

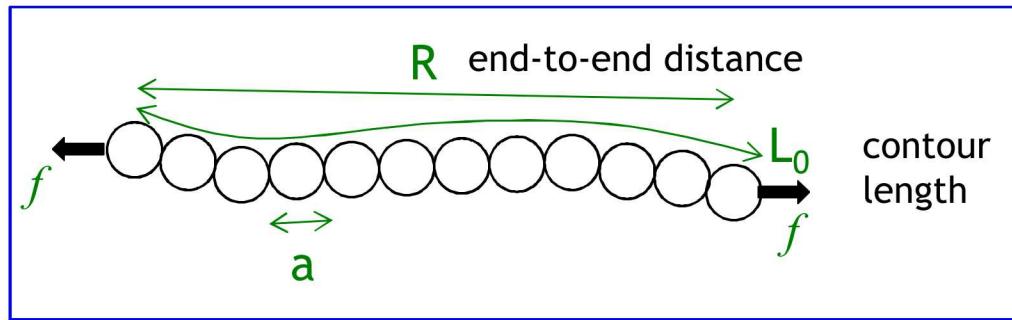


Recent Results in Polyelectrolyte Simulations

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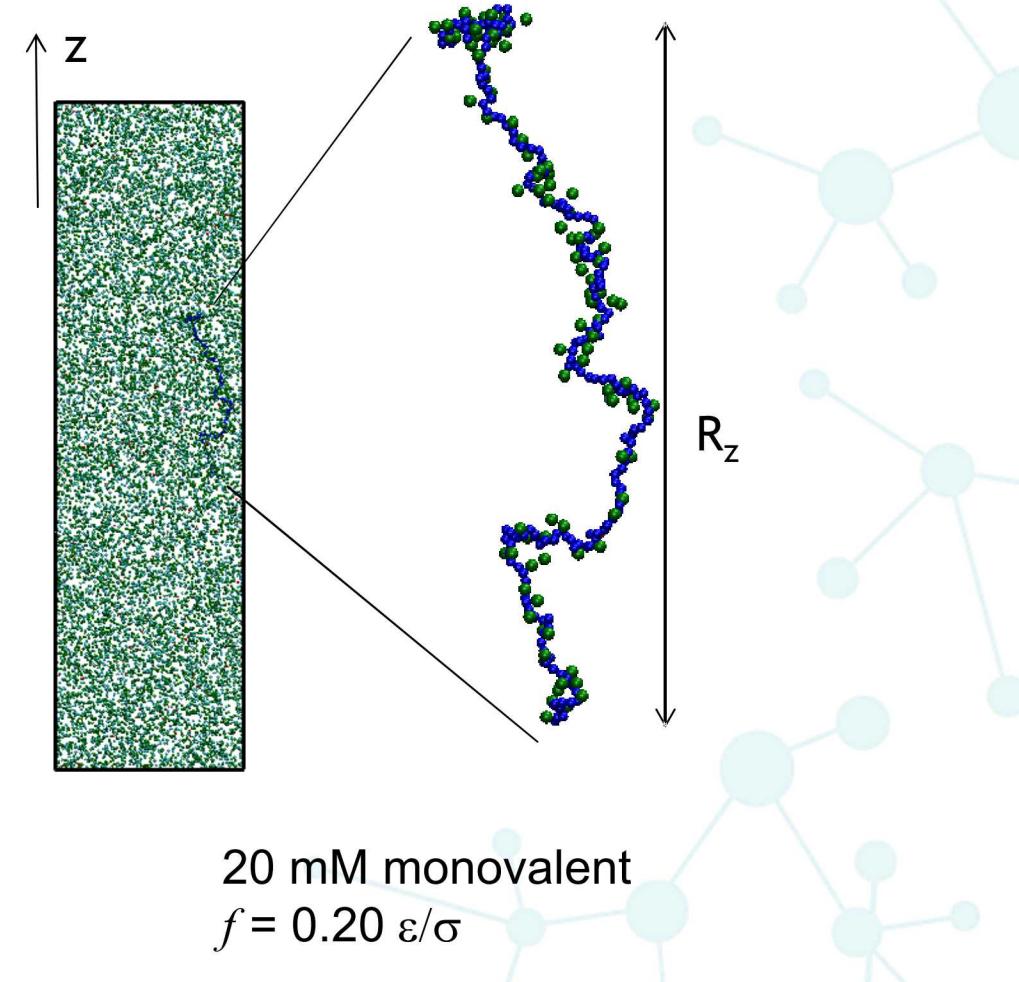
Simple Model for Polyelectrolytes

“All ion” simulations: MD
bead (spring) flexible polyelectrolyte
 $F = \text{bond} + \text{entropy} + \text{electrostatics}$



every bead in polymer is charged
 $a = 6.4 \text{ \AA} = 0.96 \sigma$ (ssDNA spacing)
 $l_B = 1.065 \sigma$ (Bjerrum length) [dielectric continuum]
 $N = 200$
 $L_0 = (N-1) a = 192 \sigma$

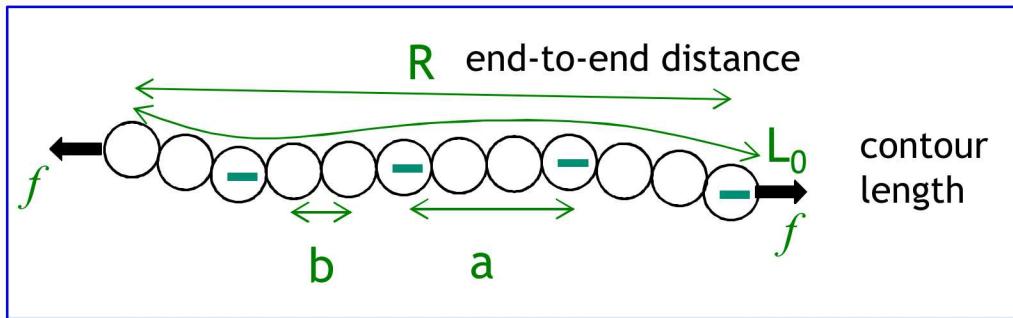
Force-extension simulations



Additional Features

$b = 0.96 \sigma$ bond length

$a = m b$ charge spacing is varied



Intrinsic stiffness:
add angle term to potential

$$U_{\text{angle}} = k_a (1 + \cos \theta) \quad \theta_0 = 180^\circ$$



Comparison of Experimental and Simulation Data (ssDNA)

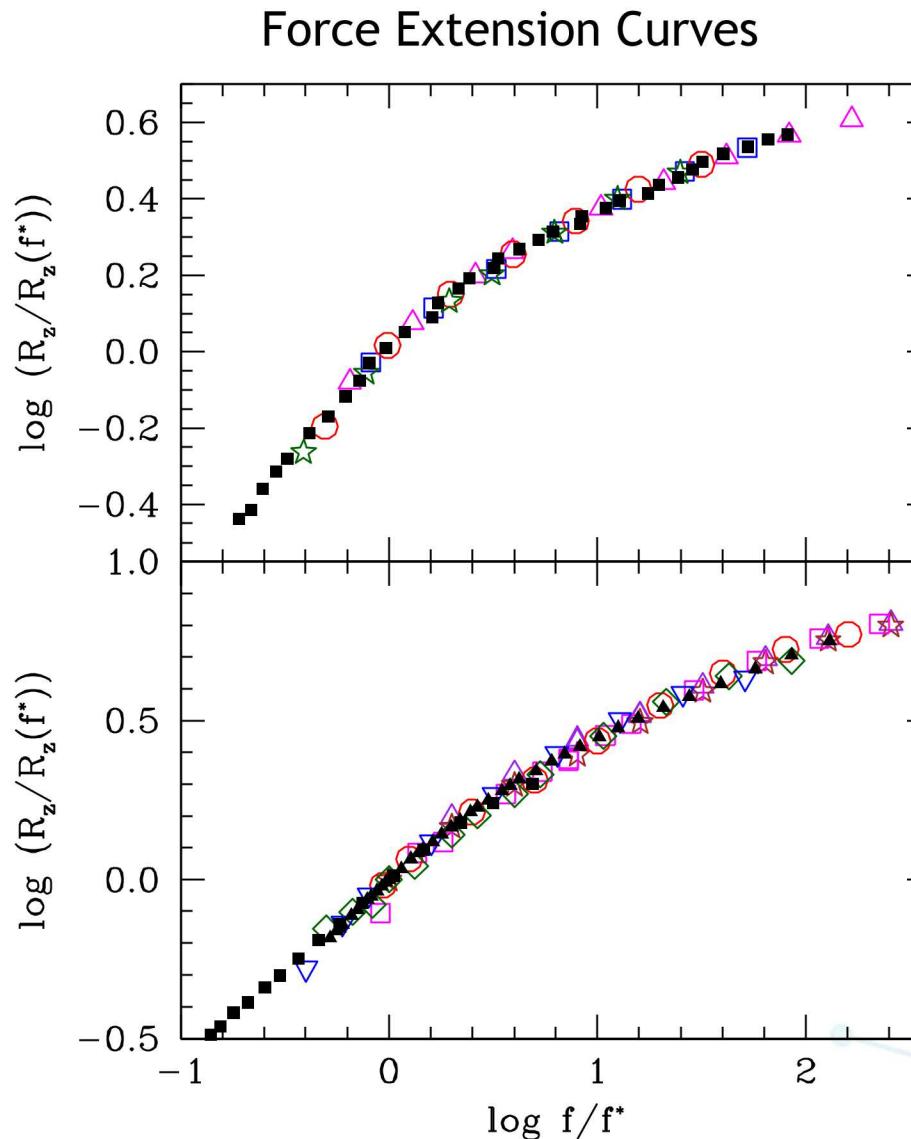


Same scaling procedure as experiments

Simple model is sufficient.

electrostatics, entropy,
connectivity

Dustin McIntosh, Omar Saleh, UCSB



monovalent

20 mM

50 mM

100 mM

200 mM

experimental data

solid points

divalent

0.2 mM

0.5 mM

1.0 mM

2.0 mM

10 mM

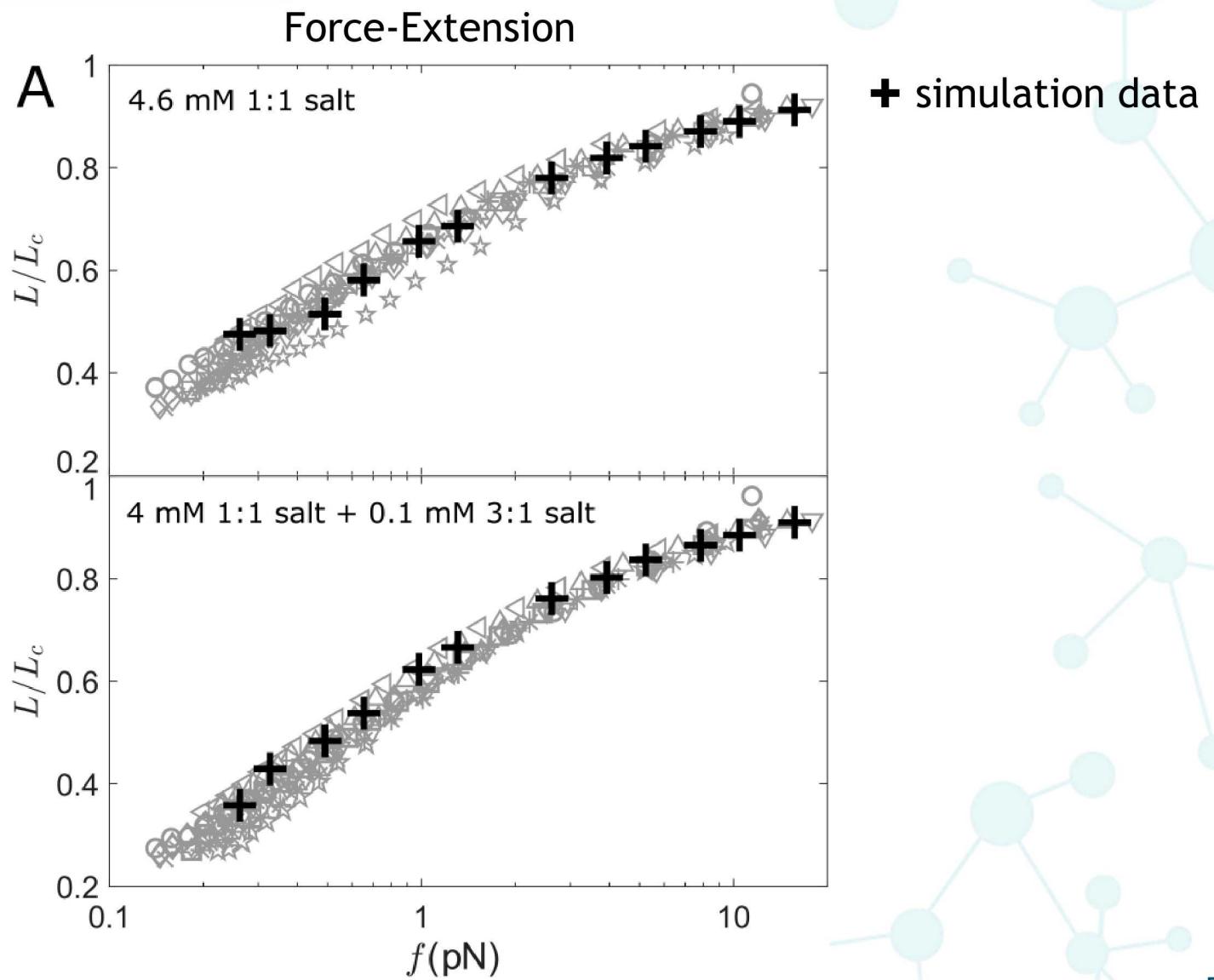
20 mM

Hyaluronic Acid

Persistence length of 5 nm

Overlap of simulation and experimental data

Simple model works for a variety of
polyelectrolytes



Pincus Tension Screening

Tension blobs (Pincus, Macromolecules 1976)

