Numerical solution of a coupled PDE and ODE system in nonlinear aeroelasticity

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Outline

I. Introduction

- Motivation

II. Nonlinear Equations of Motion

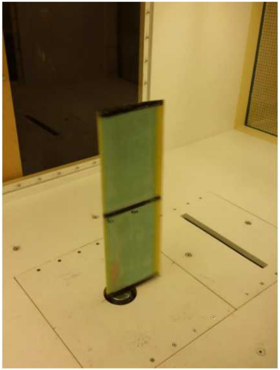
- Structural Model
- Aerodynamic Model

III. Conclusion

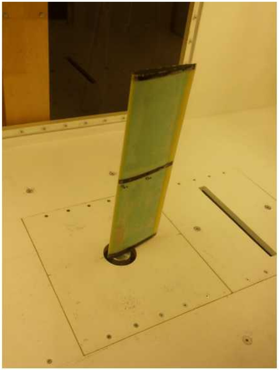
Introduction

Motivation

Limit Cycle Oscillations (LCO) were observed during wind tunnel experiments performed at the Royal Military College of Canada.



Large Amplitude Oscillations

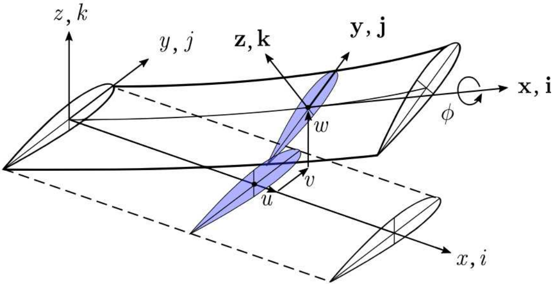


Small Amplitude Oscillations

Nonlinear Equations of Motion

Degrees of Freedom

- $u(x,t)$ Axial displacement
- $v(x,t)$ Edgewise bending (in-plane)
- $w(x,t)$ Flapwise bending (out-of-plane)
- $\phi(x,t)$ Angle of twist about elastic axis
- $\theta(t)$ Base rotation about pitch axis (not pictured)

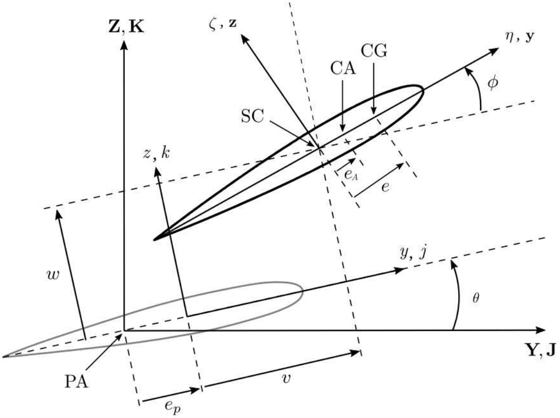


Nonlinear Equations of Motion

Degrees of Freedom

I Flexible wing rotates rigidly, about the fixed pitch axis (PA)

II Alignment of the bending axes relative to the flow direction changes with the pitch angle, θ



Nonlinear Equations of Motion

Derivation of the Equations of Motion

- Two approaches were used to independently derive the equations of motion using Hamilton's Principle, Eq.(1).
  - i The first derivation of the equations of motion followed the procedure outlined in *Hodges and Dowell [1974]* [1]. This approach relied on geometric definitions of the system and many mathematical manipulations.
  - ii The second relied instead on a fundamental understanding of the system's mechanics

$$0 = \int_{t_1}^{t_2} \{\delta(U - T) - \delta W\} dt \tag{1}$$

[1] Hodges, D.H., and Dowell, E.H.. "Nonlinear equations of motion for the elastic bending and torsion of twisted nonuniform rotor blades." (1974).

