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Exploring Uncertainties in Multi-Input-Multi-Output (MIMO) Testing

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ABSTRACT

Multi-Input-Multi-Output (MIMO) testing enables more realistic replication of true environment in dynamic testing of structures. MIMO infers inputs from the desired responses at multiple locations on the test article of interest. Errors in the estimation of inputs include but are not limited to: output measurement errors and errors in the Transfer Function (TRF) matrix that captures the input-output relationship. A systematic study involving the measurement of acceleration, control system and system response were undertaken to investigate MIMO actuation of a fixed-end cantilever beam. The response of the beam was also modeled using closed-from equations and finite element method. Various actuation schemes were used to explore the effect of location of actuators in the context of the modal response of the beam. The MIMO input derivation was modeled using the Moore-Penrose Pseudo-Inverse of a rectangular Transfer Function matrix. The results of this study quantify the sources of uncertainty in MIMO vibration testing, specifically the errors due to equipment, sensor calibration, and actuator effects on the beam. The conclusions obtained from the cantilever beam used on this study can be extrapolated to more complex geometries.

Keywords: MIMO, Multi-Input, Vibration, Testing, Uncertainty, Acceleration, Structural dynamics.

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1. INTRODUCTION

Multiple Input- Multiple Output (MIMO) testing is a critical aspect of modern vibration analysis and research in many engineering fields. MIMO can use field response data and solve for the test inputs [1]. This is useful since in most aerospace scenarios, researchers can easily collect data by attaching sensors, but do not have a way of using that data to determine which forces are acting on the structure. It is common that researchers use a simple beam to conduct MIMO analysis [2]. The analysis of dynamic response can be done using different methods including Finite Elements [3]. This field of research still poses questions regarding signals processing and the inversion of matrices [4]. Another topic of research in MIMO is the examination of what errors are likely in these tests, and how they propagate through the steps performed in both forward (derivation of responses from inputs) and backward MIMO (derivation of inputs from responses).

This paper studies these contemporary MIMO techniques and aims to investigate the errors present in a representative beam experiment, and how these errors impact the ability to accurately deduce unknowns in the test. A series of simple Multiple Input- Single Output (MISO) tests is performed on a cantilever beam, with two inputs. These tests follow examples from past efforts in MIMO to obtain transfer functions of the beam [5]. Strategic locations were chosen along the beam similar to the experiments in Thite (2012) [6]. Two actuators excited the beam and the experimental output data was collected from the tip of the beam. The estimated output was calculated from the input signals and the previously established transfer functions. The measured output was then compared to the estimated output to find a comprehensive measurement of approximate error. Analytical transfer functions for a similar beam were computed and used to find an analytical output given similar inputs to those for the experiment, using methods mentioned in Smallwood (2005) and Trudnowski (2009) [7],[8]. The calculated error representing sensor uncertainty was then added to the analytical output [9]. From these erroneous outputs, a MIMO inversion process was run to deduce the inputs and examine the error and its relation to that in the output. This virtual experiment was performed so different sources of error such as sensor calibration error could be isolated from the others without the uncertainties from coupling or distortion from the input signal. Key observations are made about sources of error from test-taking and post-processing steps.

2. QUANTIFYING UNCERTAINTIES IN INSTRUMENTATIONS

The Linear Voltage Differential Transducer (LVDT) is a displacement sensor used for measuring dynamic properties. This sensor measures displacement in a rod by inducing a voltage that is proportional to the displacement. The ratio that represents this proportionality between output voltage and input displacement is known as the sensitivity of an LVDT. This section summarizes the accuracy of this sensitivity. For each LVDT, the manufacturer provided a sensitivity value. As part of this project, the value was measured in the lab by connecting two LVDTs, one by one, to a voltmeter and collecting the voltage generated for a range of displacement. Figures 1 shows the experimental sensitivity calculation for two LVDTs for the full range of displacement.

