Scalable Iterative Solvers for Stochast SAND2019-12254C
Equations

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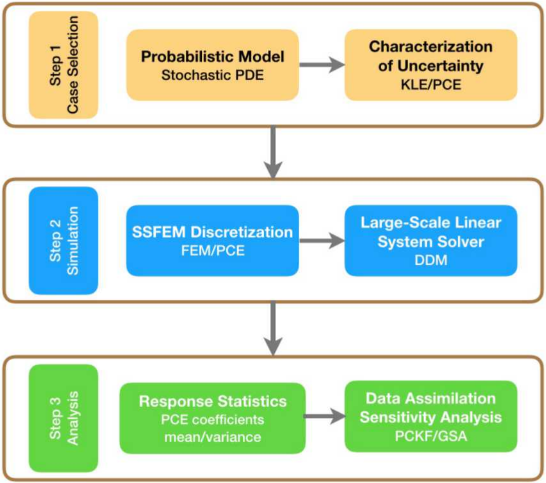
Introduction

- Motivation
  - Uncertainty quantification for high resolution numerical models.
    - *fine mesh resolution*
    - *many random parameters/variables*
- Objective
  - Develop scalable (numerical and parallel) algorithms to quantify uncertainty in large-scale computational models.
- Methodology
  - Exploit non-overlapping domain decomposition methods in conjunction with an intrusive polynomial chaos approach.

Uncertainty Quantification Framework



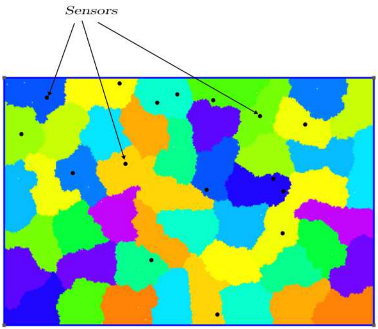
Bayesian Estimation using Nonlinear Filtering

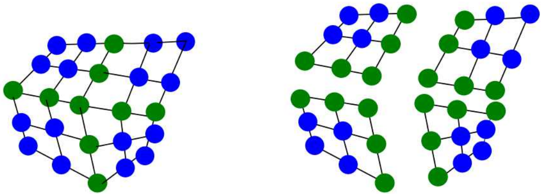
- Model Equation

$$\mathbf{u}_{k+1} = \psi_k \left(\mathbf{u}_k, \mathbf{f}_k, \mathbf{q}_k \right) \quad -- \quad \text{Forecast Step}$$

- Measurement Equation

$$\mathbf{d}_k = \mathbf{h}_k \left(\mathbf{u}_k, \epsilon_k \right) \qquad -- \quad \text{Assimilation Step}$$





- Spatial decomposition

$$\begin{bmatrix} \mathbf{A}^s_{II}(\theta) & \mathbf{A}^s_{I\Gamma}(\theta) \\ \mathbf{A}^s_{\Gamma I}(\theta) & \mathbf{A}^s_{\Gamma\Gamma}(\theta) \end{bmatrix} \left\{ \begin{array}{c} \mathbf{u}^s_I(\theta) \\ \mathbf{u}^s_\Gamma(\theta) \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{f}^s_I \\ \mathbf{f}^s_\Gamma \end{array} \right\}.$$

- Polynomial Chaos expansion

$$\sum_{i=0}^{L} \Psi_i \begin{bmatrix} \mathbf{A}^s_{II,i} & \mathbf{A}^s_{I\Gamma,i} \\ \mathbf{A}^s_{\Gamma I,i} & \mathbf{A}^s_{\Gamma\Gamma,i} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{u}^s_I(\theta) \\ \mathbf{u}^s_\Gamma(\theta) \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{f}^s_I \\ \mathbf{f}^s_\Gamma \end{array} \right\}.$$

Domain Decomposition Method for Stochastic PDEs

$$\langle \sum_{i=0}^{L} \Psi_i(\theta) \begin{bmatrix} \mathbf{A}_{II,i}^1 & \dots & 0 & \mathbf{A}_{I\Gamma,i}^1 \mathbf{R}_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \mathbf{A}_{II,i}^{n_s} & \mathbf{A}_{I\Gamma,i}^{n_s} \mathbf{R}_{n_s} \\ \mathbf{R}_1^T \mathbf{A}_{\Gamma I,i}^1 & \dots & \mathbf{R}_{n_s}^T \mathbf{A}_{\Gamma I,i}^{n_s} & \sum_{s=1}^{n_s} \mathbf{R}_s^T \mathbf{A}_{\Gamma\Gamma,i}^s \mathbf{R}_s \end{bmatrix} \sum_{j=0}^{N} \Psi_j(\theta) \begin{Bmatrix} \mathbf{u}_{I,j}^1 \\ \vdots \\ \mathbf{u}_{I,j}^{n_s} \\ \mathbf{u}_{\Gamma,j} \end{Bmatrix} \Psi_k(\theta) \rangle$$

$$= \langle \begin{Bmatrix} \mathbf{f}_I^1 \\ \vdots \\ \mathbf{f}_I^{n_s} \\ \sum_{s=1}^{n_s} \mathbf{R}_s^T \mathbf{f}_\Gamma^s \end{Bmatrix} \Psi_k(\theta) \rangle, \quad k = 0, \dots, N.$$

