

Analysis of the Dependence of Critical Electric Field on Semiconductor Bandgap



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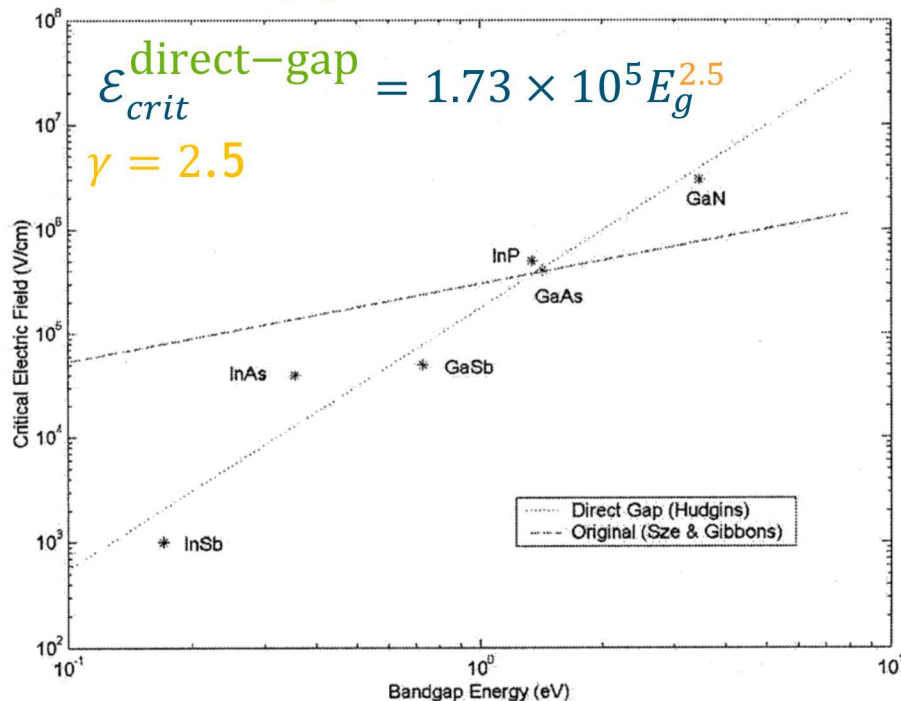


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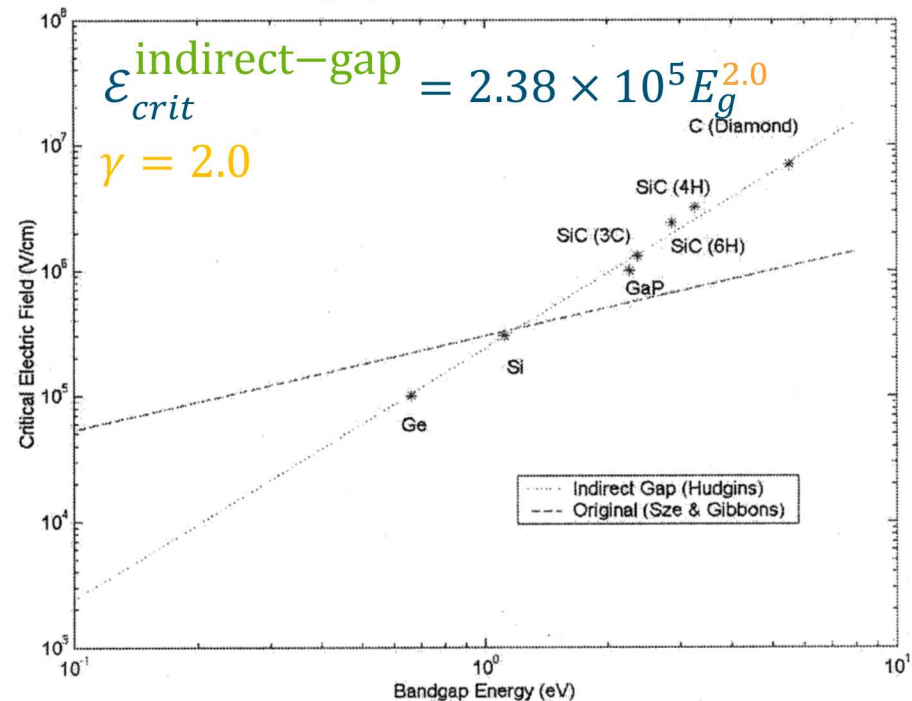
Empirical Fitting of Critical Electric Field on Semiconductor Bandgap

- Empirical fits are used to predict capabilities of emerging power electronics materials: $\mathcal{E}_{crit} \sim E_g^\gamma$
- Fits by Hudgins et al. in 2003 remain the standard

Direct Bandgap Material



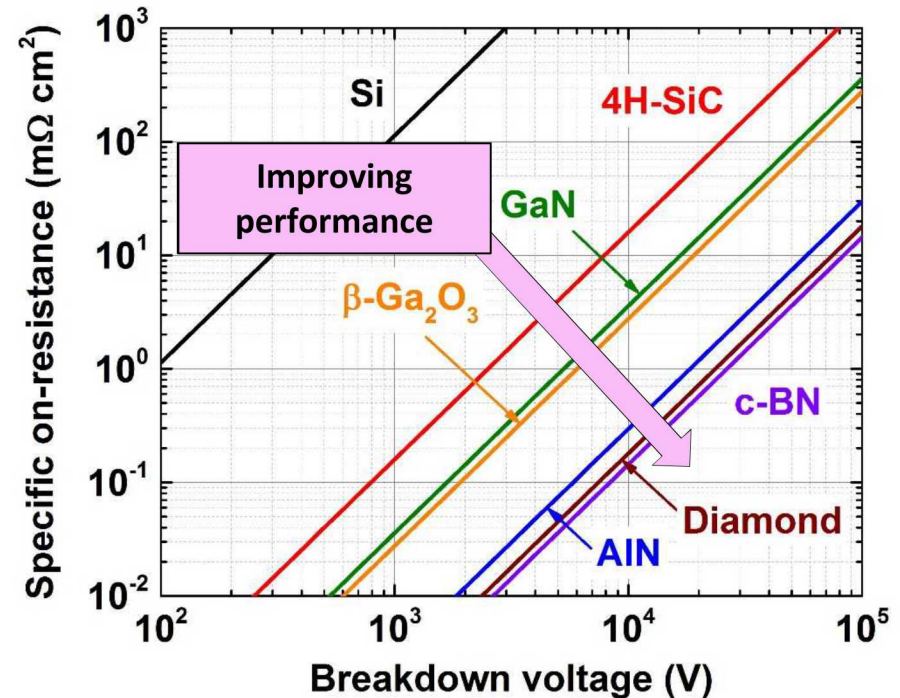
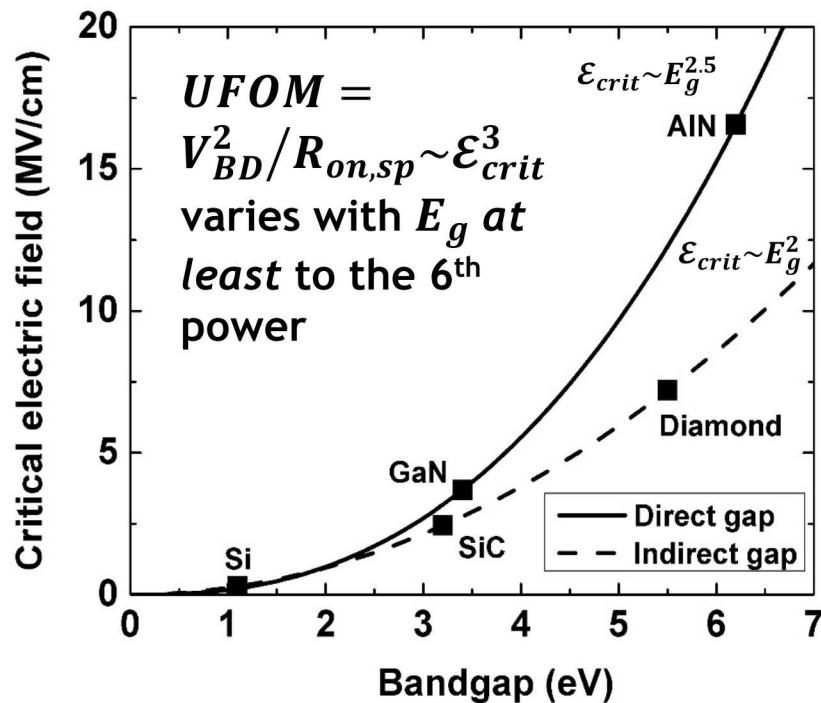
Indirect Bandgap Material



- Empirical fits to data from the literature (not existing theory)
- Not normalized to doping, device structure, temperature
- Subject to device quality