$\xi = k_B T / f$ is tensile screening length

where the excluded volume renormalization is the substitution of the Flory radius⁴ $R_F \simeq N^{\rho} a$ for the ideal chain radius $R_0 = N^{1/2} a$. However, as the chain stretches, its average monomer density decreases leading to weakening of the excluded volume effect. Thus for sufficiently large external forces, we expect to eventually recover ideal behavior with $\bar{Z} \propto N$ rather than $\bar{Z} \propto N^{2\rho}$ as given by (I.2). This crossover will be described in terms of a competition between a “tensile screening length” $\xi_t = (\beta f)^{-1}$ and the Flory radius R_F . For weak stretching $R_F/\xi_t \ll 1$, the de Gennes result (eq I.2) should be correct; for stronger stretching with $R_F/\xi_t \gg 1$, we expect to find a modified elastic behavior with $\bar{Z} \propto N$. To find the stress-strain relationship in this limit, we are tempted to employ a scaling argument. Let us assume that the average end-to-end separation may be written as

that its Laplace transform

$$\Gamma \rho(\mathbf{r}) = \sum_{N=0}^{\infty} \Gamma_N(\mathbf{r}) e^{-N\rho} \quad (\text{II.1})$$

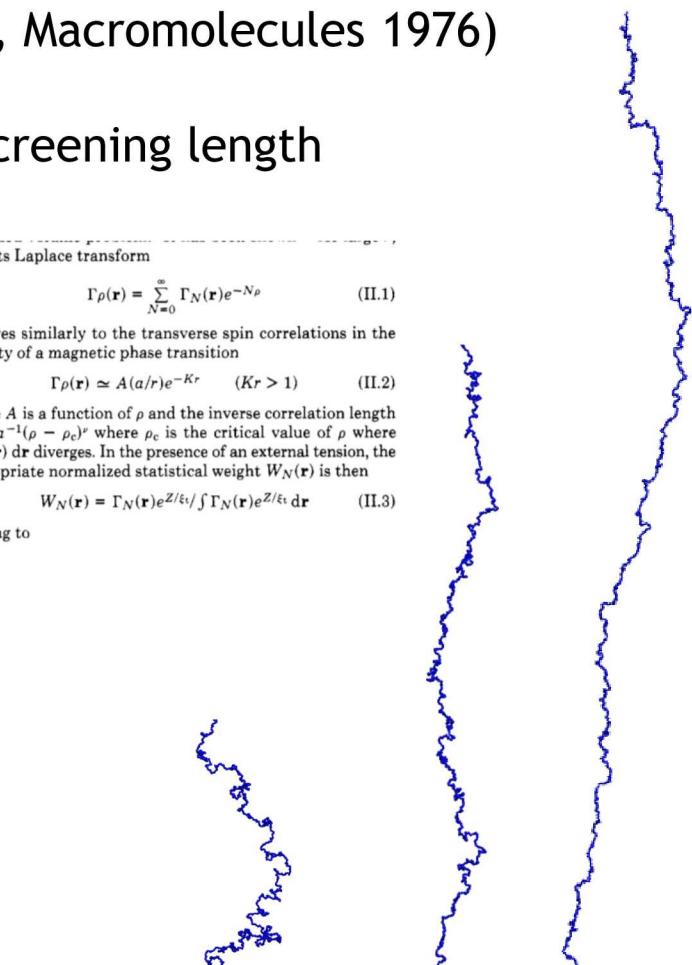
behaves similarly to the transverse spin correlations in the vicinity of a magnetic phase transition

$$\Gamma \rho(\mathbf{r}) \simeq A(a/r)e^{-Kr} \quad (Kr > 1) \quad (\text{II.2})$$

where A is a function of ρ and the inverse correlation length $K = a^{-1}(\rho - \rho_c)^\nu$ where ρ_c is the critical value of ρ where $\int \Gamma \rho(\mathbf{r}) d\mathbf{r}$ diverges. In the presence of an external tension, the appropriate normalized statistical weight $W_N(\mathbf{r})$ is then

$$W_N(\mathbf{r}) = \Gamma_N(\mathbf{r}) e^{Z/\xi_t} / \int \Gamma_N(\mathbf{r}) e^{Z/\xi_t} d\mathbf{r} \quad (\text{II.3})$$