Domain Decomposition Method for Stochastic PDEs

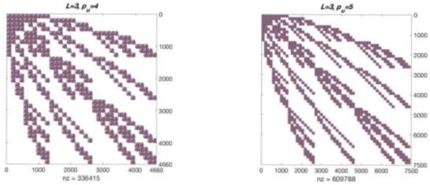
$$\begin{bmatrix} \mathcal{A}_{II}^{1} & \dots & 0 & \mathcal{A}_{I\Gamma}^{1}\mathcal{R}_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \mathcal{A}_{II}^{n_s} & \mathcal{A}_{I\Gamma}^{n_s}\mathcal{R}_{n_s} \\ \mathcal{R}_1^T\mathcal{A}_{\Gamma I}^{1} & \dots & \mathcal{R}_{n_s}^T\mathcal{A}_{\Gamma I}^{n_s} & \sum_{s=1}^{n_s}\mathcal{R}_s^T\mathcal{A}_{\Gamma\Gamma}^{s}\mathcal{R}_s \end{bmatrix} \begin{Bmatrix} \mathcal{U}_I^1 \\ \vdots \\ \mathcal{U}_I^{n_s} \\ \mathcal{U}_\Gamma \end{Bmatrix} = \begin{Bmatrix} \mathcal{F}_I^1 \\ \vdots \\ \mathcal{F}_I^{n_s} \\ \sum_{s=1}^{n_s}\mathcal{R}_s^T\mathcal{F}_\Gamma^s \end{Bmatrix}, \qquad (1)$$

$$[\mathcal{A}_{\alpha\beta}^{s}]_{jk} = \sum_{i=0}^{L}\langle\Psi_i\Psi_j\Psi_k\rangle\mathbf{A}_{\alpha\beta,i}^{s} \quad ; \quad \mathcal{F}_{\alpha,k}^{s} = \langle\Psi_k\mathbf{f}_\alpha^s\rangle, \qquad (2)$$

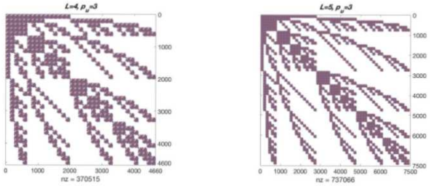
$$\mathcal{U}_I^m = (\mathbf{u}_{I,0}^m, \dots, \mathbf{u}_{I,N}^m)^T \quad ; \quad \mathcal{U}_\Gamma = (\mathbf{u}_{\Gamma,0}, \dots, \mathbf{u}_{\Gamma,N})^T.$$

Sarkar, A. Benabbou, N. and Ghanem, R., IJNME, 2009.

Block Sparsity Structure



$L = 3$ and $p_u = 4, 5$.



$p_u = 3$ and $L = 4, 5$.

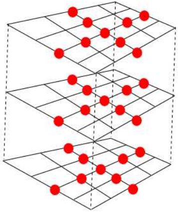
Extended Interface Problem

- The Extended Schur Complement System

$$\mathcal{S}\mathcal{U}_\Gamma = \mathcal{G}_\Gamma.$$

$$\mathcal{S} = \sum_{s=1}^{n_s} \mathcal{R}_s^T [\mathcal{A}^s_{\Gamma\Gamma} - \mathcal{A}^s_{\Gamma I}(\mathcal{A}^s_{II})^{-1}\mathcal{A}^s_{I\Gamma}]\mathcal{R}_s.$$

  - Develop parallel iterative algorithms.
  - Formulate scalable preconditioners.
  - Application to 2D and 3D Stochastic PDEs with non-Gaussian coefficients.



$$\mathcal{M}^{-1} = \sum_{s=1}^{n_s} {\mathcal{H}_f^s}^T [\mathcal{S}_f^s]^{-1} \mathcal{H}_f^s + \mathcal{H}_0^T [\mathcal{S}_c]^{-1} \mathcal{H}_0,$$

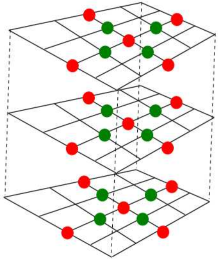
- Condition Number Bound of Deterministic System
  - One-level preconditioner

$$\kappa(M^{-1}S) \leq C \frac{1}{H^2} (1 + \log \frac{H}{h})^2$$

  - Two-level preconditioner

$$\kappa(M^{-1}S) \leq C (1 + \log \frac{H}{h})^2$$

- Partitioning the interface nodes into remaining (■) and corner(●) nodes



$$\mathcal{U}_\Gamma^s = \left\{ \begin{array}{c} \mathcal{U}_r^s \\ \mathcal{U}_c^s \end{array} \right\}$$

Probabilistic Balancing Domain Decomposition with Constraints

$$\begin{bmatrix} \mathcal{A}^s_{ii} & \mathcal{A}^s_{ir} & \mathcal{A}^s_{ic}\mathcal{B}^s_c \\ \mathcal{A}^s_{ri} & \mathcal{A}^s_{rr} & \mathcal{A}^s_{rc}\mathcal{B}^s_c \\ \sum_{s=1}^{n_s} {\mathcal{B}^s_c}^T\mathcal{A}^s_{ci} & \sum_{s=1}^{n_s} {\mathcal{B}^s_c}^T\mathcal{A}^s_{cr} & \sum_{s=1}^{n_s} {\mathcal{B}^s_c}^T\mathcal{A}^s_{cc}\mathcal{B}^s_c \end{bmatrix} \begin{Bmatrix} \mathcal{X}^s \\ \mathcal{U}^s_r \\ \mathcal{U}_c \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathcal{F}^s_r \\ \sum_{s=1}^{n_s} {\mathcal{B}^s_c}^T\mathcal{F}^s_c \end{Bmatrix}$$

$$\boxed{\mathcal{M}^{-1}_{NNC} = \sum_{s=1}^{n_s} \mathcal{R}_s^T\mathcal{D}_s{\mathcal{R}^r_s}^T[\mathcal{S}^s_{rr}]^{-1}\mathcal{R}^r_s\mathcal{D}_s\mathcal{R}_s + R_0^T[F_{cc}]^{-1}R_0,}$$

$$R_0 = \sum_{s=1}^{n_s} {\mathcal{B}^s_c}^T(\mathcal{R}^c_s - \mathcal{S}^s_{cr}[\mathcal{S}^s_{rr}]^{-1}\mathcal{R}^r_s)\mathcal{D}_s\mathcal{R}_s.$$

