

Implicit and Hybrid Techniques for the Simulation of High-Density Electrode Plasmas for Pulsed Power Applications

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Topics

- Motivation for Implicit/Hybrid Modeling
- PIC algorithm advances including Magnetic Implicit with Poisson Correction.
- Fast fluid technique and Particle Migration Hybrid.
- OpenMP threading.
- Application to Simplified Convolute geometry to assess speed up and accuracy.
- Summary and Conclusions

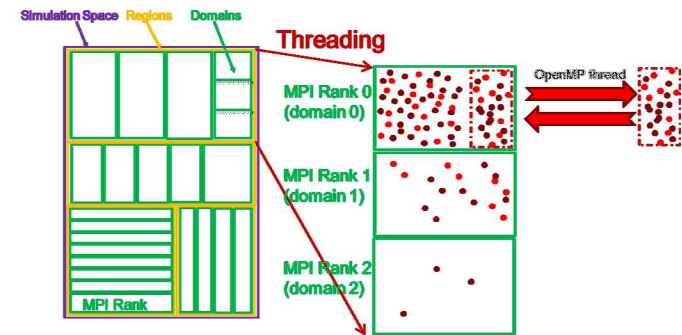
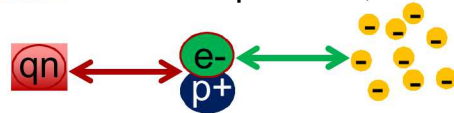
Motivation for Implicit/Hybrid Power Flow Simulation

- Power flow, charged particle emission, current sheaths:
 - Non-neutral, nonMaxwellian, turbulent, nonlinear instability growth.
 - $< 10^{15} \text{ cm}^{-3}$ density -> Fully Kinetic physics.
- Evolving Contaminant Plasmas:
 - Non-neutral physics such as bipolar flow, nonlinear instability growth.
 - Detailed chemistry: Breakdown, charge exchange, etc.
 - $< 10^{19} \text{ cm}^{-3}$ density -> Multi Fluid through kinetic (hybrid).
- MITL Metal Substrate:
 - Mostly quasi-neutral with *kinetic impacts*.
 - EOS, solid density -> Single Fluid or Multi Fluid (hybrid).
- Liner Evolution:
 - Mostly quasi-neutral with EOS, maybe with Hall like physics.
 - Magnetized Shocks, mix.
 - \gg solid density, but features such as laser heating, *beams* and *fusion product transport* requiring kinetic effects -> Hybrid.

All regimes can benefit from some hybrid description, PIC techniques being developed in the Chicago and LSP codes offer a integrated method to model both kinetics and fluids!

Chicago* is a FDTD Toolkit of Field/Particle Advanced Techniques for Plasma Simulation

- Fully electromagnetic and relativistic.
- 3D orthogonal grid 1st or 2nd order (partial-cell conformal boundaries with particles*) surfaces.
- **All** plasma descriptions use PIC techniques, and can be combined in **Particle Migration Hybrid** operation.
- Kinetic, multi-fluids (inertia and charge separation).
- Quasi-neutral with multiple ions, new Hall implementation.#



Chicago uses a two-level spatial decomposition *plus* multiple threads/domain.

	Solution technique	Speed	$\lambda_{\text{Debye}} / \Delta x$	$\omega_c \Delta t$	$\omega_p \Delta t$	Use/Comments
MITL Sheaths	Explicit mom. conserving	fast	< 1	< 2	< 2	No particle self force. Must resolve Debye length.
	Explicit energy conserving	fast	>> 1	< 1—2	< 1—2	Best energy and momentum conservation with second order CIC.
	Direct Implicit	slower	>> 1	< 1	>> 10	Cyclotron orbit growth. Best conservation with Poisson Correction on, CIC.
Electrode plasmas	Magnetic Implicit	slower	>> 1	< 3-12	>> 10	Accurate orbits. Limited to moderate ω_c with large ω_p .
HEDP plasmas	Multi-Fluids	faster	>> 1	< 1-12	>> 10	Lagrangian or Eulerian. EOS/Radiation available. Better long time conservation. Implicit, CIC.
	Quasi-Neutral (Resistive MHD)	fast	>> 1	>> 1	>> 1	EOS, Radiation, Kinetic ions available. Hall physics tested in 2D. No electron inertia.

