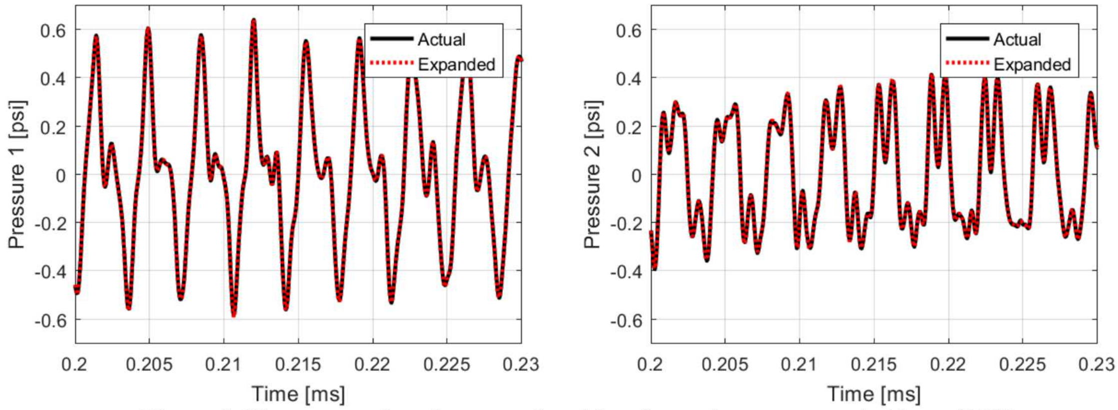

Figure 5: Time expansion shown as time histories at the two expanded b-set DOF

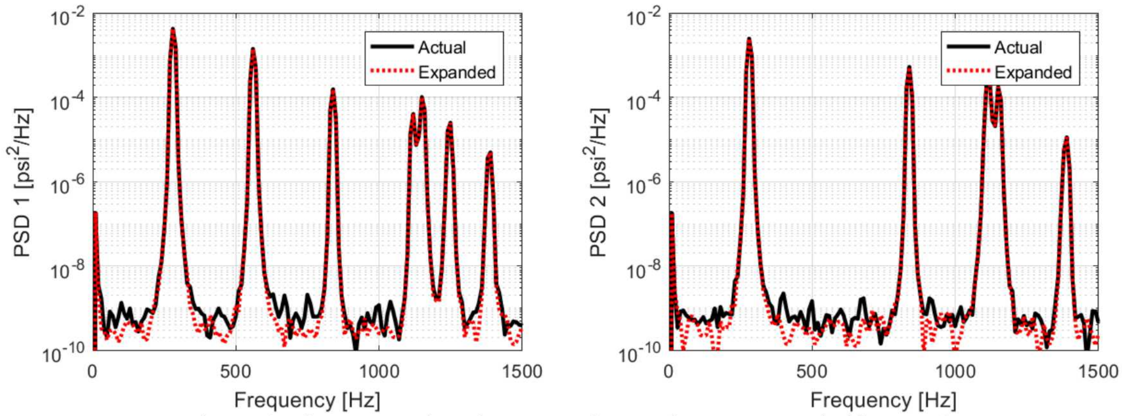

Figure 6: Time expansion shown as PSDs at the two expanded b-set DOF

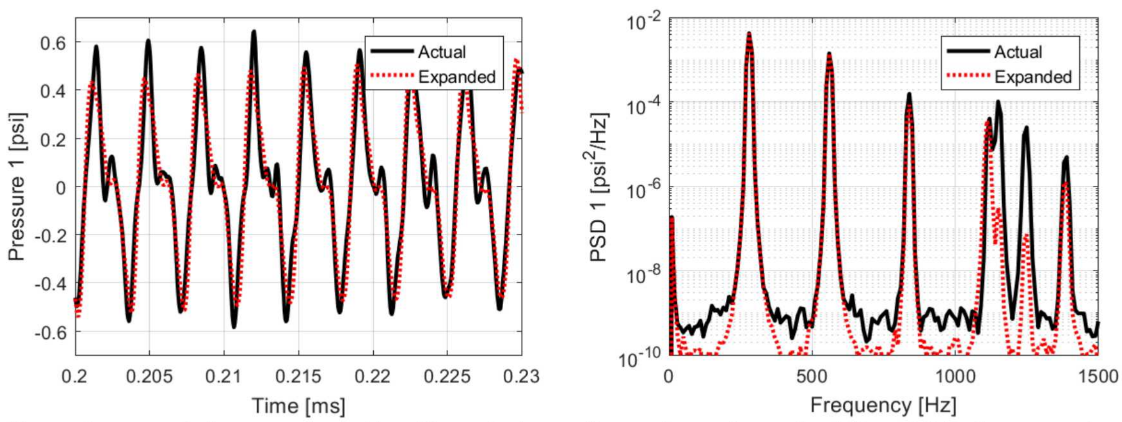

Figure 7: Example expanded response results with mode truncation, using only modes 1,2, and 3 in the expansion. Left: Time response at one b-set DOF. Right: PSD at one b-set DOF.