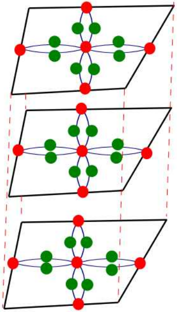
$$F_{cc} = \sum_{s=1}^{n_s} {\mathcal{B}^s_c}^T(\mathcal{S}^s_{cc} - \mathcal{S}^s_{cr}[\mathcal{S}^s_{rr}]^{-1}\mathcal{S}^s_{rc})\mathcal{B}^s_c,$$

$$\mathcal{S}^s_{\alpha\beta} = \mathcal{A}^s_{\alpha\beta} - \mathcal{A}^s_{\alpha i}[\mathcal{A}^s_{ii}]^{-1}\mathcal{A}^s_{i\beta},\ [\mathcal{A}^s_{\alpha\beta}]_{jk} = \sum_{i=0}^{L}\langle\Psi_i\Psi_j\Psi_k\rangle\mathbf{A}^s_{\alpha\beta,i}$$

Probabilistic Dual Primal Domain Decomposition

$$
\begin{bmatrix}
\mathcal{A}_{ii}^{s} & \mathcal{A}_{ir}^{s} & \mathcal{A}_{ic}^{s}\mathcal{B}_{c}^{s} & 0 \\
\mathcal{A}_{ri}^{s} & \mathcal{A}_{rr}^{s} & \mathcal{A}_{rc}^{s}\mathcal{B}_{c}^{s} & \mathcal{B}_{r}^{s\,T} \\
\sum_{s=1}^{n_s} \mathcal{B}_{c}^{s\,T}\mathcal{A}_{ci}^{s} & \sum_{s=1}^{n_s} \mathcal{B}_{c}^{s\,T}\mathcal{A}_{cr}^{s} & \sum_{s=1}^{n_s} \mathcal{B}_{c}^{s\,T}\mathcal{A}_{cc}^{s}\mathcal{B}_{c}^{s} & 0 \\
0 & \sum_{s=1}^{n_s} \mathcal{B}_{r}^{s} & 0 & 0
\end{bmatrix}
\begin{Bmatrix}
\mathcal{U}_{i}^{s} \\ \mathcal{U}_{r}^{s} \\ \mathcal{U}_{c} \\ \Lambda
\end{Bmatrix}
=
\begin{Bmatrix}
\mathcal{F}_{i}^{s} \\ \mathcal{F}_{r}^{s} \\ \sum_{s=1}^{n_s} \mathcal{B}_{c}^{s\,T}\mathcal{F}_{c}^{s} \\ 0
\end{Bmatrix},
$$

$$
(\bar{F}_{rr} + \bar{F}_{rc}[\bar{F}_{cc}]^{-1}\bar{F}_{cr})\Lambda = \bar{d}_{r} - \bar{F}_{rc}[\bar{F}_{cc}]^{-1}\bar{d}_{c},
$$



$$\mathcal{M}^{-1} = \sum_{s=1}^{n_s} \mathcal{H}_f^{s\,T} [\mathcal{S}_f^s]^{-1} \mathcal{H}_f^s + \mathcal{H}_0^T [\mathcal{S}_c]^{-1} \mathcal{H}_0,$$

	$\mathcal{H}_f^s$	$[\mathcal{S}_f^s]^{-1}$	$\mathcal{H}_0$	$[\mathcal{S}_c]^{-1}$
a	$\mathcal{R}_s^r \mathcal{D}_s \mathcal{R}_s$	$[\mathcal{S}_{rr}^s]^{-1}$	$\sum_{s=1}^{n_s} \mathcal{B}_c^{s\,T} (\mathcal{R}_s^c - \mathcal{S}_{cr}^s [\mathcal{S}_{rr}^s]^{-1} \mathcal{R}_s^r) \mathcal{D}_s \mathcal{R}_s$	$\sum_{s=1}^{n_s} \mathcal{B}_c^{s\,T} (\mathcal{S}_{cc}^s - \mathcal{S}_{cr}^s \left[\mathcal{S}_{rr}^s\right]^{-1} \mathcal{S}_{rc}^s) \mathcal{B}_c^s$
b	$\mathcal{D}_r^s \mathcal{B}_r^s$	$\left[\mathcal{S}_{rr}^s\right]^{-1}$	$\sum_{s=1}^{n_s} \mathcal{B}_c^{s\,T} \mathcal{S}_{cr} \left[\mathcal{S}_{rr}^s\right]^{-1} \mathcal{D}_r^s \mathcal{B}_r^s$	$\sum_{s=1}^{n_s} \mathcal{B}_c^{s\,T} (\mathcal{S}_{cc}^s - \mathcal{S}_{cr}^s \left[\mathcal{S}_{rr}^s\right]^{-1} \mathcal{S}_{rc}^s) \mathcal{B}_c^s$
c	$\mathcal{B}_r^{s\,T}$	$[\mathcal{S}_{rr}^s]^{-1}$	$\sum_{s=1}^{n_s} \mathcal{B}_c^{s\,T} \mathcal{S}_{cr}^s [\mathcal{S}_{rr}^s]^{-1} \mathcal{B}_r^{s\,T}$	$\sum_{s=1}^{n_s} \mathcal{B}_c^{s\,T} (\mathcal{S}_{cc}^s - \mathcal{S}_{cr}^s [\mathcal{S}_{rr}^s]^{-1} \mathcal{S}_{rc}^s) \mathcal{B}_c^s$

a) Neumann-Neumann with Coarse grid, b) Primal-Primal,c) Dual-Primal Operator.