* D. R. Welch, N. Bennett, T. C. Genoni, D.V. Rose, C. Thoma, C. Miller, and W. A. Stygar, *Electrode contaminant plasma effects in 10⁷-A Z pinch accelerators*, Phys. Rev. Accel. Beams **22**, 070401 (2019).. # C. Thoma, et al., manuscript submitted (2019).

Direct Implicit* has allowed relaxed frequency constraints by integrating over fast plasma oscillation.

$$x^{n+1} = x^n + \frac{\Delta t}{c} p^{n+1/2}$$

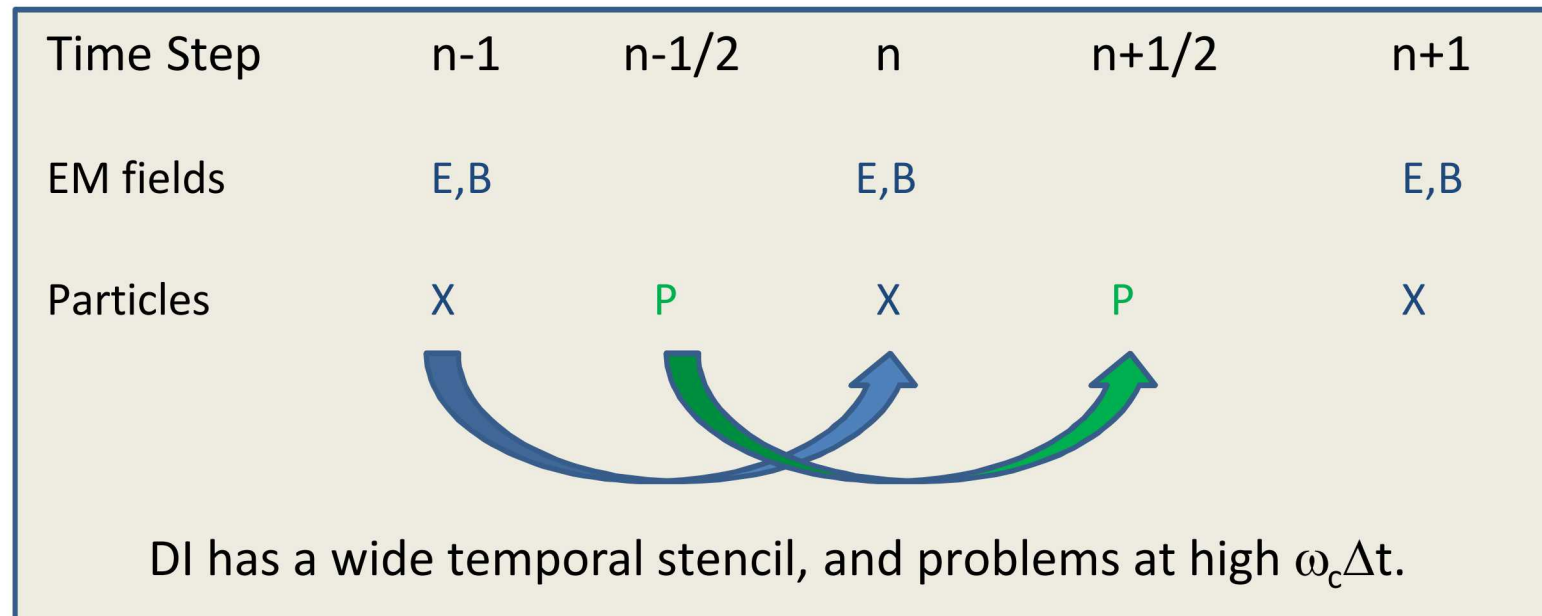
$$p^{n+1/2} = p^{n-1/2} + \frac{\Delta t}{2m} \left(E^{n-1} + E^{n+1} + \frac{p^{n-1/2} + p^{n+1/2}}{\gamma c} \times B^n \right)$$

$$E^{n+1} = E^n + \frac{\Delta t}{2} \nabla \times (B^n + B^{n+1}) - \Delta t J^{n+\frac{1}{2}} - S \cdot E^{n+1}$$

$$B^{n+1} = B^n - \frac{\Delta t}{2} \nabla \times (E^n + E^{n+1})$$

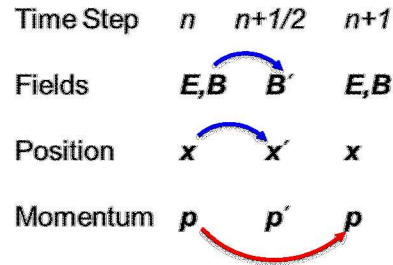
Solve for future fields via matrix inversion.

