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Calibration of Shaker Electro-Mechanical Models

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Abstract
Simple electro-mechanical models of electrodynamic shakers are useful for predicting shaker electrical requirements in vibration testing. A lumped parameter, multiple degree-of-freedom model can sufficiently capture most of the shaker electrical and mechanical features of interest. While several model parameters can be measured directly or obtained from a specifications sheet, others must be inferred from an electrical impedance measurement. Here, shaker model parameters are determined from electrical impedance measurements of a shaker driving a mass. Then, parameter sensitivity is explored to determine a model calibration procedure where model parameters are determined using manual and automated selection methods. The model predictions are then compared to test measurements. The model calibration procedure described in this work provides a simple, practical approach to developing predictive shaker electromechanical models which can then be used in test design and assessment simulations.

Keywords: electro-mechanical model, electrodynamic shaker, model calibration

1 Introduction

An electrodynamic shaker can be approximated as a coupled electro-mechanical multiple degree-of-freedom (MDOF) system [1]. A simplified MDOF model of an electrodynamic shaker can be used to assess the effects of connecting a shaker to a structure or to estimate the voltage and current requirements to excite a structure. The MDOF model of the shaker includes mechanical parts (springs, masses, and dampers) and electrical parts (inductors and resistors). There is also a coupling part, sometimes called a gyrator, which links or couples the mechanical dynamic system to the electrical dynamic system. This coupling is the electro-magnetic force. The current in the electrical circuit produces a field which reacts with the magnet in the shaker body, generating a force which drives the mechanical system (and vice versa).

Some of the model parameters can be directly measured or obtained from a specification sheet, although some cannot and must be inferred from a dynamic measurement of the electrical impedance. With the measured electrical impedance, unknown model parameters can be tuned to calibrate the shaker model. This tuning process can be done manually by first examining the sensitivity of the electrical impedance to each parameter and then making appropriate changes to affect the impedance in particular frequency ranges. Alternatively, the model parameters can be tuned using a parameter search algorithm. This paper shows model calibration done manually and also with a simple Monte Carlo parameter search.

2 Shaker Electro-Mechanical Model

Shaker electro-mechanical models have been derived by various authors over the years using various techniques. Tiwari et al. presents a lumped parameter model of a shaker and provides detailed methods for obtaining various unknown parameters using measurements with specific shaker configurations and reduced expressions for the model applicable to specific frequency ranges [1]. Lang also provides several different test conditions to isolate particular behavior to extract shaker parameters [2]. Smallwood used a two-port impedance model method to represent a shaker, with the entries in the impedance matrix coming from measurements of specific frequency response functions (FRFs) or impedances [3]. Varoto and Oliveira present analytical expressions for a lumped parameter model that also includes terms which account for an amplifier operating in current mode or voltage mode [4]. The approach used here is similar to that given by Lang and Snyder, which presents a matrix form of the lumped parameter model [5]. Mayes et al. used a similar approach with the lumped parameter

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model equations put in matrix form and then cast in a component mode synthesis problem to connect the shaker model to a model of a system [6]. The set of equations of motion is formed as a matrix equation which can then be solved, giving the shaker response per input FRFs. As there are no external forces, the input is the applied voltage and the response of interest is the shaker head response.

Development of the shaker model begins by examining a typical electrodynamic shaker. Here, the focus is on modal shakers, though similar methods could be used to model large, vibration test shakers as well [3]. A cutaway of a modal shaker is shown in Figure 1. The body of the shaker contains some stationary magnets. A lightweight armature attaches to the body with several flexures which are springs that allow the armature to translate primarily along the axis of the shaker body. The armature is surrounded by a coil through which current flows, generating the transient field which interacts with the magnets to provide the motive force on the armature. A stinger connects the armature to the device under test (DUT).

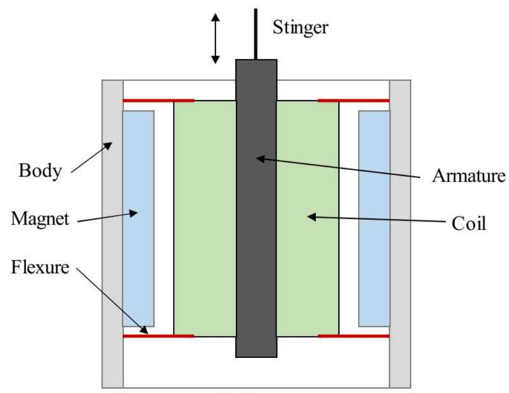

Figure 1: Shaker components

Figure 2 shows the components of the MDOF system used in this work. The mechanical component, shown on the left, has three masses connected by springs and dampers. Mass M_1 is the armature, mass M_2 is the shaker body, and mass M_3 is the stinger. The stinger mass can represent the mass loading of an attached DUT. The flexures are represented by spring K_{12} and the stinger stiffness is represented by spring K_{13}. If the shaker armature had additional dynamics of interest, or if there was a table or shaker head, additional springs and masses could be added to M_1 to represent these features. This model form with three masses works well for a typical modal shaker in a typical frequency range (i.e. around 1500 Hz). The mechanical component is driven by a force between the body and armature, which is the electromotive force (EMF) of the coil in the magnetic field. The electrical component, shown on the right, has a voltage source which is provided by the amplifier, in series with a resistor and inductor with represent the coil. Then, there is a voltage source which represents the back electromotive force; this is the coupling of the mechanical component back into the electrical component. That is, motion of the mechanical component introduces a voltage in the electrical component through this back EMF source. This coupling is represented in the model by the force factor, BL, which links the current in the electrical domain to the force and relative velocity in the mechanical domain.