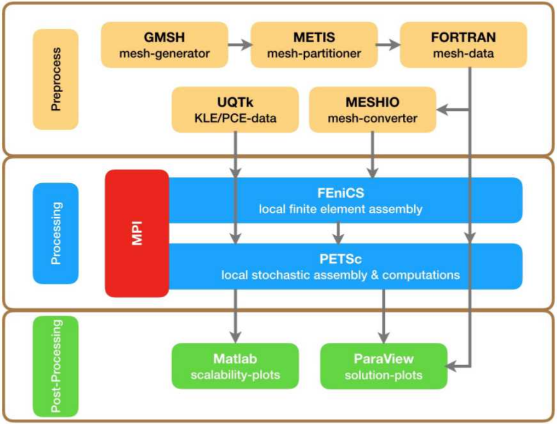
Investigated numerical and parallel scalabilities:

Subber, W. and Sarkar, A., JCP, 2014

Subber, W. and Sarkar, A., CMAME, 2013

Desai, A., Khalil, M., Pettit, C., Poirel, D. and Sarkar, A., CMAME 2017

Implementational Framework



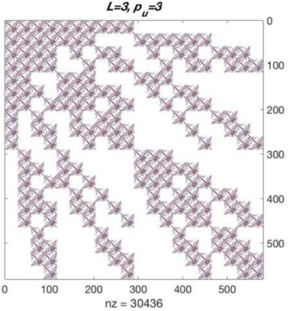
- Model Problem:

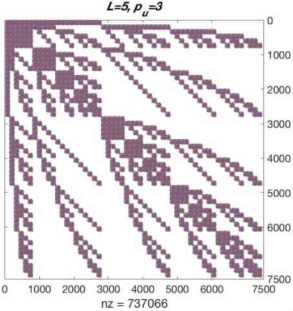
$$\begin{aligned} -\nabla \cdot \left(\; c_d(\mathbf{x},\theta) \;\; \nabla u(\mathbf{x},\theta) \; \right) &= F(\mathbf{x}), & \Omega \times \mathcal{W}, \\ u(\mathbf{x},\theta) &= 0, & \delta\Omega \times \mathcal{W}, \end{aligned}$$

- Diffusion coefficient c_d modelled as a lognormal process with the underlying a Gaussian process having mean μ, variance σ^2 and exponential covariance function C (on a 2D domain).

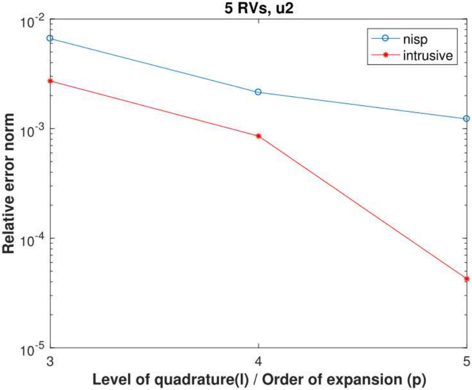
$$C(x_1, y_1; x_2, y_2) = \sigma^2 \; e^{-|x_2 - x_1|/b_1 - |y_2 - y_1|/b_2},$$

Block Sparsity Structure



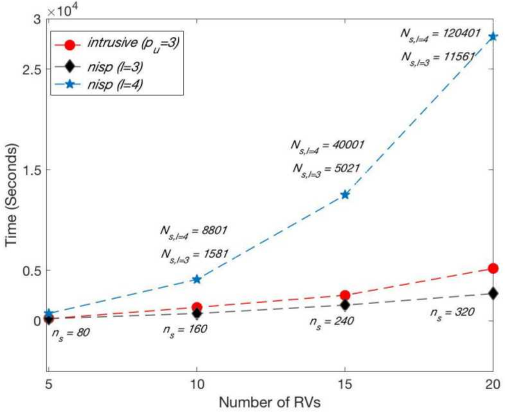


Errors Analysis of PCE Coefficients of Solution Process: Intrusive Vs Non-Intrusive



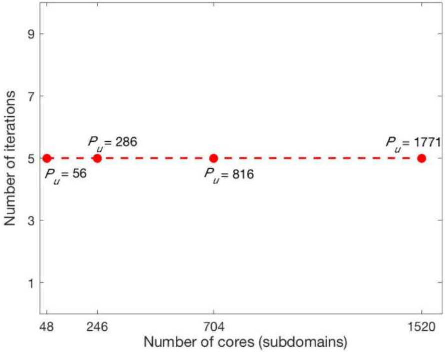
Relative error norm $= \frac{\|u_{iH} - u_{ih}\|}{\|u_{iH}\|}$, 5 random variables, error in $(\hat{\mathbf{u}}_2)$, coarse mesh $(N \approx 150)$

Scalability against Stochastic Dimensions: Intrusive vs Non-Intrusive (Sparse Grid)