3 Figures of Merit

Figures of merit often rely on parameters predicted from empirical fits

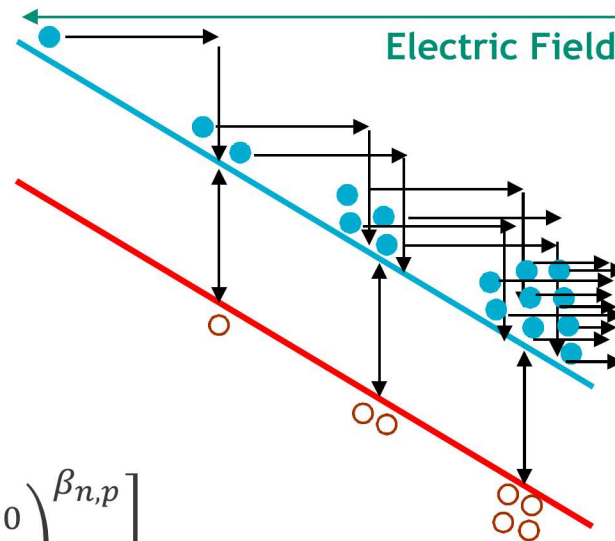


- UFOM only valid for non-punch-through device structures
- UFOM only valid for vertical devices

4 Impact Ionization

Impact ionization drives avalanche breakdown in a device

Charge carriers accelerated under high electric fields lose energy and ionizing more electron-hole pairs, resulting in multiplication of charge, leading to rapid increase in current & avalanche breakdown



Carrier ionization rates are given by:

$$\alpha_{n,p} = \alpha_{n0,p0} \cdot \exp \left[- \left(\frac{\mathcal{E}_{n0,p0}}{\mathcal{E}} \right)^{\beta_{n,p}} \right]$$

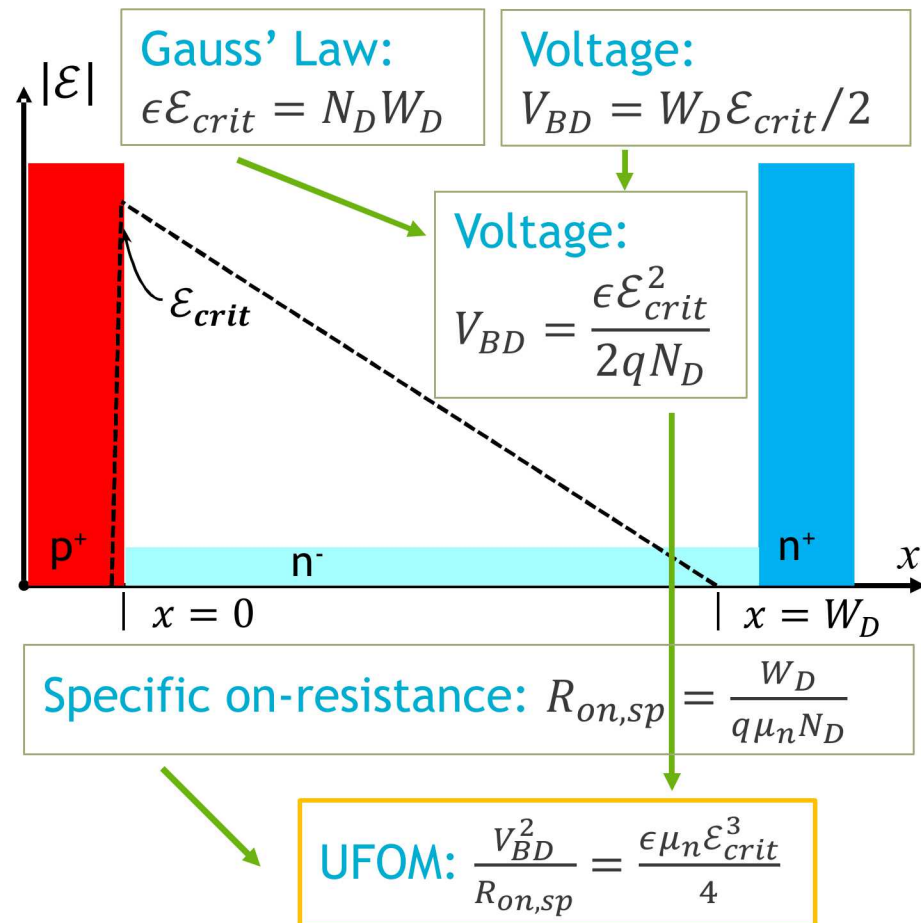
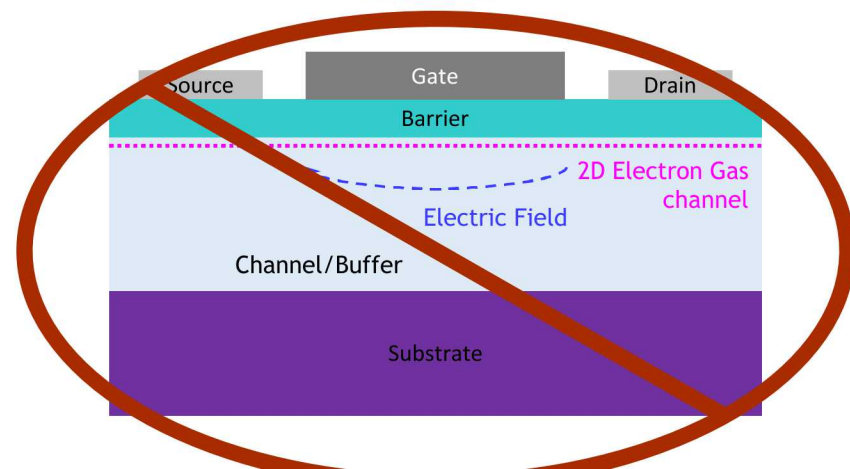
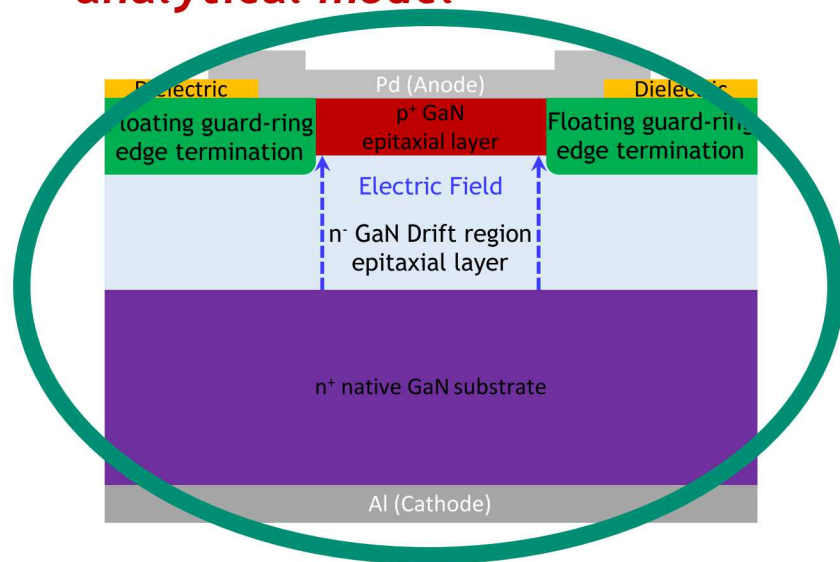
Multiplication factor describes increase in current under impact ionization:

$$M_n = \frac{\exp \left[\int_0^{x_D} \left(\alpha_n(x) - \alpha_p(x) \right) dx \right]}{1 - \int_0^{x_D} \alpha_n(x) \cdot \exp \left[\int_0^x \left(\alpha_p(x') - \alpha_n(x') \right) dx' \right] dx}$$

Avalanche occurs when denominator approaches zero

Critical Electric Field and Device Performance

The critical field is defined as the maximum electric field that leads to avalanche breakdown in a 1D analytical model

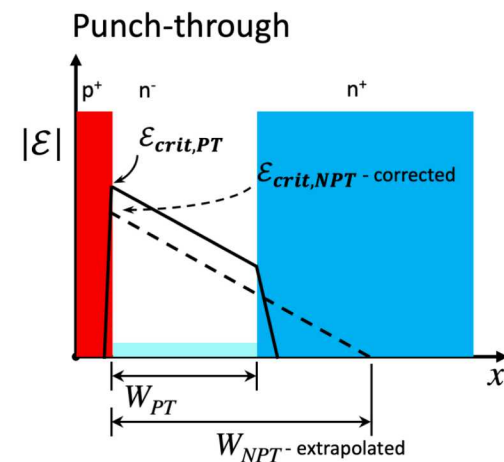
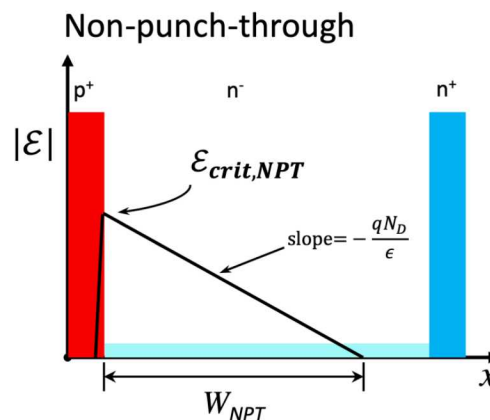
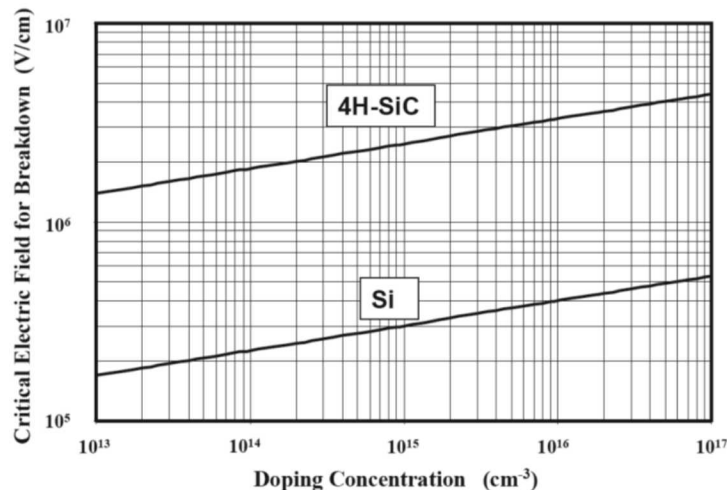


