

Practical Considerations for Force Appropriation Testing of a Bolted Structure

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1 Abstract

Many structural assemblies contain nonlinearities in their dynamic response due to frictional contact at joints. The authors have studied methods to extract and identify this nonlinear phenomenon with traditional modal testing techniques (i.e. hammer impulse, shaker excitation). Previously, a cursory investigation was completed using force appropriation techniques to extract nonlinear modal information from a jointed structure. This work continues the investigation of force appropriation techniques and their applicability to nonlinear bolted assemblies, specifically on a structure that demonstrates weakly nonlinear frequency dependence and a strong amplitude dependence for dissipation. This work further explores force appropriation testing by investigating the repeatability of nonlinear characteristics when multiple tests are conducted to various amplitudes. Additionally, different response sensors and drive point locations are utilized to excite and control the structure during the test. The results from these various runs are evaluated and practical recommendations are provided for the selection of optimal excitation and response measurement locations.

Keywords – nonlinear normal modes, force appropriation, nonlinear structural dynamics, experimental techniques, modal analysis

2 Approach

The objective of this work is to explore some of the practical aspects of nonlinear force appropriation testing of a free-free bolted structure. The selected structure for this study is referred to as the Cylinder Plate Beam (CPB) whose nonlinear characteristics have been studied in previous works [1, 2, 3]. The testing implemented in this work closely follows that described in [4] which derived a testing technique to excite and isolate nonlinear normal modes (NNMs). The basis of this method is founded in the concept that for sinusoidal excitation of an NNM, the excitation force will be 90 degrees out of phase with the response acceleration. The theory states that this quadrature criterion must hold for all degrees of freedom and all harmonics, i.e. all degrees of freedom must be excited simultaneously. However, Peeters showed that NNM isolation may be approximated by using single-point excitation [4, 5]. These two principles (the quadrature criterion and single-point excitation approximation) are the guiding concepts for the testing technique utilized in this work. For the excitation of the CPB, a modal shaker is used with a closed-loop control system that excites and tracks the first bending mode of the beam using the force measured at the drive point and a single response accelerometer as feedback. The structure is excited with sinusoidal input at increasing levels in order to obtain the nonlinear characteristics as a function of amplitude. At a given excitation amplitude, the controller adjusts the fundamental frequency of the input until quadrature (or some other target phase as described later) is achieved between the measured force and response acceleration. At this point, a trigger is sent to the data acquisition computer which records a frame of data of the force and all 84 response accelerometer channels. The excitation amplitude is then increased, and this process repeats for the desired number of amplitude levels. Force is a difficult quantity to set in shaker tests as it is a function of the shaker armature motion and the motion of the structure at the drive point. Therefore, excitation amplitude levels are referenced in terms of the voltage amplitude supplied to the shaker amplifier.

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Many tests were conducted on the CPB and it was prudent to evaluate the repeatability of the results of these force appropriation tests since joint settling can alter the linear dynamics of a structure [6]. For the repeatability study, the CPB was first assembled and a series of burst random tests were conducted at various forcing levels in an initial attempt to settle the joints. A series of force appropriation tests were then conducted in the following sequence of maximum voltage levels:

1. 3 runs to 0.105 V
2. 3 runs to 0.205 V
3. 3 runs to 0.305V
4. 1 run to 0.205 V
5. 3 runs to 0.405 V
6. 1 run to 0.205 V

Thus, a total of 14 separate force appropriation test runs were conducted. Each started at 0.005 V and incremented in very small steps to the maximum voltage listed. At each voltage increment, quadrature was achieved before moving to the next. At each step where the previous maximum voltage was exceeded (e.g. from step 2 to 3 where the maximum voltage was increased from 0.205V to 0.305V), the test at the new maximum was conducted three times to evaluate the repeatability as the excitation amplitude is increased. Throughout the test sequence, there are two tests where the maximum voltage was reduced to 0.205V (steps 4 and 6). These reduction steps were performed to evaluate if the structure responds in a similar manner even when the exact test is not repeated.