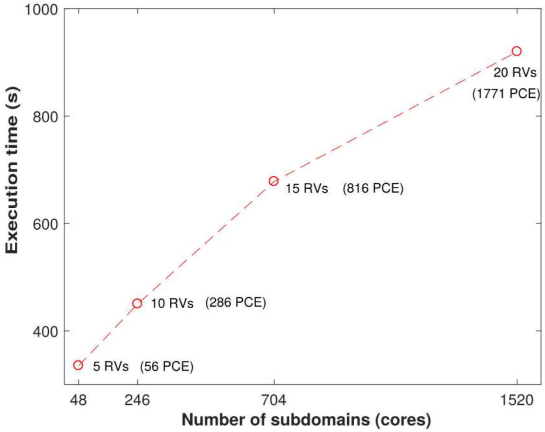
Fixed mesh resolution (52704 nodes and 105410 elements) and third order PCE for intrusive. Smolyak sparse grid with $l = 3$ and $l = 4$ for non-intrusive.

Scalability against Number of Random Variables: NNC/BDDC



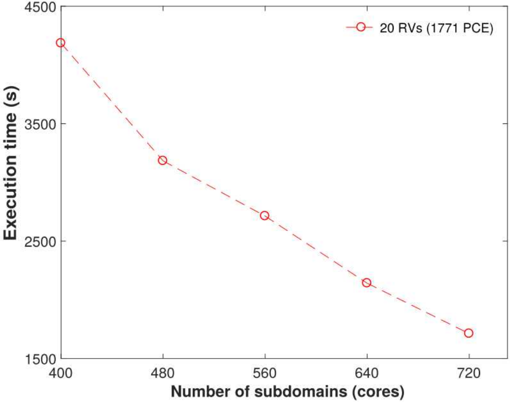
Fixed mesh resolution (52704 nodes and 105410 elements), fixed problem size per subdomain ($\approx$ 60,000) and third order PCE (linear system of order max. $\approx$ 93 million)

Scalability against Number of Random Variables: NNC/BDDC



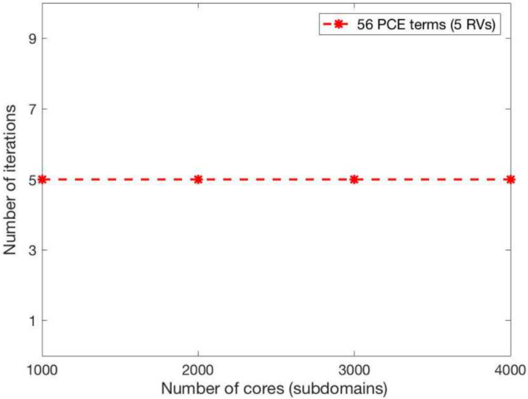
Fixed mesh (52704 nodes and 105410 elements), fixed problem size per subdomain ($\approx$ 60,000) and third order PCE

Parallel Scalability (Strong): NNC/BDDC



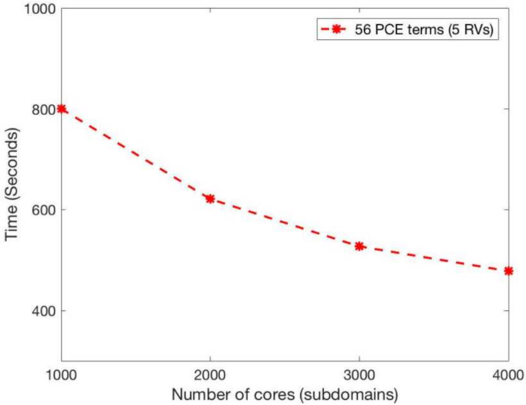
Fixed global problem, mesh with (52704 nodes and 105410 elements) and number of PCE terms $P_u = 1771$.

Scalability using Large-Scale HPC Cluster



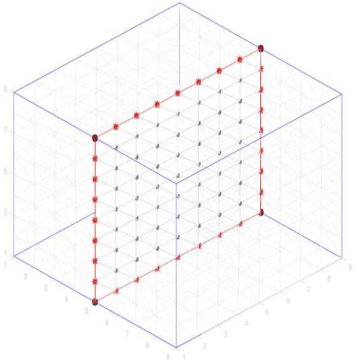
For the fixed mesh resolution (0.332 million nodes and 0.664 million elements.) and fixed number of PCE terms ($P_u = 56$).

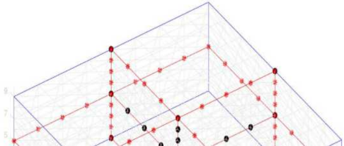
Scalability using Large-Scale HPC Cluster



For the fixed mesh resolution (0.332 million nodes and 0.664 million elements.) and fixed number of PCE terms ($P_u = 56$).

Probabilistic Coarse Grid in Three Dimensions:





Deterministic Setting: Condition Number Bound Vertex vs Wirebasket-based Methods

Ref. Book by Smith, Bjorstad and Gropp, 2004

For the vertex-based method in two dimensions

$$\kappa \leq C(1 + \log(H/h))^2,$$

For the vertex-based method in three dimensions

$$\kappa \leq C(H/h)(1 + \log(H/h)).$$

For the wirebasket-based methods in three dimensions

$$\kappa \leq C(1 + \log(H/h))^2.$$

Probabilistic BDDC/NNC using Extended Wirebasket-based Coarse Grid

$$\mathcal{F}_{WW}\, \mathcal{U}_W = d_W,$$

$$\mathcal{F}_{WW} = \sum_{s=1}^{n_s} {\mathcal{B}_W^s}^{\mathrm{T}} \left(\mathcal{S}_{WW}^s - \mathcal{S}_{WF}^s [\mathcal{S}_{FF}^s]^{-1} \mathcal{S}_{FW}^s \right) \mathcal{B}_W^s,$$

$$d_W = \sum_{s=1}^{n_s} {\mathcal{B}_W^s}^{\mathrm{T}} \left(f_W^s - \mathcal{S}_{WF}^s [\mathcal{S}_{FF}^s]^{-1} f_F^s \right).$$