Unipolar Figure of Merit (UFOM)

- $\mathcal{E}_{crit}$ is obtained from V_{BD}
- $R_{on,sp}$ is extracted from forward-current slope
- Carrier mobility is often taken from literature

Procedure for Renormalization of $\mathcal{E}_{crit}$ values

- UFOM assumes a given doping and field profile
- Valid comparison of critical electric field between devices requires two major kinds of corrections:
 - Doping:
 - Critical electric field is doping-dependent
 - Devices of different doping levels must be compared to an equivalent doping level
 - Structure
 - Electric fields in non-punch-through (NPT) and punch-through (PT) style devices are not directly comparable
 - To equate, PT devices are transformed into an “equivalent” NPT device (integrated E-field curve is constant)



Correction for Doping

Analytical solution for renormalization requires two critical assumptions so the impact ionization integrals in the multiplication factor equation can be evaluated

1. Electron & hole ionization rates are equal: $\alpha_n = \alpha_p$
2. Ionization rate follows a power-law form: $\alpha = \alpha_0 \mathcal{E}^\delta$
 - with most-widely accepted : $\delta = 7$

Thus correction for different doping profiles follows:

$$\int_0^{x_{D1}} \alpha(x) dx = \int_0^{x_{D2}} \alpha(x) dx = 1$$

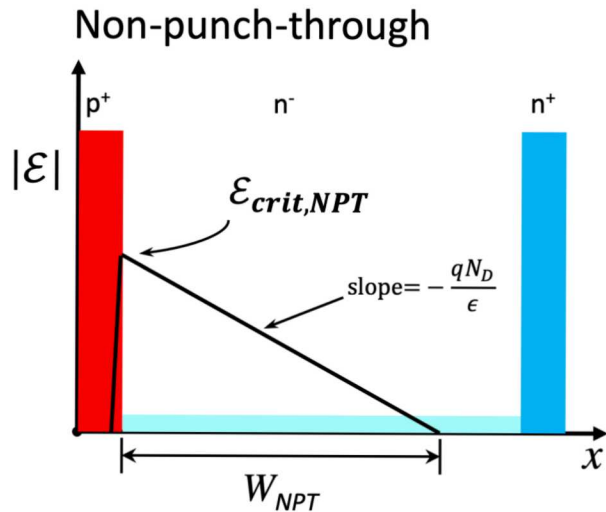


$$\frac{1}{N_{D1}} \int_{\mathcal{E}_{crit1}}^0 \alpha(\mathcal{E}) d\mathcal{E} = \frac{1}{N_{D2}} \int_{\mathcal{E}_{crit2}}^0 \alpha(\mathcal{E}) d\mathcal{E}$$



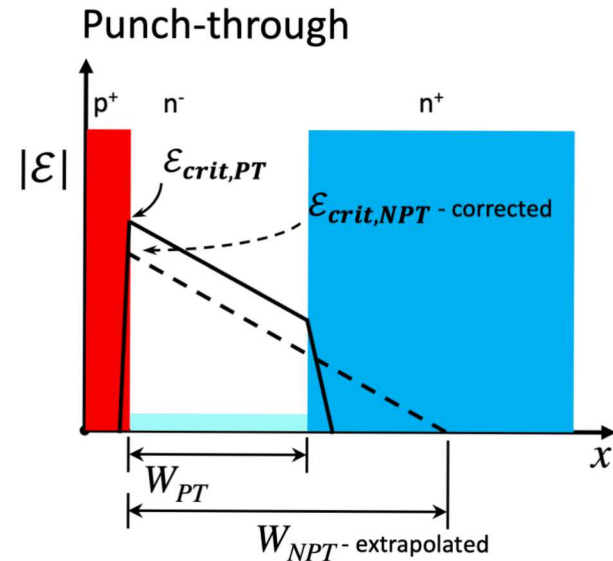
$$\frac{\mathcal{E}_{crit1}}{\mathcal{E}_{crit2}} = \left(\frac{N_{D1}}{N_{D2}} \right)^{1/(\delta+1)}$$

Correction for NPT & PT Device Structure



$$V_{BD,NPT} = \frac{\epsilon}{2qN_D} \mathcal{E}_{crit,NPT}^2$$

$$\mathcal{E}(x) = \mathcal{E}_{crit,NPT} - \frac{qN_D}{\epsilon} x = \frac{qN_D}{\epsilon} (W_{NPT} - x)$$



$$V_{BD,PT} = \mathcal{E}_{crit,PT} W_{PT} - \frac{qN_D}{2\epsilon} W_{PT}^2$$

$$\mathcal{E}(x) = \mathcal{E}_{crit,PT} - \frac{qN_D}{\epsilon} x \mid x \leq W_{PT}$$

Equating the impact ionization integrals for different electric field profiles:

$$\int_0^{x_{NPT}} \alpha(x) dx = \int_0^{x_{PT}} \alpha(x) dx \quad \longrightarrow \quad \int_{\mathcal{E}_{crit,NPT}}^0 \alpha(\mathcal{E}) d\mathcal{E} = \int_{\mathcal{E}_{crit,PT}}^{\mathcal{E}_{crit,PT} - \frac{qN_D}{\epsilon} W_{PT}} \alpha(\mathcal{E}) d\mathcal{E}$$

$$\frac{\mathcal{E}_{crit,NPT}}{\mathcal{E}_{crit,PT}} = \left[1 - \left(1 - \frac{qN_{D,PT}}{\epsilon} \frac{W_{PT}}{\mathcal{E}_{crit,PT}} \right)^{\delta+1} \right]^{1/(\delta+1)}$$

Combined Corrections

Doping and device structure corrections for $\mathcal{E}_{crit}$ combine into:

$$\frac{\mathcal{E}_{crit,NPT}}{\mathcal{E}_{crit,PT}} = \left(\frac{N_{D,NPT}}{N_{D,PT}} \right)^{1/8} \left[1 - \left(1 - \frac{q N_{D,PT}}{\epsilon} \frac{W_{PT}}{\mathcal{E}_{crit,PT}} \right)^8 \right]^{1/8}$$

- Using this equation gives us the ability to equate $\mathcal{E}_{crit}$ for devices with different doping and field profiles

Reference:	DRIFT-LAYER MATERIAL	N_D (cm ⁻³)	W_{PT} (μm)	V_{BD} (V)	$\mathcal{E}_{crit,PT}$ - measured (MV/cm)	$\mathcal{E}_{crit,NPT}$ - corrected (MV/cm)	W_{NPT} - extrapolated (μm)	$\mathcal{E}_{crit,NPT}$ - normalized to $N_D=10^{16}$ cm ⁻³ (MV/cm)
Armstrong et al.	GaN	3×10^{15}	30	3930	2.09	2.09	40.10	2.43
Allerman et al.	Al _{0.3} Ga _{0.7} N	5×10^{16}	4.3	1627	5.76	5.76	6.26	4.71
Ohta et al.	GaN	9×10^{15}	22	4700	3.86	3.86	24.64	3.91
Hu et al.	GaN	2.5×10^{15}	8	1406	1.93	1.88	25.46	2.23
Nishikawa et al.	GaN	2×10^{16}	0.225	52.375	2.37	1.98	5.68	1.81
Nishikawa et al.	Al _{0.29} Ga _{0.71} N	2×10^{16}	0.225	83.75	3.76	3.00	8.16	2.75
Nishikawa et al.	Al _{0.34} Ga _{0.66} N	2×10^{16}	0.225	91.125	4.14	3.23	8.72	3.00
Nishikawa et al.	Al _{0.46} Ga _{0.54} N	2×10^{16}	0.225	112.5	5.04	3.90	10.26	3.58
Nishikawa et al.	Al _{0.52} Ga _{0.48} N	2×10^{16}	0.225	138.75	6.21	4.69	12.20	4.30
Nishikawa et al.	Al _{0.57} Ga _{0.43} N	2×10^{16}	0.225	181.5	8.11	5.94	15.30	5.45

- A. M. Armstrong et al., *Elec. Lett.* **52**, 1170 (2016)
- A. A. Allerman et al., *Elec. Lett.* **52**, 1319 (2016)
- H. Ohta et al., *IEEE Elec. Dev. Lett.* **36**, 1180 (2015)
- Z. Hu et al., *App. Phys. Lett.* **107** (2015)
- A. Nishikawa et al., *Jap. Jour. of App. Phys.* **46**, 4B (2007)