4 Transmissibility Expansion for Acoustic DOF in a Large Domain Model

Expansion of a large domain using transmissibilities is demonstrated using a model of a 21x25x30 foot reverberation chamber. Due to the size of this domain, the element size was limited to 4 inches, which limits the maximum frequency of the analysis to around 500 Hz. Three independent inputs were provided by surface velocities at small patches in three of the bottom corners of the chamber model, which could represent three loudspeaker inputs, as shown in Figure 8. Locations of 20 a-set DOF are shown as red dots in the left image of Figure 8. These locations were chosen at random in this case, rather than with the EFI algorithm. Two sets of expansion DOF were chosen, representing two different use cases. The first, the b-set DOF, represents points on the wetted surface of a cylindrical test article located in one corner of the chamber, useful for cases where the as-tested pressure on a test article is desired. The second, the c-set DOF, are just random points throughout the entire chamber domain, which provides an indication of the quality of the expansion over the entire space. These two sets of expansion DOF are shown in the center and right images in Figure 8.

The chamber has modes, just like the shell cavity, which can be viewed in terms of a pressure mode shape. Figure 9 shows two such modes of the chamber. The difference between the shell cavity and this chamber is the modal density. Where the shell cavity has just 4 modes below 1000 Hz, the chamber has more than 40,000 modes, as shown in Figure 10. This extreme number of modes is what motivated the use of transmissibilities instead of a modal projection for expansion of this system.

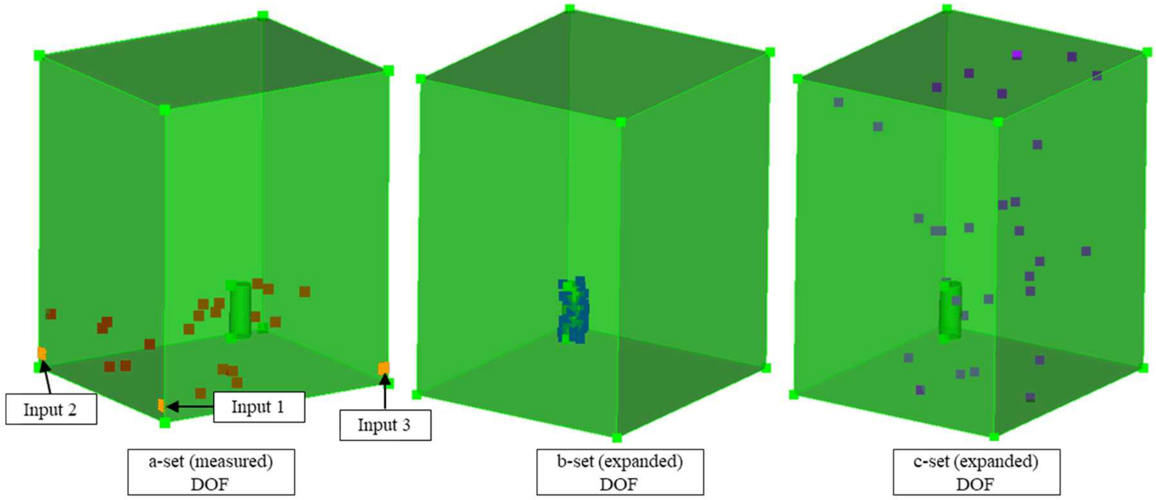

Figure 8: Acoustic chamber model showing the three input locations and a-set DOF (left), b-set DOF located on the surface of a cylindrical test article (center) and c-set DOF distributed throughout the chamber volume (right)