Fluctuations in E reduced by $1+1/2 \omega_p^2$



*Cohen, Langdon, Friedman, J. Comp. Phys. **46**, 15 (1982);
D. R. Welch, et al., Nucl. Inst. Meth. Phys. Res. A **464**, 134 (2001).

Magnetic Implicit* algorithm further relaxes constraints and enables realistic power flow, diode simulations.



MI calculates all particle attributes and EM fields at full time steps after initial half step advance for centering. *Energy conserving* with 2nd order cloud gives superior results:

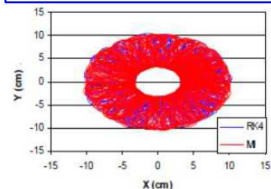
$$\omega_p \Delta t \gg 1 \text{ and } \omega_c \Delta t < 10$$

2 Step Particle advance

- 1st step pushes x and p with E^n and $B^{n+1/2}$ and $\langle S \rangle$ constructed at x' .
- 2nd step after fields are advanced to $n+1$, x and p are pushed from n to $n+1$ positions. Corrects for errors in mirror force and transverse magnetic field gradient drift.

$$p^{n+1} = p^n + \frac{q\Delta t}{2m} \left[(E^n + E^{n+1}) + \frac{(p^n + p^{n+1})}{\gamma^{n+1/2} c} \times B^{n+1/2} \right],$$

$$x^{n+1} = x^n + \Delta t \left(\frac{p^n + p^{n+1}}{\gamma^n + \gamma^{n+1}} + \Delta v_{drift} \right).$$



Benchmarked* against 4th order RK solution of complex Field Reversed Configuration orbits.

Implicit EM advance with Poisson correction

- For calculation of susceptibility $\langle S \rangle$ and magnetic rotation matrix $\langle T \rangle$ at x' , B is advanced $\frac{1}{2}$ step with explicit Faraday's Law,

$$\frac{B^{n+1/2} - B^n}{\Delta t} = -\frac{1}{2} \nabla \times E^n.$$

- After 1st particle advance, fields advanced with implicit term for J at $n+1$

$$\frac{\partial E}{\partial t} = \nabla \times B - J - \langle S \rangle \cdot E'_{n+1} \quad \text{Implicit electromagnetics}$$

$$\nabla \cdot (1 + \langle S \rangle) \cdot \nabla \psi_{err} = \rho_{n+1}^0 - \nabla \cdot (1 + \langle S \rangle) \cdot E'_{n+1} = \rho_{err}, \quad \text{Poisson Correction}$$

$$E_{n+1} = E'_{n+1} - \nabla \psi_{err}, \quad \text{Corrected field applied to particle in second push}$$

*T. C. Genoni, R. E. Clark and D. R. Welch, The Open Plasma Physics Journal **3**, 36 (2010).
D. R. Welch, et al., Phys. Rev. Accel. Beams **22**, 070401 (2019).

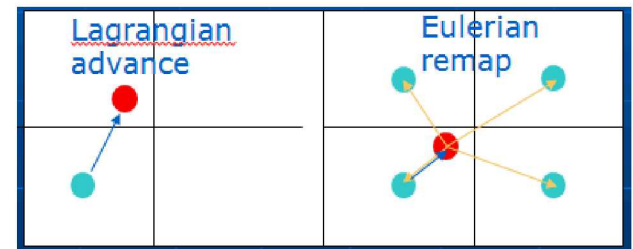
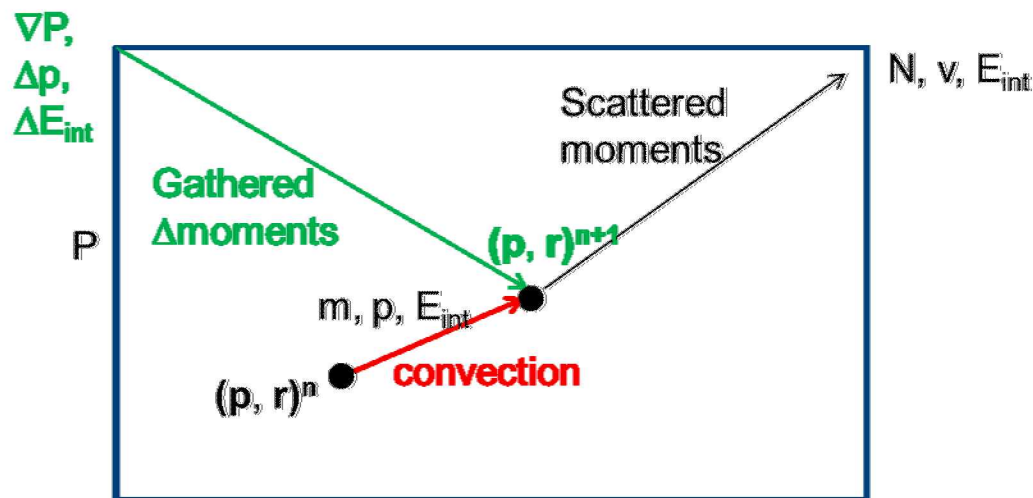
Basic PIC Fluid Technique* with optional EOS is computational faster with fewer particles.