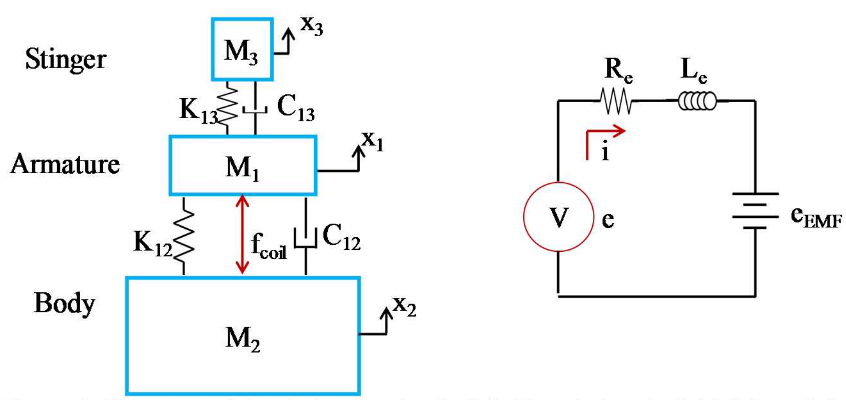

Figure 2: Elements of the shaker mechanical (left) and electrical (right) models

2.1 Equations of Motion

With the mechanical and electrical component models formed and their parts defined, the equations of motion can be written. There is an equation of motion for each mechanical DOF (each mass), and one equation of motion for the electrical circuit. These four equations can be put in matrix form in the typical mass, damping, and stiffness matrix form as shown below:

$$\left[M_{shk}\right]=\begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

$$\left[C_{shk}\right]=\begin{bmatrix} \left(C_{12}+C_{13}\right) & -C_{12} & -C_{13} & 0 \\ -C_{12} & C_{12} & 0 & 0 \\ -C_{13} & 0 & C_{13} & 0 \\ BL & -BL & 0 & L_e \end{bmatrix}, \tag{2}$$

$$\left[K_{shk}\right]=\begin{bmatrix} \left(K_{12}+K_{13}\right) & -K_{12} & -K_{13} & -BL \\ -K_{12} & K_{12} & 0 & BL \\ -K_{13} & 0 & K_{13} & 0 \\ 0 & 0 & 0 & R_e \end{bmatrix}, \tag{3}$$

This system of equations has inputs of force on each mass and voltage on the circuit and outputs of displacement of each mass and current in the circuit. Thus, the system of equations can be written as:

$$\left[M_{shk}\right]\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{i} \end{Bmatrix}+\left[C_{shk}\right]\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{i} \end{Bmatrix}+\left[K_{shk}\right]\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ i \end{Bmatrix}=\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ e \end{Bmatrix} \quad . \tag{4}$$

Direct frequency response can then be used to solve this matrix equation and give the output/input FRFs for all outputs and all inputs.

$$\left[H\right]=\left[-\omega^2\left[M_{shk}\right]+j\omega\left[C_{shk}\right]+\left[K_{shk}\right]\right]^{-1} \quad . \tag{5}$$

However, as there are no external forces, $f_1=f_2=f_3=0$, so the only FRF of interest is with the voltage input, e, which is the fourth column of $\left[H\right]$. The result is a four output, one input FRF matrix which is evaluated at each frequency line of interest. This FRF matrix is essentially an impedance model of the coupled electro-mechanical shaker system.

2.2 Shaker Force from the Stinger Spring

The force applied by the shaker to the DUT is the force in the spring between masses M_1 and M_3. This force does not come directly out of this model but can be determined given the displacement of those two masses, x_1 and x_3, and the stiffness of the stinger spring, k_{13}. The stinger force/input voltage FRF, H_{f_se}, can be obtained using the two displacement/voltage FRFs, H_{x_1e}, H_{x_3e}:

$$H_{f_se}=K_{13}\left(H_{x3e}-H_{x1e}\right) \quad . \tag{6}$$

Note that this is nothing more than Hooke's law, $f = k\Delta x$, written in terms of FRFs.