Nonlinear Equations of Motion

System of Nonlinear Coupled Differential Equations

$$\Big\{EI_{\mathbf{z}}\big(v''''\big) + \big(EI_{\mathbf{z}} - EI_{\mathbf{y}}\big)\big(w''''\phi + 2w'''\phi' + w''\phi''\big) - EAe_A{}^2\big(v'''' + w''''\phi - 2w'''\phi' + w''\phi''\big)\Big\} \tag{2a}$$

$$+ \Big\{\bar{m}\ddot{v} - \bar{m}\ddot{\theta}(e\phi + w) - \bar{m}\dot{\theta}^2((v + e_p) + e) - 2\bar{m}\dot{\theta}(e\dot{\phi} + \dot{w})\Big\} = \bar{W}_v\,,$$

$$\Big\{EI_{\mathbf{y}}\big(w''''\big) + \big(EI_{\mathbf{z}} - EI_{\mathbf{y}}\big)\big(v''''\phi + 2v'''\phi' + v''\phi''\big) + EAe_A{}^2\big(v''''\phi + 2v'''\phi' + v''\phi''\big) + EC_1{}^*\big(\phi''''\big)\Big\} \tag{2b}$$

$$+ \Big\{\bar{m}\ddot{w} + \bar{m}e\ddot{\phi} + \bar{m}\ddot{\theta}((v + e_p) + e) - \bar{m}\dot{\theta}^2(e\phi + w) + 2\bar{m}\dot{\theta}(\dot{v})\Big\} = \bar{W}_w\,,$$

$$\Big\{\big(EI_{\mathbf{z}} - EI_{\mathbf{y}}\big)\big(v''w''\big) - GJ\big(\phi''\big) - EAe_A{}^2\big(v''w''\big) + EC_1{}^*\big(\phi''''\big) + EC_2{}^*\big(w''''\big)\Big\} \tag{2c}$$

$$+ \Big\{\bar{I}_0(\ddot{\theta} + \ddot{\phi}) + \bar{m}e\ddot{w} + \bar{m}e\ddot{\theta}(v + e_p) - \bar{m}e\dot{\theta}^2(w) + 2\bar{m}e\dot{\theta}(\dot{v})\Big\} = \bar{W}_\phi\,,$$

$$I_\theta\,\ddot{\theta} + D_\theta\,\dot{\theta} + K_\theta\,\theta + \int_0^S \Big\{\bar{I}_0(\ddot{\theta} + \ddot{\phi}) - \bar{m}\ddot{v}(e\phi + w) + \bar{m}\ddot{w}((v + e_p) + e) + \bar{m}e\ddot{\phi}(v + e_p) \tag{2d}$$

$$+\bar{m}\ddot{\theta}w^2 + 2\bar{m}\dot{\theta}w\dot{w} + \bar{m}\ddot{\theta}(v + e_p)^2 + 2\bar{m}\dot{\theta}(v + e_p)\dot{v} + 2\bar{m}e\ddot{\theta}w\phi + 2\bar{m}e\dot{\theta}\dot{w}\phi + 2\bar{m}e\dot{\theta}w\dot{\phi} + 2\bar{m}e\ddot{\theta}(v + e_p) + 2\bar{m}e\dot{\theta}\dot{v}\Big\}$$

Nonlinear Equations of Motion

System of Nonlinear Coupled Differential Equations

I The inertial nonlinearities seen in this system are atypical in structural dynamics problems

II Must solve the system of equations in their implicit form; no convenient way to obtain a state-space model

III In order to solve the system of equations numerically, two discretization schemes were adopted:

  i Galerkin method

  ii Finite difference method

Figure: Basis functions (natural vibration modes) used in the Galerkin method

Free Vibration Results

Sample results free vibration results

- Flapwise bending and pitch results for no airflow, comparing Galerkin projection and finite difference solutions

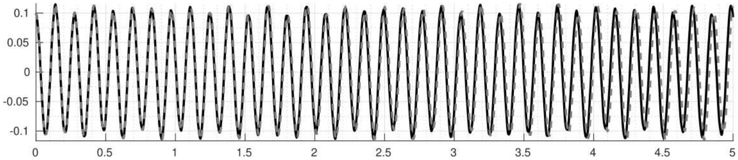


Figure: Flapwise bending displacement at the tip (normalized by the span) vs time (s)

