



# Recent Results in Polyelectrolyte Simulations

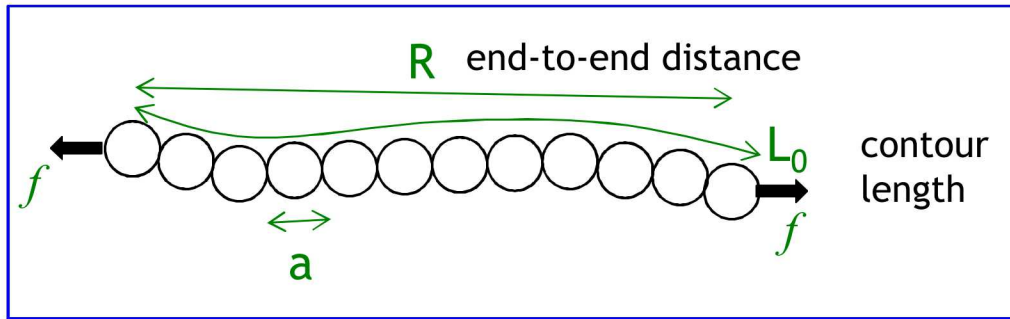
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**CINT**  
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# Simple Model for Polyelectrolytes

“All ion” simulations: MD

bead (spring) flexible polyelectrolyte

$F = \text{bond} + \text{entropy} + \text{electrostatics}$



every bead in polymer is charged

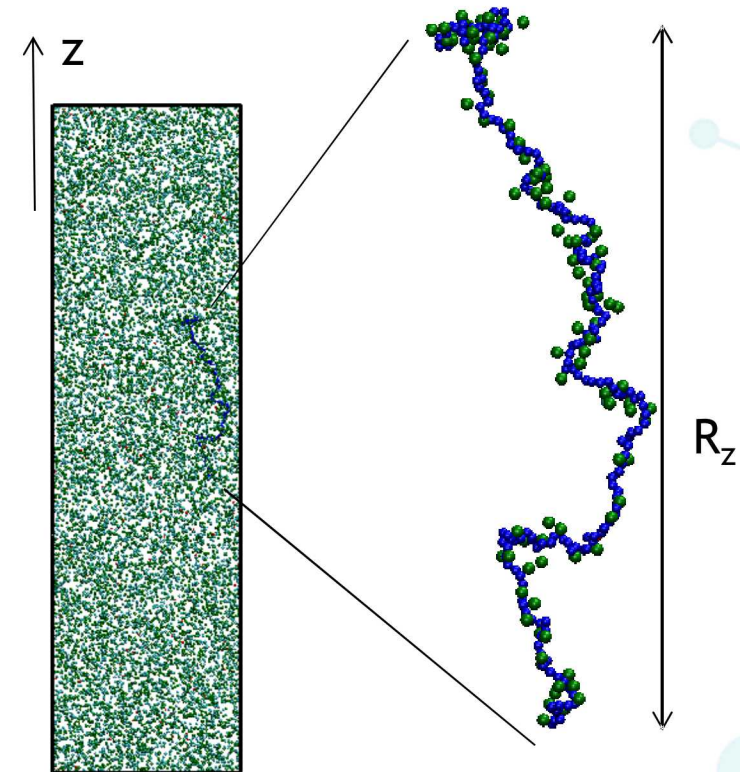
$a = 6.4 \text{ \AA} = 0.96 \sigma$  (ssDNA spacing)

$l_B = 1.065 \sigma$  (Bjerrum length) [dielectric continuum]

$N = 200$

$L_0 = (N-1) a = 192 \sigma$

Force-extension simulations

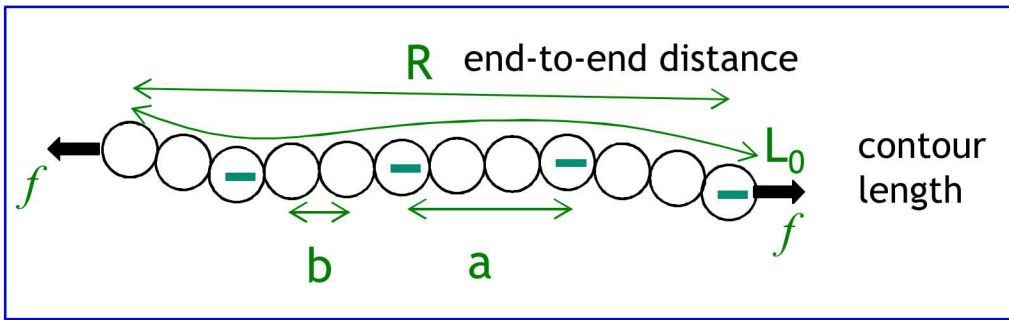


20 mM monovalent  
 $f = 0.20 \epsilon/\sigma$

# Additional Features

$b = 0.96 \sigma$  bond length

$a = m b$  charge spacing is varied



Intrinsic stiffness:  
add angle term to potential

$$U_{\text{angle}} = k_a (1 + \cos \theta) \quad \theta_0 = 180^\circ$$



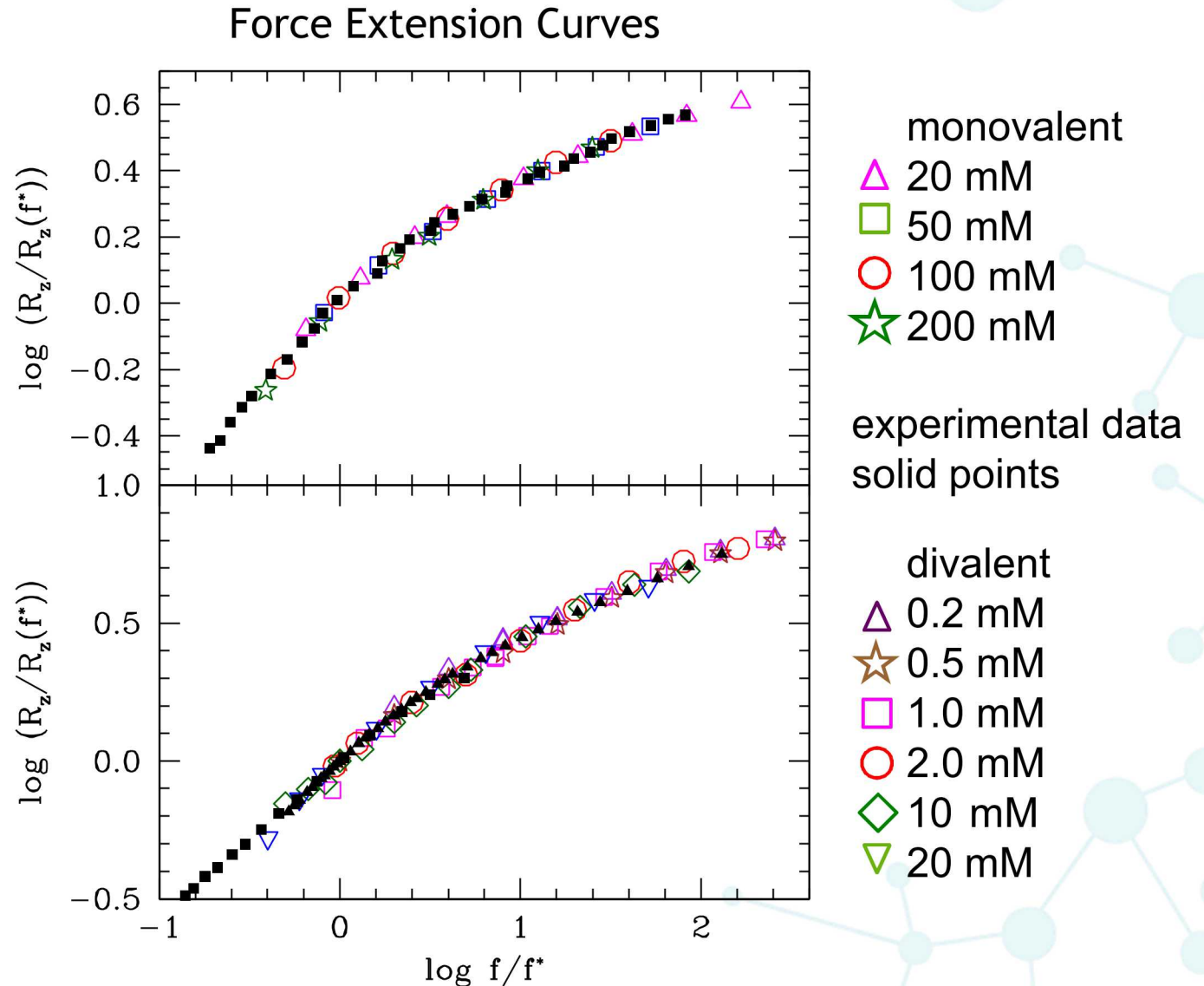
# Comparison of Experimental and Simulation Data (ssDNA)

Same scaling procedure as experiments

Simple model is sufficient.