Modified BDDC/NNC Preconditioner:

$$\mathcal{M}_{NNW}^{-1} = \sum_{s=1}^{n_s} \mathcal{R}_s^{\mathrm{T}} \mathcal{D}_s ({\mathcal{R}_s^F}^{\mathrm{T}} [\mathcal{S}_{FF}^s]^{-1} \mathcal{R}_s^F) \mathcal{D}_s \mathcal{R}_s + \mathcal{R}_0^{\mathrm{T}} [\mathcal{F}_{WW}]^{-1} \mathcal{R}_0.$$

$$\mathcal{R}_0 = \sum_{s=1}^{n_s} {\mathcal{B}_W^s}^{\mathrm{T}} (\mathcal{R}_s^W - \mathcal{S}_{WF}^s [\mathcal{S}_{FF}^s]^{-1} \mathcal{R}_s^F) \mathcal{D}_s \mathcal{R}_s,$$

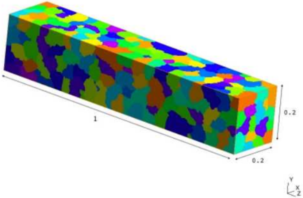
Numerical Experiments: Wirebasket based BDDC/NNC solver

- Diffusion equation

$$
\begin{aligned}
-\nabla \cdot \left(\, c_d(\mathbf{x}, \theta) \;\; \nabla u(\mathbf{x}, \theta) \, \right) &= F(\mathbf{x}), & \Omega \times \mathcal{W}, \\
u(\mathbf{x}, \theta) &= 0, & \delta\Omega \times \mathcal{W},
\end{aligned}
$$

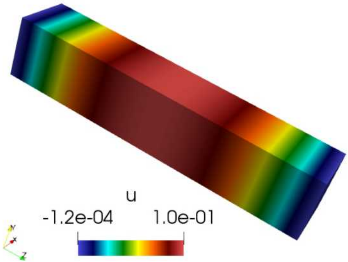
- Diffusion coefficient c_d - lognormal process having underlying a Gaussian process with exponential covariance C

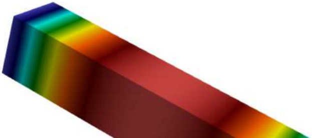
$$
C(x_1, y_1, z_1; x_2, y_2, z_2) = \sigma^2 e^{-|x_2 - x_1|/b_x - |y_2 - y_1|/b_y - |z_2 - z_1|/b_z}.
$$



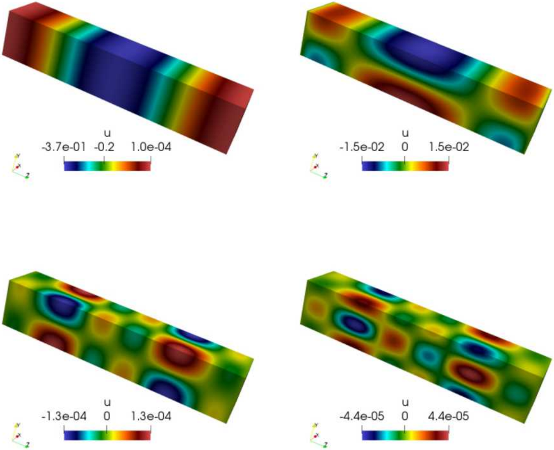
Characteristics of the Solution Process:

Diffusion Equation





Characteristics of the Solution Process: Diffusion Equation



Selected PCE coefficients.

- Linear Elasticity

$$
\begin{aligned}
-\nabla \cdot \sigma\big(\mathcal{U}(\mathbf{x},\theta)\big) &= F(\mathbf{x}) \quad \mathit{in} \quad \mathcal{D},\\
\sigma\big(\mathcal{U}(\mathbf{x},\theta)\big)\cdot \hat{\mathbf{n}} &= b_T \quad \mathit{on} \quad \Gamma_1 = \delta\mathcal{D}\backslash\Gamma_0,\\
\mathcal{U}(\mathbf{x},\theta) &= 0 \quad \mathit{on} \quad \Gamma_0.
\end{aligned}
$$

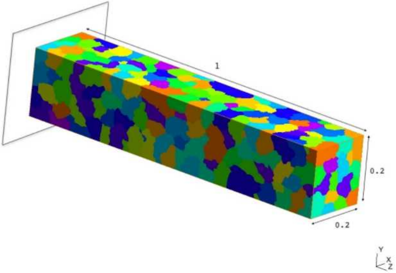
Stress tensor σ:

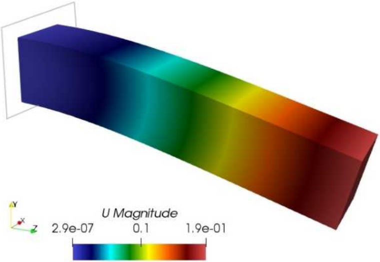
$$
\sigma\big(\mathcal{U}(\mathbf{x},\theta)\big) = \lambda\big(\nabla\cdot\mathcal{U}(\mathbf{x},\theta)\big)I + 2\mu\epsilon\big(\mathcal{U}(\mathbf{x},\theta)\big),
$$

where $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$ are Lamé constants.