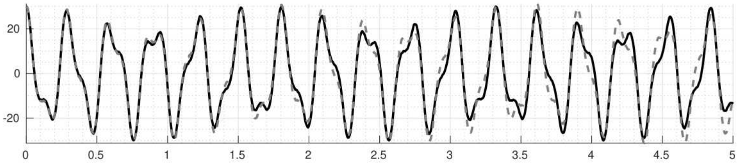


Figure: Base pitch rotation (deg) vs time (s)

Free Vibration Results

Sample results free vibration results

- Edgewise bending and flexible twist angle results for no airflow, comparing Galerkin projection and finite difference solutions

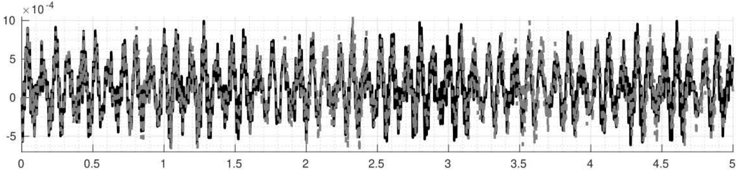


Figure: Edgewise bending displacement at the tip (normalized by the span) vs time (s)

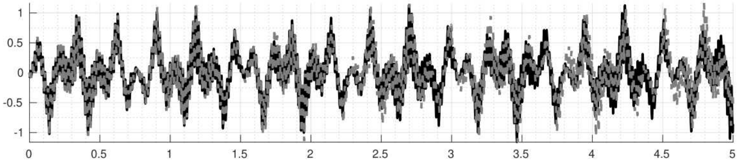


Figure: Twist angle at the tip (deg) vs time (s)

Alternate Form

I The Galerkin method produced consistent results regardless of the form that was used

II The finite difference scheme only matched when the form of Eq.(5) was used

$$I_\theta \ddot{\theta} + D_\theta \dot{\theta} + K_\theta \theta + \int_0^S \Big\{ \bar{I}_0 (\ddot{\theta} + \ddot{\phi}) - \bar{m}\ddot{v}(e\phi + w) + \bar{m}\ddot{w}((v + e_p) + e) + \bar{m}e\ddot{\phi}(v + e_p) \tag{3}$$

$$+ \bar{m}\ddot{\theta}w^2 + 2\bar{m}\dot{\theta}w\dot{w} + \bar{m}\ddot{\theta}(v + e_p)^2 + 2\bar{m}\dot{\theta}(v + e_p)\dot{v} + 2\bar{m}e\ddot{\theta}w\phi + 2\bar{m}e\dot{\theta}\dot{w}\phi + 2\bar{m}e\dot{\theta}w\dot{\phi} + 2\bar{m}e\ddot{\theta}(v + e_p) + 2\bar{m}e\dot{\theta}\dot{v} \Big\} dx = \bar{W}_\theta \tag{4}$$

$$I_\theta \ddot{\theta} + D_\theta \dot{\theta} + K_\theta \theta + \int_0^S \Big\{ \bar{I}_0 (\ddot{\theta} + \ddot{\phi}) - \bar{m}\ddot{v}(e\phi + w) + \bar{m}\ddot{w}((v + e_p) + e) + \bar{m}e\ddot{\phi}(v + e_p)$$

$$+ \frac{\partial}{\partial t}\Big(\bar{m}\dot{\theta}w^2 + \bar{m}\dot{\theta}(v + e_p)^2 + 2\bar{m}e\dot{\theta}w\phi + 2\bar{m}e\dot{\theta}(v + e_p) \Big) \Big\} dx = \bar{W}_\theta \,.$$

Free Vibration Results

Sample results free vibration results

- Bending and pitch results for no airflow, comparing Galerkin projection and finite difference solutions

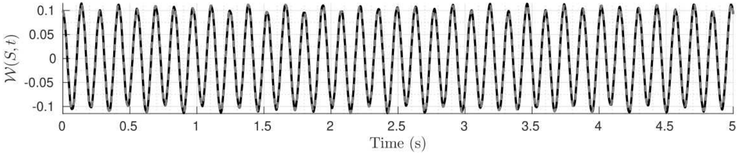


Figure: Flapwise bending displacement (normalized by the span) vs time (s)