2.3 Features of the Impedance and FRF Curves

The electrical impedance is the ratio of the voltage and current: $Z_{ei} = e / i$. The coupled electro-mechanical shaker system has some interesting features in the impedance. There are two peaks in the impedance, one at the suspension mode (low frequency) and one at the stinger mode (high frequency). The suspension mode is the mass of the armature and stinger moving opposite the shaker body through the flexure springs. The stinger mode is the mass of the stinger moving opposite the armature through the stinger spring. The frequency of these peaks is dependent on the mass and stiffness and the width of the peaks is affected by the damping. Between the two peaks, the real part is dominated by the resistor in the electrical component and the imaginary part is dominated by the inductor. The resistance only affects the real part and is constant with frequency whereas the inductance only affects the imaginary part and the effect is proportional to frequency.

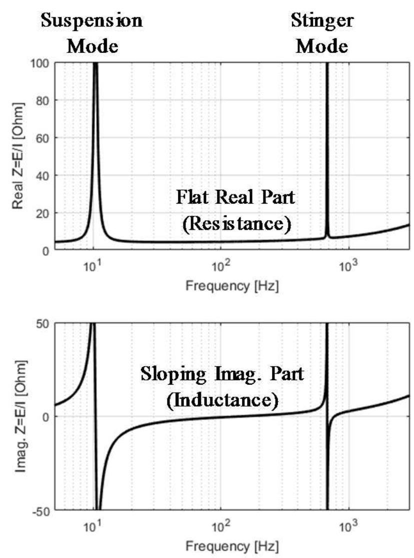

Figure 3: Example electrical impedance from the shaker model. Top: Real part. Bottom: Imaginary part.

2.4 Model Parameter Sensitivity

To tune the model, it is useful to understand how the various model parameters affect the impedance. The figures below show how each parameter affects the impedance by plotting impedance with each parameter at nominal, lower, and higher values. For all parameters except the inductance, lower is one half the nominal value and higher is twice the nominal value. For the inductance lower is one fifth and higher is five times the nominal value. Nominal model parameters are given in Table 1. Note that the stinger mass is large, 2.6 kg, representing a large mass at the end of the shaker, which is representative of the experiment described in the next section.

Table 1: Nominal model parameter values

Parameter	Value	Unit
M_1	0.44	kg
M_2	15	kg
M_3	2.6	kg
K_{12}	1.10E+04	N/m
K_{13}	1.46E+07	N/m
C_{12}	17	N/(m/s)
C_{13}	10	N/(m/s)
R_e	4	Ohm
L_e	6.00E-04	Henry
BL	35	-

Increasing the armature mass (M_1) shifts the modes lower in frequency, Figure 4. In this case because the stinger has a large mass attached, changing the armature mass changes the stinger mode frequency more than the suspension mode frequency. Similarly, increasing the stinger mass (M_3) decreases the mode frequencies. The amplitudes of the peaks are not affected, nor is the impedance between the peaks. Increasing the flexure or stinger stiffness increases the mode frequencies, as expected, as seen in Figure 5. The flexure stiffness only affects the suspension mode and the stinger stiffness only affects the stinger mode. The damping C_{12} and C_{13} affect the amplitude and width of the peaks near the suspension and stinger modes, respectively.

The resistance (R_e) only affects the real part of the impedance as it is a constant over all frequencies as seen in Figure 7. The inductance (L_e) only affects the imaginary part and is proportional to frequency. Increasing the resistance increases the real part of the impedance over the entire bandwidth and increasing the inductance increases the imaginary part of the impedance, particularly at high frequency. The force factor (BL) affects both the real and imaginary parts of the impedance over the entire bandwidth and has effects similar the damping in that it affects the amplitude and width of the peaks, see Figure 8. Effects of all the model parameters on the impedance is summarized in Table 2.

Table 2: Summary of parameter effects on the electrical impedance

Parameter	Effect of Increasing Parameter on Impedance
M_1	Decreases frequency of suspension and stinger modes
M_2	Decreases frequency of suspension mode
M_3	Decreases frequency of suspension and stinger modes
K_{12}	Increases frequency of suspension mode
K_{13}	Increases frequency of stinger mode
C_{12}	Reduces amplitude at suspension mode
C_{13}	Reduces amplitude at stinger mode
R_e	Increases real part, constant with frequency
L_e	Increases imaginary part, proportional with frequency
BL	Increases amplitude and peak width at suspension and stinger modes