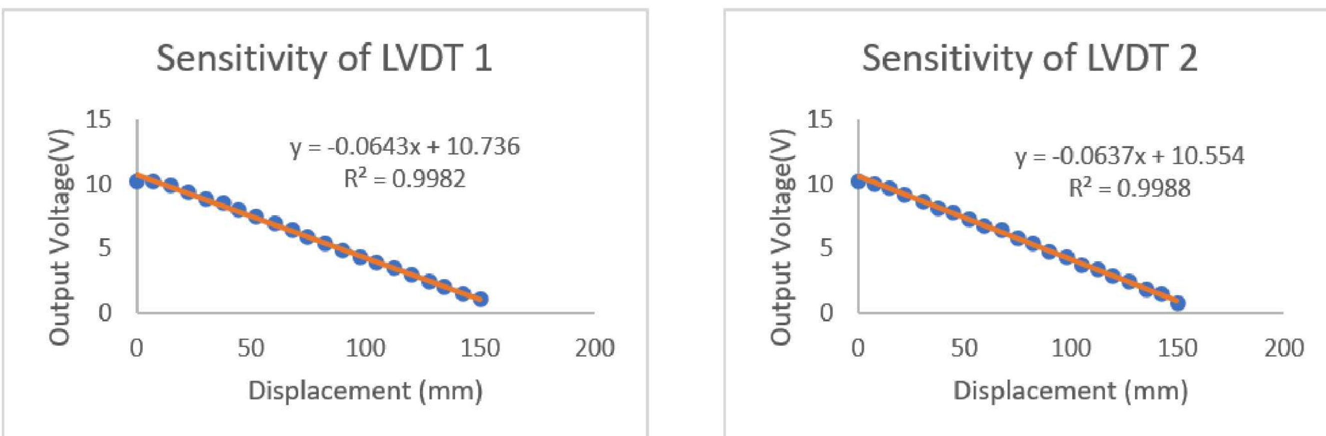

Figure 1: LVDT Sensitivity

According to these graphs, the measured sensitivities of LVDT 1 and 2 are 64.32 and 63.74 mV/mm respectively. The manufacturer's specifications are 65.42 and 65.21 respectively. Thus, in both LVDTs there is a flat 1% disagreement when comparing manufacturer-given properties to lab measurements. This generalized disagreement is combined with the error from the tip response and added to the analytical output.

Here, the calibration instrument used to measure displacement has a precision of 0.001 mm and the voltmeter used has a precision of 0.01 V. Thus, the maximum error possible in these calculations is 0.01V/10V or 0.1%. Since the error found in Figures 1 and 2 show a 1% deviation from manufacturer values and the max error of the calculation itself is only 0.1%, it can be concluded that the sensitivities determined by this calculation are significantly different from the manufacturer and it was decided to account for them in experimental testing.

It is important to note that the slope of the line of best fit are not consistent throughout the displacement range. Additional measures were taken to calculate sensitivities for a smaller range of displacement across the entire range. The results are summarized in Table 1. The results show that the best ranges of displacements to use are: 15 to 150 mm for LVDT 1 and 7.5 to 150 mm for LVDT 2. Based on these results, experimental work avoided using the first 15 mm of the rod's displacement from the neutral position of the rod.

Table 1: Summary of correlations for different ranges of displacement

Range of Displacement	Linear Correlation LVDT 1	Linear Correlation LVDT 2
0-50mm	0.9646	0.9786
7.5-57.5mm	0.9964	0.9995
15-65mm	0.9982	0.9998
22.5-72.5mm	0.9982	0.9990
30-80mm	0.9983	0.9987
37.5-87.5mm	0.9986	0.9988
45-95mm	0.9986	0.9985
52.7-102.5mm	0.9987	0.9985
60-110mm	0.9991	0.9977
67.5-117.5mm	0.9990	0.9978
75-125mm	0.9990	0.9981
82.5-132.5mm	0.9990	0.9977
90-140mm	0.9993	0.9977
97.5-147.5mm	0.9993	0.9978
105-150mm	0.9993	0.9977