electrostatics, entropy,  
connectivity

Dustin McIntosh, Omar Saleh, UCSB

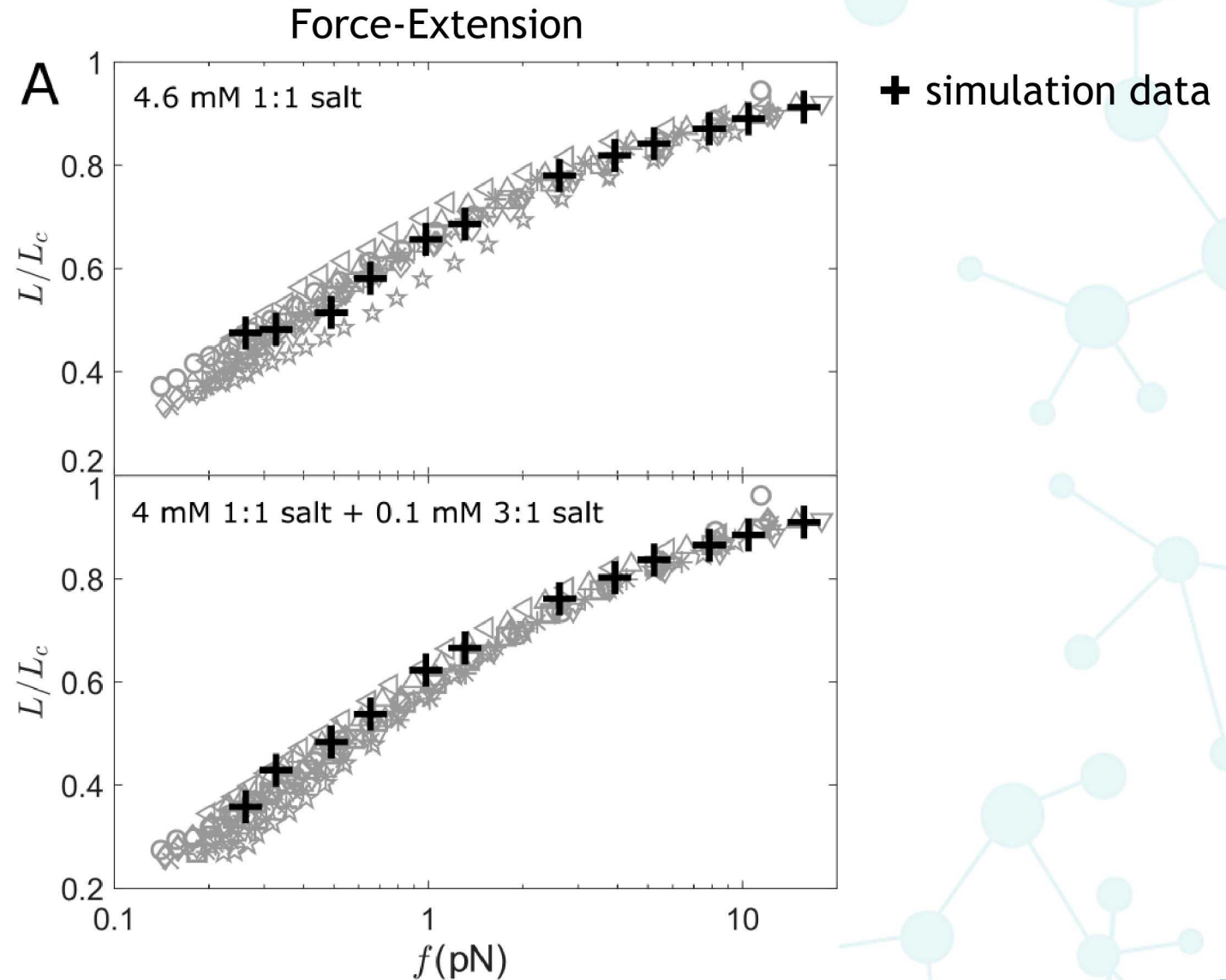


# Hyaluronic Acid

Persistence length of 5 nm

Overlap of simulation and experimental data

Simple model works for a variety of polyelectrolytes



# Pincus Tension Screening

Tension blobs (Pincus, Macromolecules 1976)

$\xi = k_B T / f$  is tensile screening length

where the excluded volume renormalization is the substitution of the Flory radius<sup>4</sup>  $R_F \approx N^a$  for the ideal chain radius  $R_0 = N^{1/2}a$ . However, as the chain stretches, its average monomer density decreases leading to a weakening of the excluded volume effect. Thus for sufficiently large external forces, we expect to eventually recover ideal behavior with  $\bar{Z} \propto N$  rather than  $\bar{Z} \propto N^{2\nu}$  as given by (I.2). This crossover will be described in terms of a competition between a "tensile screening length"  $\xi_t = (\beta f)^{-1}$  and the Flory radius  $R_F$ . For weak stretching  $R_F/\xi \ll 1$ , the de Gennes result (eq I.2) should be correct; for stronger stretching with  $R_F/\xi \gg 1$ , we expect to find a modified elastic behavior with  $\bar{Z} \propto N$ . To find the stress-strain relationship in this limit, we are tempted to employ a scaling argument. Let us assume that the average end-to-end separation may be written as

that its Laplace transform

$$\Gamma\rho(\mathbf{r}) = \sum_{N=0}^{\infty} \Gamma_N(\mathbf{r}) e^{-N\rho} \quad (\text{II.1})$$

behaves similarly to the transverse spin correlations in the vicinity of a magnetic phase transition

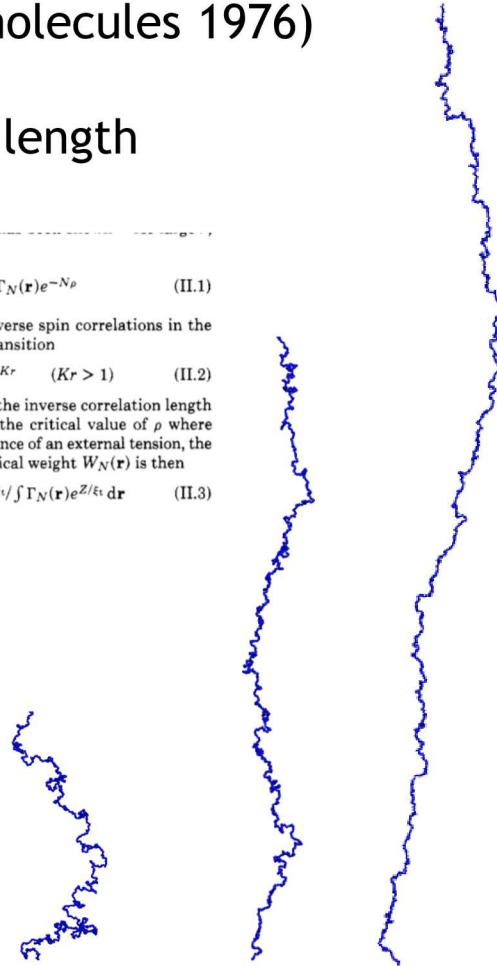
$$\Gamma\rho(\mathbf{r}) \approx A(a/r)e^{-Kr} \quad (Kr > 1) \quad (\text{II.2})$$

where  $A$  is a function of  $\rho$  and the inverse correlation length  $K = a^{-1}(\rho - \rho_c)^{\nu}$  where  $\rho_c$  is the critical value of  $\rho$  where  $\int \Gamma\rho(\mathbf{r}) d\mathbf{r}$  diverges. In the presence of an external tension, the appropriate normalized statistical weight  $W_N(\mathbf{r})$  is then