leading to

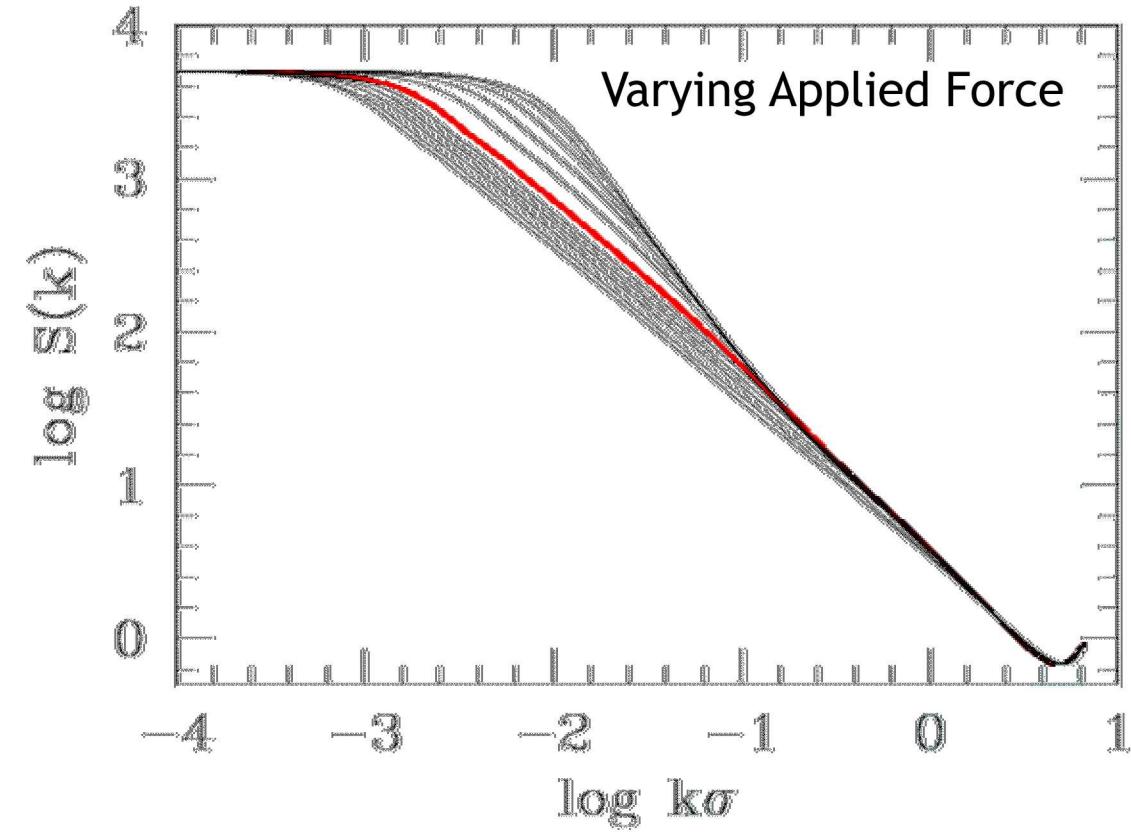


$fL_B/kT = 0.027$

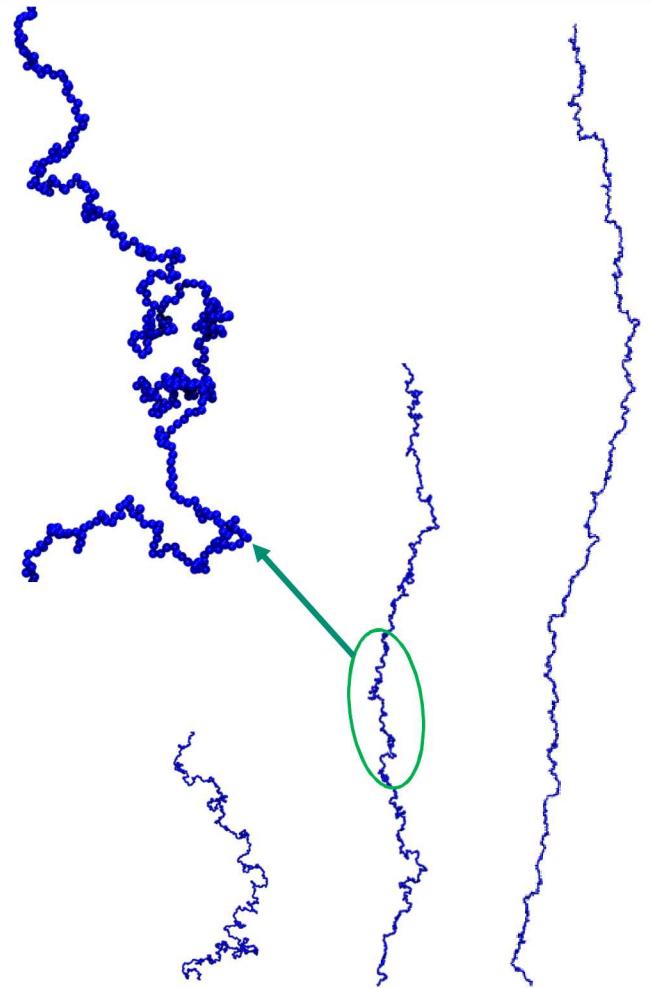
0.112

0.27

$N=5000$ ssDNA
200 mM



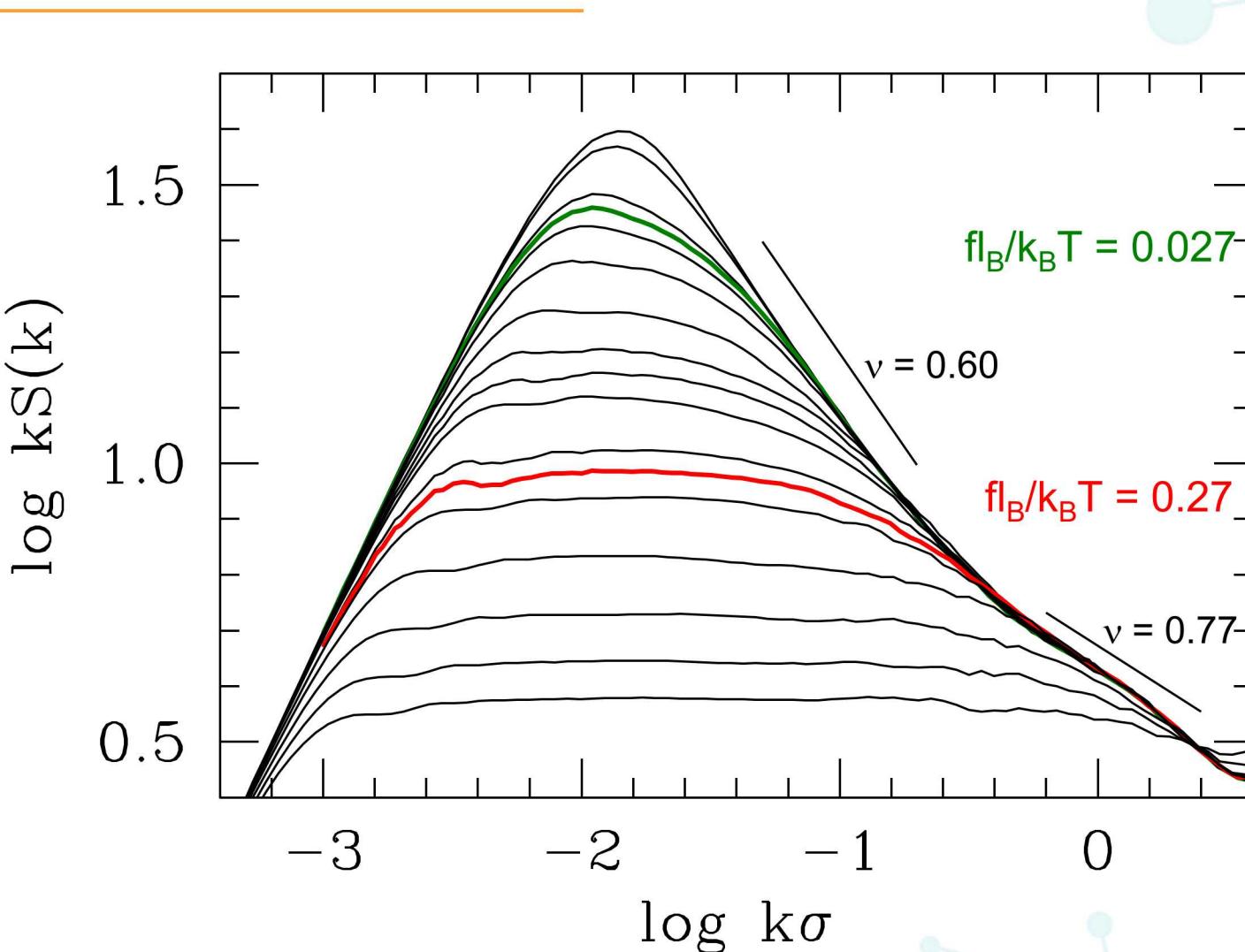
Structure Factor for N=5000 at 200 mM



$f l_B / k_B T = 0.027$

0.112

0.27



Region

Rotational

Pincus

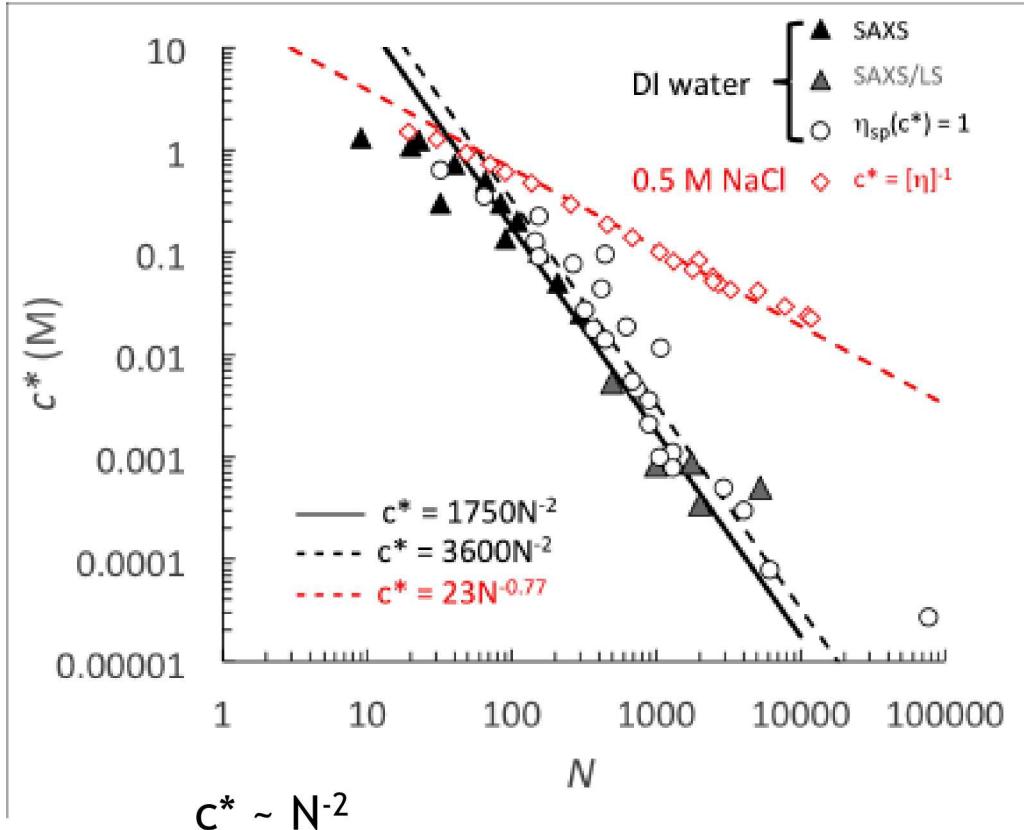
logarithmic

Polyelectrolytes in Solution: Overlap concentration



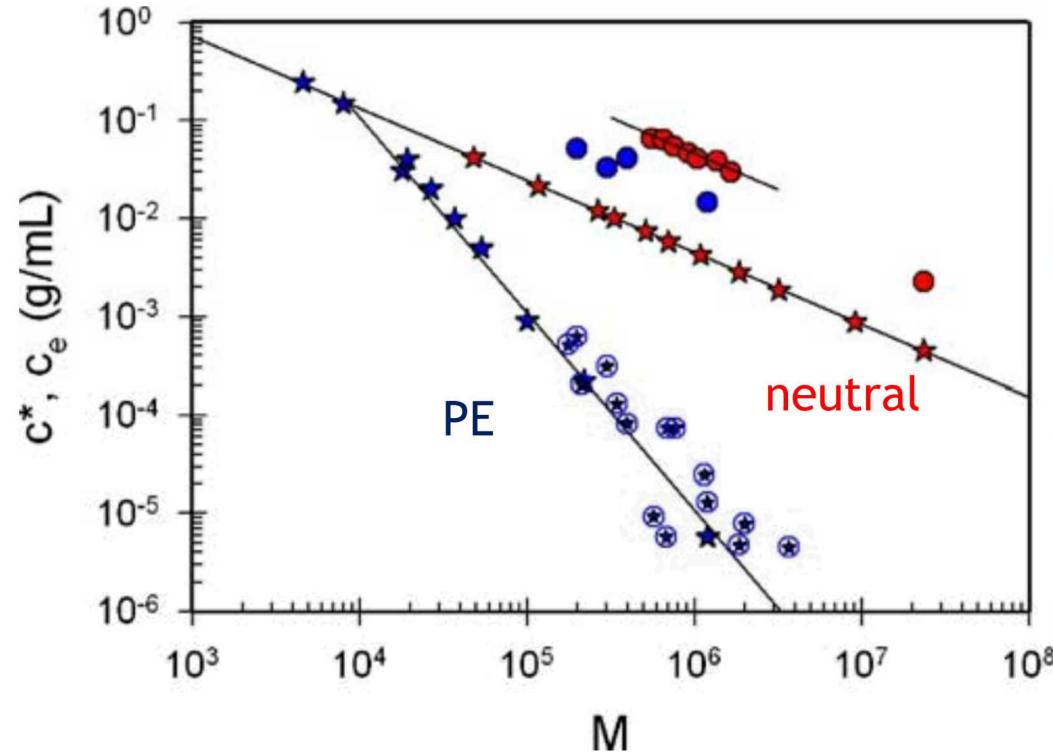
Los Alamos
National Laboratory
EST. 1943

Lopez 2019



if $R \sim N^{1/3}$ and $c^* = MN/V \simeq MN/(M\pi R^3/6)$, then $c^* \sim N/N^3 = N^{-2}$

Colby Rheol Acta 2010



Simulations of Polyelectrolyte Solutions

“All ion” MD simulations

$M = 27, \dots, 800$ number of chains

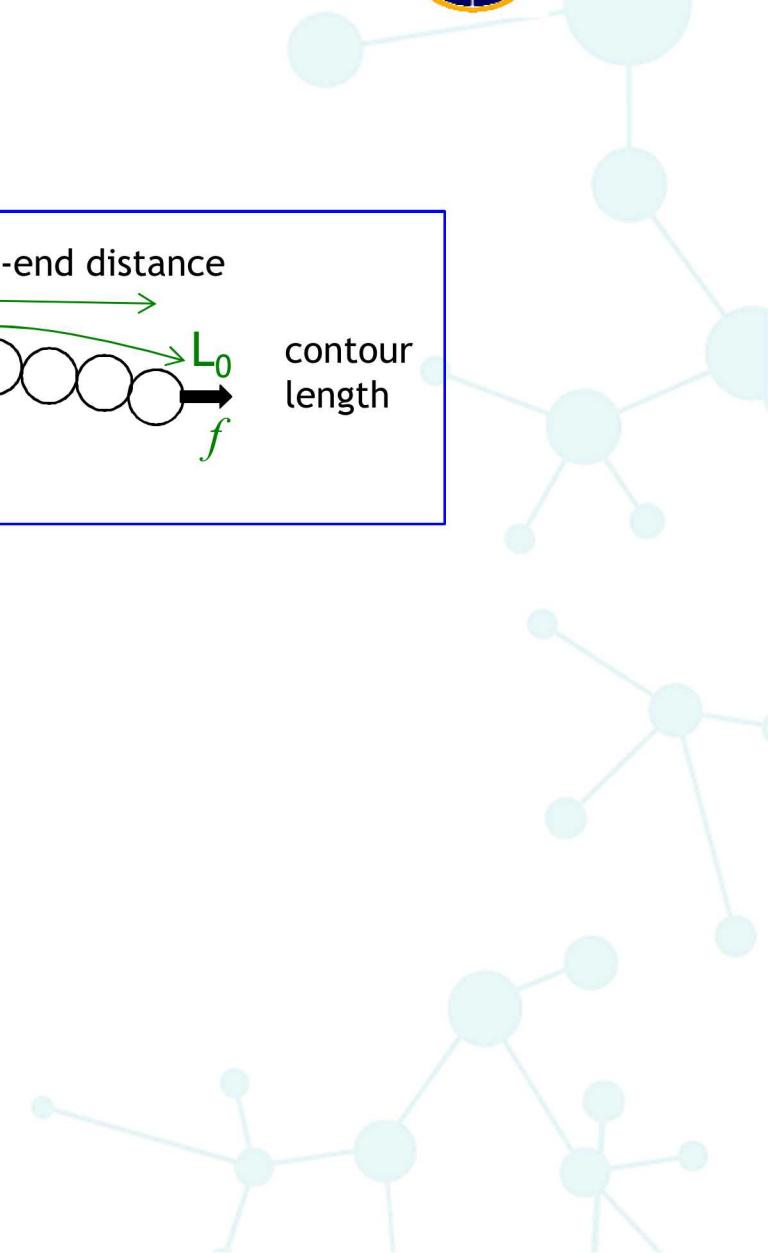
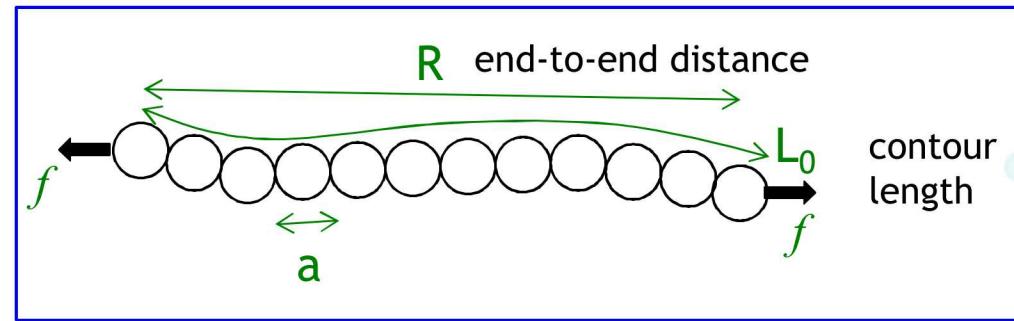
salt free