Updated ε_{crit} vs. E_g Values



	Semiconductor	Bandgap at 300 K (eV)	Type	Size	Hudgins	Wang	Our research
Well-established	InSb	0.17	Direct	-	1×10^3	1×10^3	^a
	InAs	0.354	Direct	-	4×10^4	4×10^4	^a
	Ge	0.661	Indirect	2.5×10^5	1×10^5	1×10^5	2.0×10^5
	GaSb	0.726	Direct	-	5×10^4	5×10^4	^b
	In _{0.53} Ga _{0.47} As	0.74	Direct	-	-	-	2.84×10^5
	Si	1.12	Indirect	4.37×10^5	3×10^5	3×10^5	3.7×10^5
	InP	1.344	Direct	-	5×10^5	5×10^5	5.0×10^5
	GaAs	1.424	Direct	4.98×10^5	4×10^5	6×10^5	5.4×10^5
Widebandgap	GaP	2.26	Indirect	7.59×10^5	1×10^6	1×10^6	^b
	3C-SiC	2.36	Indirect	-	1.3×10^6	1×10^6	^c
	6H-SiC	3.0	Indirect	-	2.4×10^6	5×10^6	2.9×10^6
	4H-SiC	3.23	Indirect	-	3.18×10^6	-	3.2×10^6
Ultra-Widebandgap	GaN	3.45	Direct	-	3×10^6	5×10^6	3.9×10^6
	Al _x Ga _{1-x} N	3.45-6	Direct	-	-	-	^d
	β -Ga ₂ O ₃	4.7	Direct	-	-	15×10^6	^d
	C (diamond)	5.5	Indirect	-	5.7×10^6	21.5×10^6	10.1×10^6

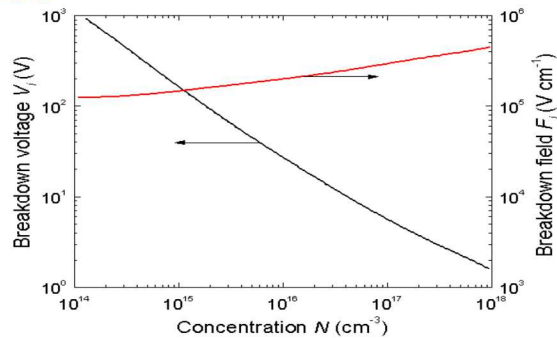
- ^a tunneling cannot be decisively excluded as a contributing factor in carrier multiplication
- ^b no reliable data in the literature was found
- ^c insufficient device data to perform normalization
- ^d no experimental data confirming temperature-dependent behavior indicative of true avalanche breakdown

- S. M. Sze and G. Gibbons, *App. Phys. Lett.* 8, 111 (1966)
- Hudgins et al., *IEEE Trans. on Pow. Elec.* 18, No. 3 (2003)
- L.-M. Wang, *25th IEEE Int. Conf. on Microelec.* (2006)

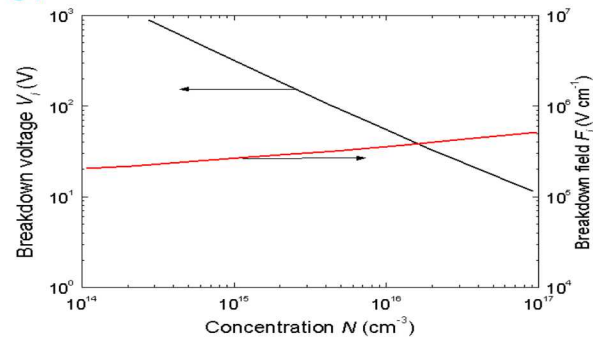
Well-Established Semiconductors

- Impact ionization coefficients and $\mathcal{E}_{crit}$ vs. doping is known for many of the well-established semiconductors
- Compiled in Semiconductor Parameters Volumes 1 & 2 edited by Levinshtein, Rumyantsev & Shur

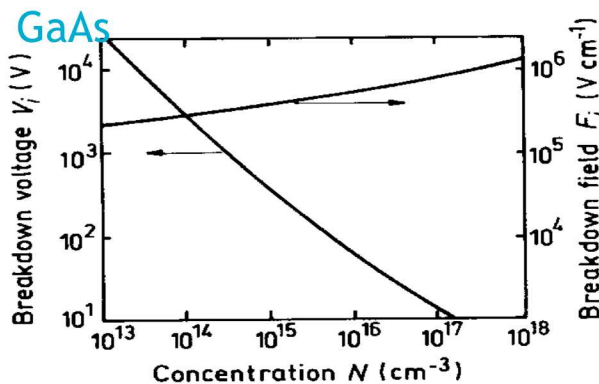
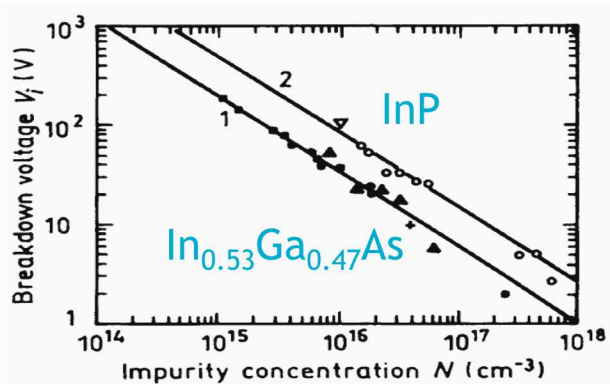
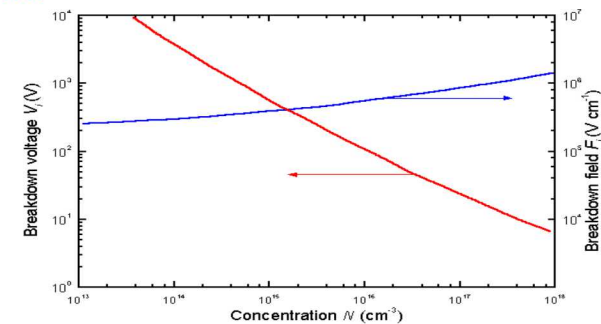
Ge