3. INSTRUMENTATION UNCERTAINTIES IN ANALYTICAL MIMO SYSTEM

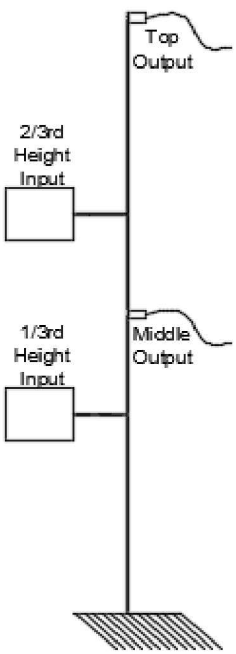

Figure 2: Analytical Model Setup

The results of examining the LVDT error is used as a general error for any sensor in testing. To isolate the error due to a miscalibration in sensors, a Finite Element Model representing a cantilever beam with assumed Euler Bernoulli Properties was built to represent a simple experiment. This analytical model assumed two inputs and two outputs, as shown in Figure 2. Two outputs were used to ensure that any error is due to the factor being examined, rather than a procedural error from the inherent inaccuracies of using a MISO rather than a MIMO system. This is due to the fact that every point in the frequency domain has an infinite number of solutions when solving a MISO problem, due to having more variables to solve for than equations. Band-limited white noise was generated as the input and was multiplied by the analytical transfer functions to produce an analytical output at the tip and the middle of the beam.

This output was multiplied by a factor or 1.01 and 0.99 to simulate a 1% deviation in sensor calibration for a high estimation and low estimation respectively, replicating a percentile deviation as investigated in Section 2. While not descriptive of the type of error investigated in Section 2, the analytical output was additionally perturbed by adding or deducting up to 1% of the measured signal at every point, creating a random alteration of the data to represent noise and small-scale non-linearities. The inversion process was then performed with the modified outputs as described by Equation 1.

$$In = TF^{+}Out \quad [1]$$

where In = Input Signal Matrix,

TF = Transfer Function Matrix,

Out = Output Signal Matrix,

And $^{+}$ denotes the Moore-Penrose Pseudo-Inverse of a matrix

The inverse process was also performed with an unadulterated output signal to use as a standard control. A sample of these results are shown in Figure 3, which depicts the ASD of the 1/3rd height input for each miscalibration case, with an arbitrary frequency range zoomed in for closer examination. The pattern observed is consistent across the frequency spectrum and between the two input signals. The flat percentile deviations each lead to an expected consistent miscalculation of the original signal, with an overestimated output resulting in too high of an input, and too low of a signal resulting in a too low of an estimated input. Even though the maximum value of change added to the output for the random perturbations never exceeded the level of the flat deviations, the input signal estimated from it commonly jumps beyond the bounds of those estimated from the flat rates. However, all of them remain close enough to the original signal that they are nearly indistinguishable from an overall view.