- Young's modulus E - lognormal stochastic process (as before).

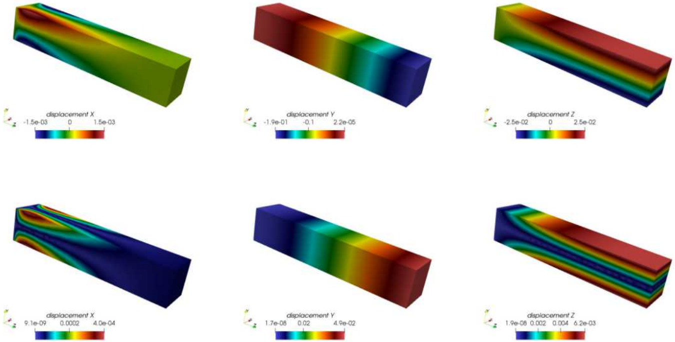
Characteristics of the Solution Process:





Characteristics of the Solution Process:

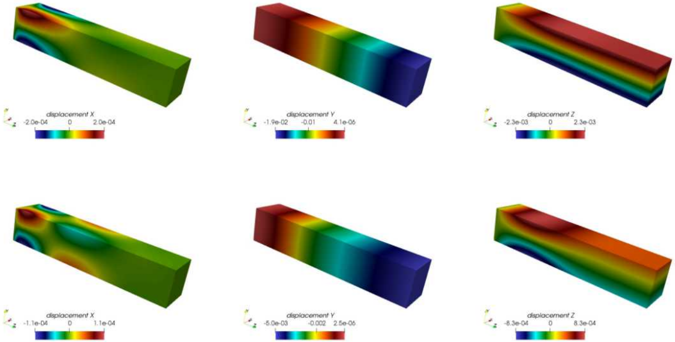
Linear Elasticity



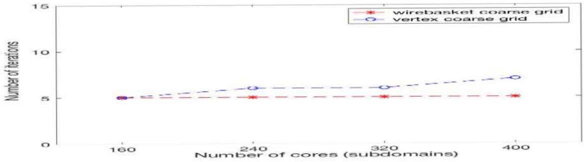
x, y and z components of the mean and standard deviation.

Characteristics of the Solution Process:

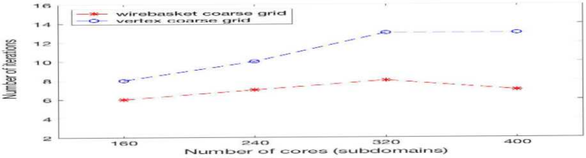
Linear Elasticity



x, y and z components of the selected PCE coefficients.

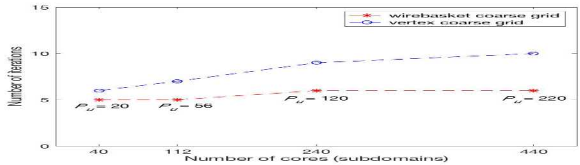


Diffusion

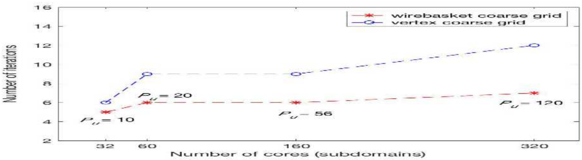


Elasticity

Iteration count versus number of subdomains for the fixed mesh resolution with fixed number of PCE terms.

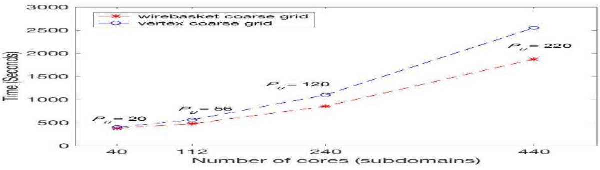


Diffusion

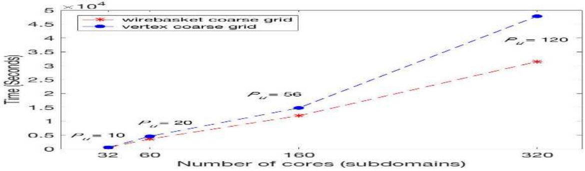


Elasticity

Iteration count versus number of subdomains for fixed problem size per subdomain with increasing number of PCEs (fixed mesh resolution).

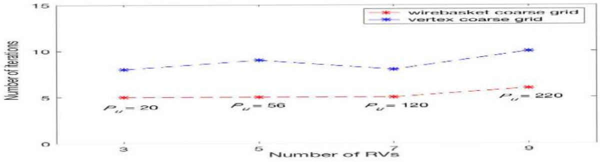


Diffusion

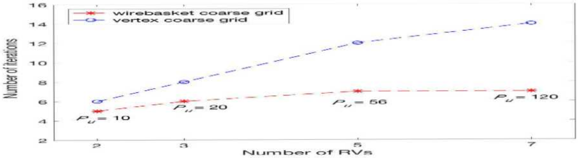


Elasticity

Execution time versus number of subdomains for fixed problem size per subdomain with increasing number of PCEs (fixed mesh resolution).