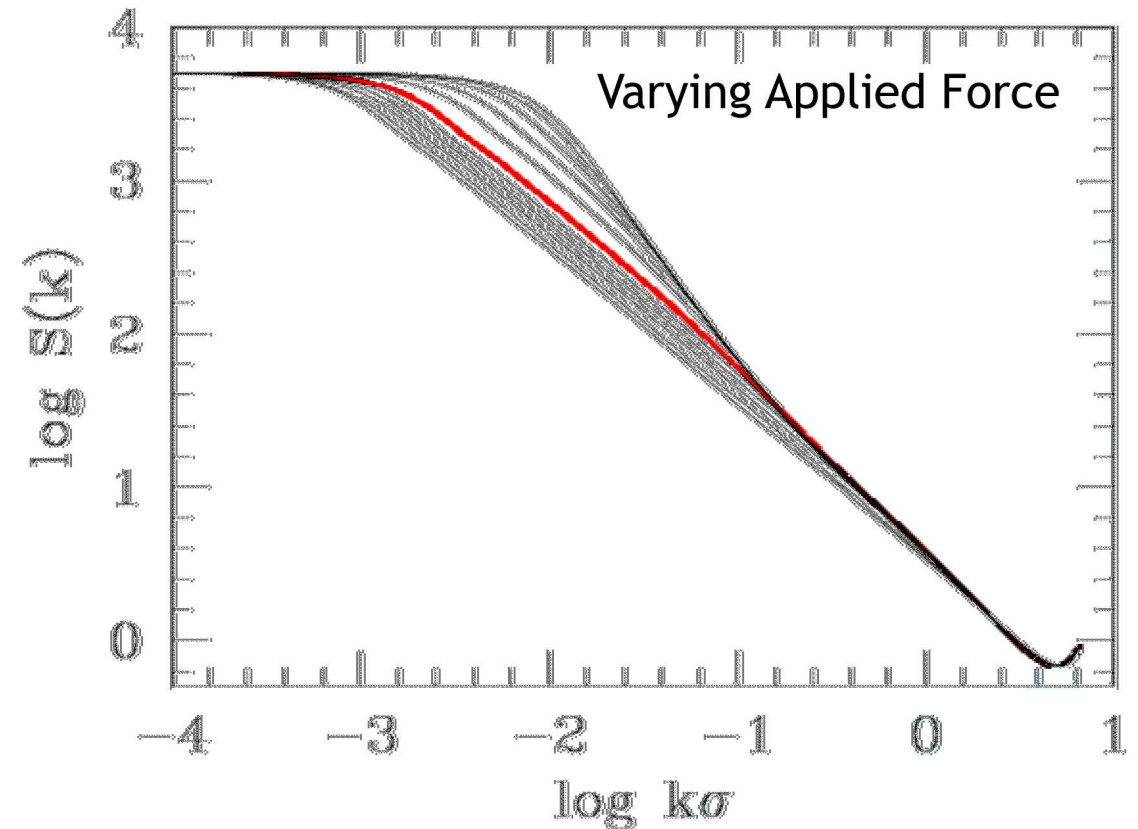
$$W_N(\mathbf{r}) = \Gamma_N(\mathbf{r}) e^{Z/\xi_t} / \int \Gamma_N(\mathbf{r}) e^{Z/\xi_t} d\mathbf{r} \quad (\text{II.3})$$

leading to

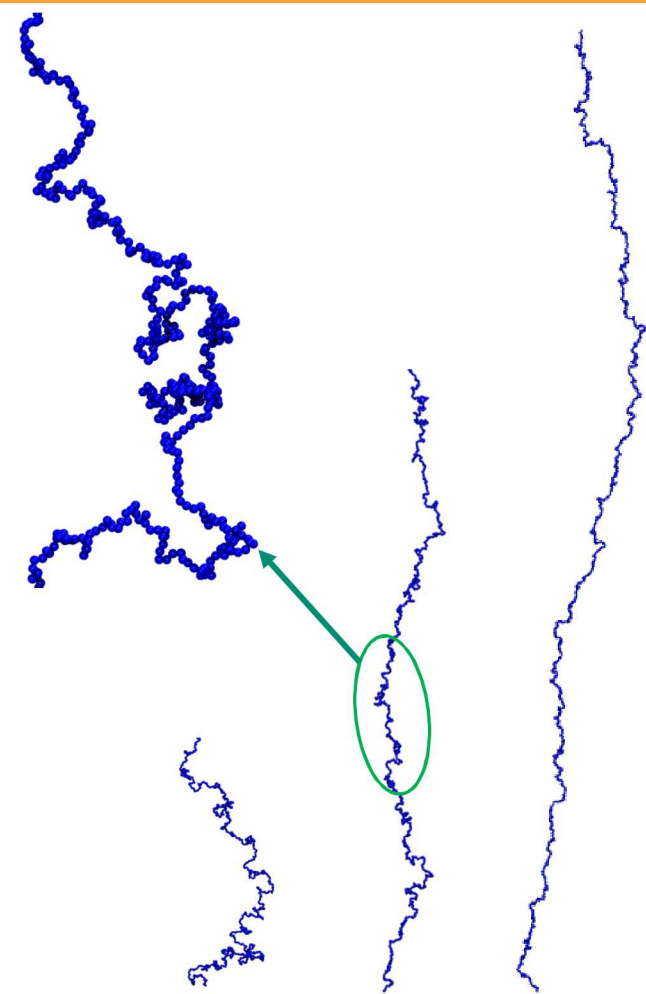
$f l_B / kT = 0.027 \quad 0.112 \quad 0.27$