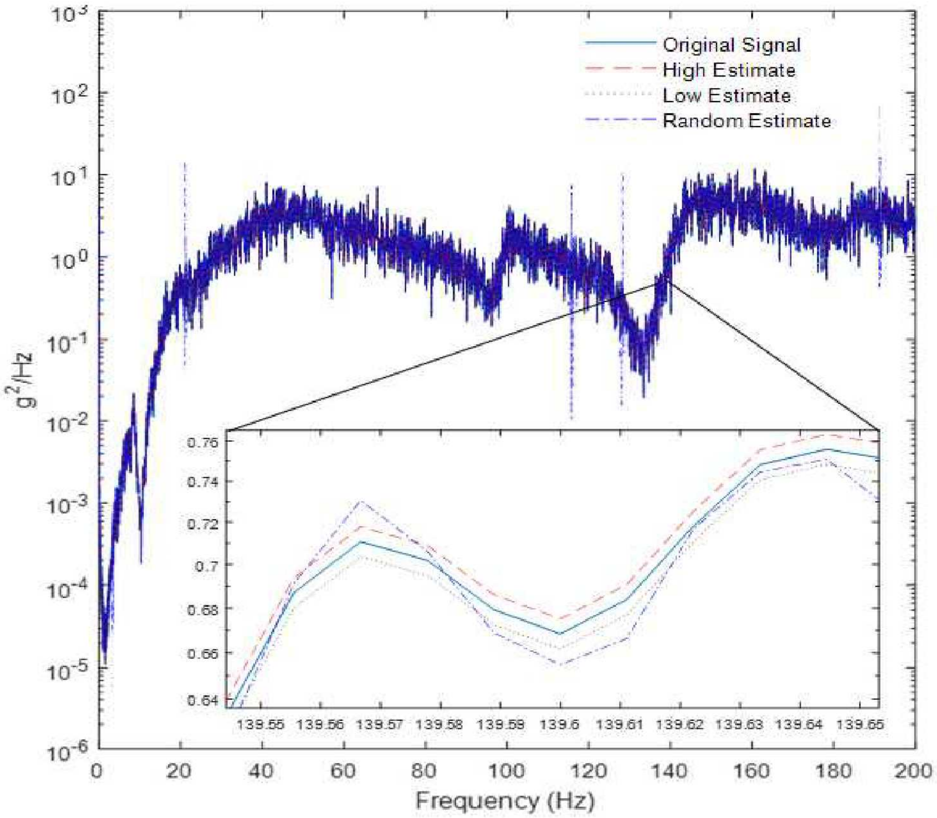

Figure 3: Calculated Inputs for 1/3rd Height

The Error across the frequency range of 0-200Hz is summarized for each signal alteration by taking the average absolute log of the ratio between the erroneous signal and the measured, as described by Equation 2.

$$Error = \log\left(\frac{Estimated\ Signal}{Measured\ Signal}\right) \quad [2]$$

The results for each case are tabulated in Table 2. This table shows that the error for a flat error in sensor calibration is very small, and nearly identical for either direction of misestimation. The random noise creates more than three times the error, even though the magnitude of the noise was at most equal to that of the flat error. The reference input calculated from the unaltered analytical output had an error of effectively zero, which is as would be expected.

Table 2: Error- Average Absolute Log of Ratio

	1/3rd Height Input	**2/3rd Height Input**
Sensor Underestimates	0.0044	0.0044
Sensor Overestimates	0.0043	0.0043
Sensor Randomly Perturbed	0.0135	0.0142

4. ACTUATOR ATTACHMENT UNCERTAINTIES IN EXPERIMENTAL MIMO SYSTEM

Noting the resistance to displacement of the stinger, experiments were performed which quantified a source of error in the use of The SmartShaker actuators to excite the beam. It was found that the application of the Stingers caused substantial changes in the natural frequencies of the beam. These tests focused on the addition of the exciter at the 1/3rd height mark, recognizing that the farther up the beam it was located, the more it would restrain the natural response of the beam and alter the Transfer Function. The test setups are depicted in Figure 4.