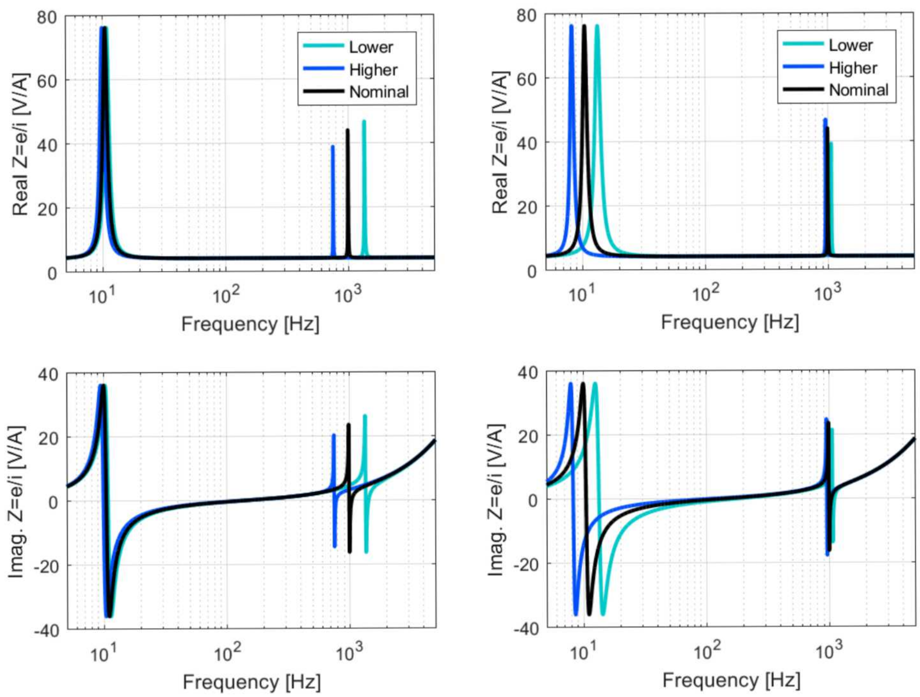

Figure 4: Model impedance sensitivity to M_1 mass (left) and M_3 mass (right).
Top: Real part. Bottom: Imaginary part.

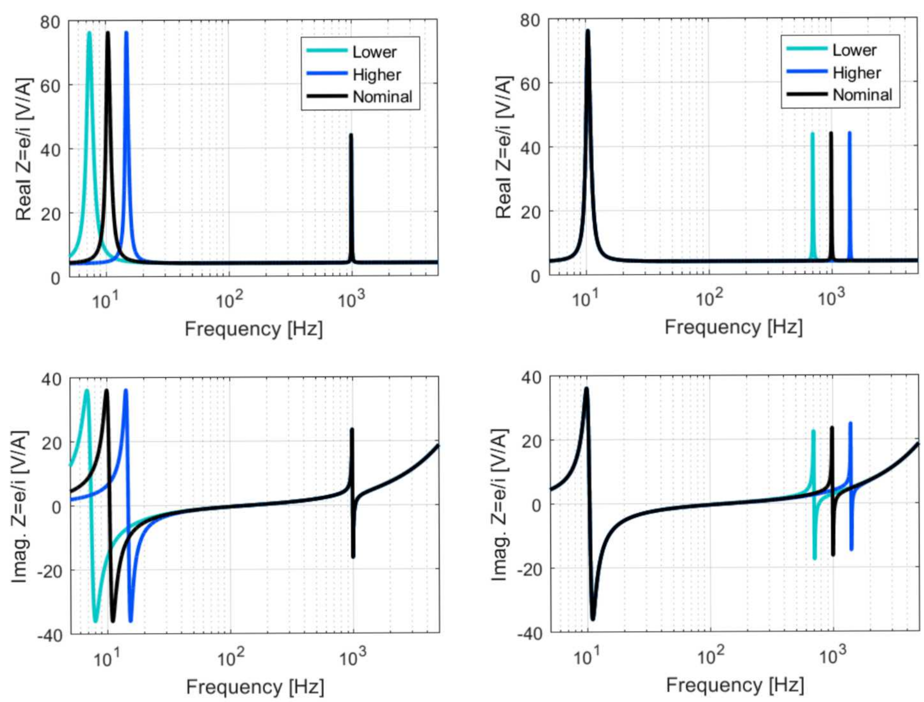

Figure 5: Model impedance sensitivity to K_{12} spring (left) and K_{13} spring (right) values.
Top: Real part. Bottom: Imaginary part.