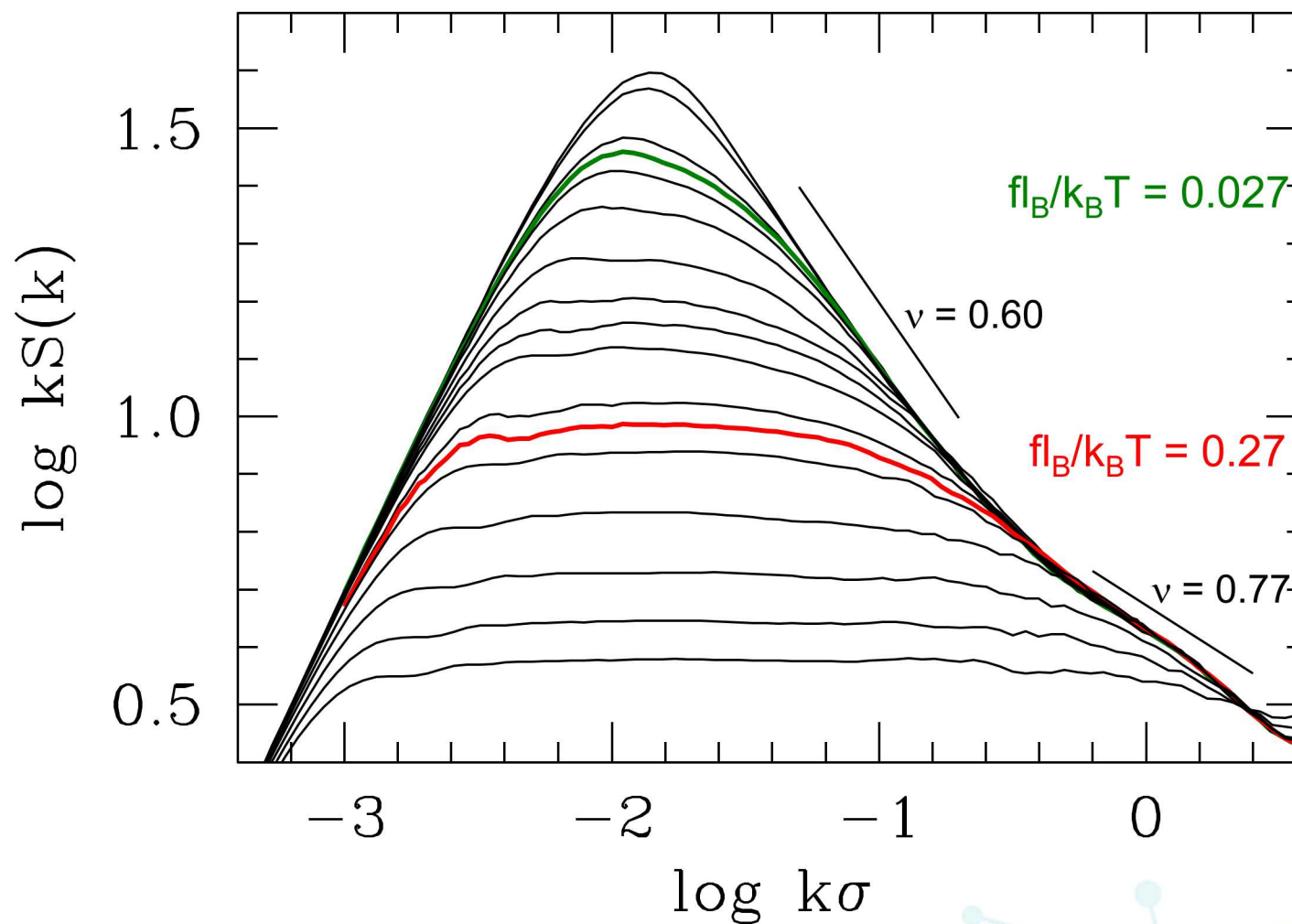
N=5000 ssDNA  
200 mM



# Structure Factor for $N=5000$ at 200 mM



$f l_B / kT = 0.027$    0.112   0.27



Region

Rotational

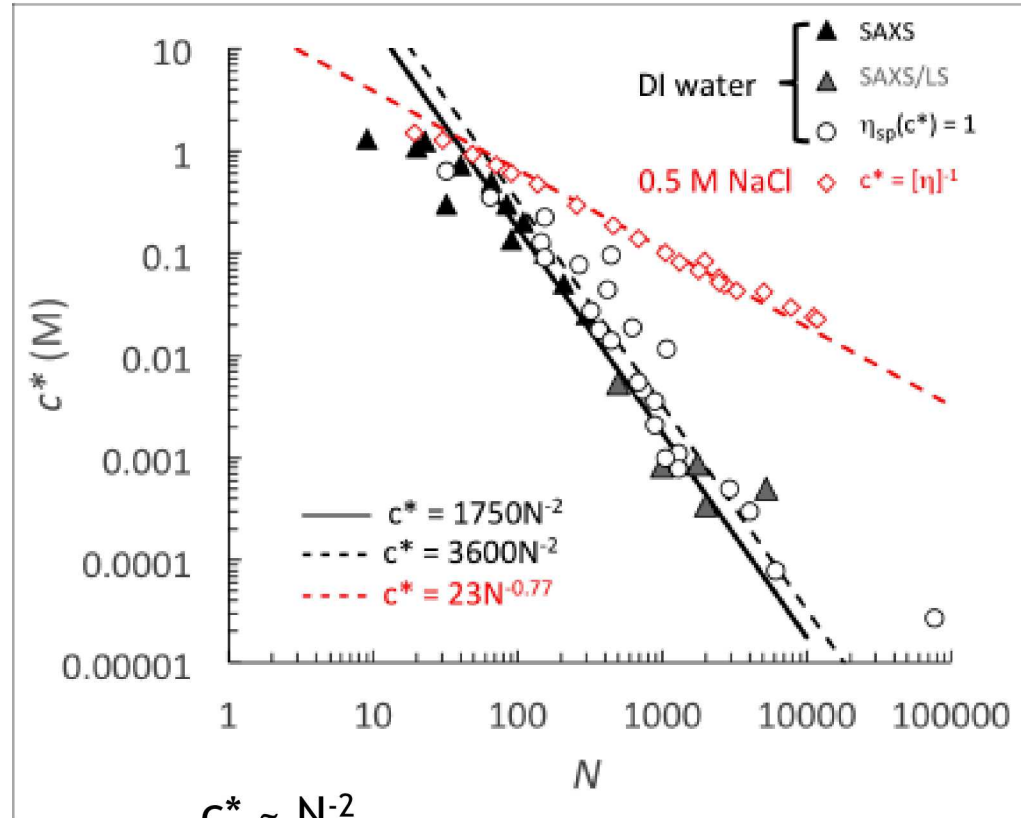
Pincus

logarithmic



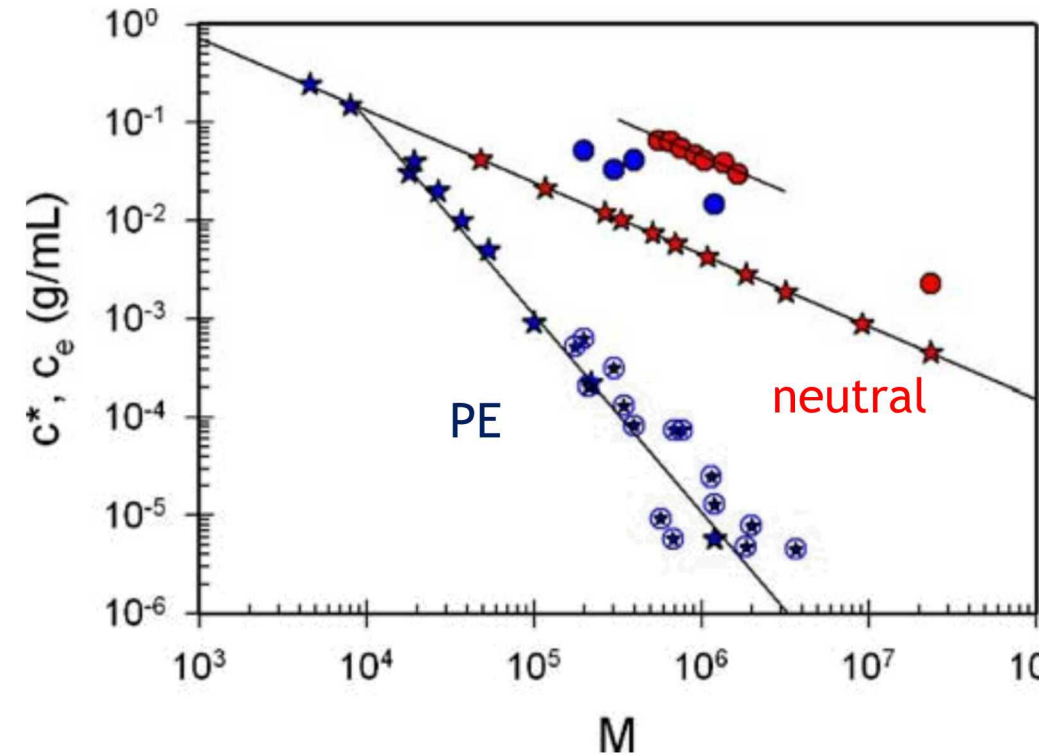
# Polyelectrolytes in Solution: Overlap concentration

Lopez 2019



if  $R \sim N^1$  and  $c^* = MN/V \simeq MN/(M\pi R^3/6)$ , then  $c^* \sim N/N^3 = N^{-2}$