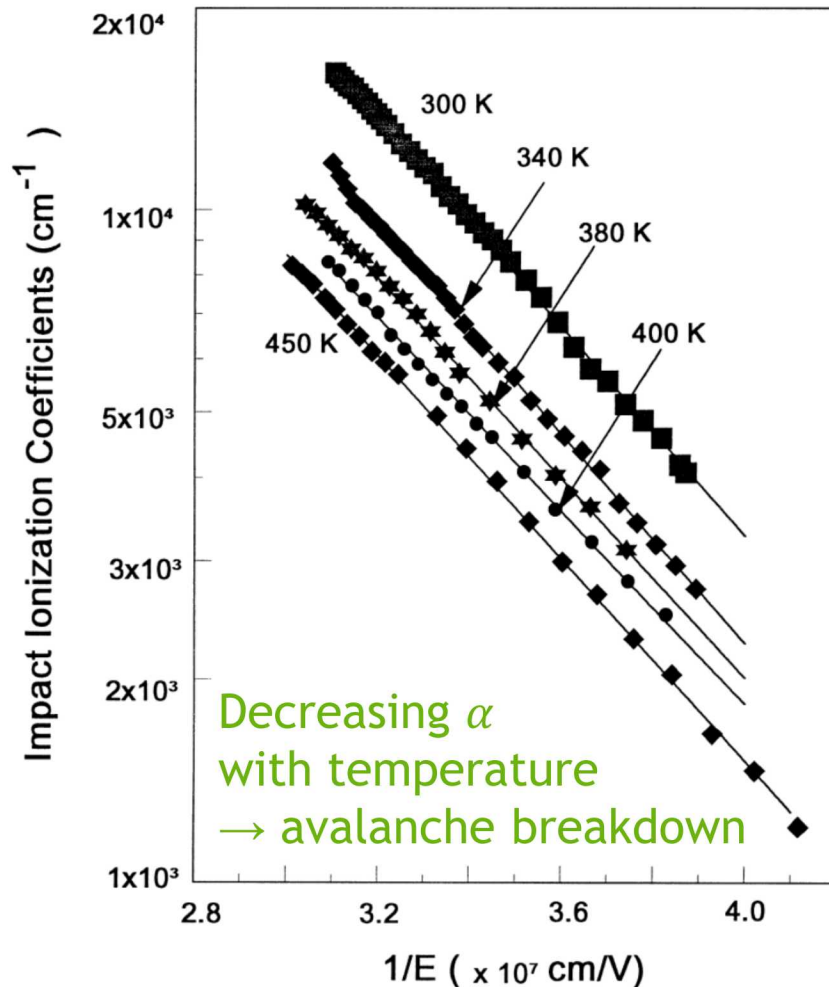
Si



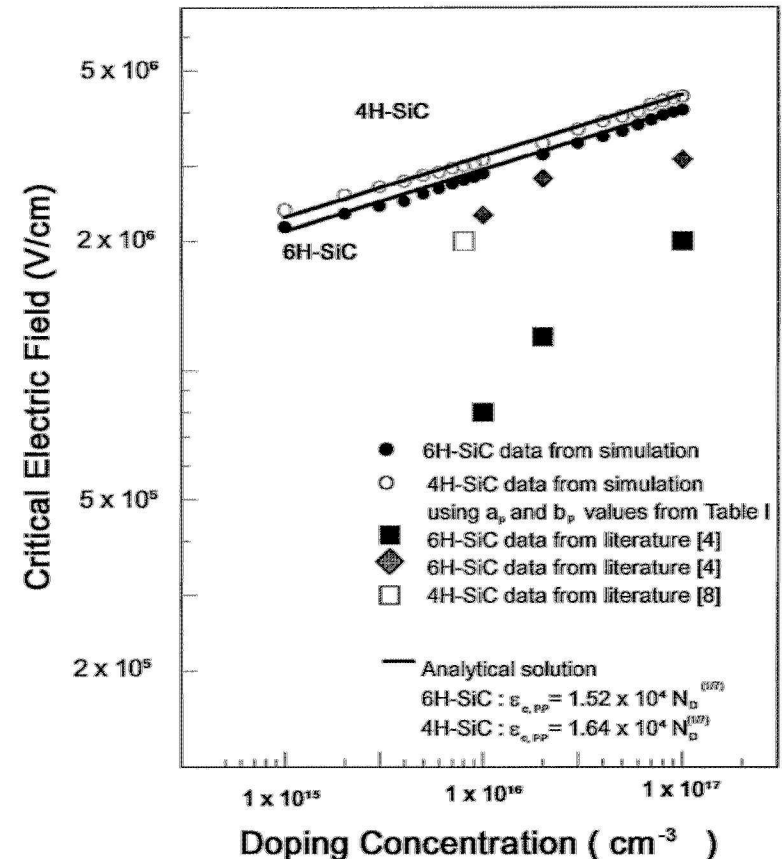
InP



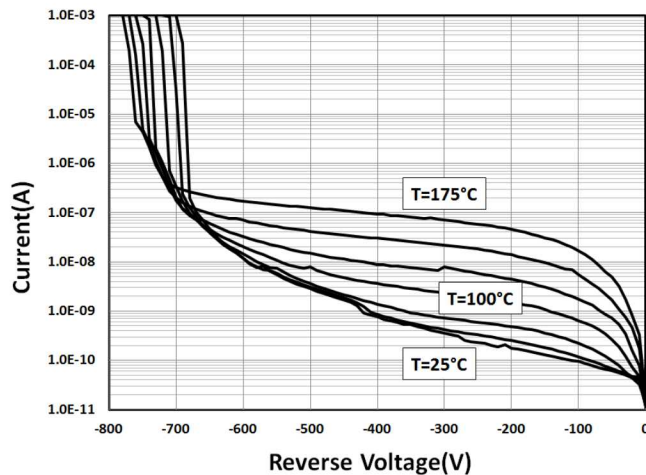
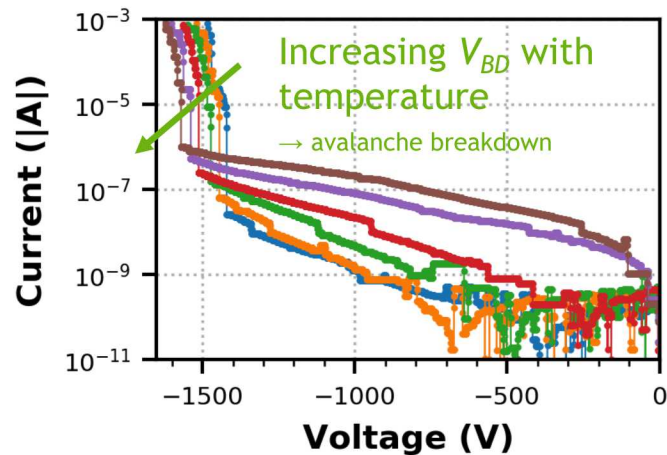
- M. Levinshtein, S. Rumyantsev, M. Shur, *Semiconductor Parameters Vol. 1* (1996)
- M. Levinshtein, S. Rumyantsev, M. Shur, *Semiconductor Parameters Vol. 2* (1999)

α_p Impact Ionization Data

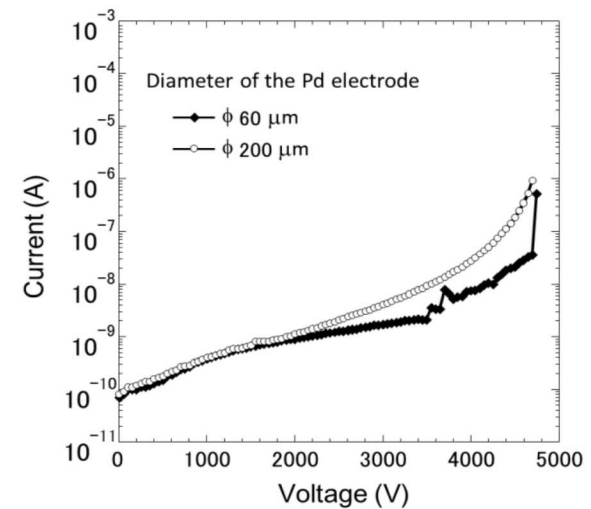
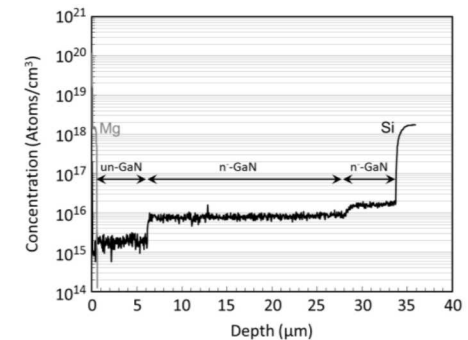
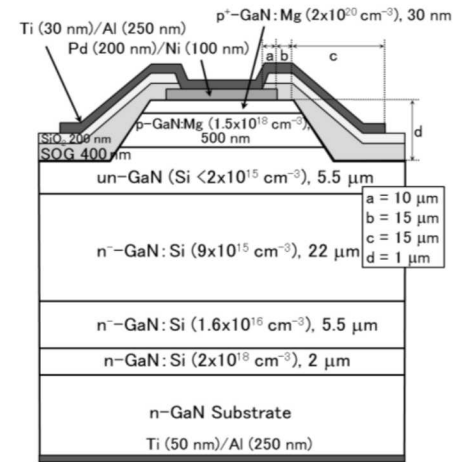
Critical electric field vs. doping



Breakdown vs. Temperature Data



Best Device Reported in Literature

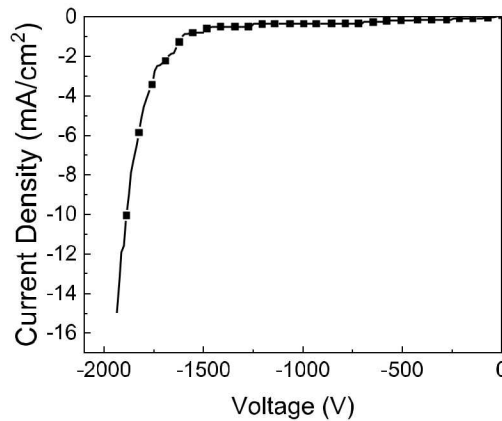
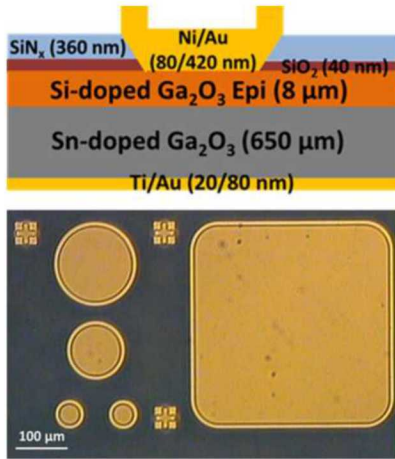


- O. Slobodyan et al., *MRS. Comm.* **8**, 4 (2018)
- I. Kizilyalli et al., *IEEE Trans. On Elec. Dev.* **60**, 10, (2013)
- Ohta et al., *IEEE Elec. Dev. Lett.* **36**, 11, (2015)

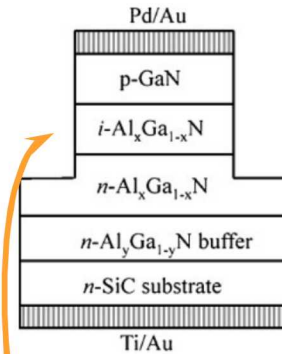
UWBG Semiconductors: $\beta\text{-Ga}_2\text{O}_3$ & $\text{Al}_x\text{Ga}_{1-x}\text{N}$

$\beta\text{-Ga}_2\text{O}_3$ has predicted $\mathcal{E}_{crit} \sim 7 \text{ MV/cm}$