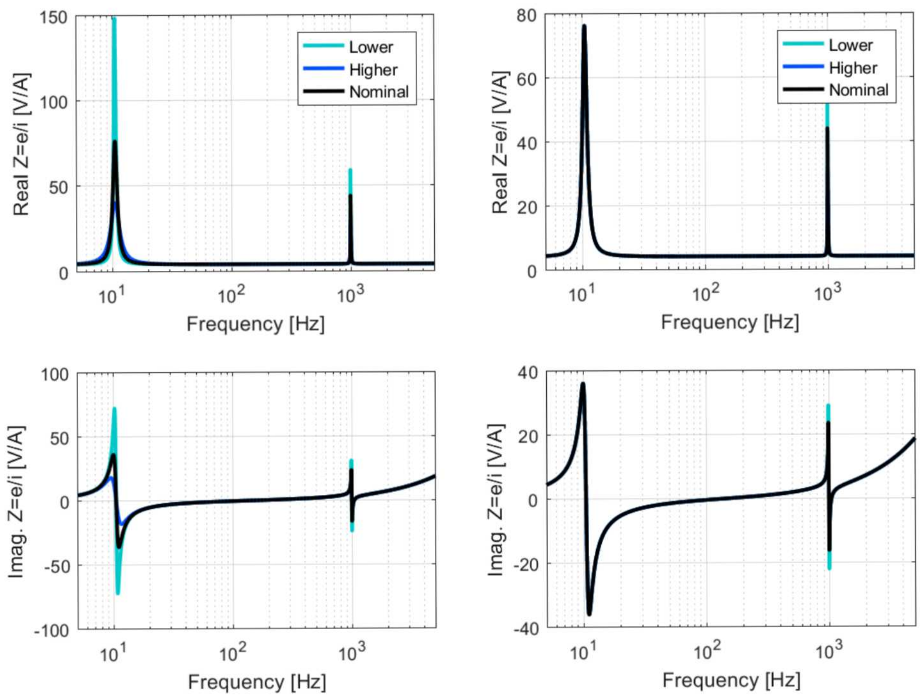

Figure 6: Model impedance sensitivity to C_{12} damper (left) and C_{13} damper (right) values. Top: Real part. Bottom: Imaginary part.

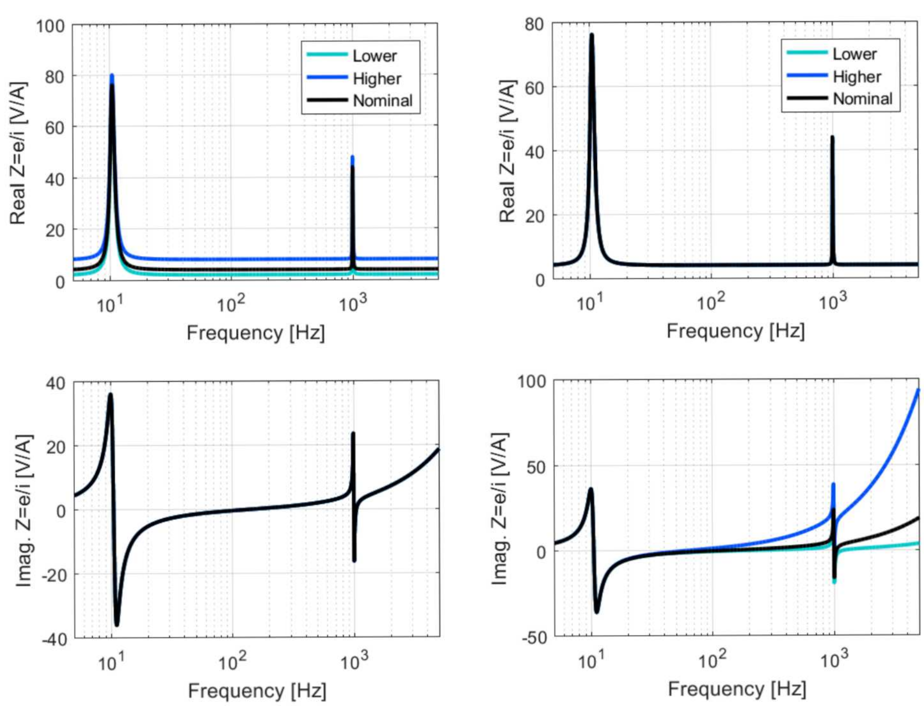

Figure 7: Model impedance sensitivity to resitor R_e (left) and inductor L_e (right) values. Top: Real part. Bottom: Imaginary part.

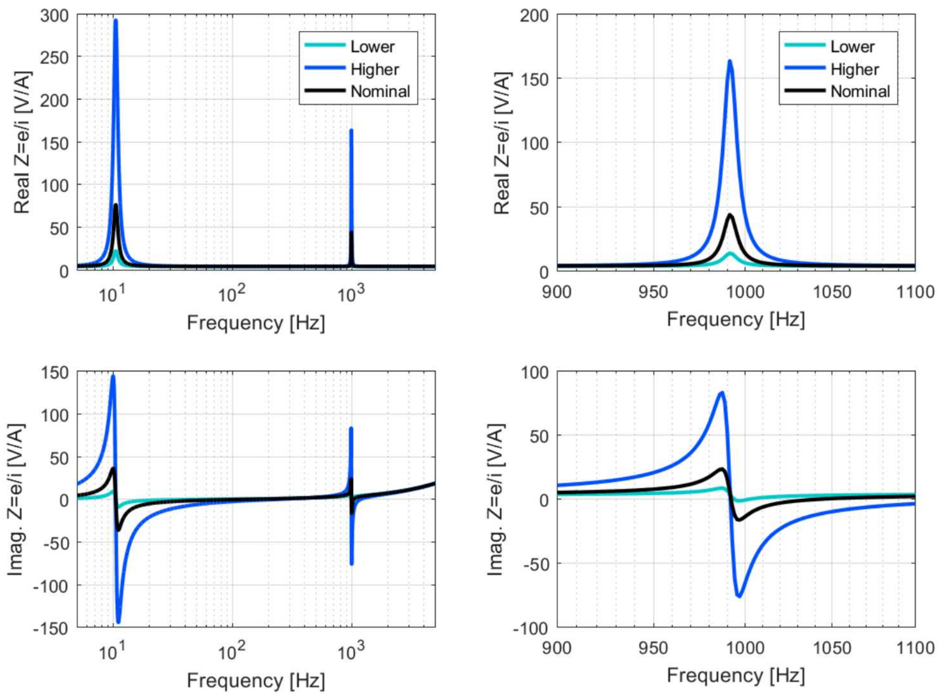

Figure 8: Model impedance sensitivity to BL, left full frequency range, right: zoomed to high frequency range. Top: Real part. Bottom: Imaginary part.