Colby Rheol Acta 2010





# Simulations of Polyelectrolyte Solutions

“All ion” MD simulations

$M = 27, \dots, 800$  number of chains

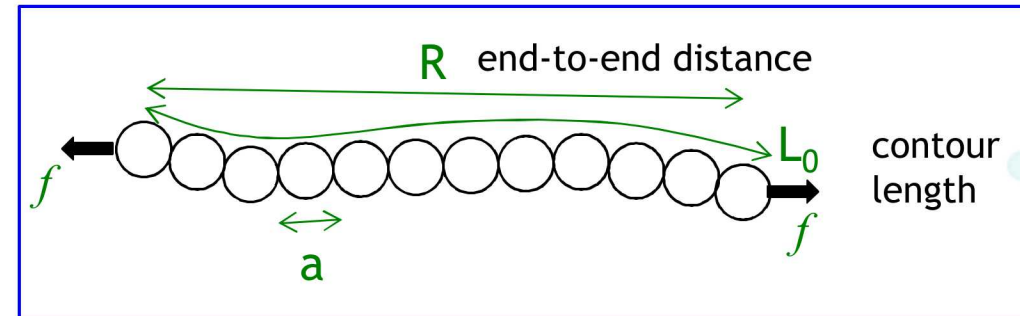
salt free

Bjerrum length = charge separation

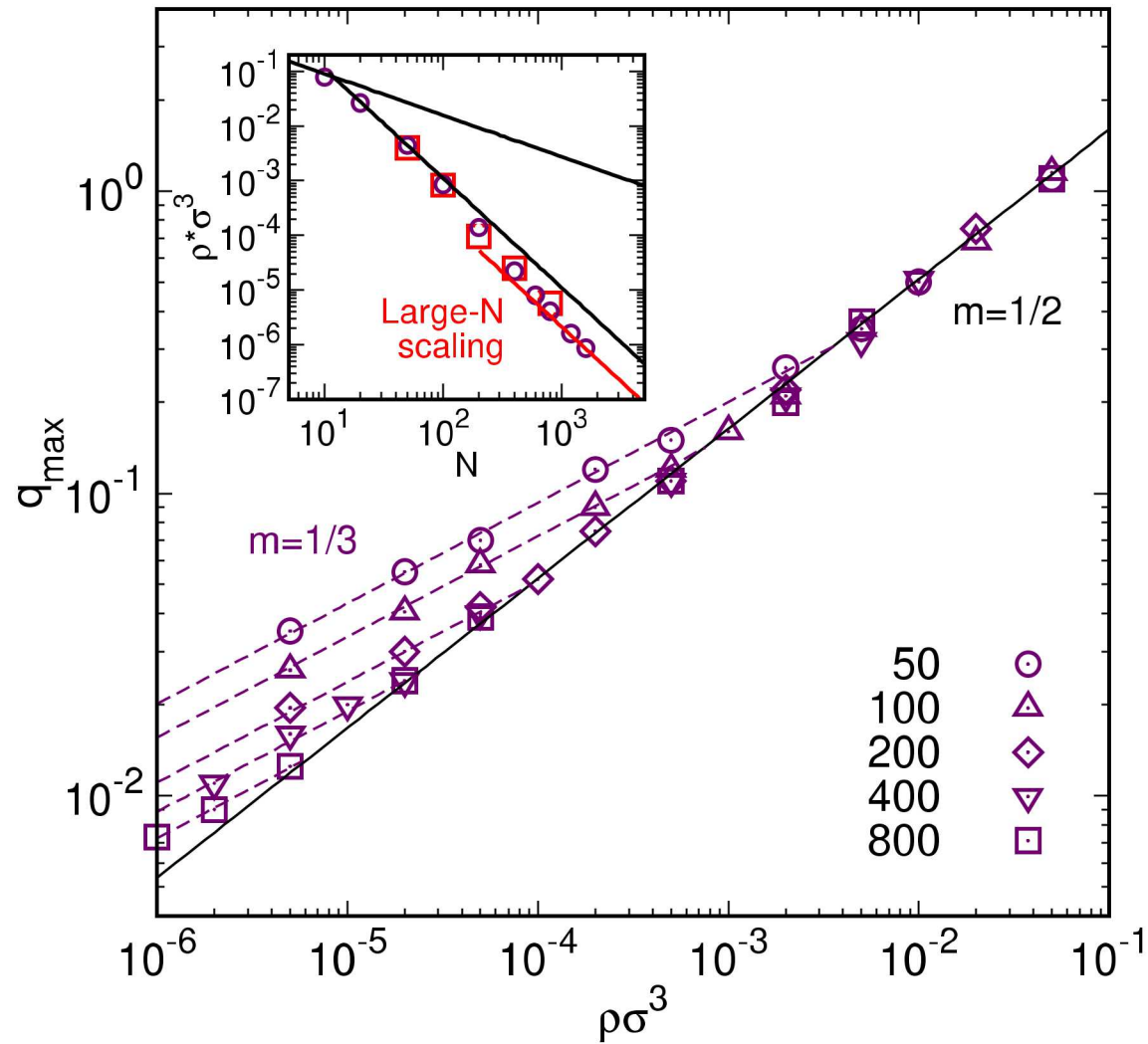
strong polyelectrolyte

$F = \text{bond} + \text{entropy} + \text{electrostatics}$

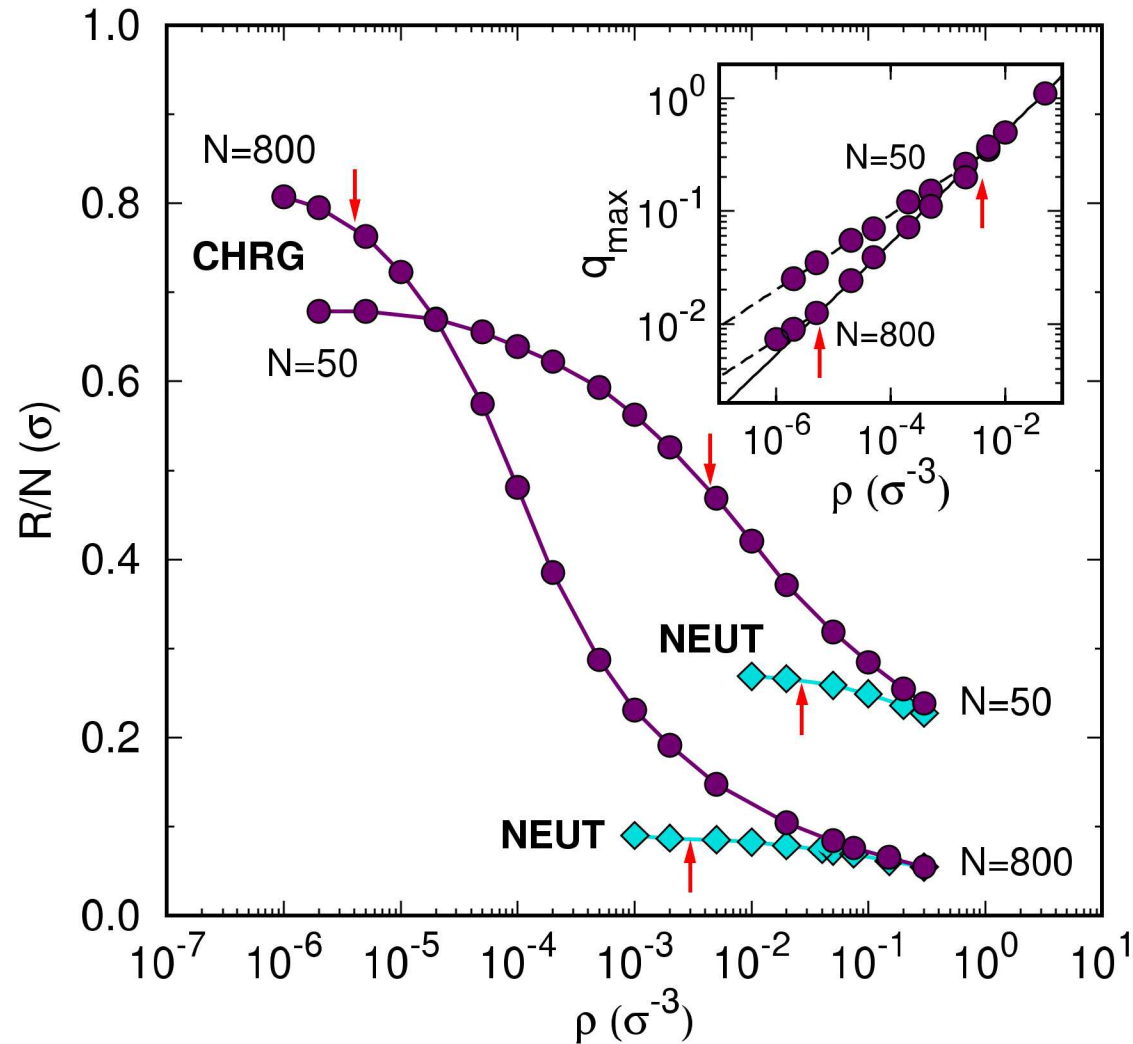
very large  $N$ : up to 1600



# Overlap from Scattering Data in Simulations



# End-to-End Distance



## Overlap Methods

### Scattering:

transition between  $q^{1/2}$  and  $q^{1/3}$

### Packing:

contact between spheres of diameter  $R$   
 packing fraction = random close packed  
 $0.64 = \phi = M \pi R^3 / 6V = \rho \pi R^3 / 6N$