- Lagrangian fluid macroparticles *convect moments* (charge, mass, momentum and energy) with little diffusion.
- Fluid moments scattered to grid where interactions, transport, etc. calculated.
- Δp , ΔE_{int} gathered back to particles.
- Pushed with collective velocity.
- More diffusive with Eulerian remap, but 1 particle per cell.

$$n_i \frac{dU_i}{dt} = -n_i T_i \nabla \cdot \mathbf{v}_i + \sum_j \frac{2m_j n_j}{m_i \tau_{ji}} (T_j - T_i) + \nabla \cdot \kappa \nabla T_i$$

$$+ \sum_j v_{ji} \frac{m_i m_j}{m_i + m_j} \left(\frac{p_i}{m_i} - \frac{p_j}{m_j} \right)^2 + \dot{R}$$

$$\frac{d\mathbf{p}_i}{dt} = \frac{q}{m} (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \frac{\nabla P_i}{nm_i} + \sum_j (p_i - p_j) \mathbf{v}_{ij}$$



*J. U. Brackbill, Comp. Phys. Comm **48**, 25-38 (1988); D. R. Welch, Phys. Plasmas **16**, 123102 (2009); C. Thoma, Phys. Plasmas **18**, 103507 (2011).

PIC representations of kinetic, multi-fluid, and quasi-neutral macroparticles can be combined → **Particle Migration Hybrid**

- Advanced Implicit Method for kinetic/multi-fluid PIC.

$$\frac{\partial E}{\partial t} = \nabla \times B - J - \langle S \rangle \cdot E \quad \text{Implicit electromagnetics}$$

$$\nabla \cdot (1 + \langle S \rangle) \cdot \nabla \psi_{err} = \rho_{n+1}^0 - \nabla \cdot (1 + \langle S \rangle) \cdot E'_{n+1} = \rho_{err}, \quad \text{Poisson Correction}$$

$$E_{n+1} = E'_{n+1} - \nabla \psi_{err}, \quad \text{Corrected field applied to particle in second push}$$

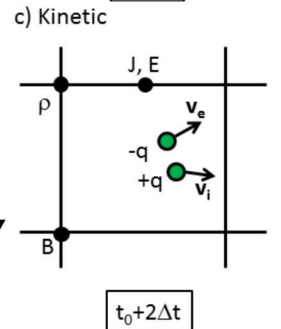
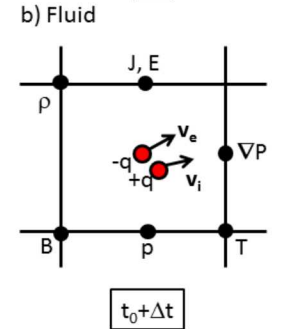
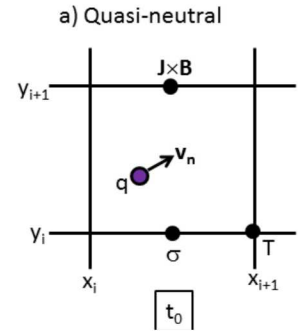
- PIC fluids have equations of motion and energy. Macroparticles carry internal energy.

$$\frac{dp_i}{dt} = \frac{q}{m} (E + v_i \times B) - \frac{\nabla P_i}{nm_i} + \sum_j (p_i - p_j) v_{ij}$$

- Advanced *Multi-Ion* Quasi-Neutral PIC algorithm includes effects from kinetic and multi-fluid particles.