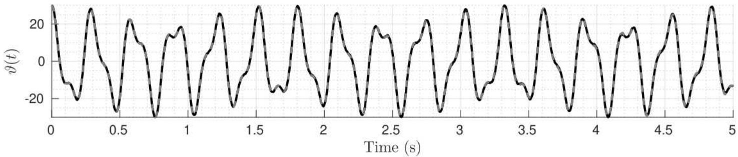


Figure: Base pitch rotation (deg) vs time (s)

Free Vibration Results

Sample results free vibration results

- Bending and pitch results for no airflow, comparing Galerkin projection and finite difference solutions

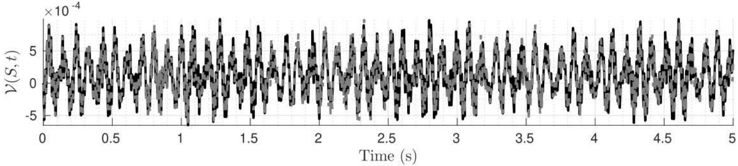


Figure: Edgewise bending displacement (normalized by the span) vs time (s)

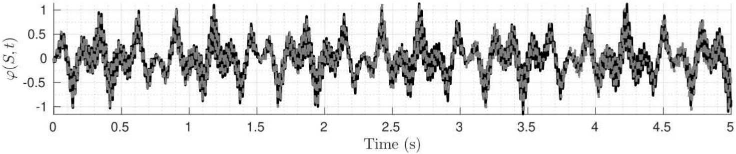


Figure: Twist angle at the tip (deg) vs time (s)

Aerodynamic Model

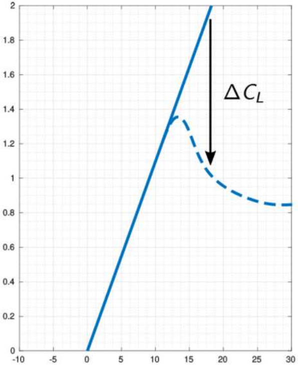


Figure: Illustration of the static lift curve (C_L vs α)

Aerodynamic Forces

I The drag force, lift force and moment are defined as:

$$D = \frac{1}{2}\rho U^2 cs C_D \quad (5)$$

$$L = \frac{1}{2}\rho U^2 cs C_L \quad (6)$$

$$M = \frac{1}{2}\rho U^2 c^2 s C_M \quad (7)$$

II Where the drag, lift, and moment coefficients (C_D, C_L, and C_M) can be defined according to a number of aerodynamic theories

Post-Flutter LCO Results

Sample results for linear unsteady model

- Bending and pitch results for an airspeed (U_∞) 10% above the flutter speed

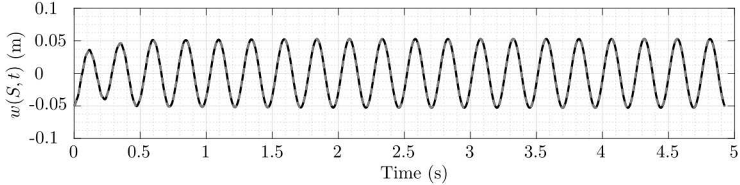


Figure: Bending Displacement (m) vs Time (s)

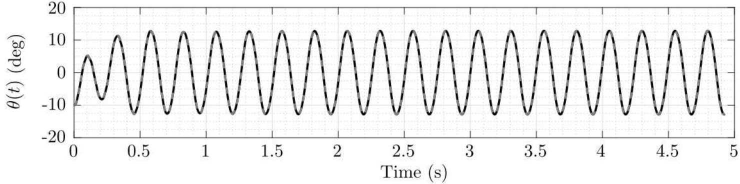


Figure: Pitch Rotation (deg) vs Time (s)

Future Work

I Perform a more systematic analysis of the effects of the various nonlinear terms in the structural equations of motion

II Explore nonlinear aerodynamic models and assess their suitability for the current problem

III Extend the work done by our group in statistical inference and model selection for rigid airfoils to the case of a flexible cantilever wing

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III Department of National Defence