### Comparison:

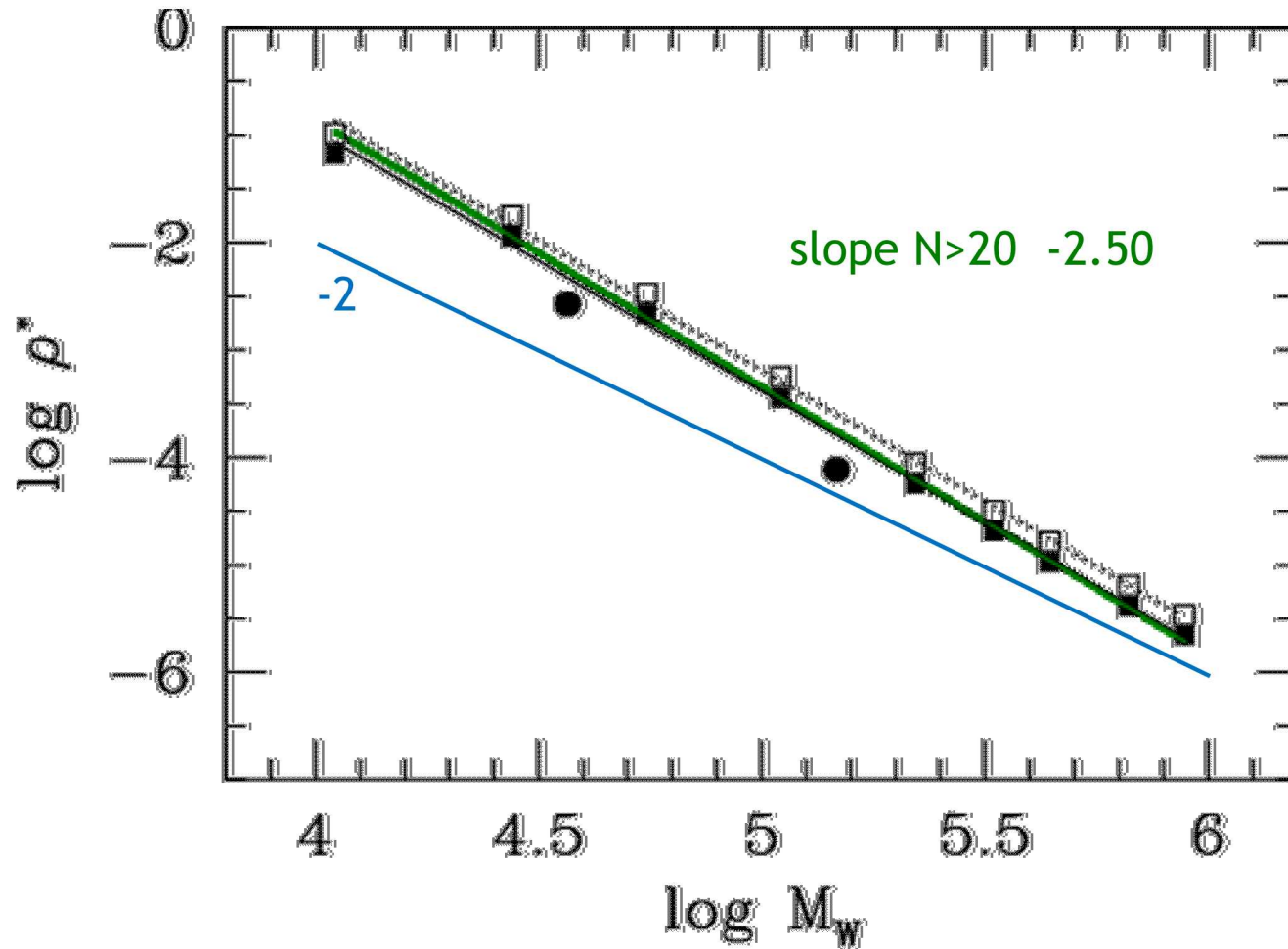
$N=50$

chain expands below  $\rho^*$   
 chain bends in 3 linear segments at  $\rho^*$

$N=800$

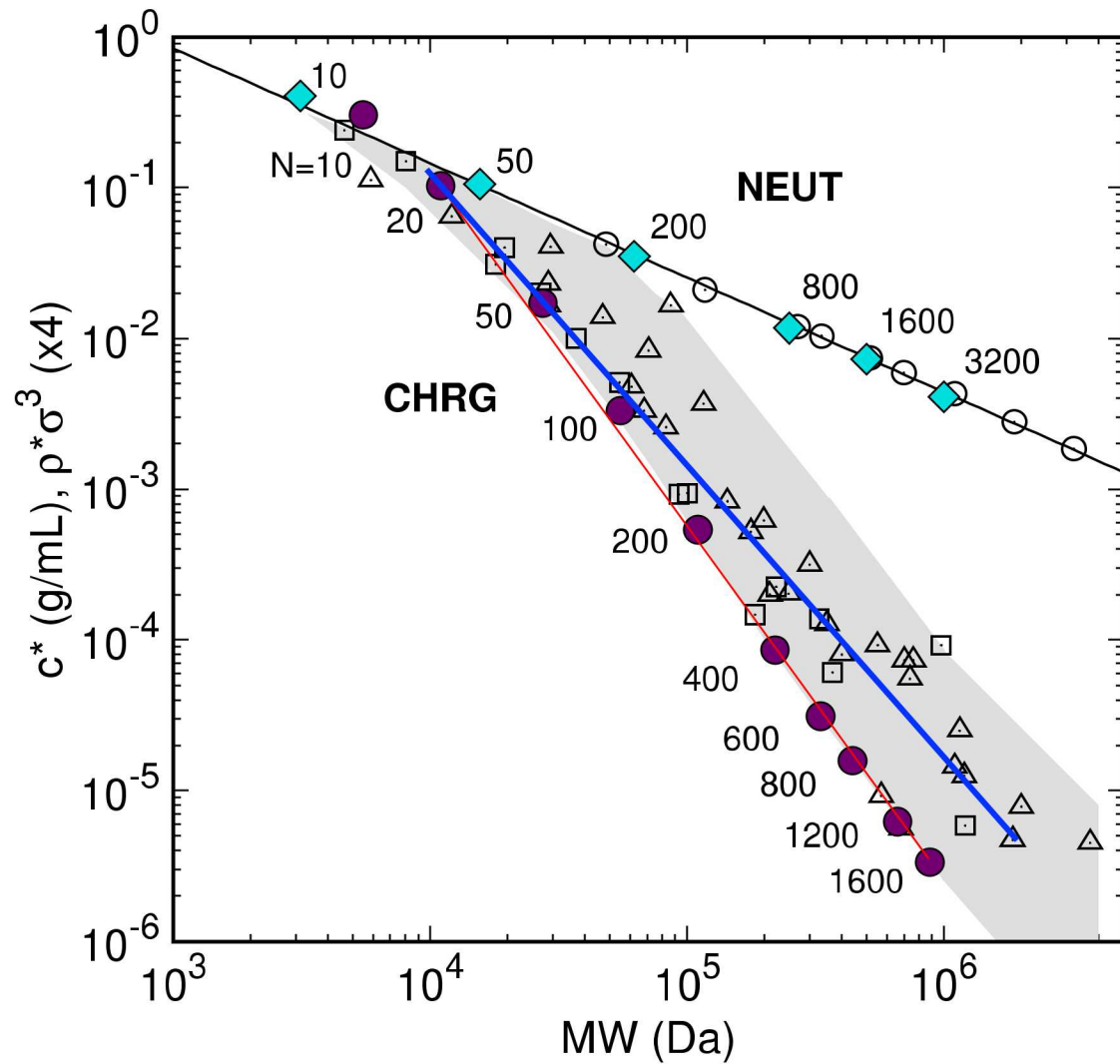
chain barely expands below  $\rho^*$   
 chain is 1 linear segment at  $\rho^*$