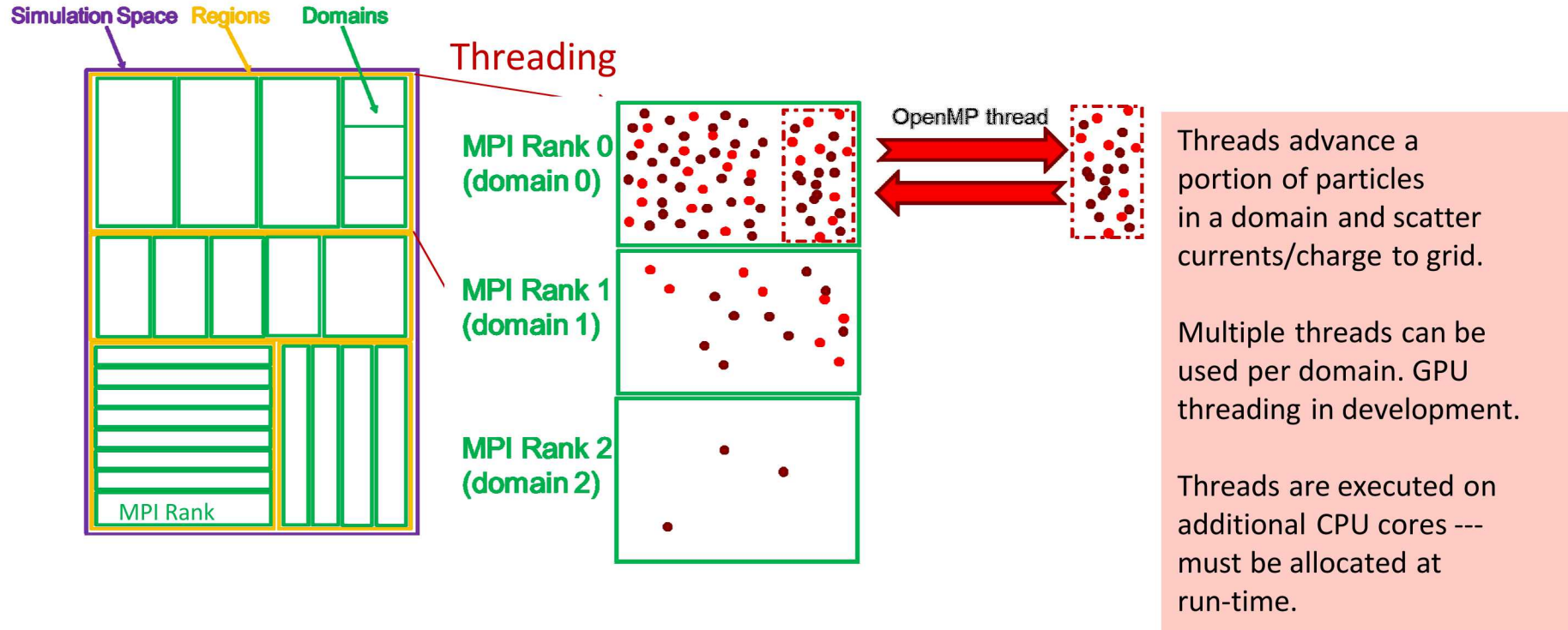
$$\begin{aligned} n_K m_K \frac{D\vec{v}_K}{Dt} = & \left(\frac{\bar{Z}_K n_K}{n_e} \right) \vec{J}_{MHD} \times \vec{B} + e \bar{Z}_K n_K (\vec{v}_K - \vec{v}_+) \times \vec{B} \\ & - \left[\nabla \tilde{P}_K + \left(\frac{\bar{Z}_K n_K}{n_e} \right) \left(\nabla \tilde{P}_e - \sum_K \bar{Z}_K \frac{m_e}{m_K} \nabla \tilde{P}_K \right) \right] + \beta_o \bar{Z}_K n_K \nabla T_e \left(1 - \frac{(n_e)_L}{n_e} F_\beta \right) \\ & - n_K m_K \sum_{M'} v_{KM'} (\vec{v}_K - \vec{v}_{M'}) - n_K m_K v_{Ke} (\vec{v}_K - \vec{v}_+) + \bar{Z}_K n_K m_e \sum_{M'} v_{eM'} (\vec{v}_{M'} - \vec{v}_+) \\ & - n_e m_e \left[\left(\frac{\bar{Z}_K n_K}{n_e} \right) \sum_{M'} v_{eM'} - v_{eK} \right] (\vec{v}_e - \vec{v}_+) \end{aligned}$$

- Seamless *migration* between particles of one EOM to another (*Particle Migration Hybrid*, *PMH*). Particles may migrate wholesale or particle by particle based on energy, perveance, Mach number, etc.