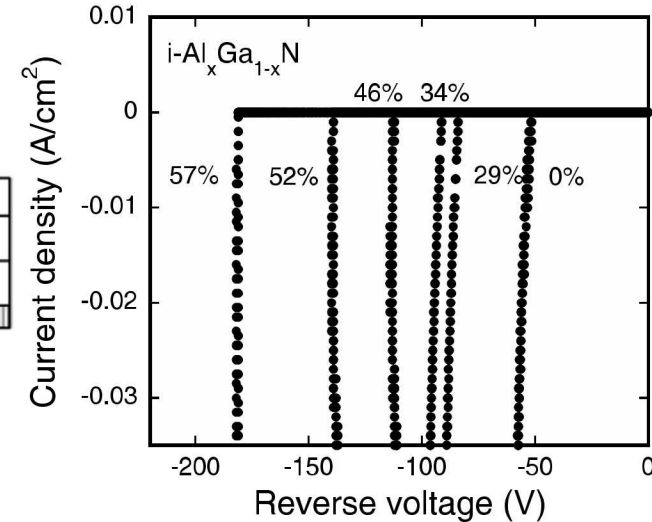
AlN has predicted $\mathcal{E}_{crit} \sim 10 \text{ MV/cm}$ and GaN $\mathcal{E}_{crit} \sim 3.9 \text{ MV/cm}$ with $\text{Al}_x\text{Ga}_{1-x}\text{N}$ alloys values falling in-between



$$\mathcal{E}_{crit} = 2.8 \text{ MV/cm}$$



Drift layer = 225 nm



$$\mathcal{E}_{crit}^{0\%} = 1.81 \text{ MV/cm}$$

$$\mathcal{E}_{crit}^{29\%} = 2.75 \text{ MV/cm}$$

$$\mathcal{E}_{crit}^{34\%} = 2.96 \text{ MV/cm}$$

$$\mathcal{E}_{crit}^{46\%} = 3.58 \text{ MV/cm}$$

$$\mathcal{E}_{crit}^{52\%} = 4.30 \text{ MV/cm}$$

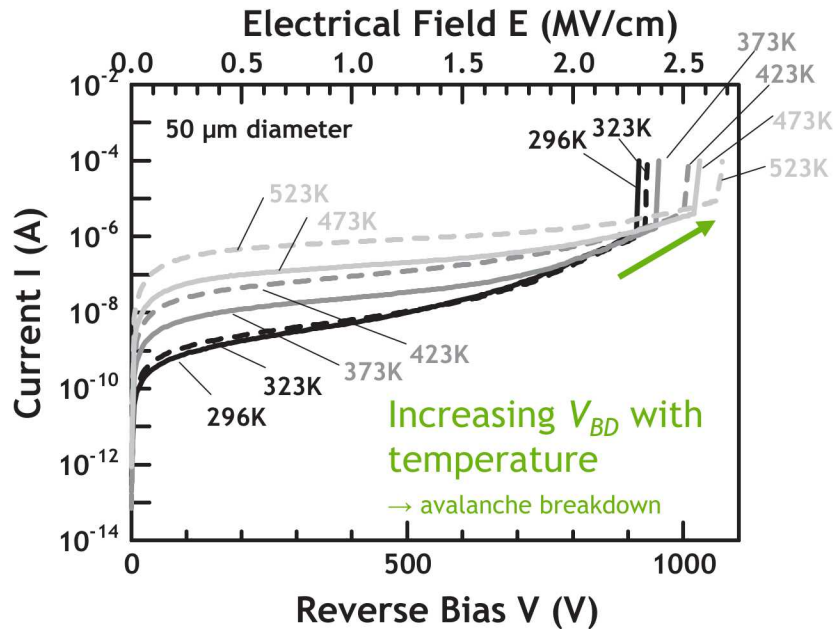
$$\mathcal{E}_{crit}^{57\%} = 5.45 \text{ MV/cm}$$

No *current* temperature dependence of breakdown voltage or ionization coefficients support avalanche breakdown in $\beta\text{-Ga}_2\text{O}_3$ or $\text{Al}_x\text{Ga}_{1-x}\text{N}$

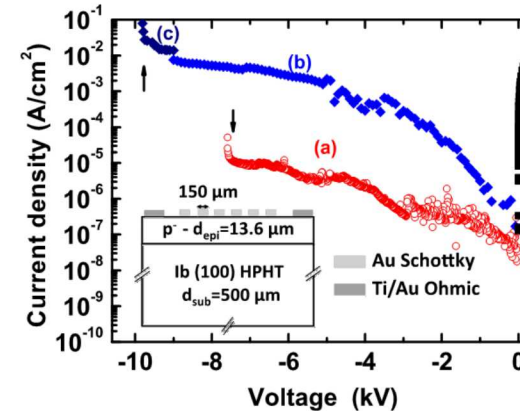
- J. Yang et al., *ECS Jour. of Sol. State Sci. and Tech.* **8**, 7 (2019)
- A. Nishikawa et al., *Jap. Jour. of App. Phys.* **46**, 4B (2007)

UWBG Semiconductor: C (diamond)

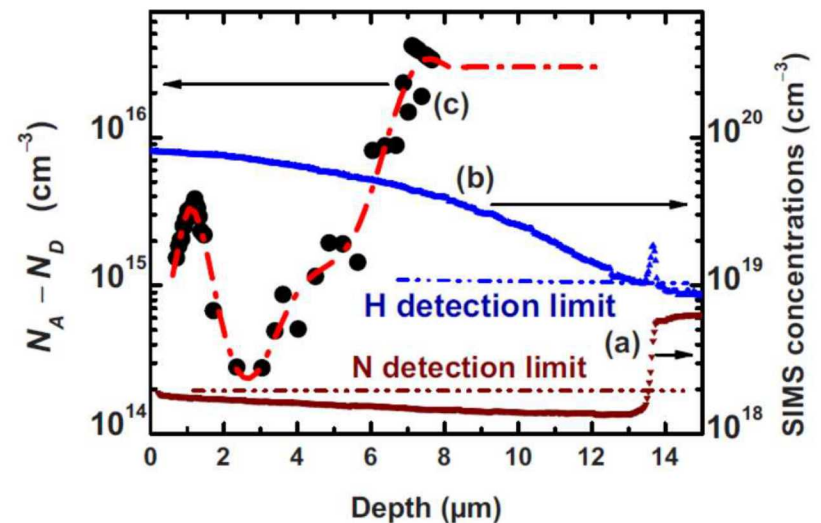
Breakdown vs. Temperature Data



Best Device Reported in Literature



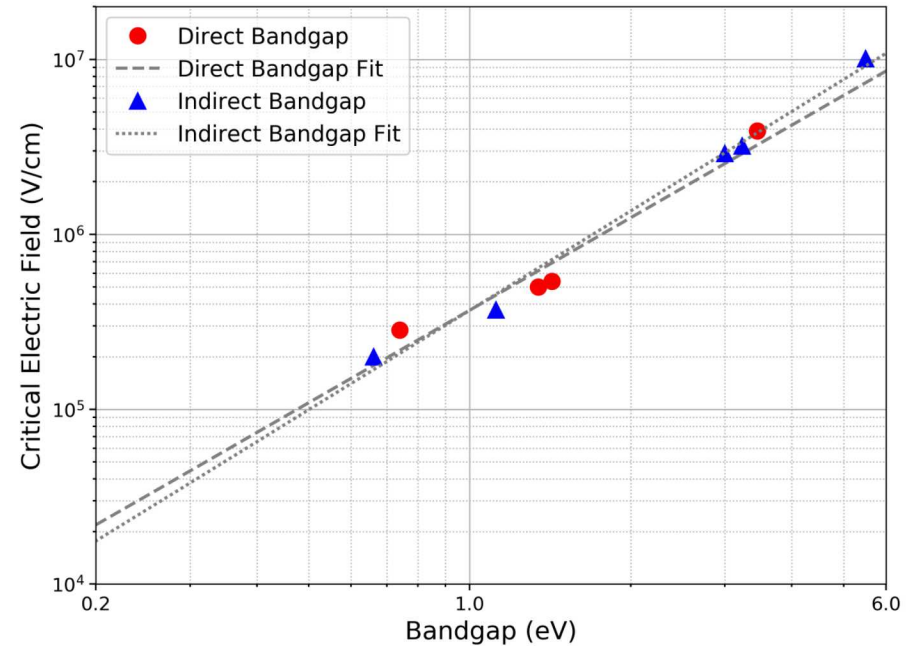
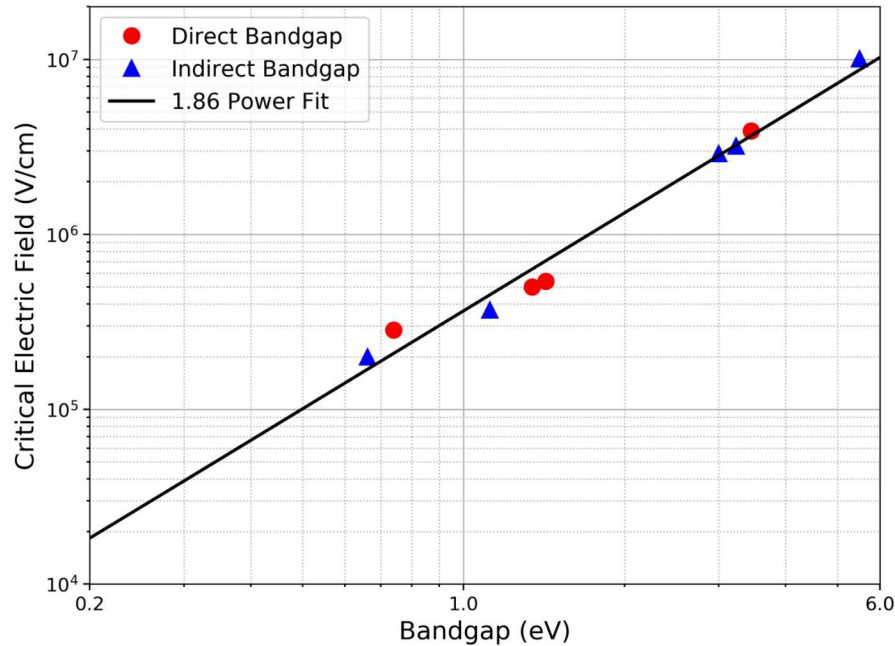
But... a complicated doping profile