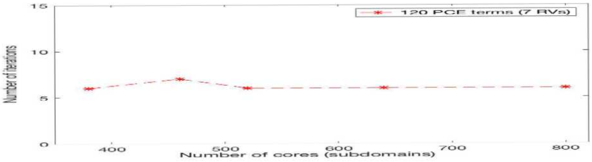


Diffusion

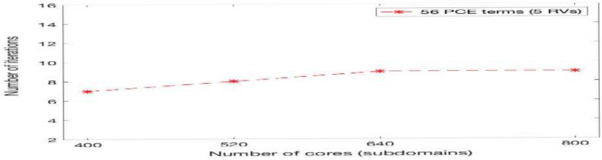


Elasticity

Iteration count versus number of PCE terms for the fixed mesh resolution with fixed number of subdomains.

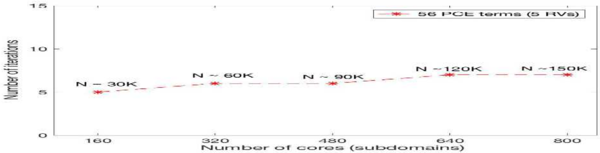


Diffusion

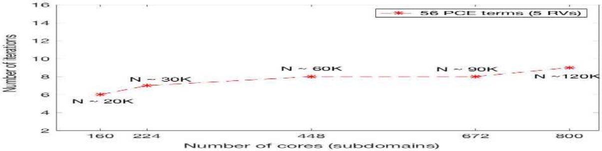


Elasticity

Iteration count versus number of subdomains for the fixed mesh resolution with fixed number of PCE terms.

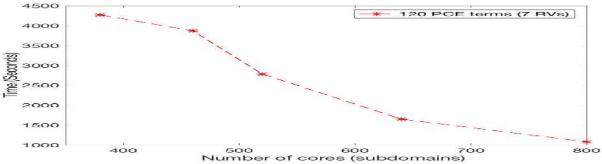


Diffusion

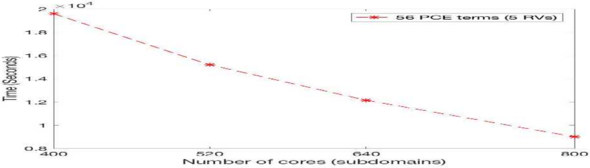


Elasticity

Iteration count versus number of subdomains for the fixed problem size per core with increasing mesh resolution (fixed number of PCE terms).

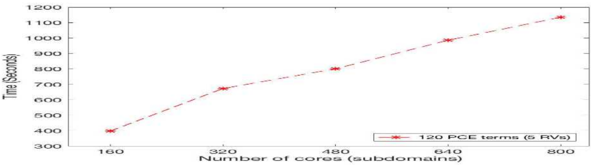


Diffusion

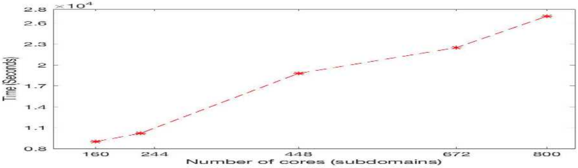


Elasticity

Execution time versus number of subdomains with the fixed mesh resolution and the number of PCE terms.

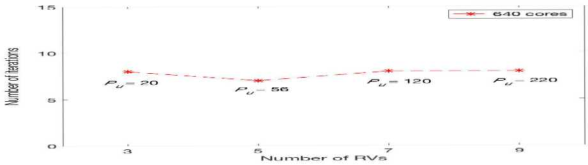


Diffusion

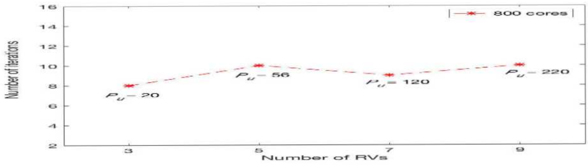


Elasticity

Execution time versus number of subdomains for the fixed problem size per core with increasing mesh resolution (fixed number of PCE terms).

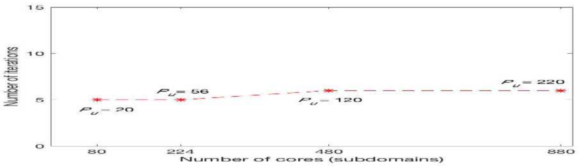


Diffusion

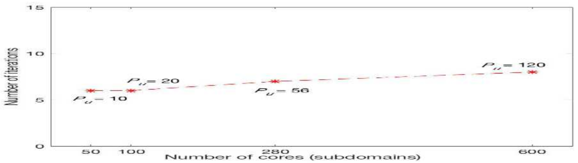


Elasticity

Iteration count versus number of PCE terms for the fixed mesh resolution with fixed number of subdomains.

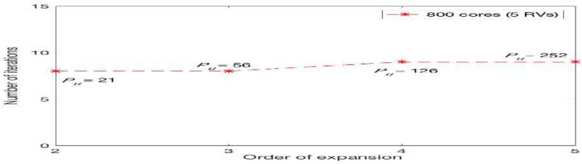


Diffusion

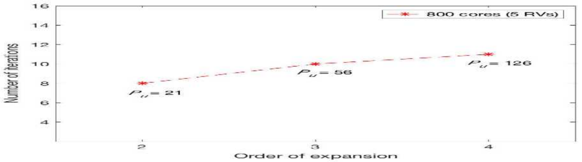


Elasticity

Iteration count versus number of subdomains for the fixed problem size per core with increasing number of PCE terms (fixed mesh resolution).



Diffusion



Elasticity

Iteration count versus order of expansion for the fixed mesh resolution with fixed number of subdomains (fixed number of RVs).

Conclusion

- Adaptation of two-level iterative substructuring techniques for SPDEs in order to handle large number of random variables.
- Development of the wirebasket preconditioner for SPDEs in three dimensions.

Acknowledgment

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    - **Natural Sciences and Engineering Research Council of Canada**
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