3 Experiment to Determine Shaker Properties

The specification sheet and some basic mass measurements provided estimates of the masses, and the coil resistance could be measured with a multimeter, but many of the model parameters had to be inferred from a measurement of the electrical impedance so an experiment was devised. The objective of this experiment was to drive the shaker with a voltage from an amplifier and measure the response and force at the stinger and the current and voltage supplied to the shaker from the amplifier. In this way, the electrical impedance can be obtained and used to calibrate the model parameters.

3.1 Test Setup

The test setup consisted of a 100 pound-force Modal Shop shaker set horizontally with a 1.5 mm steel wire stinger connecting the shaker armature to a 2.6 kg mass. The mass shifts the stinger mode frequency down to a sufficiently low frequency where it can be easily measured and also is representative of stinger mode frequencies encountered in typical modal tests. This mass was suspended from strings splayed out at an angle to try and keep the mass centered and moving in just one direction. A load cell and accelerometer on the mass measure the applied force and the acceleration of the mass. A photo of the test setup is shown in Figure 9.

In addition to the mechanical measurements, the current and voltage to the shaker was also measured. This was done using a 20:1 voltage divider and a Hall effect clamp on the cable at the output of the amplifier supplying the shaker. A diagram of the electrical measurement equipment is shown in Figure 10.