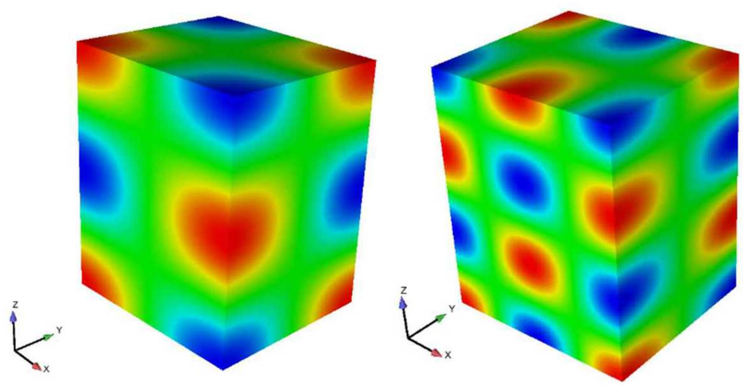

Figure 9: Example pressure mode shapes of the acoustic chamber

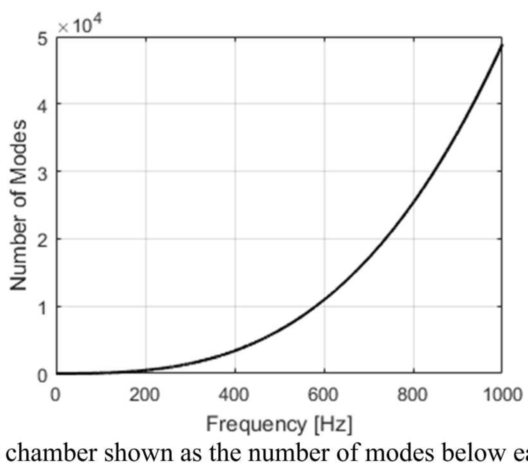

Figure 10: Modal density of the chamber shown as the number of modes below each frequency from 0 to 1000 Hz

Three independent, white noise inputs were applied at the three surface patches in terms of an acoustic velocity and used in a transient simulation of the chamber model to obtain the pressure response at the a-, b-, and c-DOFs. Next, the pressure response/input surface velocity FRFs were created using time responses of the pressure at the a-, b-, and c-DOF and the applied surface velocity. To get accurate FRF estimates, 30 seconds of simulated response was needed. With approximately 0.1 percent Rayleigh damping assigned to the air in the model, it took around 4-5 seconds for the levels to become stationary in the chamber. Also, it was observed that the FRF estimates were sensitive to the block size used. With a short block size, such as one or two seconds, the FRFs were not converged and resulted in inaccurate expanded responses. Five to seven seconds was found to give acceptable results in this case, though further study of this phenomenon is needed. The FRFs were used to create transmissibility matrices relating the response at the b- and c- DOFs to the a-DOFs due to these three input locations. Next, the PSD of the a-DOF response of this transient simulation was used along with the transmissibility matrices to expand to the b- and c-DOF. Results of expansion of two example b- and c-DOF are shown in Figure 11 and Figure 12. The response at all these locations is matched very well over this frequency range. So, with 20 a-set DOF this transmissibility-based expansion works well. Next, the a-set was reduced to determine how well the expansion works with a very reduced set of DOF. Figure 13 shows expansion results at one b-set DOF and one c-set DOF using just four a-set DOF. Even with the number of a-set DOF being thousands of times less than the number of modes, this expansion works properly. It should be noted that three a-set DOF was also used, with decent results, though increasing to four provided more accurate results.

The similarities in the expanded and actual response in Figure 11, Figure 12, and Figure 13 highlight the benefit of using transmissibility for expansion of this high modal density system. Often the acoustic pressure on a test article during an acoustic environment test in a reverberation chamber is desired but cannot be measured. This transmissibility expansion technique was able to take a very small number of measurements and accurately expand to many locations on the surface of a test article or anywhere else in the chamber.

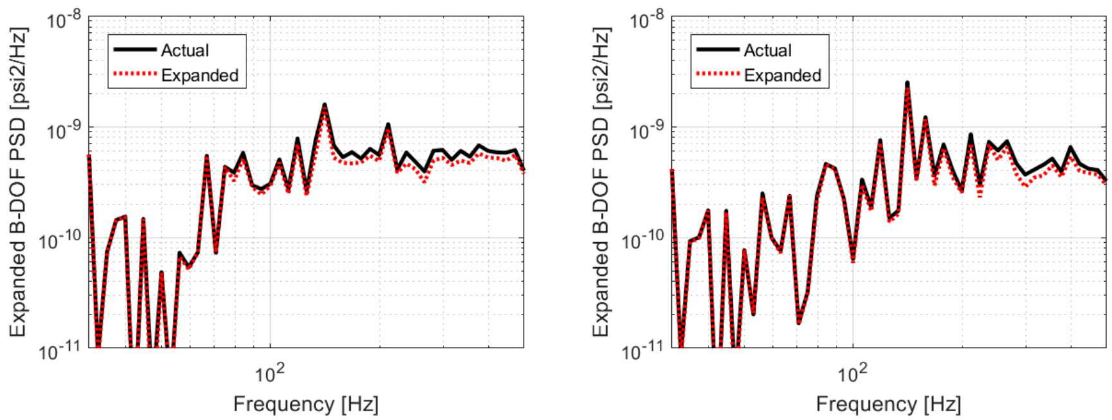

Figure 11: Actual and expanded response at the expanded b-set DOF located on the surface of a cylindrical test article, shown in 12th octaves