# Overlap concentration: Simulation



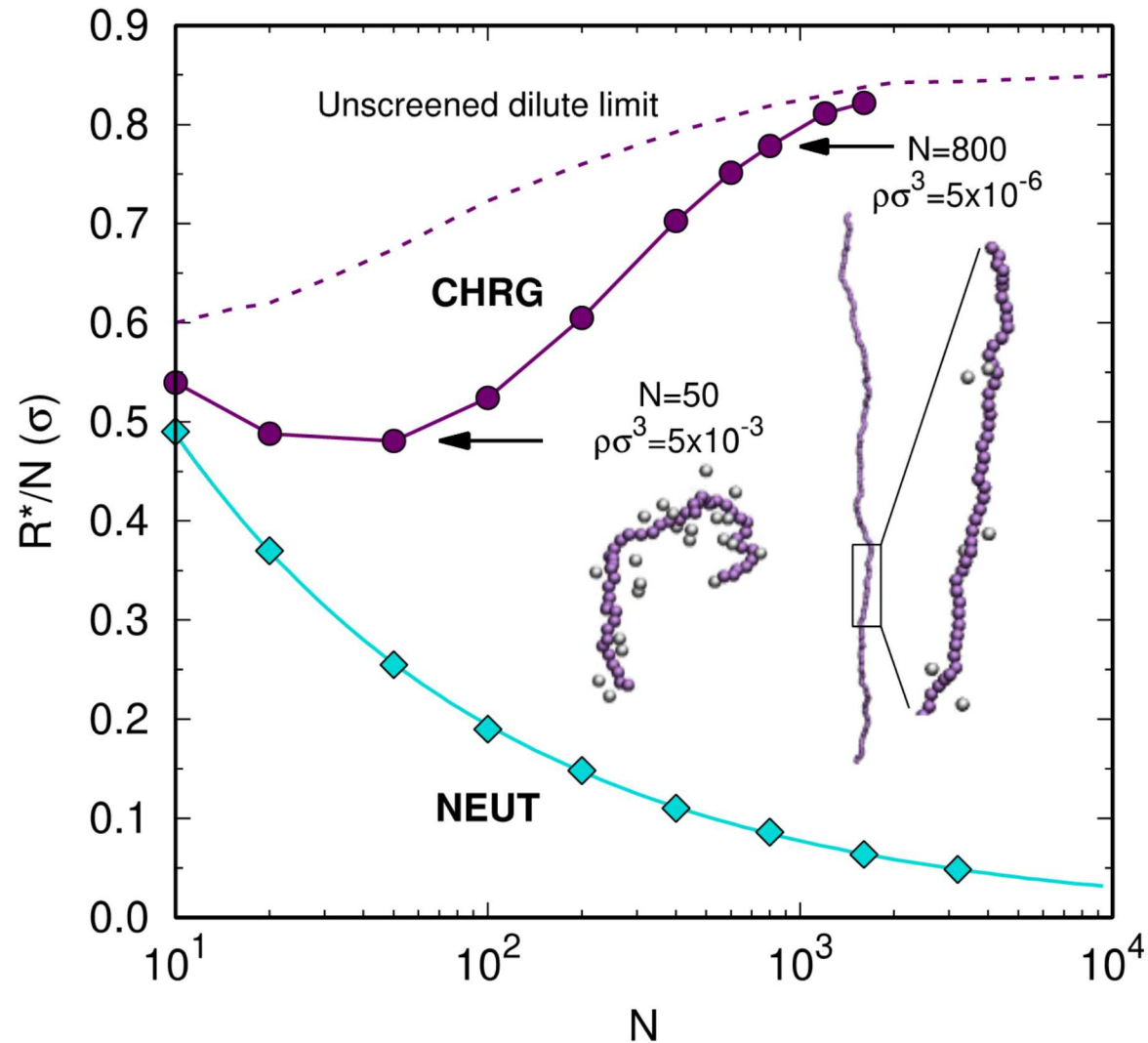


# Overlap Concentration



Map 1: 3 NaPSS monomers to 1 bead

# End-to-End Distance at Overlap vs. N



$N=50$  not fully stretched at  $\rho^*$

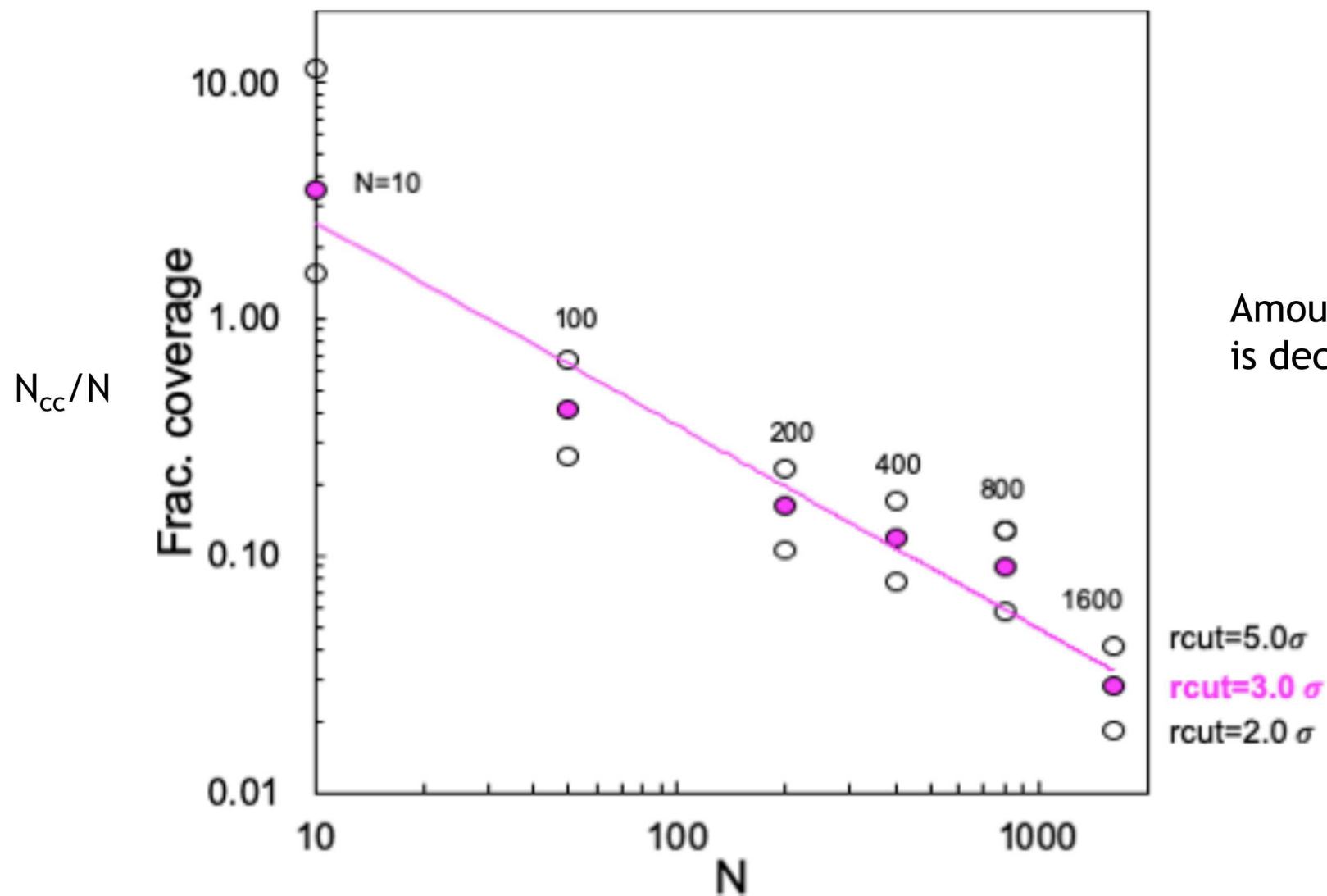
$N=800$  fully stretched at  $\rho^*$

$R/N$  not constant  $\Rightarrow R \sim N^1$

By  $N \simeq 800$  getting close to upper limit

Large  $N$  limit does not occur until large  $N$

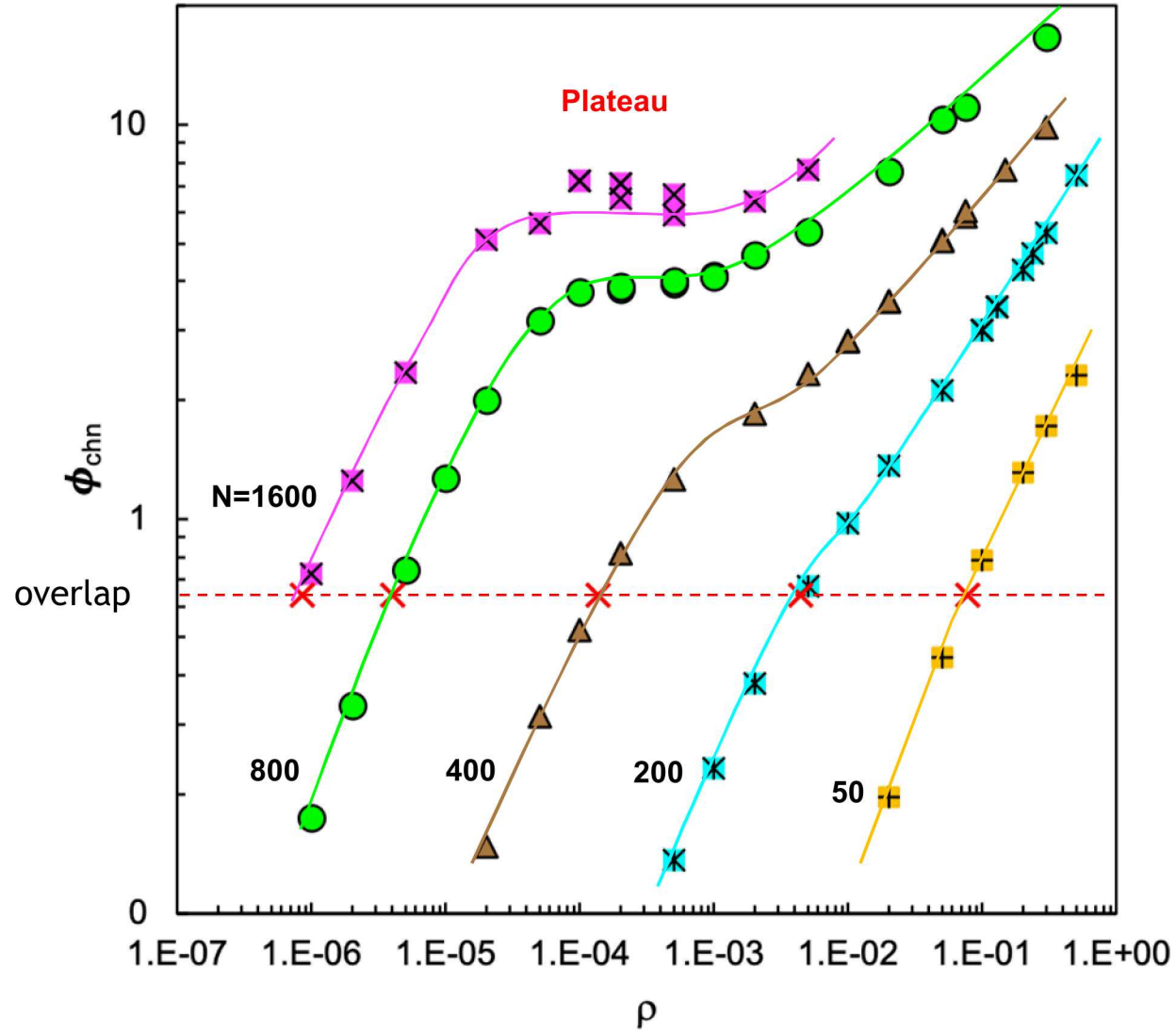
# Counterions



Amount of condensed counterions is decreasing with  $N$  and  $\rho^*$

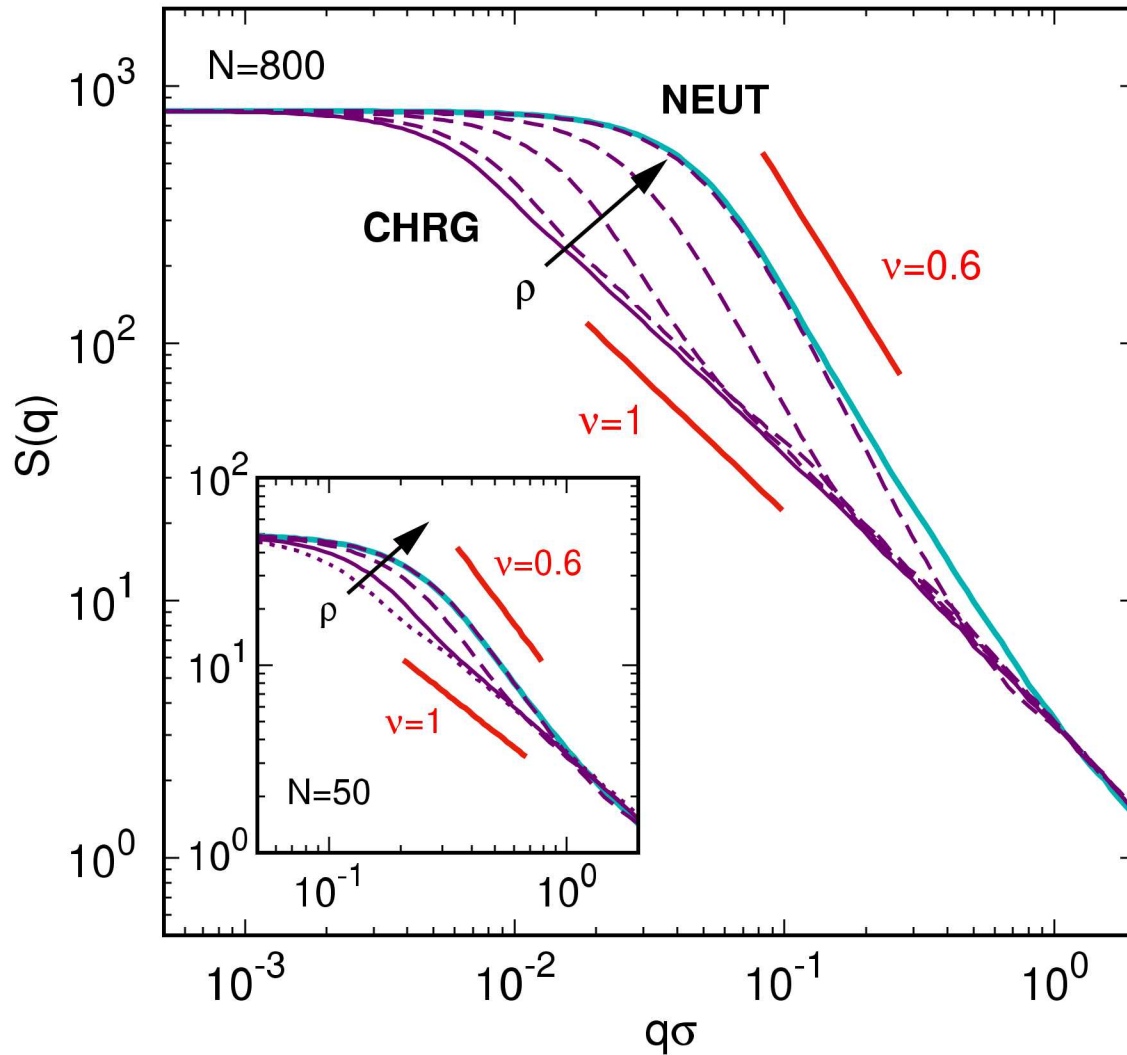
# Packing of chains

$$\phi = M \pi R^3 / 6V$$