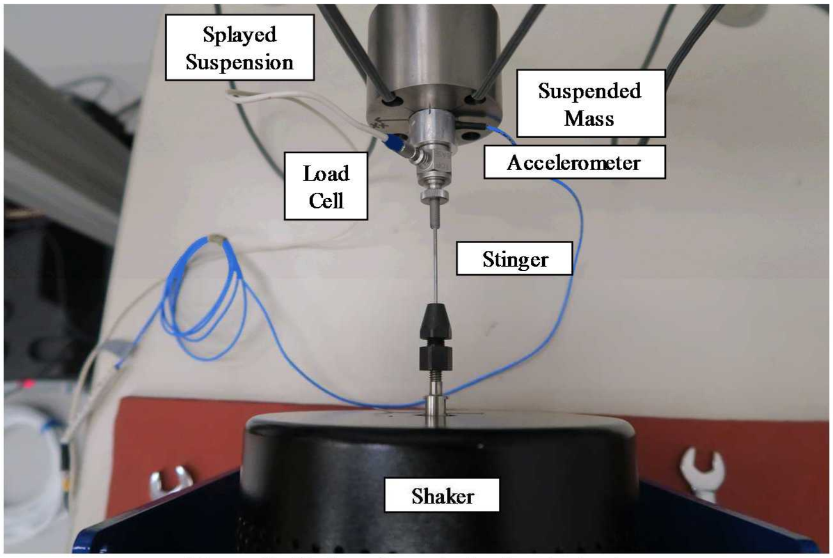

Figure 9: Shaker characterization test setup

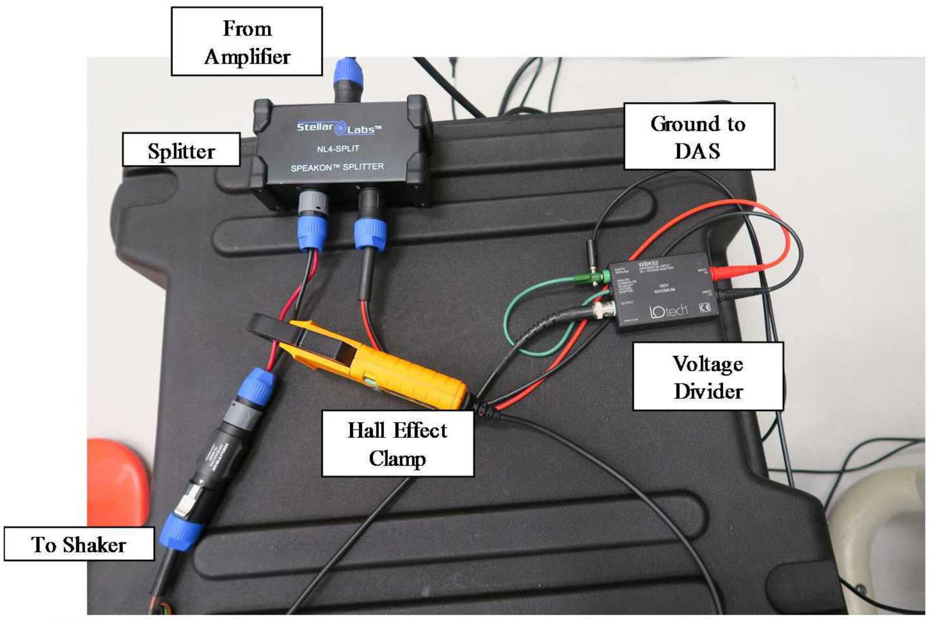

Figure 10: Electrical measurements coming out of the amplifier to the shaker

3.2 Model Calibration - Process

As previously shown, the different model parameters affect the impedance in different ways. As such, the model parameters can be tuned based on the measurements in a systematic way. Also, some parameters are known or can be measured in different ways, for example the masses can be determined with a scale and the resistance can be measured with a multimeter.

Here is the best approach to calibrating the shaker electro-mechanical model:

- Measure or obtain from specification sheet the shaker armature and body mass and measure the stinger hardware, force gauge, and test mass
- Measure the resistance with a multimeter
- Guess all other values
- Tune the suspension stiffness, K_{12}, based on the frequency of the low-frequency mode
- Tune the stinger stiffness, K_{13}, based on the frequency of the high-frequency mode
- Tune the inductance, L_e, based on the slope of the imaginary part of the impedance
- Tune C_{12}, C_{13}, and BL to match the amplitude of the impedance

3.3 Manual Tuning

The first step was approximate manual tuning or calibrating of the model parameters based on the impedance measurement. This was quite effective and brought the model impedance very near the measured impedance. Understanding how each parameter affects the model impedance made the manual tuning process possible and quick. Only the damping and force factor terms were difficult to tune because their effects are largely similar; changing the damping may yield similar results to changing the force factor, so it is difficult to determine which is the proper value of each parameter.

3.4 Fine Tuning with a Parameter Search

To refine the calibration process, a parameter search was used to rapidly implement many combinations of parameter values and compare the model impedance to the test impedance. In this way, model parameters were determined which provide a very good match to the measured impedance, with a minor exception. It was observed that the real part of the impedance was proportional to frequency, much like the effect the inductor has on the imaginary part. The cause of this is not known, but similar effects were seen in the work by Mayes et al. [6]. There, they used a complex inductance value to account for the frequency-dependent real part. The same approach was used here and this greatly improved the real part of the model impedance.

The parameter search used here was implemented as a simple Monte Carlo search wherein multiple parameters could vary over a range from the nominal value. For example, the BL parameter could have seven allowable values, three lower than the nominal and three higher than the nominal. Then, all combinations of all parameters were put in a list where each set of parameters represents a different model configuration. Of course, with many parameters and many possible values for each parameter, this list is too long to evaluate each combination of parameters. Instead, a random subset of the configurations was selected for evaluation. A more sophisticated search algorithm could be used, but this was easy to implement and effective at fine tuning the parameter values after the manual tuning got the model close. The results of each configuration were compared against the test and the best was selected. Defining best is difficult here as the impedance is complex valued and broadband. For this work, an L2 norm of the error in the real and imaginary parts of the impedance was used. Then, the L2 norms of the real and imaginary parts were averaged to form a single, scalar error metric and select the best set of parameters to match the test measured impedance. Figure 11 shows the model predictions compared with the test measurement using baseline parameters, parameters from a manual tuning, and parameters further refined using the Monte Carlo parameter search method. Table 3 provides the parameters for each data set. Even a manual tuning resulted in a very good match to the test measurement, with the parameter search only slightly refining some parameters. The baseline values, particularly the stiffnesses, needed changing to get the model close. Thus, a simple experiment can be very effective in providing model calibration data to dramatically improve the shaker electro-mechanical model.

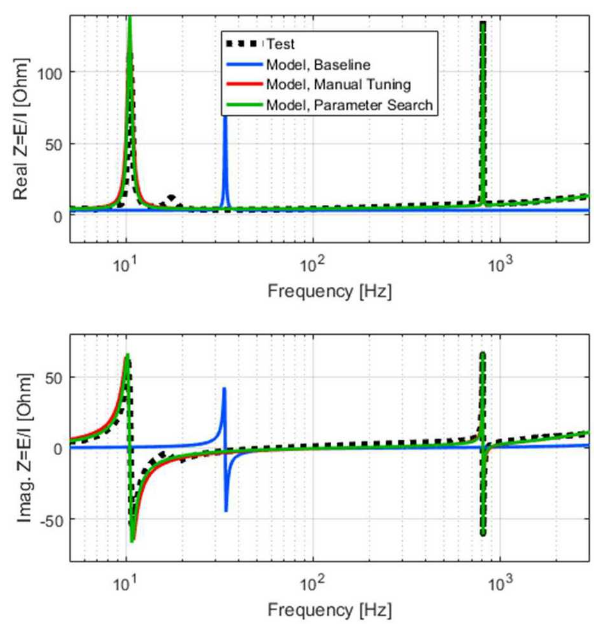

Figure 11: Model predictions compared with test measurements using different parameter values before and resulting from model calibration