The second aspect addressed is the influence of rigid body modes on the dynamics of the structure during the force appropriation tests. Several previous works on NNMs have focused on fixed systems [4, 5, 7, 8, 9]. Reference [10] contains a numerical demonstration of NNM techniques on a free-free bladed disk structure. However, the only mention of rigid body modes is a statement that they are not influenced by the nonlinearity. While this is true, the work contained herein shows how the rigid body modes can influence the results of an NNM test which, depending on the post-processing technique, could give erroneous nonlinear trends. To illustrate this, consider the linear FRF equation for a single input-output pair:

$$\frac{\ddot{X}_i}{F_j}(\omega) = H_{ij}(\omega) = \sum_{r=1}^N -\frac{\omega^2 \phi_{ir} \phi_{jr}}{\omega_r^2 - \omega^2 + i2\zeta_r \omega_r \omega} = \sum_{k=1}^6 \phi_{ik} \phi_{jk} + \sum_{r=7}^N -\frac{\omega^2 \phi_{ir} \phi_{jr}}{\omega_r^2 - \omega^2 + i2\zeta_r \omega_r \omega} \quad (1)$$

The first term on the right-hand side of this equation represents the contribution of the rigid body modes to the dynamics. The angle of this equation represents the phase difference between the acceleration response and the force. When the structure is excited at the r^{th} elastic resonant frequency, ω_r , the phase angle is given by

$$\angle H_{ij}(\omega_r) = \tan^{-1} \left(\frac{\left(\frac{\phi_{ir} \phi_{jr}}{2\zeta_r} \right)}{\sum_{k=1}^6 \phi_{ik} \phi_{jk}} \right) \quad (2)$$

This equation assumes the contribution of nearby elastic modes is negligible, either by poor excitation or large separation in frequency. Note that the denominator of (2) is the rigid body dynamics. Thus, if the rigid body shape values are large relative to the elastic shape values, then the phase angle at resonance will not be 90°. The damping ratio, ζ_r , also influences the value of this angle, but is not the focus of this study; we are interested in how the selection of drive point and response degrees of freedom (DOFs) can influence the results of the force appropriation tests via the rigid body modes. Multiple force appropriation tests are conducted with various drive point and response DOFs used in the controller along with different target phases based on (2) to illustrate how these can influence the test results.

3 Results

This section provides the test results and conclusions to the two tests series mentioned above: the repeatability study and the rigid body dynamic influence demonstration. The repeatability test sequence listed above was performed and the results were compared via frequency energy plots (FEP) computed for each of the tests. This provides a measure

of the nonlinear stiffness of the mode. The results show that, so long as the previously achieved maximum response level is not exceeded, the nonlinear stiffness is repeatable.

For the rigid body study, multiple force appropriation tests were conducted with different combinations of drive point and response feedback DOFs and target phases defined by (2). The deflection shapes of the structure for each of these tests were compared to the linear shape of the target mode and the FEPs were compared against each other to evaluate the influence of either the drive point, response DOF, or target phase. The FEP did not vary for any combination, but the deflection shapes saw a noticeable change when a poorly responding DOF was chosen as the drive point (i.e. exciting at a node of the mode). The reason for this latter trend is that, while the elastic modes are poorly excited, the rigid body modes dominate the response and distort the deflection shape. The consequence of this is that the measured physical response is not comprised purely of the motion associated with the target elastic mode. Thus, if nonlinear parameter estimation is computed based on physical responses, the resulting parameters could be incorrect.

In summary, some practical recommendations for performing force appropriations testing to establish NNM characteristics are provided:

1. Do not exceed previously achieved maximum response levels to retain repeatability. Once maximum levels are changed, break-in test runs to the new level are recommended.
2. Do not excite at a node of the mode of interest – while the FEP is not affected, the deflection shapes are contaminated by the rigid body modes.

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4 References

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