Bjerrum length = charge separation

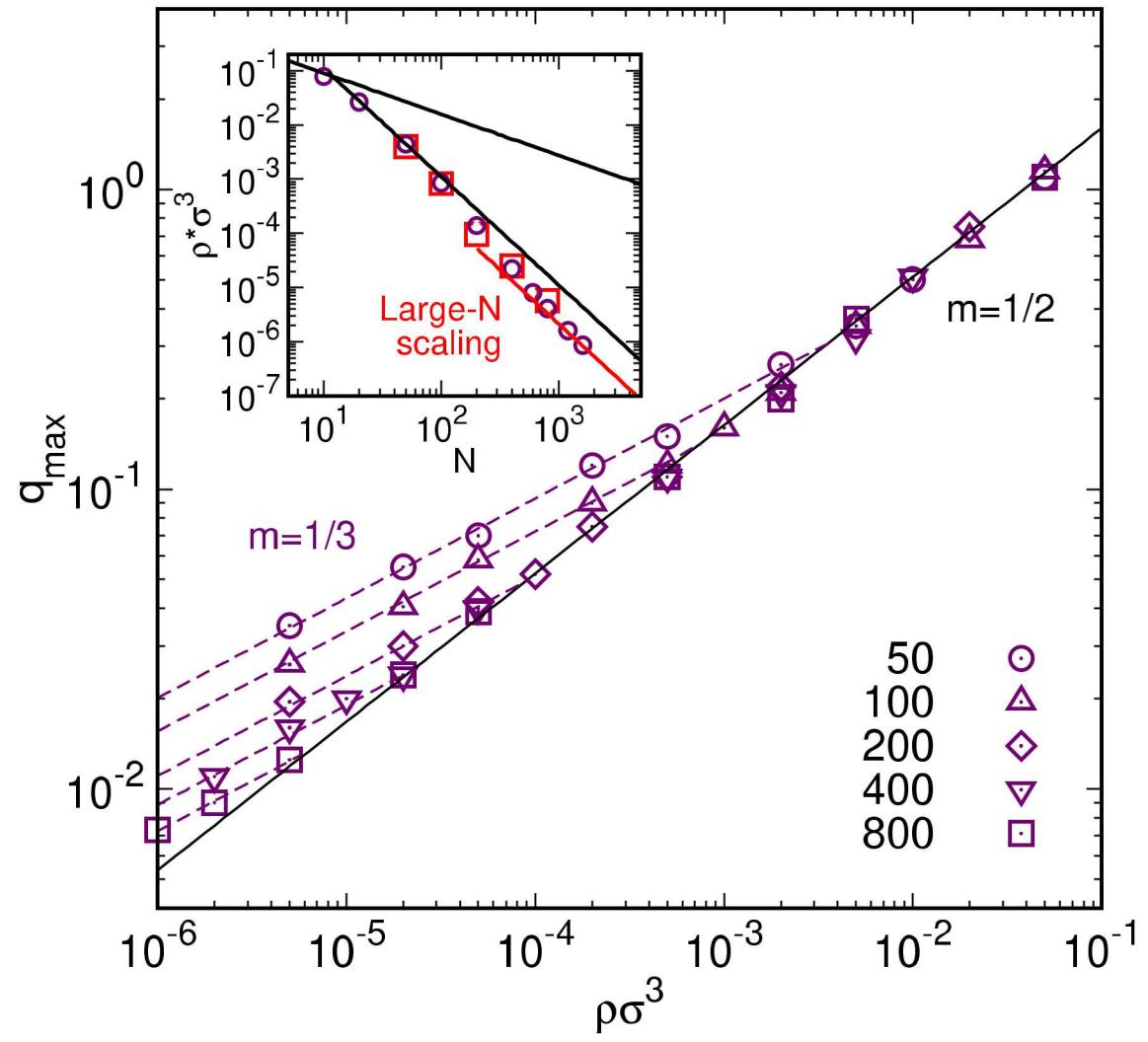
strong polyelectrolyte

$F = \text{bond} + \text{entropy} + \text{electrostatics}$

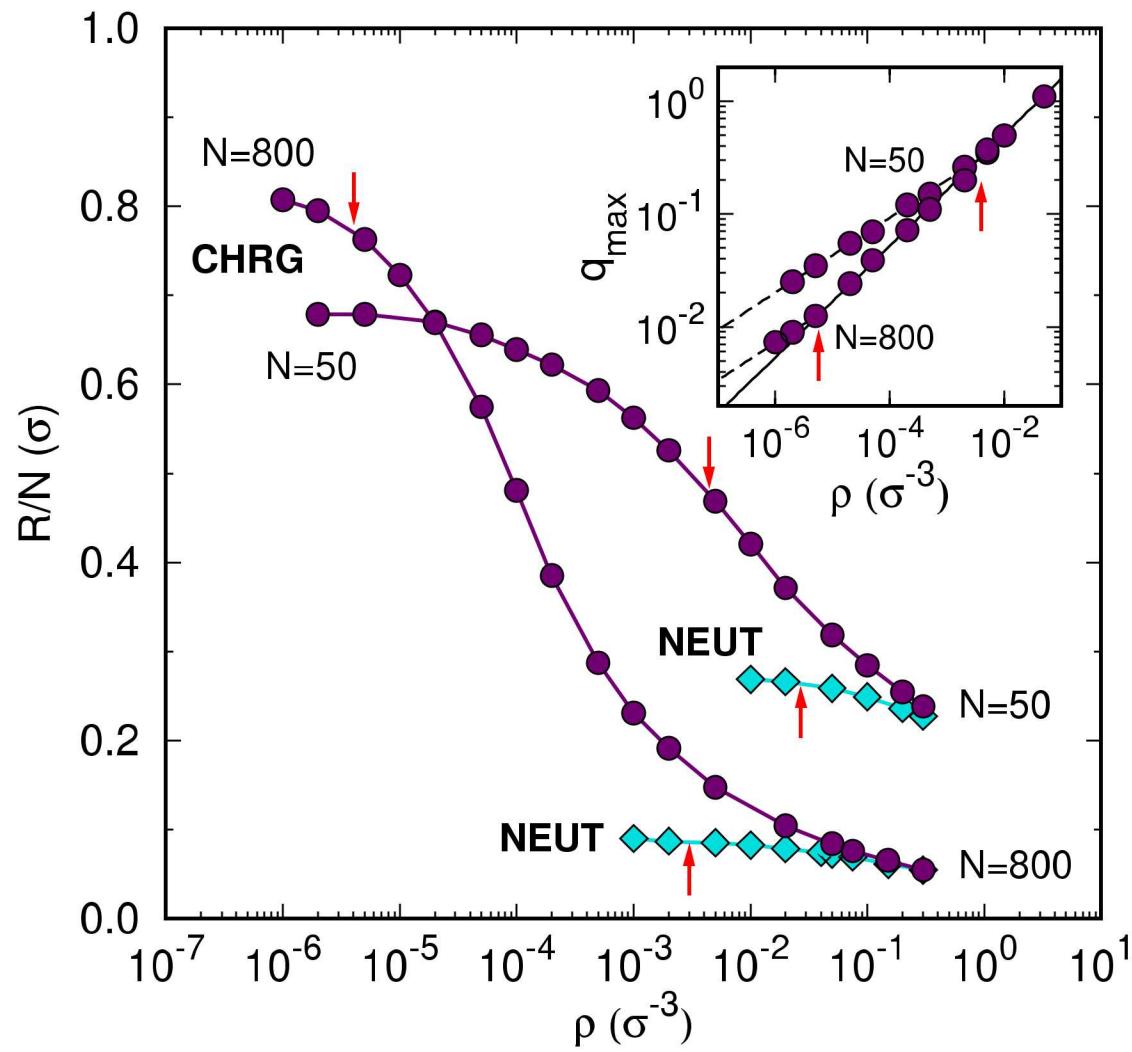
very large N : up to 1600



Overlap from Scattering Data in Simulations



End-to-End Distance



Overlap Methods

Scattering:

transition between $q^{1/2}$ and $q^{1/3}$

Packing:

contact between spheres of diameter R
packing fraction = random close packed
 $0.64 = \phi = M \pi R^3 / 6V = \rho \pi R^3 / 6N$

Comparison:

$N=50$

chain expands below ρ^*

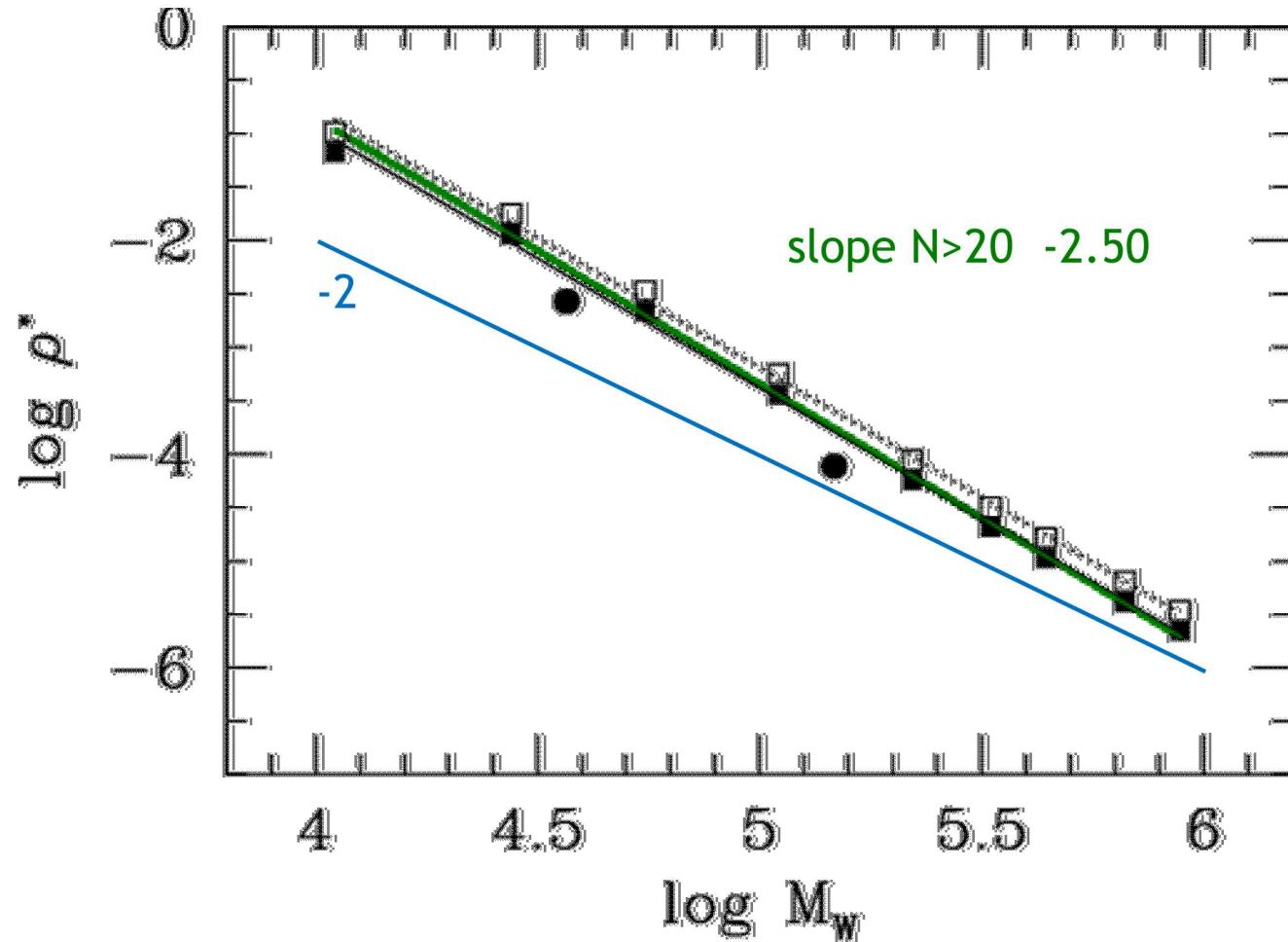
chain bends in 3 linear segments at ρ^*

$N=800$

chain barely expands below ρ^*

chain is 1 linear segment at ρ^*

Overlap concentration: Simulation



squares

$N=20, \dots, 1600$

Map 1: 3 NaPSS monomers to 1 bead
volume of monomer

Map 2: units map of volume

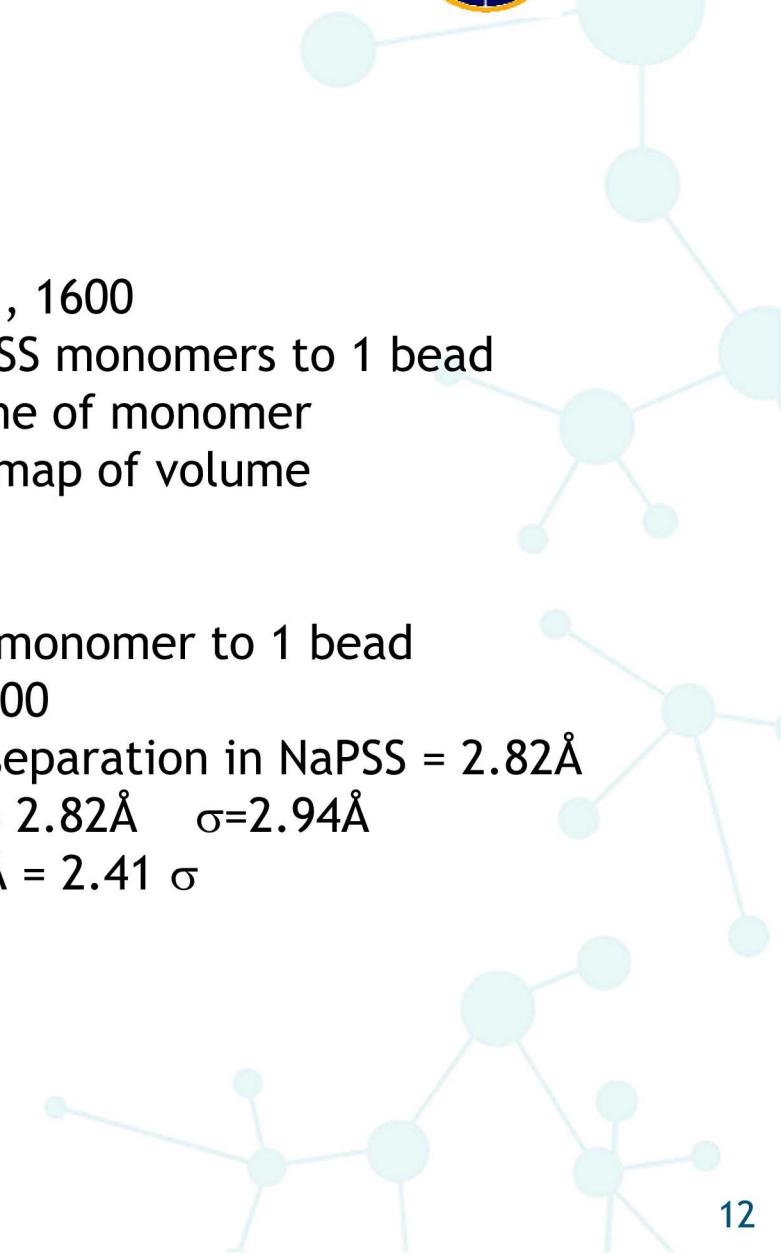
Circles:

1 NaPSS monomer to 1 bead
 $N=200, 800$

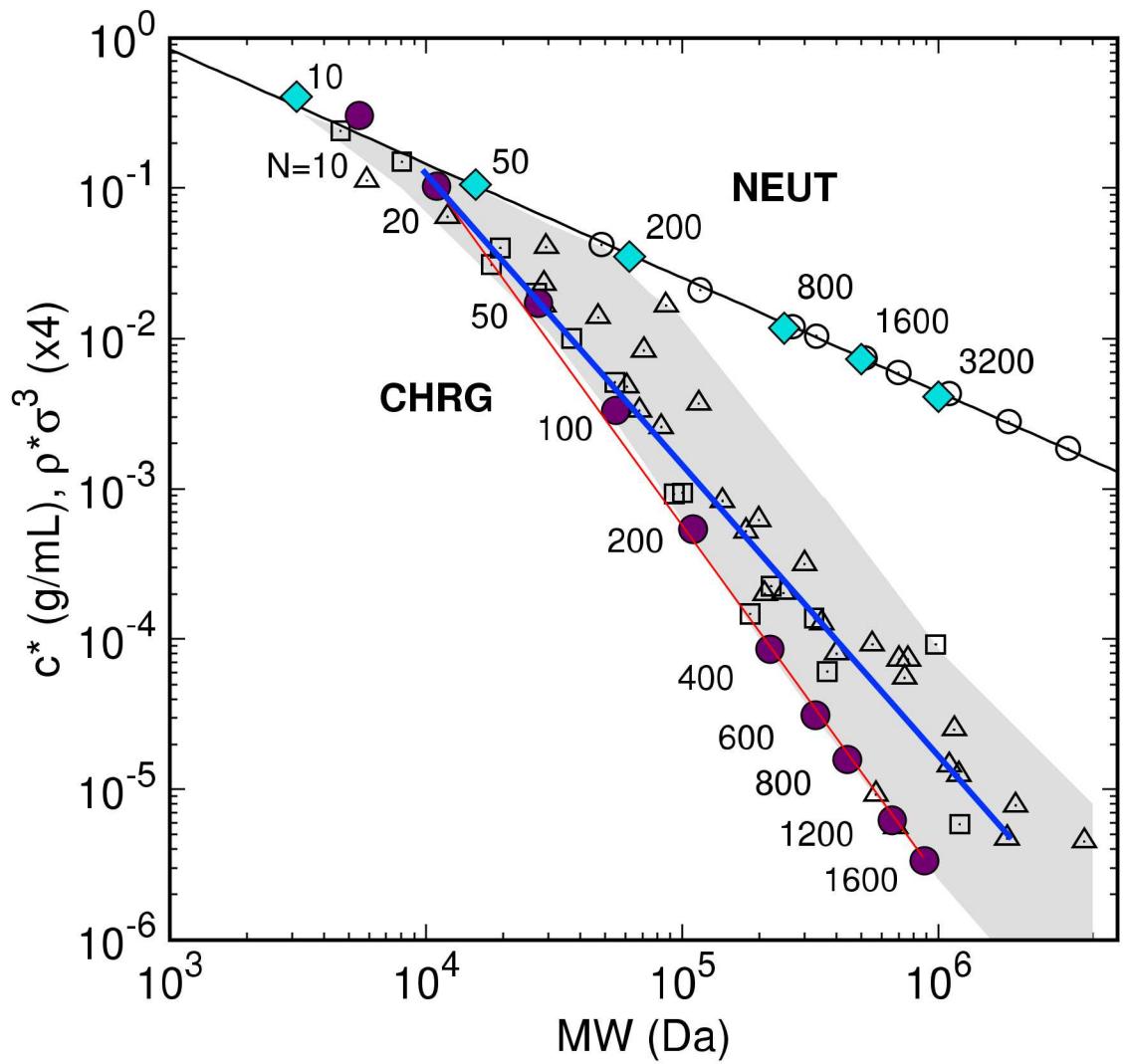
charge separation in NaPSS = 2.82 Å

$0.96 \sigma = 2.82 \text{ \AA}$ $\sigma = 2.94 \text{ \AA}$

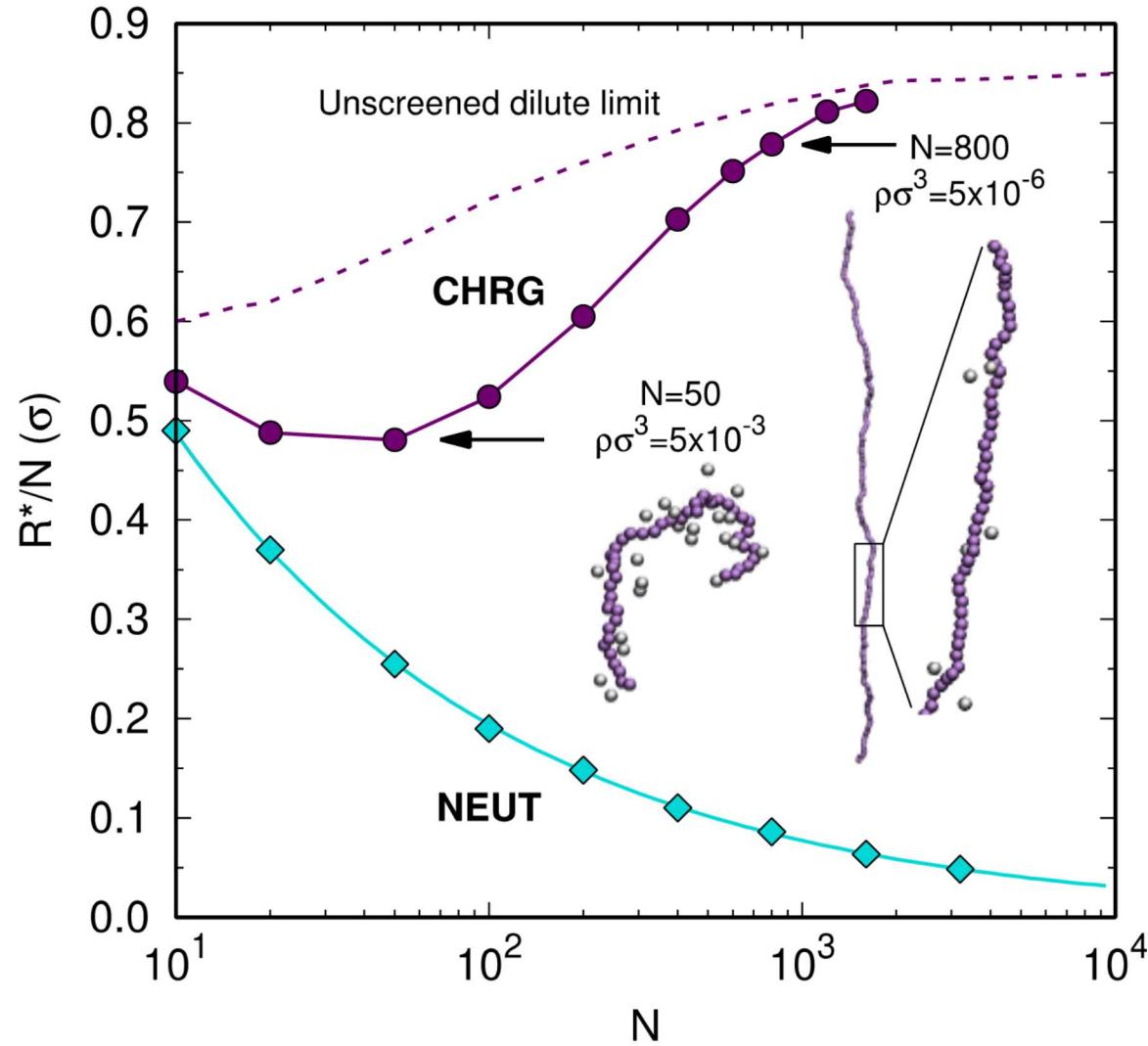
$l_B = 7.1 \text{ \AA} = 2.41 \sigma$



Overlap Concentration



End-to-End Distance at Overlap vs. N



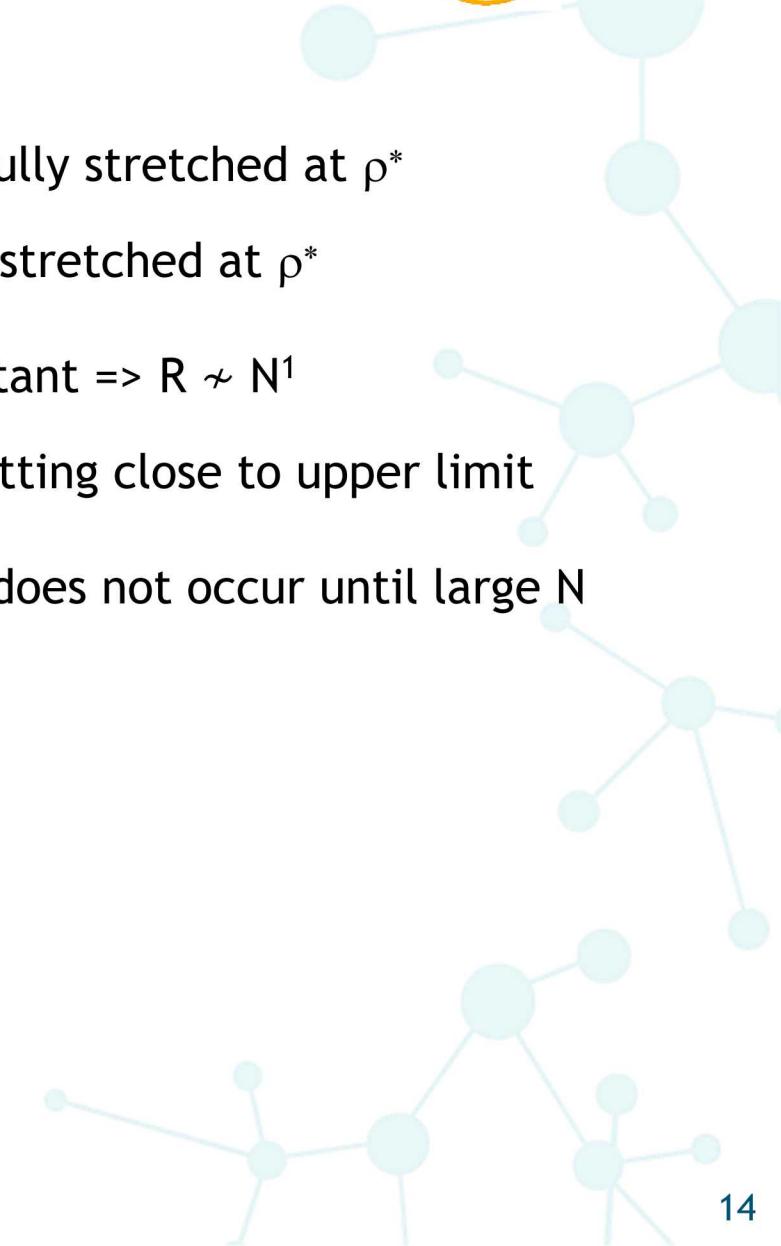
$N=50$ not fully stretched at ρ^*

$N=800$ fully stretched at ρ^*

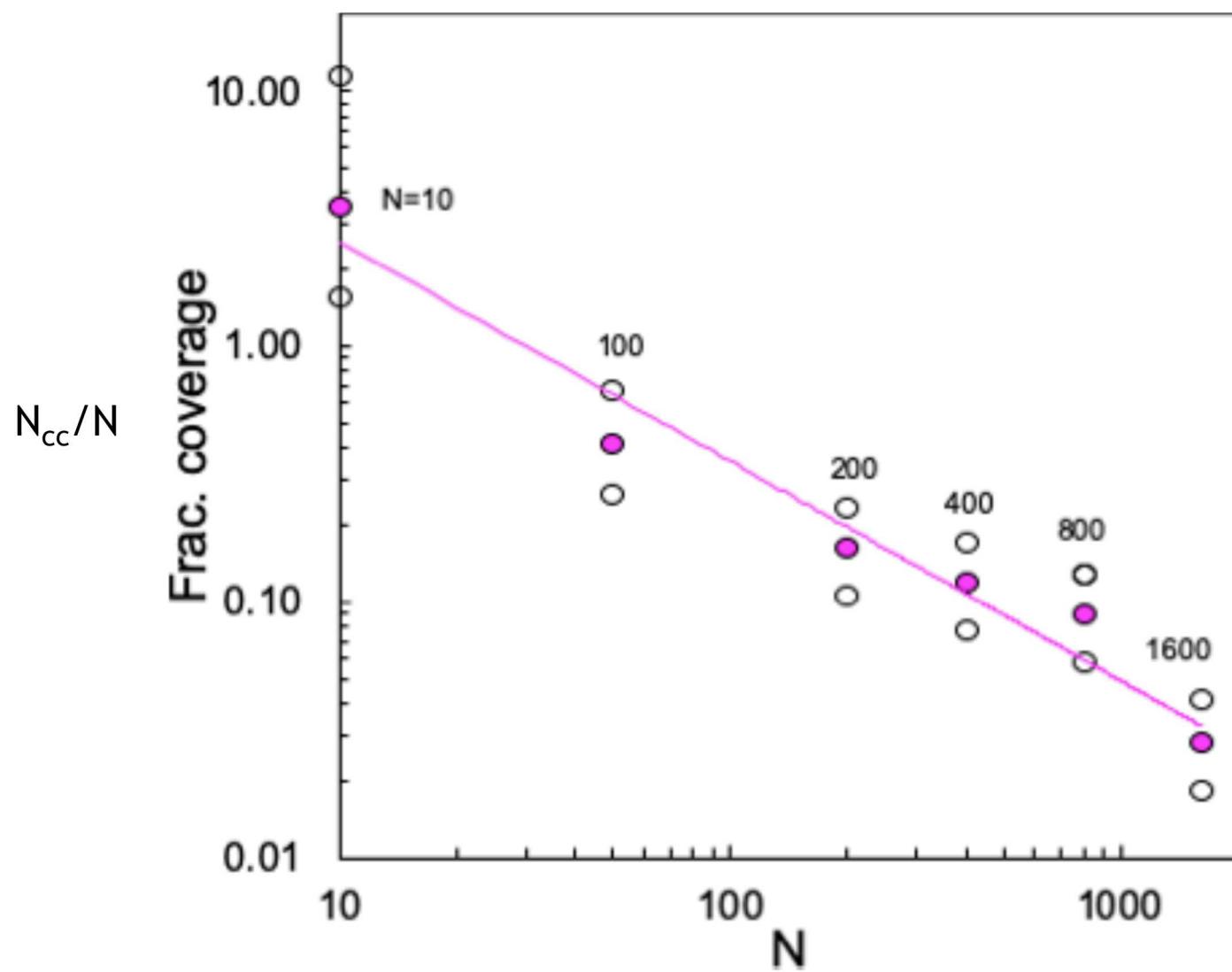
R/N not constant $\Rightarrow R \sim N^1$