- M. Suzuki et al., *Phys. Stat. Sol. A* 210, 10 (2013)
- P.-N. Volpe et al., *App. Phyl. Lett.* 97, (2010)

Fitting of Updated Critical Electric Field vs. Semiconductor Bandgap Values

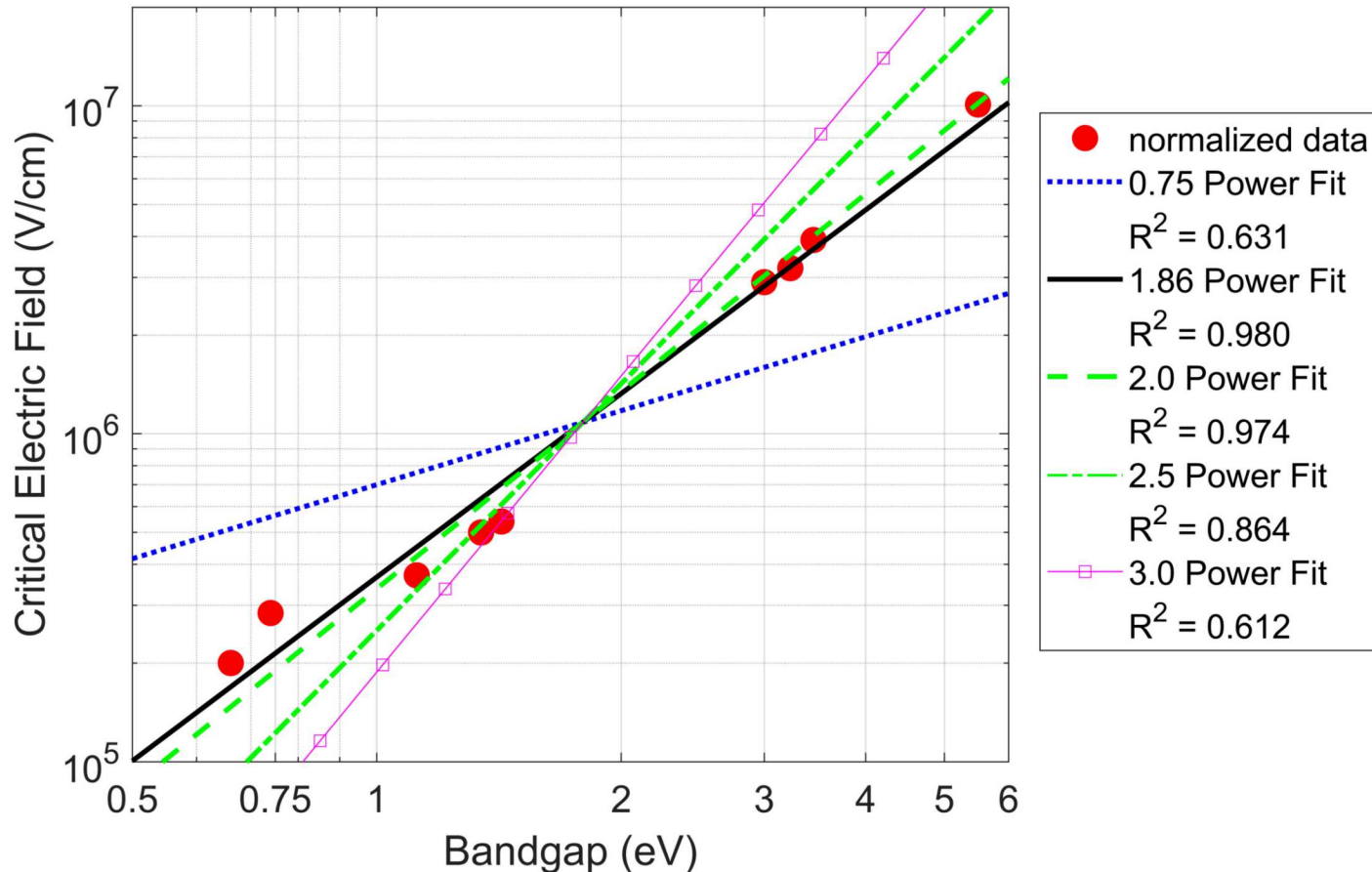
- Updated power-law fit to new critical electric field values
- Best-fit power-law slope $\mathcal{E}_{crit} \sim E_g^\gamma : \gamma = 1.86$



Semiconductor	Type	Bandgap (eV)	Our research
Ge	Indirect	0.661	2.0×10^5
$\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$	Direct	0.74	2.84×10^5
Si	Indirect	1.12	3.7×10^5
InP	Direct	1.344	5.0×10^5
GaAs	Direct	1.424	5.4×10^5
6H-SiC	Indirect	3.0	2.9×10^6
4H-SiC	Indirect	3.23	3.2×10^6
GaN	Direct	3.45	3.9×10^6
C (diamond)	Indirect	5.5	1.01×10^7

Critical Field Values	Slope	R ²
All	1.86	0.980
Direct Bandgap	1.76	0.947
Indirect Bandgap	1.89	0.992

Comparison of Updated Empirical Fit with Historical Fits



- For normalized data, the proposed 1.86 fit is much better than most historical fits to date
- This fit should more accurately predict material performance capabilities

Simulation: Ridley's Lucky-electron

- To provide fundamental basis for $\mathcal{E}_{crit}$, developed a phenomenological model for breakdown
 - Ionization integral transformed from integral over space $\rightarrow$ integral over E-field
 - Ionization rate as a function of field is inserted into integral
- Based on the fundamental avalanche model developed by B.K. Ridley ("lucky-drift" model)
 - Higher fidelity in high/low field regimes (compared to Shockley's "lucky electron" or Chenoweth models)
 - More tractable for calculation than Monte Carlo full band approximations

Ridley's Lucky-Drift Model:

$$\alpha\lambda = \frac{1}{x} \left\{ e^{-x} + \left(\frac{e^{-2rx^2}}{1-2rx} \right) + P_T \left[e^{-x(1-\zeta)} + \left(\frac{e^{-2rx^2(1-\zeta)} - e^{-x(1-\zeta)}}{1-2rx} \right) \right] \right\}$$

$$x = \frac{E_{th}}{e\mathcal{E}\lambda} \quad \zeta = \frac{P_T}{2rx^2} \quad P_T = 1 - e^{-2rx(x-3)} \Big|_{x \geq 3}$$

Lucky-drift can be described by three variables:

- E_{th} , threshold energy required to initiate avalanche
- r , ratio of energy between electron-phonon interaction and electron-electron momentum transfer
- λ , mean free path of electron

Approach is to transform E_{th} , r , and λ into functions dependent on E_g

Determining E_g dependence of Threshold Energy and Phonon Collision loss for Ridley's Lucky Electron Model

E_{th}

- Energy hot carrier must possess to create an electron-hole pair
- Many assume ionization can be initiated by any electron with energy $> E_g$
- Actual $E \geq 1.5E_g$ as derived by Maes et al.

r

- Average energy loss per phonon collision to the threshold energy

$$r = \frac{\hbar\omega}{[2n(\omega) + 1]E_{th}}$$

$\hbar$ is the reduced Planck constant
 ω is phonon angular frequency,
 $n(\omega)$ is the quantization number.