Particle Migration Hybrid

New 3 level decomposition for parallel execution in Chicago with regions, domains, and threads.



Load Balancing accomplished by adjusting domain volumes and number of threads per MPI Rank according to computational load.

Demonstration in simplified convolute.

Simulation Volume/Grid.

- -1.94—6.34 cm; 457 radial cells (200 μm minimum)
- 0-- $\pi/12$; 20 azimuthal cells (uniform)
- 2.9—27 cm; 289 axial cells (100 μm minimum)

Simulation Boundaries.

- Driven by 4 level Z circuit.
- Load is a 4.86-nH inductor (inner MITL) to a short.

Simulation Decomposition – 272 domains

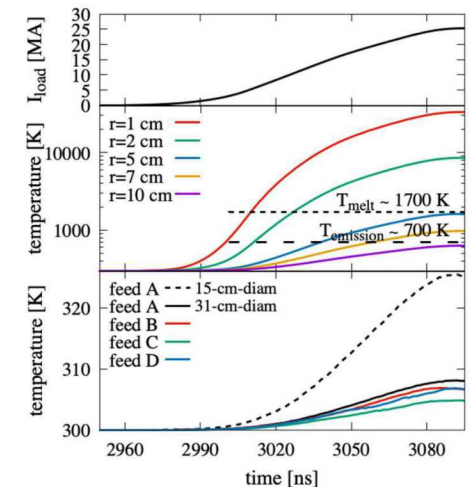
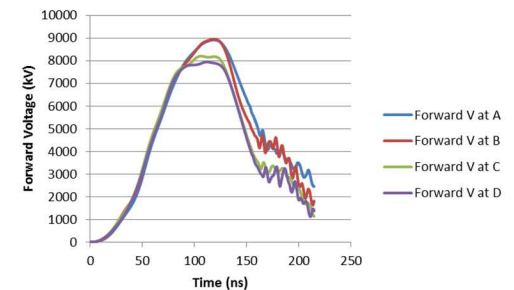
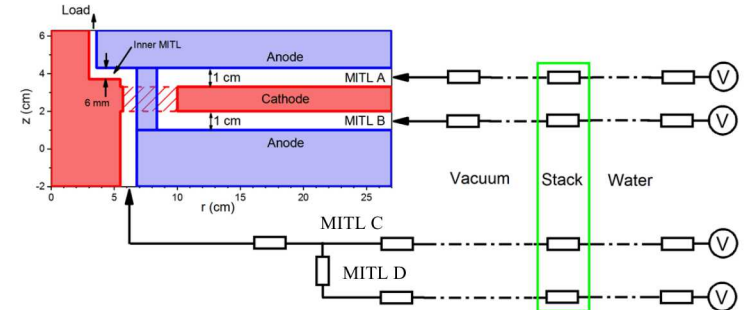
- 68 regions, 4 domain/region, 1 or 2 threads per domain.

SS Electrode material Heating – Ohmic (Knoepfel) and charged particle Impact.

$$\Delta T(t) \approx \frac{1.273 \mu_0 j^2(t)}{2c_v}$$

Plasma Generation and interactions.

- Space charge limited emission off A and K.
- Contaminant plasma desorption from A and K.
 - Temkin H_2O thermal desorption: $p = 10^{-5}$ Torr vacuum, $8 \times 10^{15} \text{ cm}^{-2} = 1 \text{ ML}$, .83-1.0 eV binding energy, Staged fragmentation and ionization of water into $3e^-$, 2H^+ , O^+ .
 - Fully general binary scattering between all particles.