By $N \simeq 800$ getting close to upper limit

Large N limit does not occur until large N



Counterions

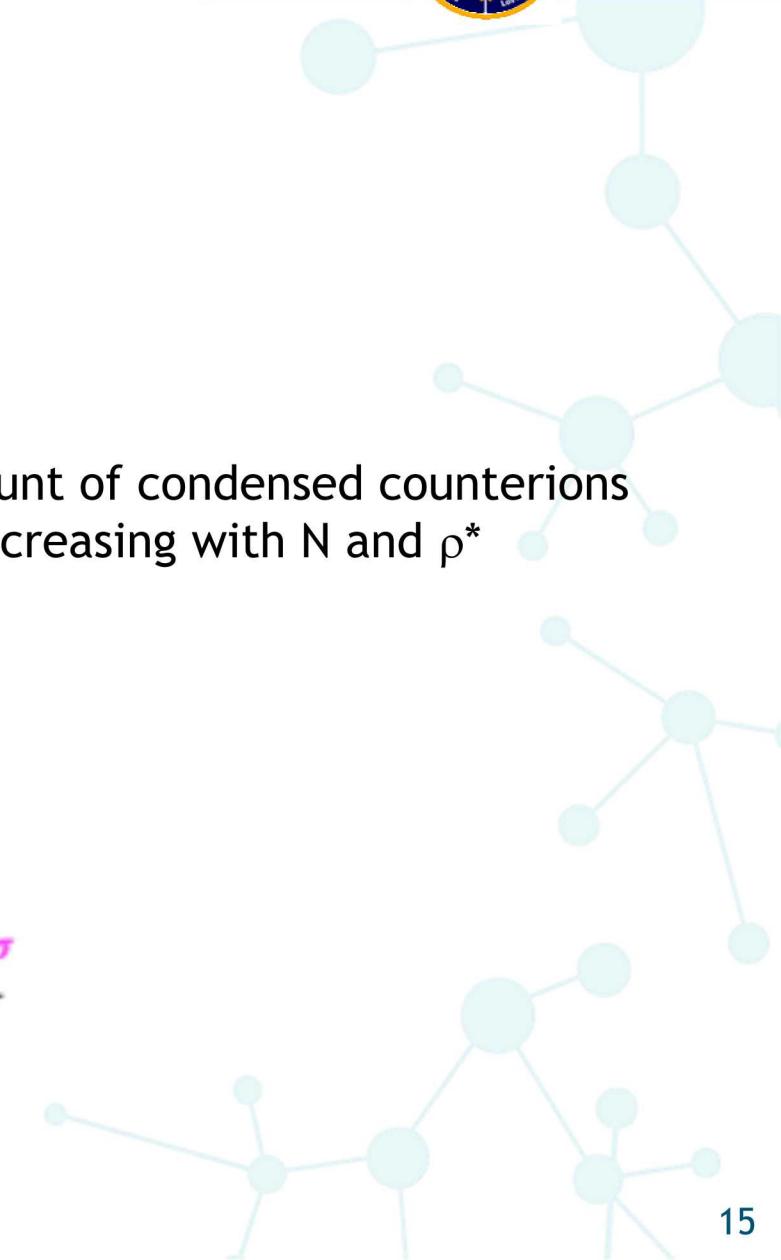


Amount of condensed counterions is decreasing with N and ρ^*

$rcut=5.0\sigma$

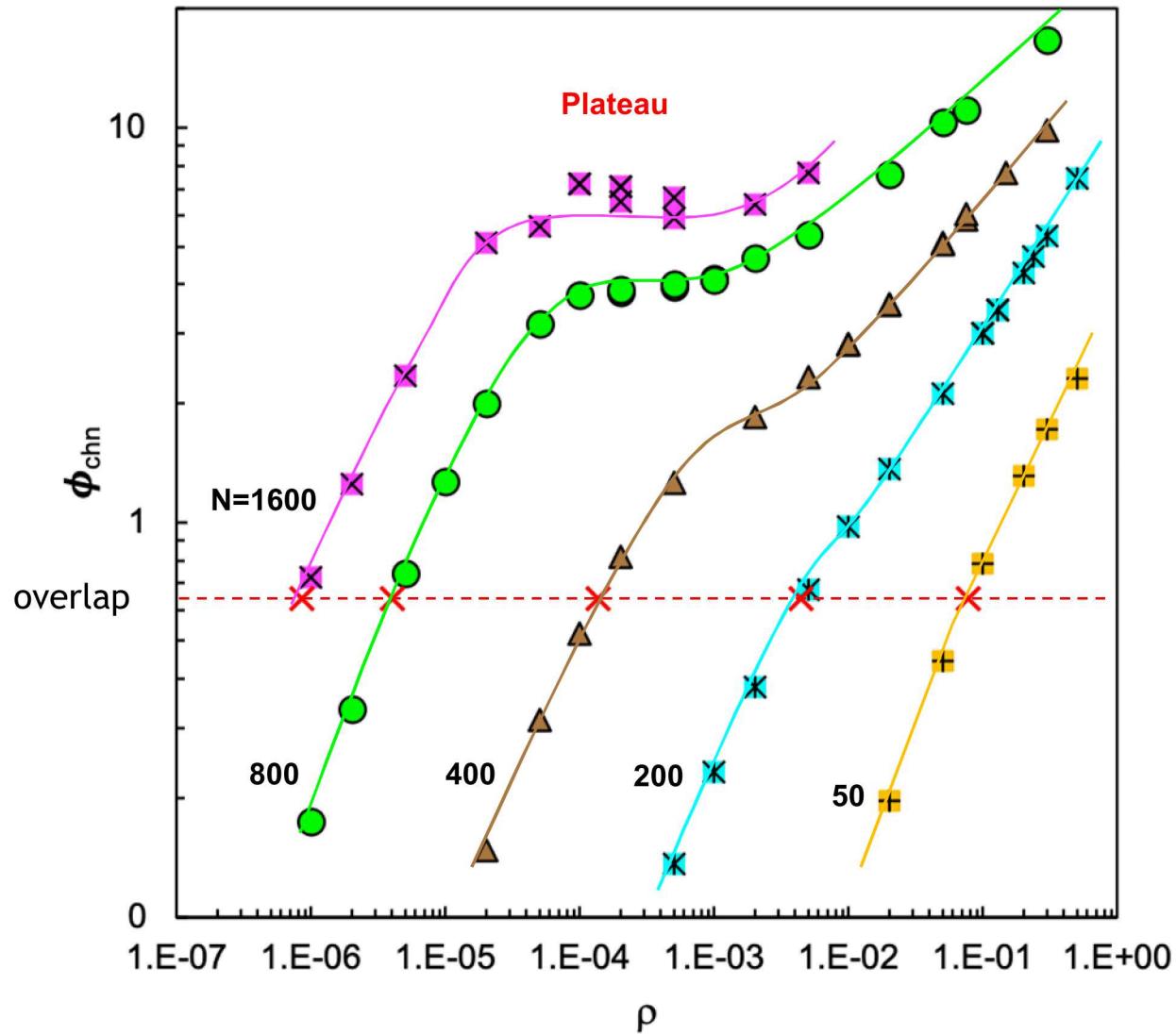
$rcut=3.0\sigma$

$rcut=2.0\sigma$

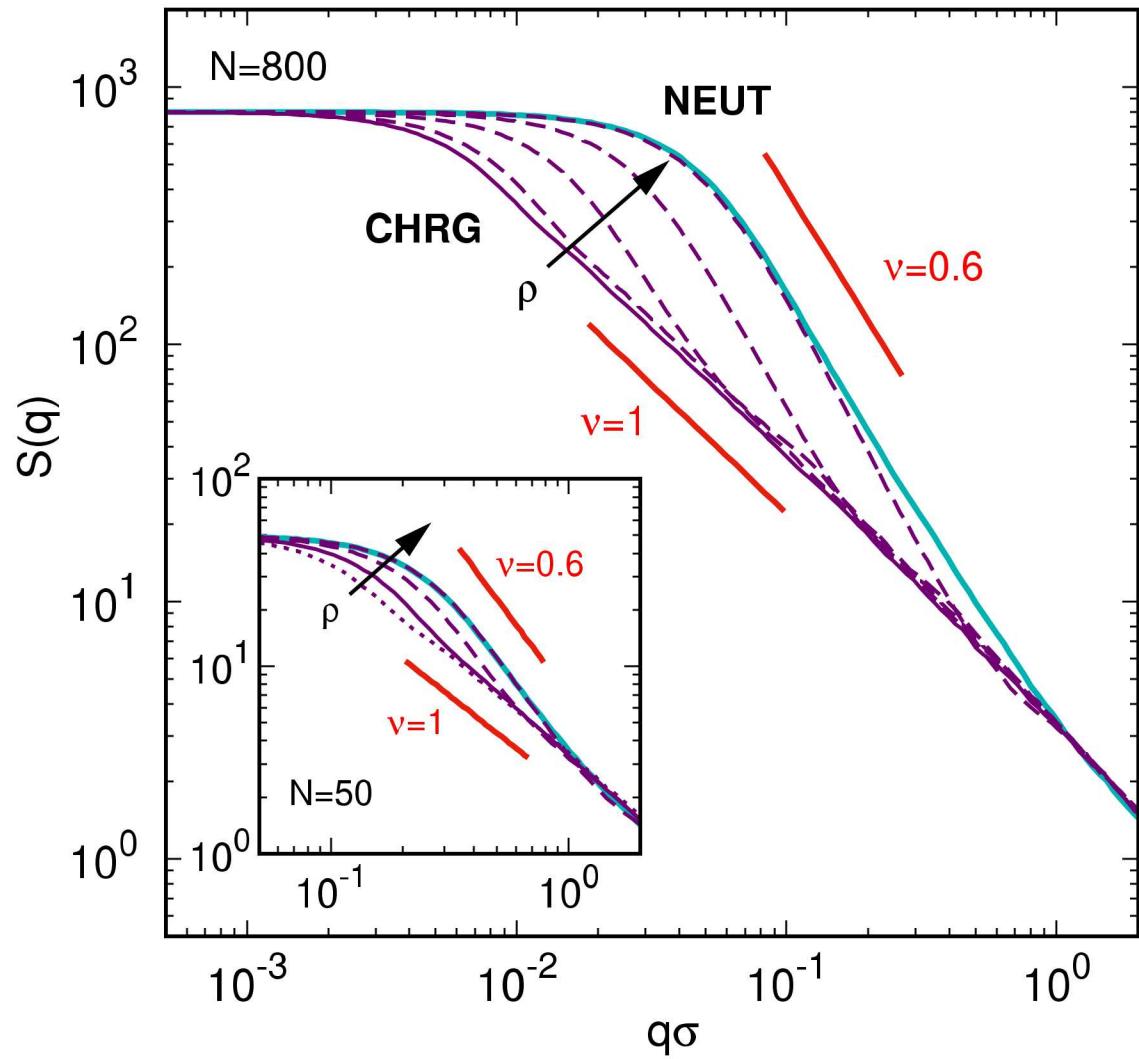


Packing of chains

$$\phi = M \pi R^3 / 6V$$



Structure as a function of density

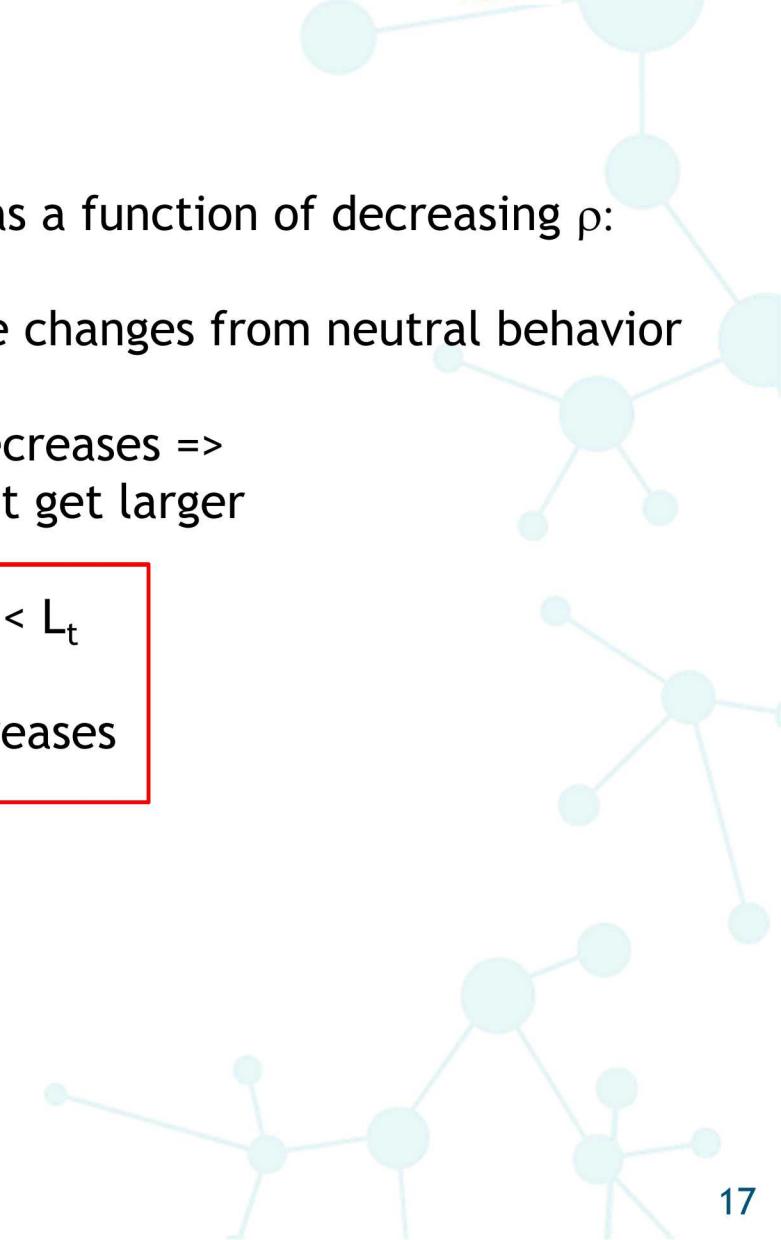


Two changes in $S(q)$ as a function of decreasing ρ :

- 1) long q dependence changes from neutral behavior toward $\nu \simeq 1$