- Only E_{th} is function of E_g , so r reduces to

$$r \propto E_g^{-1}$$

Determining E_g dependence of Mean-free-path for Ridley's Lucky Electron Model

λ

- Mean free path length of a carrier before thermal relaxation
- Assume nonpolar optical phonon scattering $\rightarrow \tau$ is energy independent and depends inversely on the effective mass (m^*)

For a carrier at E_{th}

$$\lambda = v_g \cdot \tau$$

$$\text{with } v_g(E) = \sqrt{\frac{2E}{m^*}} \text{ and } \tau \propto \frac{1}{m^{*\frac{3}{2}}}$$

$$\lambda \propto \frac{1}{m^{*\frac{3}{2}}} \cdot \sqrt{\frac{2E_{th}}{m^*}} \text{ so: } \lambda \propto \frac{1}{m^{*\frac{3}{2}}} \cdot \frac{E_{th}^{\frac{1}{2}}}{m^{*\frac{1}{2}}} \propto \frac{1}{m^{*\frac{3}{2}}} \cdot \frac{E_g^{\frac{1}{2}}}{m^{*\frac{1}{2}}}$$

A linear relationship between m^* and E_g can also be derived via Bloch Theory for a periodic potential

$$\lambda \propto \frac{1}{E_g^{\frac{3}{2}}} \cdot \frac{E_g^{\frac{1}{2}}}{E_g^{\frac{1}{2}}} \propto \frac{1}{E_g^{\frac{3}{2}}}$$

Use Si values for r , λ to calculate proportionality constants:

$$\lambda_{Si} = 7.1 \text{ nm to } 11.94 \text{ nm}$$

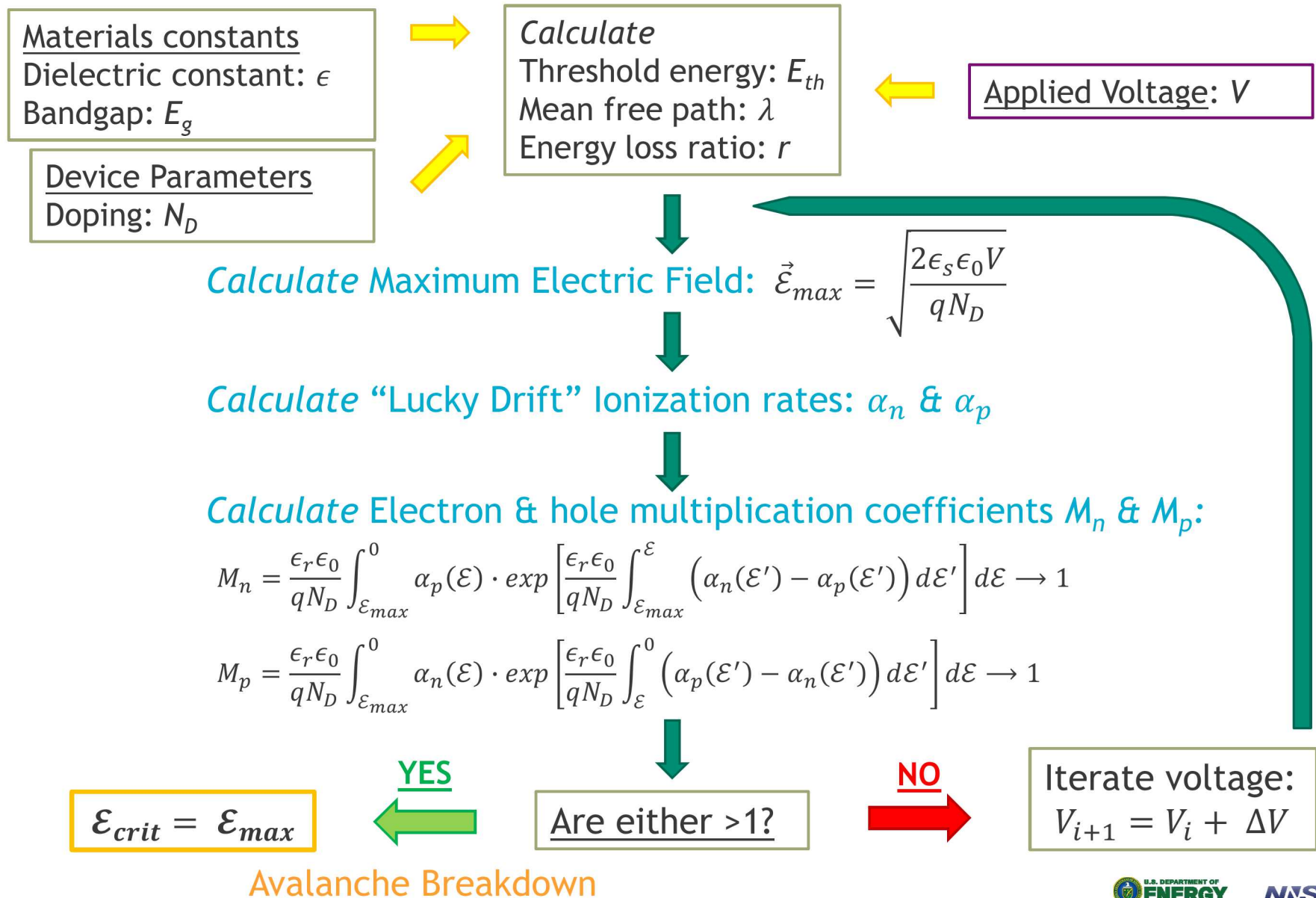
$$r_{Si} = 0.049 \text{ to } 0.063$$

Parameters for Ridley's Lucky Electron Model

Parameter	Value	Reference
Threshold Energy, E_{th}	$1.5 \cdot E_g$	Maes et al.
Ratio of average energy loss per collision to the E_{th} energy, r	$\frac{0.063}{E_g}$	Ridley
Mean relaxation length, λ	$\frac{12 \times 10^{-7}}{E_g^{3/2}}$	Several

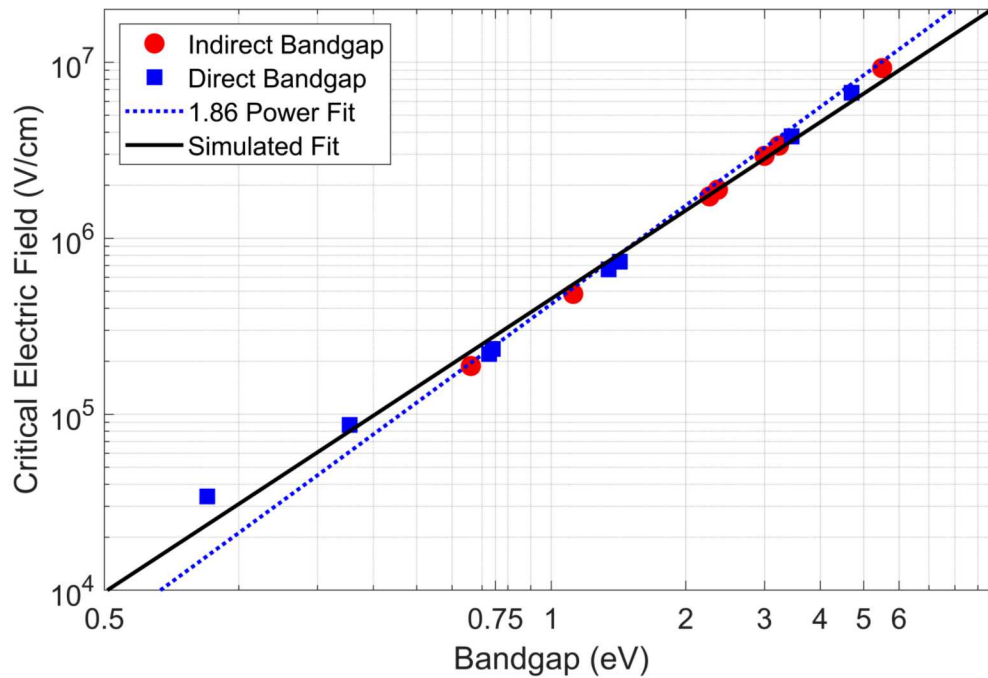
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Flow Diagram for Simulation of Ridley's Lucky Electron Model



Comparison of Empirical Fit and Simulation Results

- Empirical and simulated values are in reasonable agreement



Semiconductor	Bandgap (eV)	Our research	Simulated
InSb	0.17	-	3.41×10^4
InAs	0.354	-	8.70×10^4
Ge	0.661	2.0×10^5	1.88×10^5
GaSb	0.726	-	2.20×10^5
$\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$	0.74	2.84×10^5	2.35×10^5
Si	1.12	3.7×10^5	4.83×10^5
InP	1.344	5.0×10^5	6.65×10^5
GaAs	1.424	5.4×10^5	7.36×10^5
GaP	2.26	-	1.73×10^6
3C-SiC	2.36	-	1.89×10^6
6H-SiC	3.0	2.9×10^6	2.94×10^6
4H-SiC	3.23	3.2×10^6	3.36×10^6
GaN	3.45	3.9×10^6	3.80×10^6
$\beta\text{-Ga}_2\text{O}_3$	4.7	-	6.71×10^6
C (diamond)	5.5	1.01×10^7	9.27×10^6

Critical Field Values	Slope	R^2
Empirical	1.86	0.980
Simulation	1.67	0.990

Conclusions & Precations

- Updated critical electric field vs semiconductor bandgap values
 - Performed re-normalization to properly compare devices with different dopings and structures
 - Power-law slope of: 1.86 is best fit
 - No difference between direct against indirect bandgap semiconductors
- Performed first-principles physics simulation to determine $\mathcal{E}_{crit}$ for various semiconductors
 - Reasonable agreement with empirical data
- Future development of WBG & UWBG materials is expected to change some critical field values
- Not valid for high-electron mobility transistors (HEMTs) or lateral devices!
 - Electric field profiles are very different