Table 3: Parameter values before and resulting from model calibration

Parameter	Baseline	Manual	Search	Unit
M_1	0.44	0.44	0.44	kg
M_2	15	15	15	kg
M_3	2.6	2.6	2.6	kg
K_{12}	1.15E+05	1.10E+05	1.10E+05	N/m
K_{13}	2.50E+08	9.63E+06	9.63E+06	N/m
C_{12}	10	12	9.6	N/(m/s)
C_{13}	10	0.5	0.42	N/(m/s)
R_e	3.1	4	4	Ohm
L_e	1.00E-04	6e-4 -j5e-4	6e-4 -j5e-4	Henry
BL	30	40	36	-

3.5 Changing Model Parameters When Test Conditions Change

Ideally, the shaker model should be robust to minor changes in test setup. However, small changes can have a large effect on the shaker response. Also, it is desirable to have a shaker electro-mechanical model which can be used for a range of tests. Most parameters will not change test-to-test since they are inherent properties of the shaker, such as the armature mass or flexure stiffness. However, the stinger stiffness can change significantly depending on the stinger length or diameter. If the stinger is changed, the only model parameters that should change are the stinger stiffness (K_{13}) and damper (C_{13}). To demonstrate effects of stinger length on the impedance, three stinger lengths were used to measure the impedance. Figure 12 shows that the impedance is largely the same except for the frequency and amplitude at the stinger mode. Next, the model was changed to try to match the "long" stinger case, changing just K_{13} and C_{13} and leaving all other parameters at the values which were tuned to the "medium" stinger case. As seen in Figure 12, simply changing the stinger parameters still results in a fairly good match to the measured impedance.

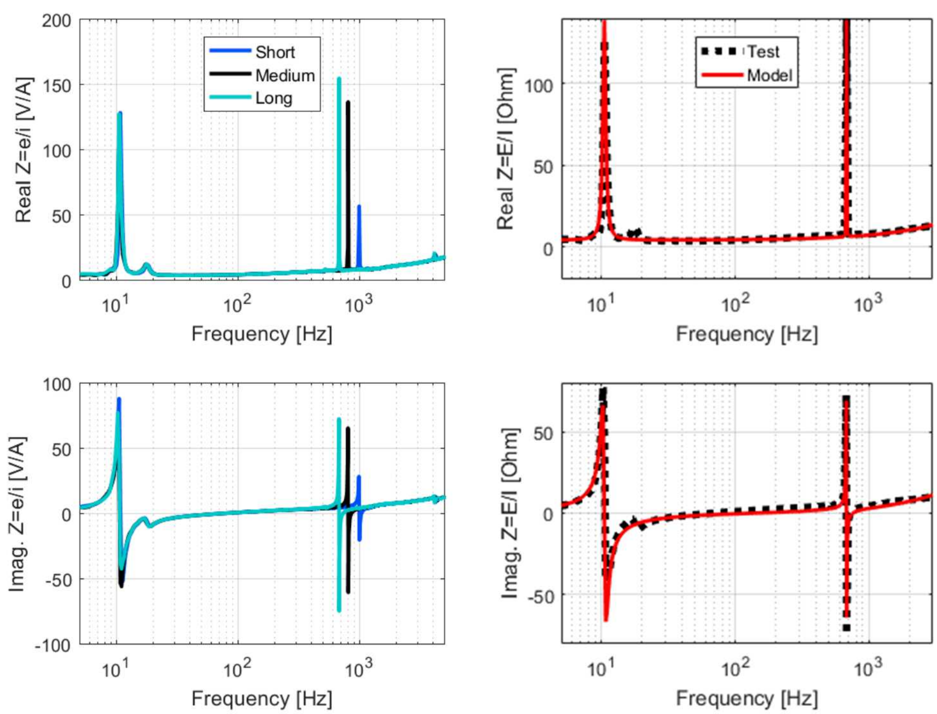

Figure 12: Left: Test-measured impedance using a shorter and longer stinger. Right: Model prediction compared to test results for the long stinger case

4 Conclusions

A simple lumped parameter model can capture the coupled electro-mechanical dynamics of typical modal shakers. Such a model could then be used to estimate the effects of the shaker on DUT dynamics or to predict the current and voltage required to achieve some DUT response. While some of the shaker electro-mechanical model parameters can be obtained from a specifications sheet or otherwise measured directly, some parameters such as the force factor and damping have to be inferred from a measurement of the shaker. Here, shaker model parameters were determined from measurements of the electrical impedance. It was found that unknown model parameters can be easily tuned using a manual tuning process and knowledge of the model's sensitivity to the various parameters. The model was found to be reasonably robust to changing test configurations, such as changing the stinger length.

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