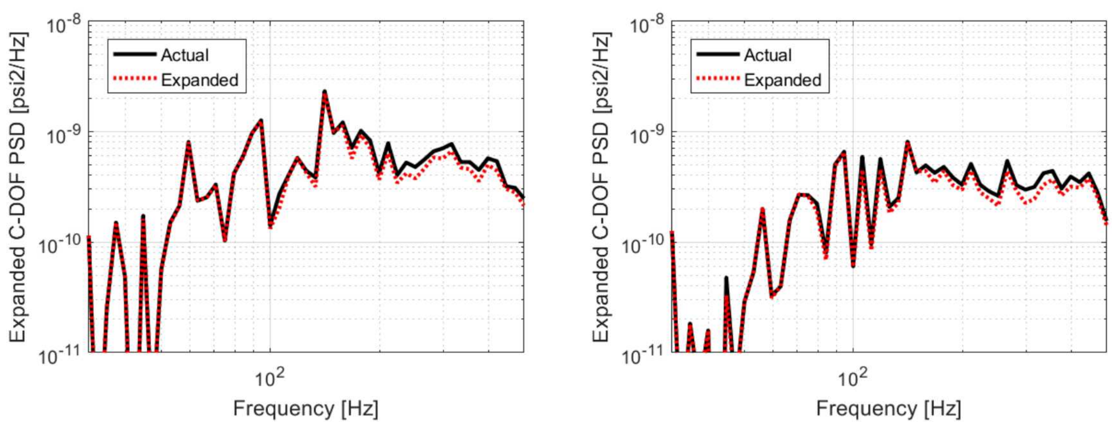

Figure 12: Actual and expanded response at the expanded c-set DOF distributed throughout the chamber volume, shown in 12th octaves

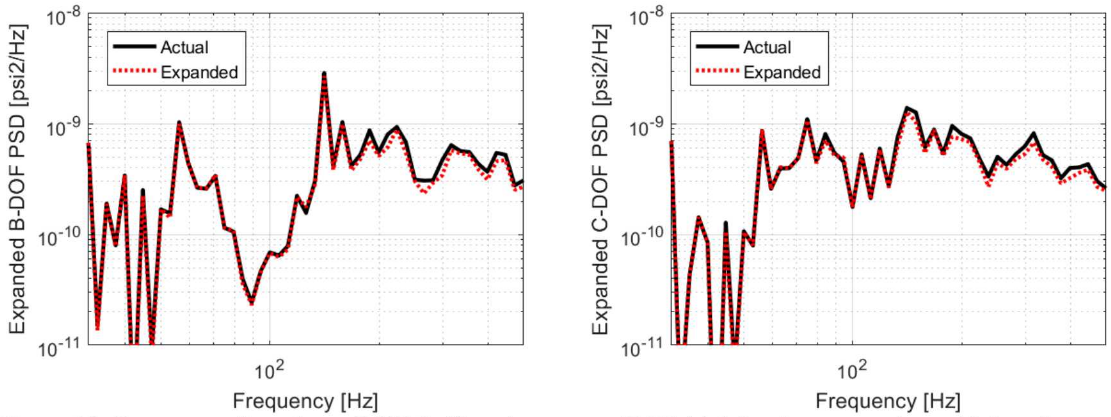

Figure 13: Response of one b-set DOF (left) and one c-set DOF (right) using expansion with just 4 a-set DOF

5 Discussion and Conclusions

Expansion techniques can make limited measurements much more useful, providing response estimates at any location on the structure, or as shown here, in the acoustic domain. Modal projection approaches, such as SEREP, can work very well provided there are enough a-set DOF for the active modes in the response. SEREP allows for shapes, time responses, or frequency quantities such as linear spectra to be expanded with no knowledge of the input forces (or their locations) required. As shown in the shell cavity example here, SEREP is also effective for interior acoustic problems with low modal density.

For problems with very large numbers of active modes, such as the reverberation chamber model used in this work, SEREP is not appropriate as the number of measurements (a-set DOF) is impractically large. Instead, transmissibilities could be used for expansion of these types of systems. The benefit of a transmissibility approach is that a much smaller number of a-set DOF are required, because the number of a-set DOF only needs to be as large as the number of independent inputs to the system. In the example shown here, four a-set DOF were used to expand response of a three-input system with thousands of modes in the bandwidth. The downside of the transmissibility approach is that the input DOF must be known.

Overall, this work provided two simple examples which represent two different, but typical, acoustic systems for which expansion would be a useful tool. The two expansion methods demonstrated here each provided accurate response estimates, indicating expansion of acoustic domains is possible, and behaves just like the expansion of structures. Future efforts will try to experimentally validate these findings using tests of small and large acoustic domains.

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