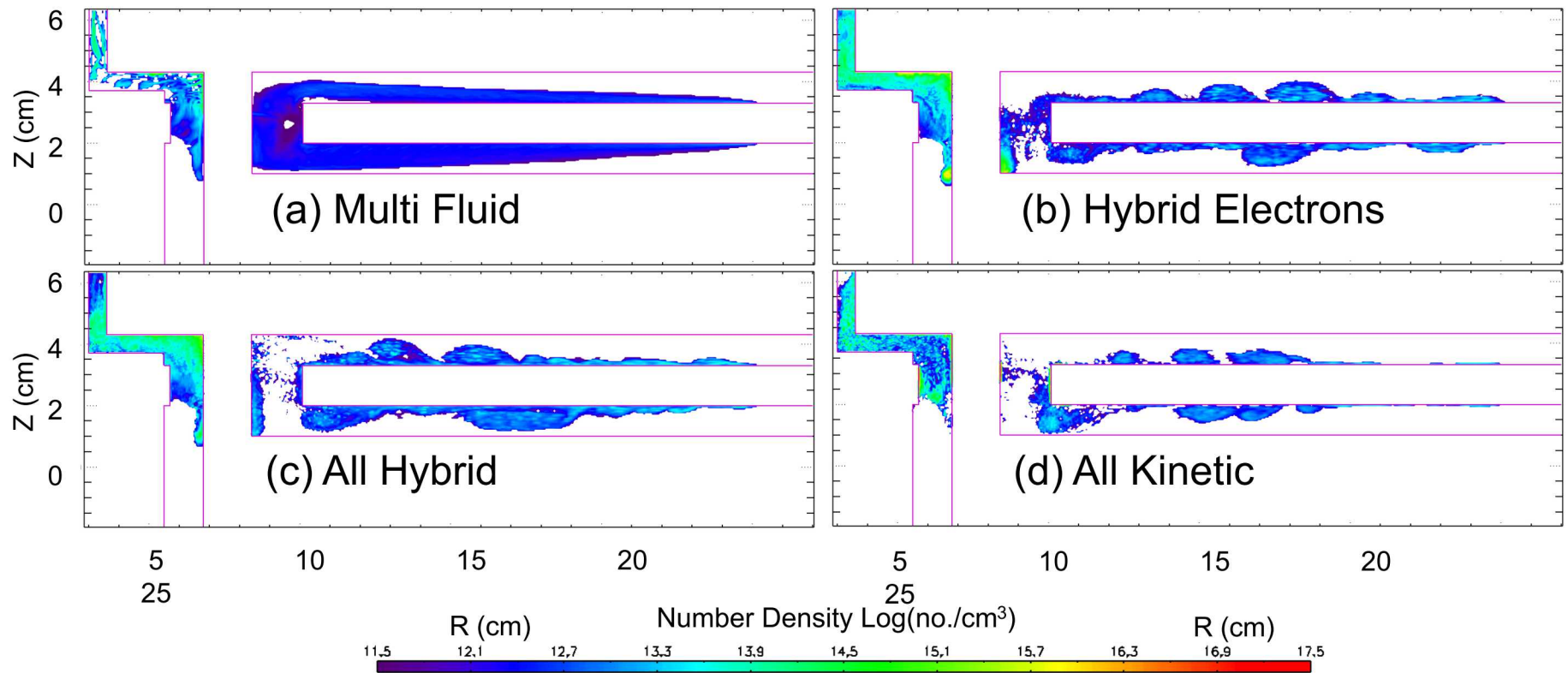


Fully kinetic, Multi-Fluid, Hybrid with Staged Ionization surface physics.

Multi Fluid shows smoother sheath behavior, hybrid closely resembles kinetic.

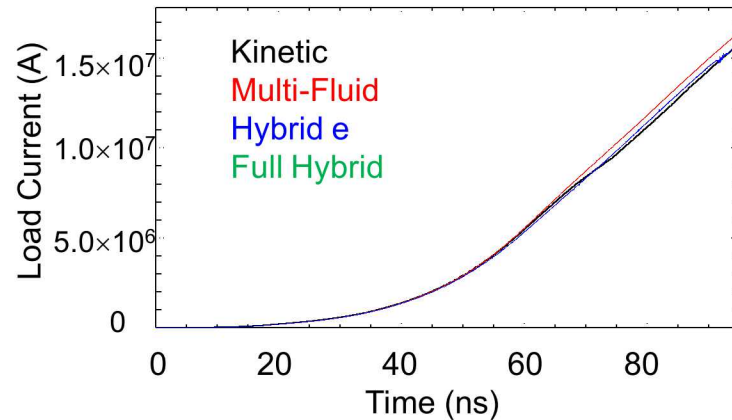
Two flavors of hybrid:

1. Hybrid electrons \rightarrow only fluid electrons transition to kinetic.
2. Full Hybrid \rightarrow all fluid electrons and ion species can transition to kinetic.