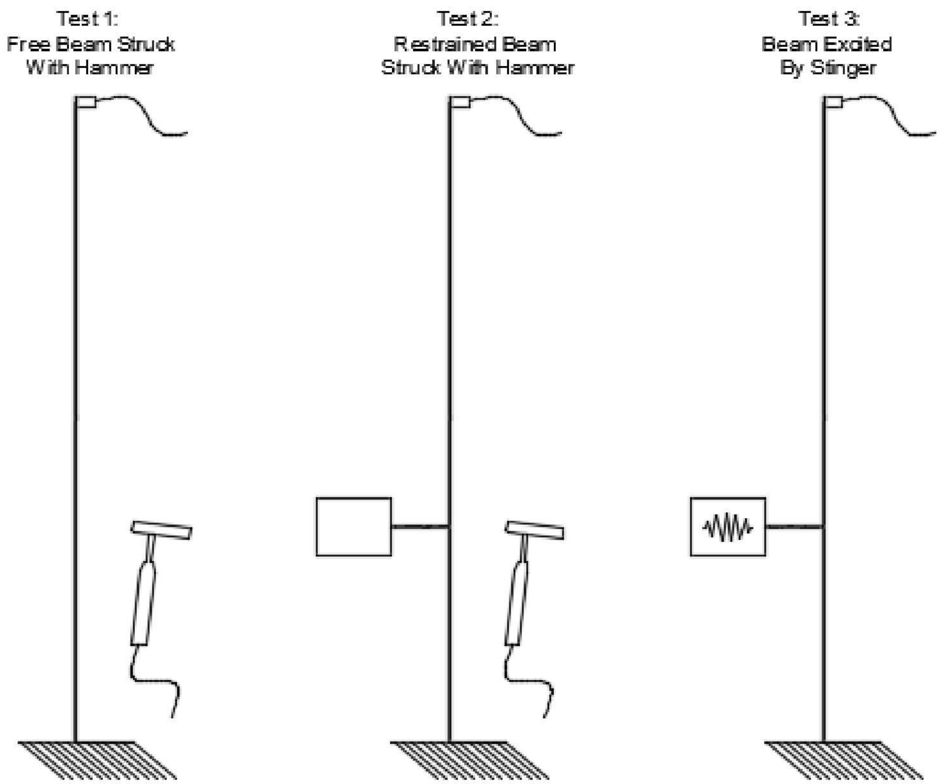

Figure 4: Test Setups for Stinger Effects

The experiments were performed by applying a series of impulses with an impact hammer at the location of interest, the 1/3rd height node, both with and without a Stinger attached at the same location. Transfer Functions were then generated from this data for the acceleration at the tip with respect to input force. Another test was run in which the dynamic impedance of the exciter was observed, as it was used to actuate the beam at a variety of frequencies, and the response of the tip with respect to the input acceleration was noted. By necessity of testing structure with two different type of inputs (hammer vs. Stinger), the format of data is different for this test from the previous two, leading to an offset in magnitude. However, the location of peak resonances still offers valuable insight into the impact of how the testing equipment works, and how it can affect the results of tests depending on the assumptions used. A summary of the three Transfer Functions calculated are depicted in Figure 5. As seen, both the attachment and the state of operation of the Stingers make a significant difference in the dynamic properties of the structure for small-scale testing such as the cantilever beam used. Larger test specimens would have lesser impact, but the difference in dynamic response must be accounted for when dealing with relatively light or flimsy objects. While it is not demonstrated in this paper, it is easy to see how this deviation will be exacerbated as the device is moved higher up the beam and closer to the top, as more of its motion is restrained and the increasing moment arm amplifies the effects of the resistance.