- 2) transition point decreases => ‘rodlike’ segment get larger

Rodlike for lengths $\ell < L_t$

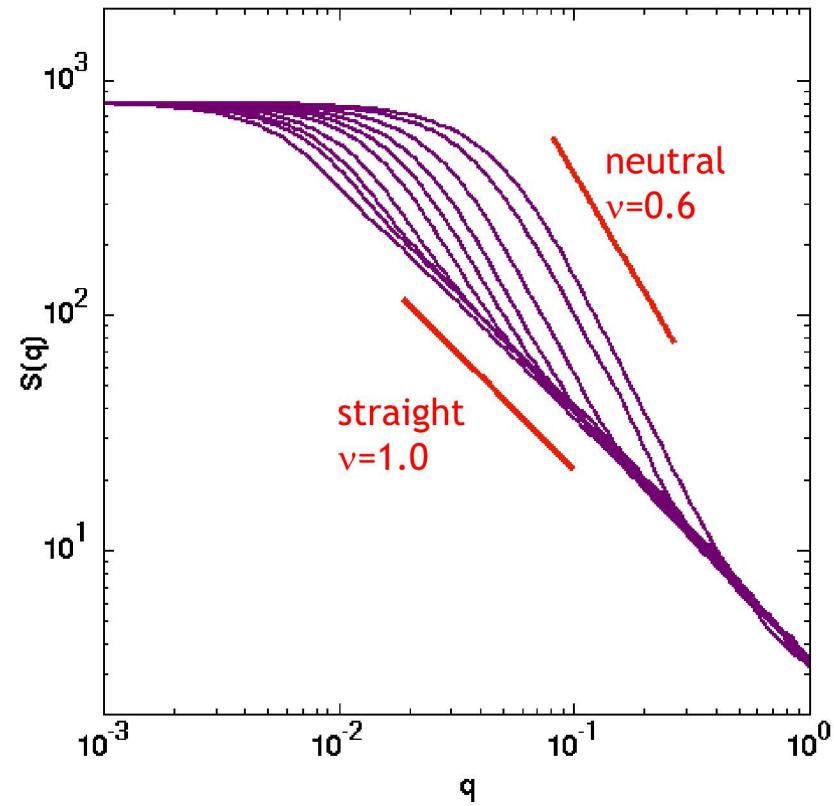
L_t increases as ρ decreases



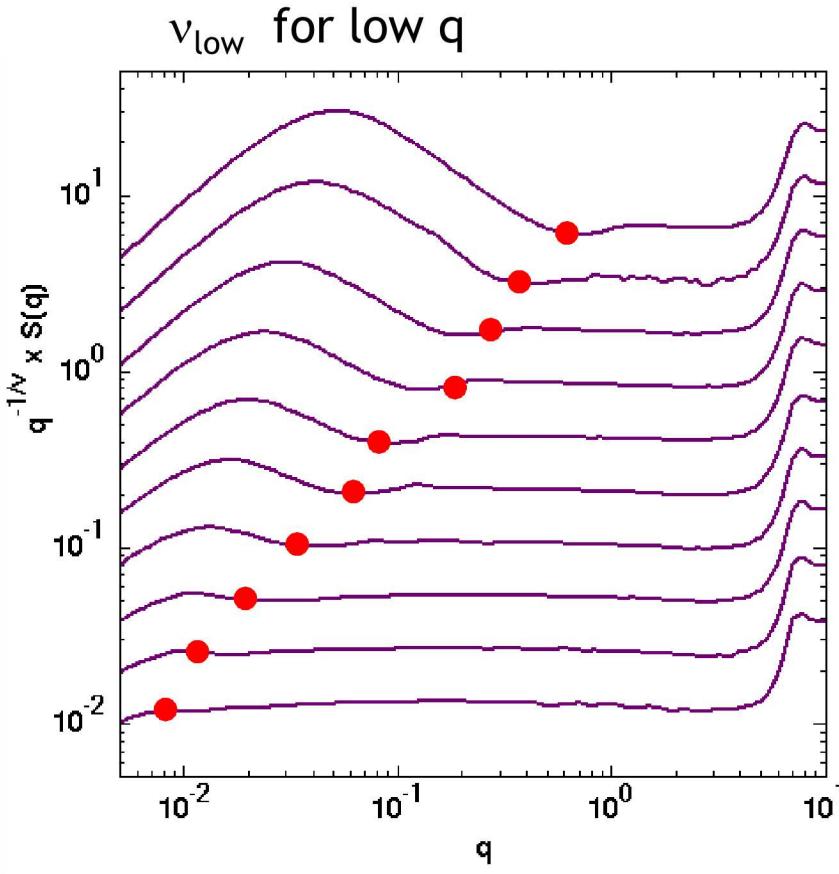
Local Chain Structure

Single chain structure factor $S(q)$
 $N=800$
density dependence

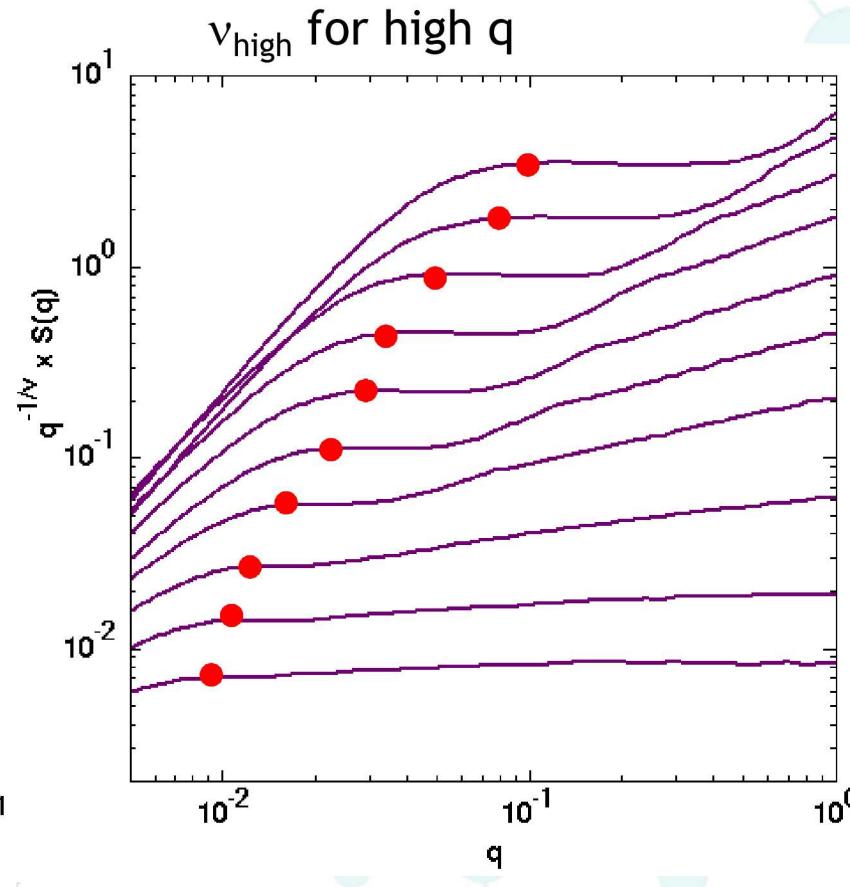
$$S(q) \sim q^{1/\nu}$$



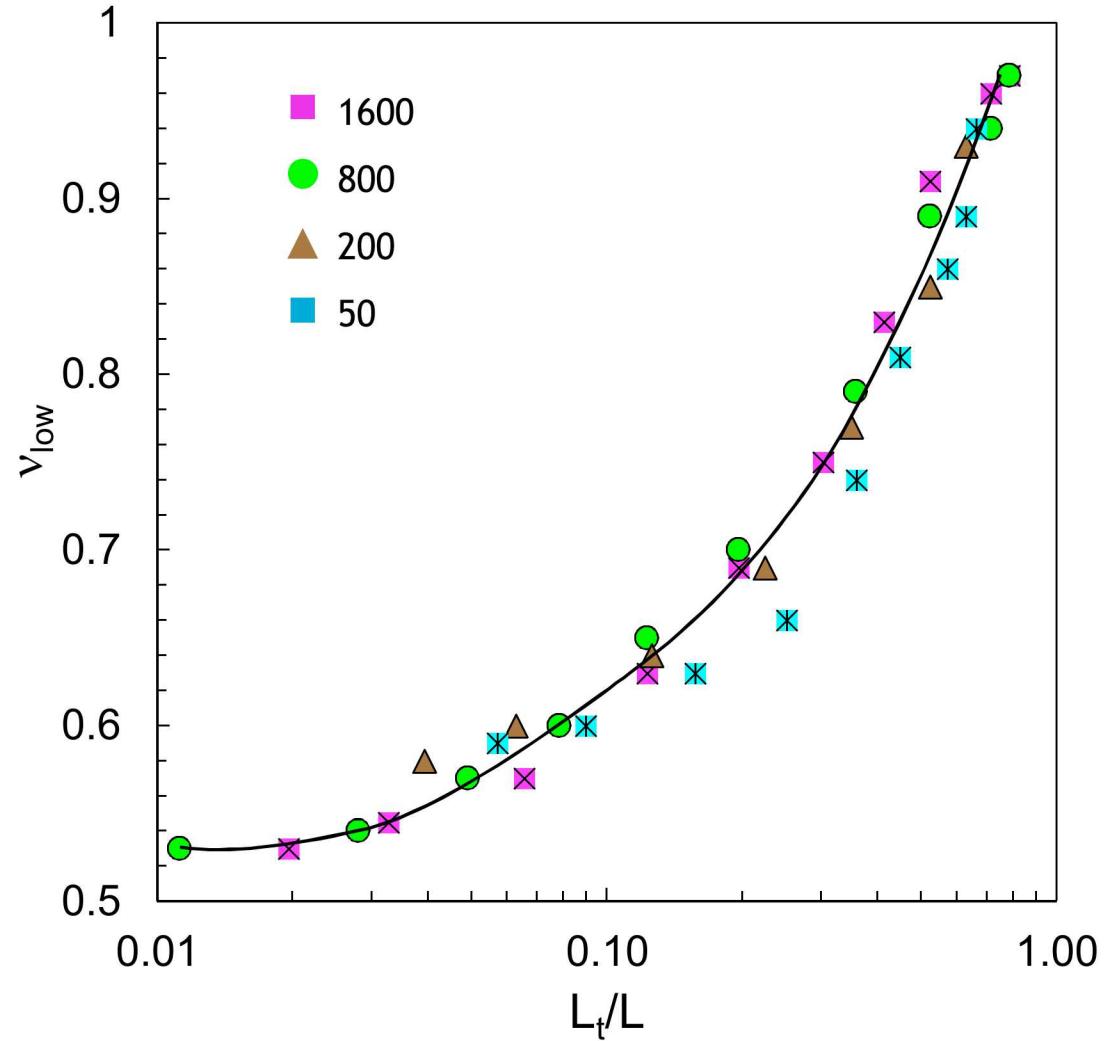
Two q -dependences (high/low)
 $q_r = 2\pi/L_r$ is transition point
 L_r is the ~straight segment length



Chain is composed of ~straight segments that form a directed walk.