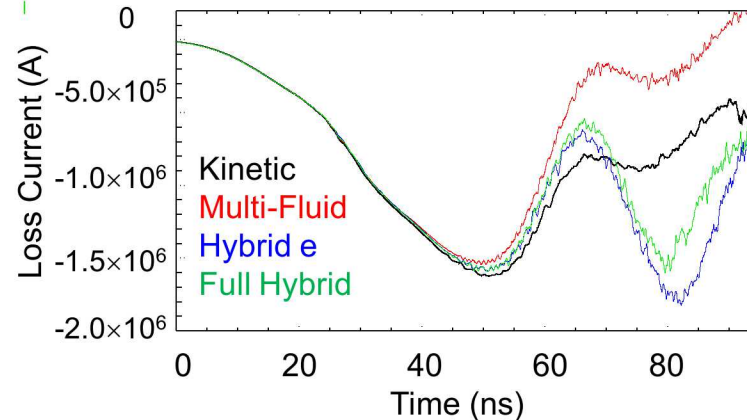


Plasma behavior with all fluids in inner MITL differs from kinetic/hybrid.

Comparison of models shows similar peak losses and load currents.



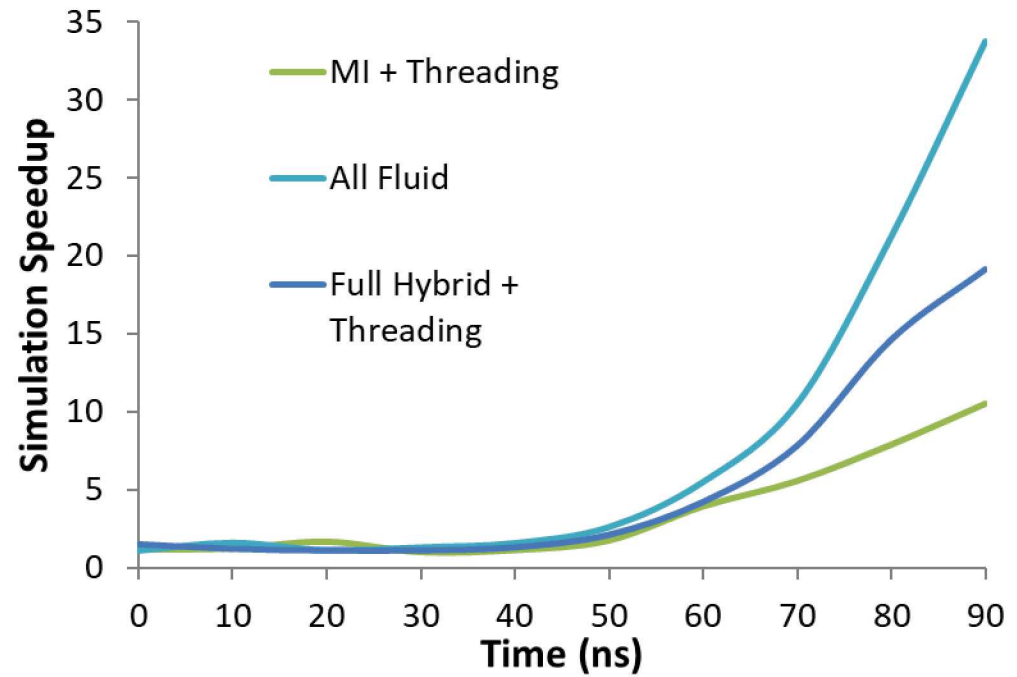
Current loss is difference of Stack and Load currents.



- Fluid simulation has no particle heating of electrodes, similar current sheath or “flow” losses, less current loss later (more turbulent transport).
- Investigating differences at 80 ns.

Speed up realized from new implicit MI algorithm, hybrid and threading.

	Total	Average	Peak
Run	Time (h)	Speedup	Speedup
DI 1 thread	314	1	1
MI 2 thread	63.49	5.0	10.5
All Fluid 1 thread	37.49	8.4	33.7
Full Hybrid 2 thread	49.03	6.4	19.1



Speedup with respect to DI 1 thread simulation.

1. MI algorithm yields 3x, threading 3x acceleration
2. Fluid/hybrid takes advantage of 9x larger time steps ($\omega_c \Delta t < 9$). Performs better at high currents.
3. Fluid speedup 34x, hybrid speedup 19x.

Summary

- Chicago and LSP have been upgraded with new implicit MI algorithm. Chicago has also been upgraded with new opnMP threading for more efficient computation. which has yield a 3—9x speed up from increases time step.
- A 3D representative Z convolute has been constructed and run with new configurations to gauge code performance and accuracy.
- Maximum speed up at high current:
 - New implicit technique is 3—9x.
 - Fully kinetic with threading is speed up 10x.
 - Hybrid technique with threading is 19x.
- We will integrating Tiling, OpenMP and GPU threads with goal of 100x speed up within 1 year.