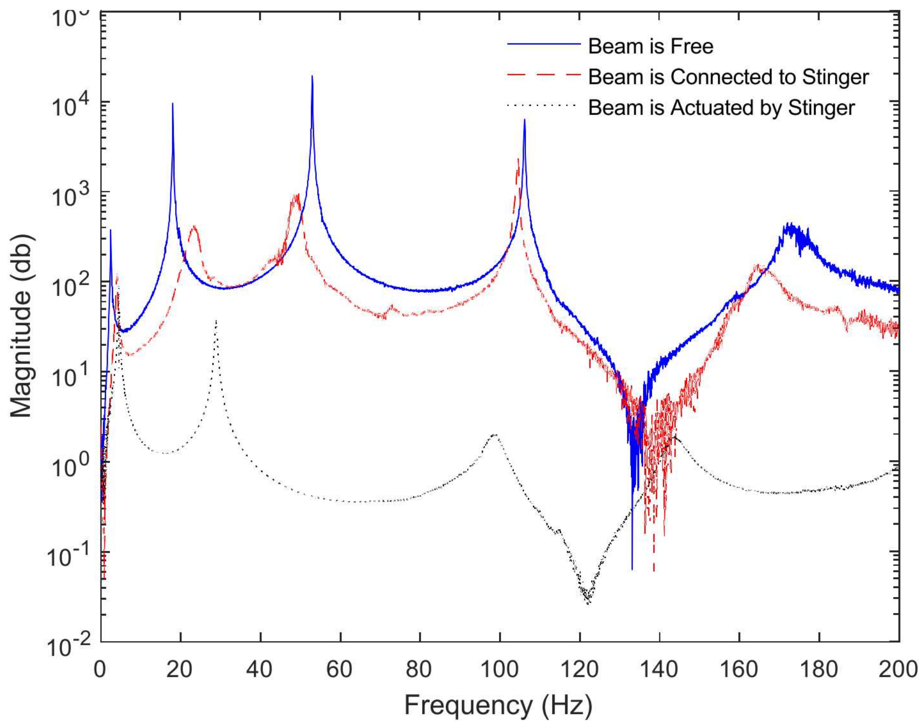

Figure 5: Compared Transfer Functions of Beam Tip with Respect to 1/3 Height Input

5. ONGOING WORK

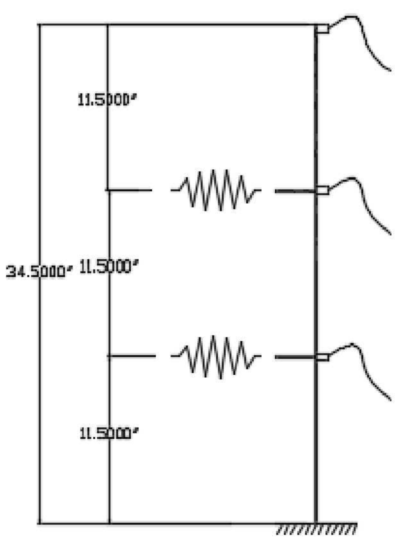

Figure 6: Experiment Setup

The primary experiments discussed were performed as a precursor to current research, examining potential sources of error in MIMO tests and how to best avoid them. Research is currently being performed on a cantilever aluminum beam of known properties with two TMS SmartShaker exciters attached at 1/3rd and 2/3rd the length of the height respectively, via stingers. Three accelerometers are also attached, one at the tip to collect the output response, and one at each input location to measure the input vibration. This setup can be seen in Figure 6. Each stinger is individually excited with band-limited white noise ranging from 1-200 Hz to establish the transfer function for the tip with respect to each input location. For this process, both exciters were left attached, even if only one was being run, to ensure consistent dynamic properties of the beam response. The shifting of Transfer Functions due to dynamic impedance provided a clear source of error for the simple test performed, as simply leaving both Stingers attached while only running one to define the patterns of beam response did not accurately define the dynamic properties of the system once both lines of input are active. Using the conclusions reached in this paper to help ensure accuracy, patterns of error inherent in the MIMO process and how it correlates with the magnitude of relevant Transfer Functions are being performed. Future research includes quantifying the different causes of uncertainty and examining how each one respectively propagates within MIMO analysis for various systems.

7. CONCLUSIONS

This paper investigates causes and influences of uncertainties in MIMO testing due to equipment use, and the nature of errors found with different tests and with different equipment will vary accordingly. This paper investigated two potential equipment-related sources of error in MIMO testing for a simply supported beam. The error due to flat rate deviation in sensors was found to be small and of consistent nature per the original deviation, while random error had more significant impact. However, even this random error was ultimately small in comparison to the scale of the original signal. It was also found that the exciters used to actuate the beam had a significant impact on the dynamic response of the system, which changed depending on if the exciter (stinger) was running or just attached. There are many other potential sources of uncertainty in MIMO testing which are independent of the equipment used, and will be studied in the future.

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