Large length scale behavior

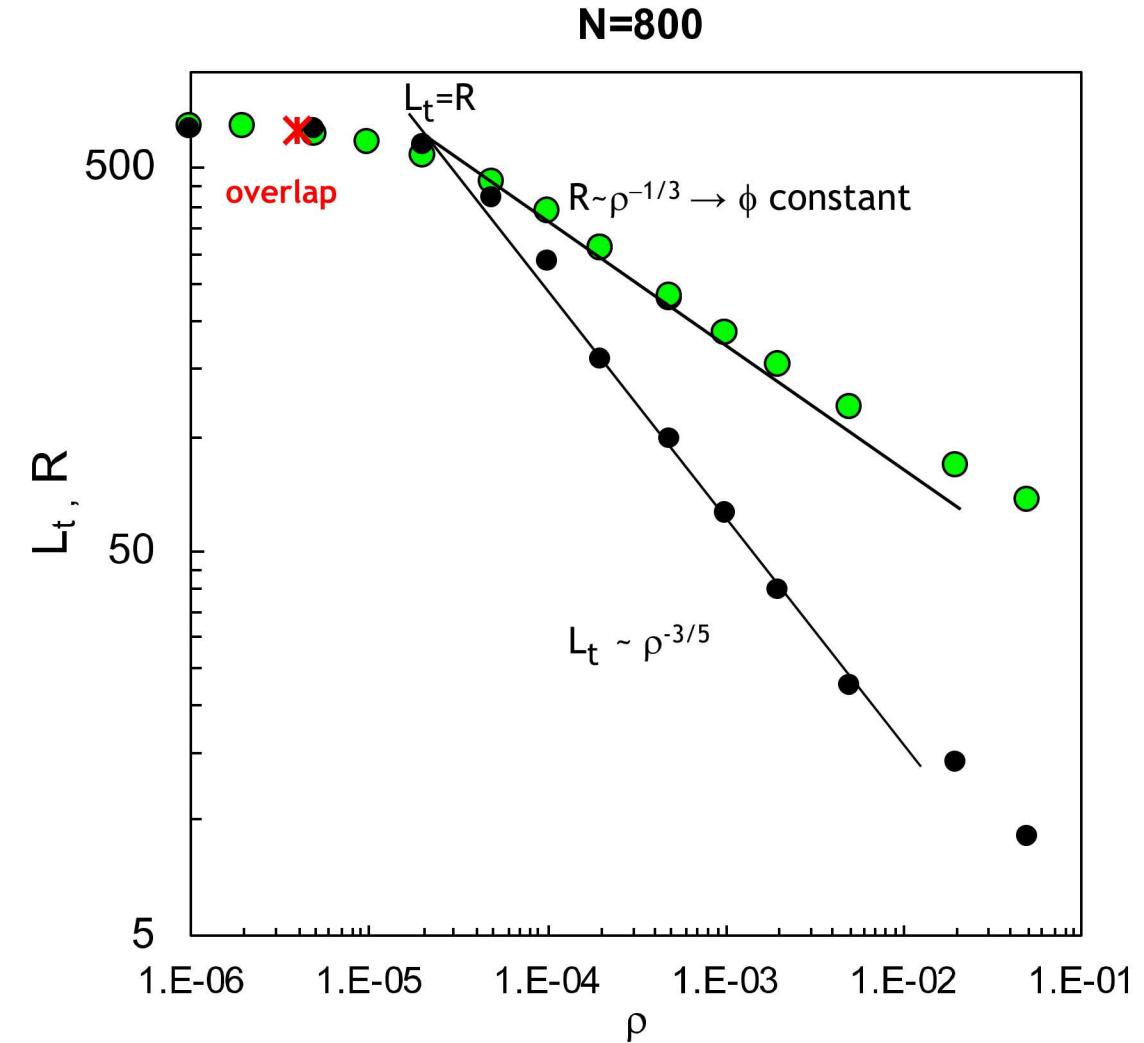
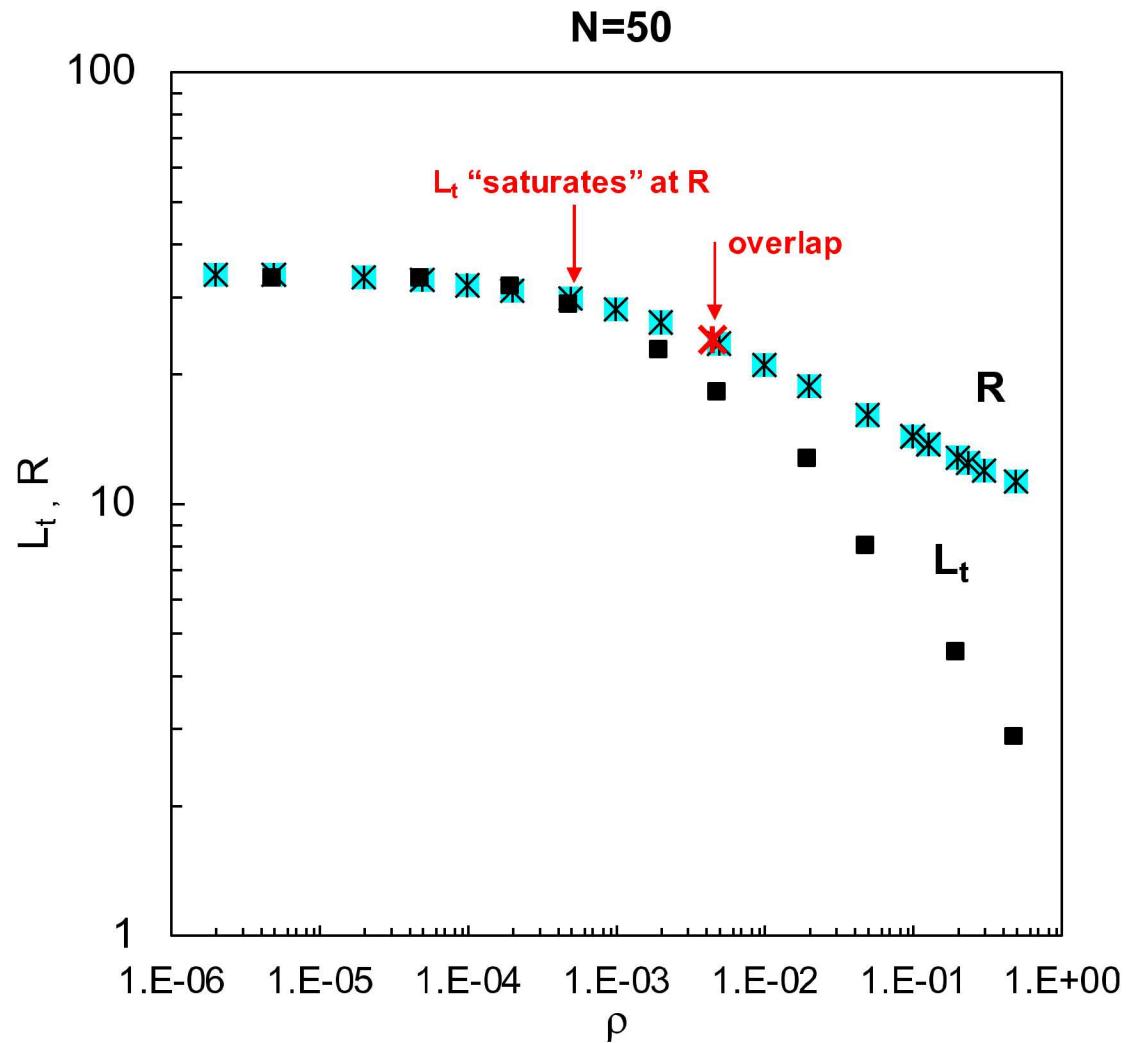


Transition from random walk at high density
to directed walk
just straight segment at large enough N

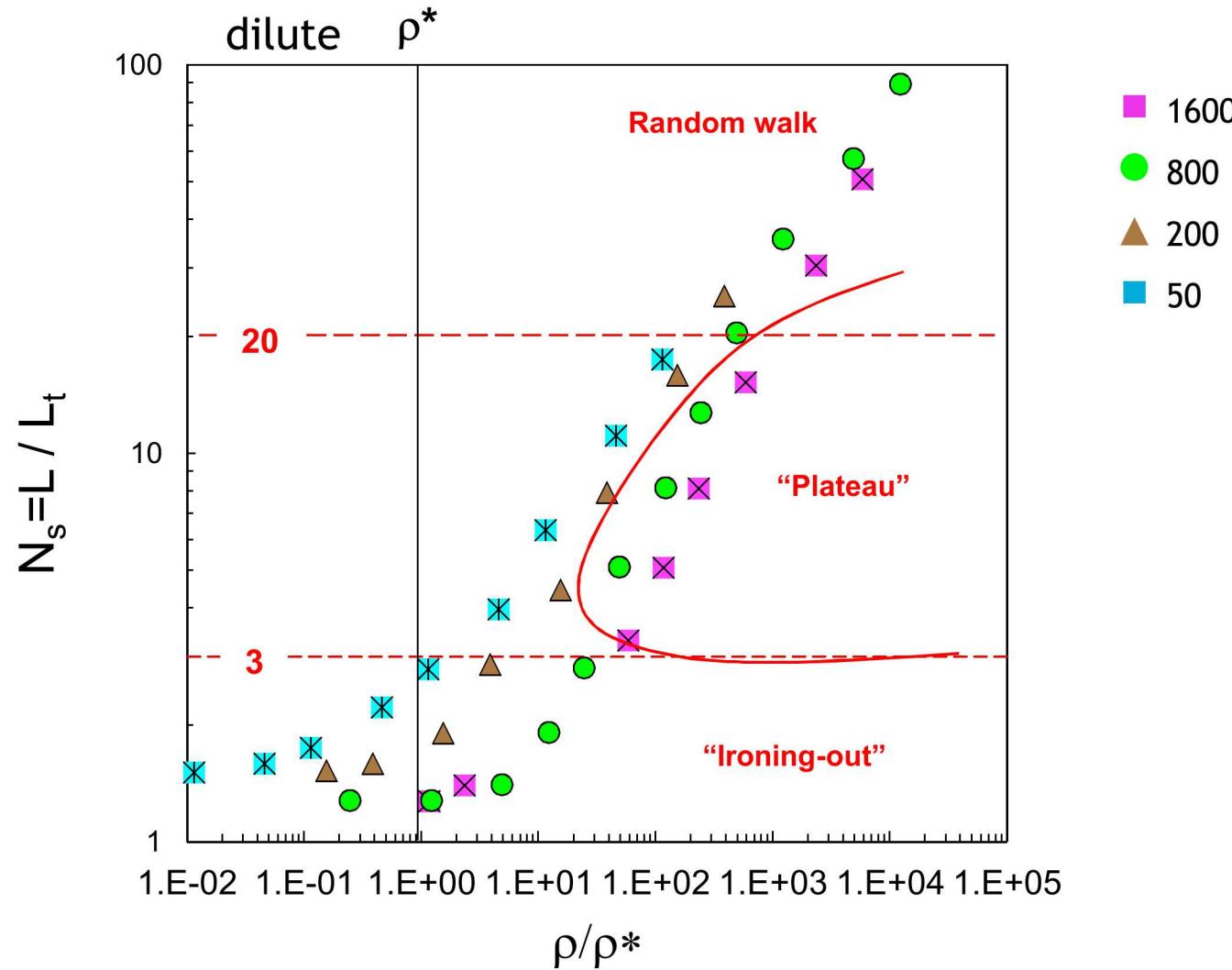
Maybe at large N ($\gtrsim 800$) overlapping behavior?

v_{high} starts at 0.90 and goes to 0.97

Segment length L_t as a function of density



Density Dependence: Number of Segments



Extended simulations out to much larger N (1600) and very much lower ρ

Find $c^* \sim N^{-2.5}$ over the range of N
reaching the unscreened dilute limit
at very large N ($= 800$)
shouldn't have expected N^{-2}

The chain structure at c^* varies significantly.
For $N \geq 800$, chains are 'fully' stretched at c^*
 $S(q) \sim q^{-1/\nu}$ with $\nu = 0.95+$

Density dependence of chain structure
Two regimes: straight segments and directed walk
Straight segment length increases with decreasing density
Results imply simple description possible

Need better experimental data for c^*
As always would like larger N for simulations



The Collaboration

Jon Bollinger, SNL

Gary Grest, SNL

Michael Rubinstein, Duke