# Structure as a function of density



Two changes in  $S(q)$  as a function of decreasing  $\rho$ :

- 1) long  $q$  dependence changes from neutral behavior toward  $\nu \simeq 1$
- 2) transition point decreases => 'rodlike' segment get larger

Rodlike for lengths  $\ell < L_t$   
 $L_t$  increases as  $\rho$  decreases

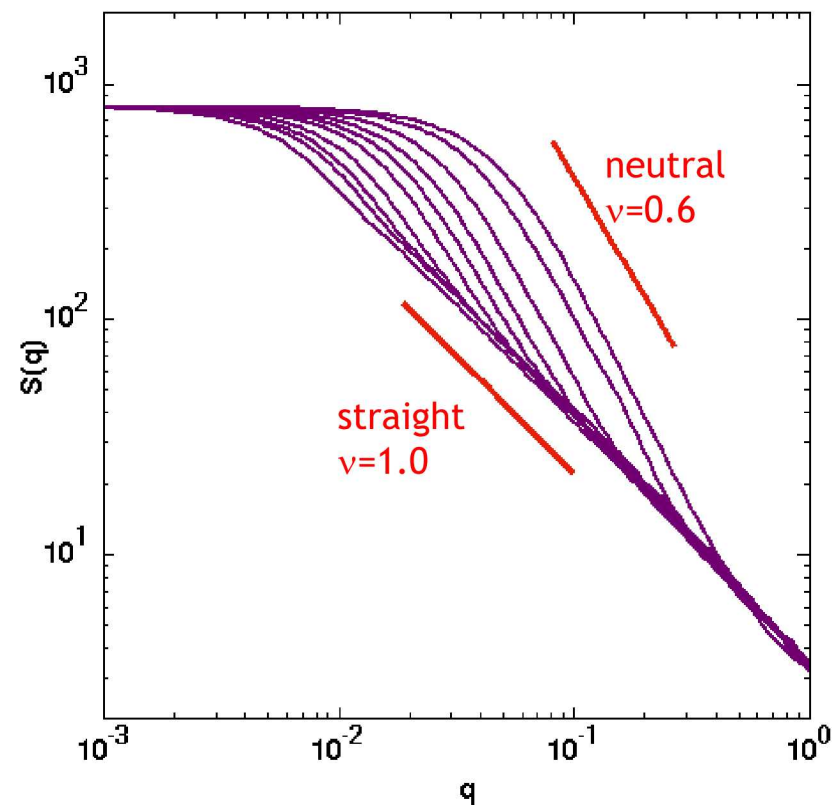
# Local Chain Structure

Single chain structure factor  $S(q)$   
 $N=800$   
density dependence

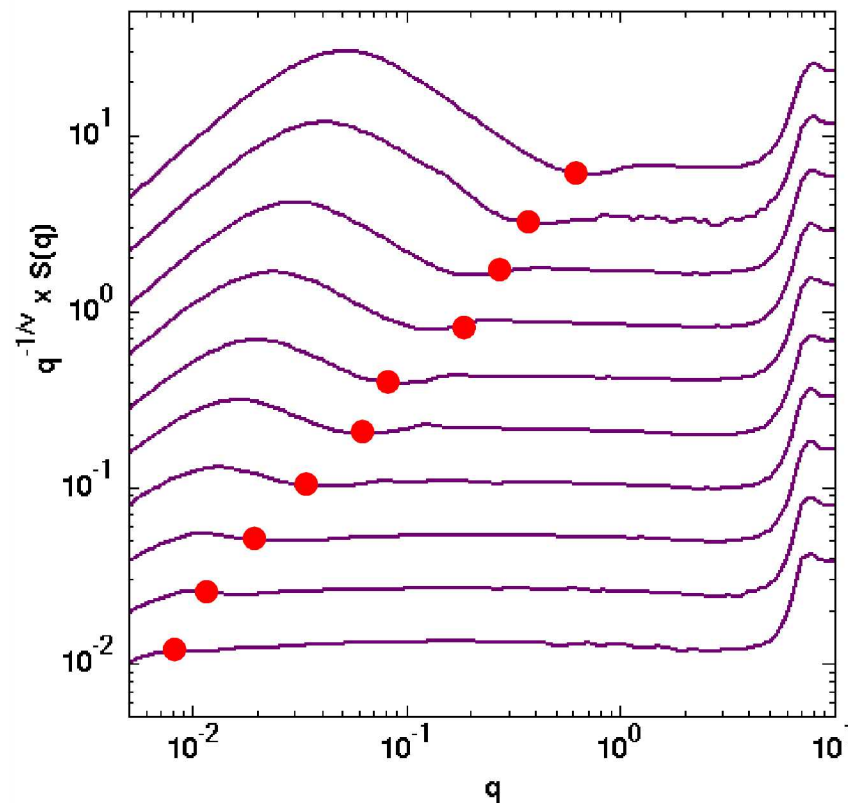
Two  $q$ -dependences (high/low)  
 $q_r = 2\pi/L_r$  is transition point  
 $L_r$  is the  $\sim$ straight segment length

Chain is composed of  $\sim$ straight segments that form a directed walk.

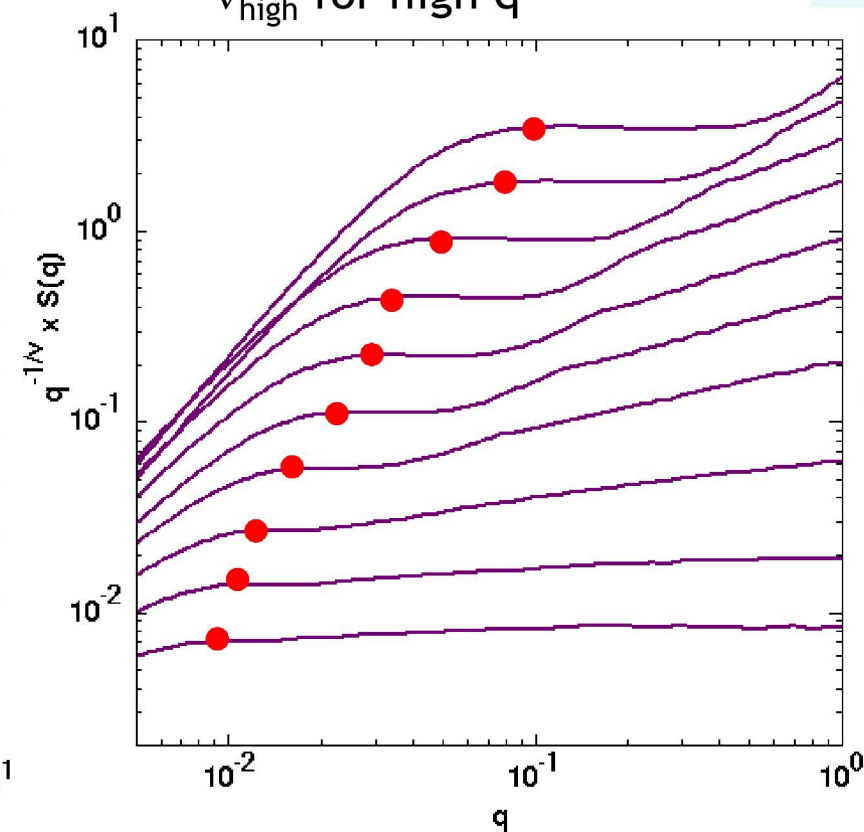
$$S(q) \sim q^{1/\nu}$$



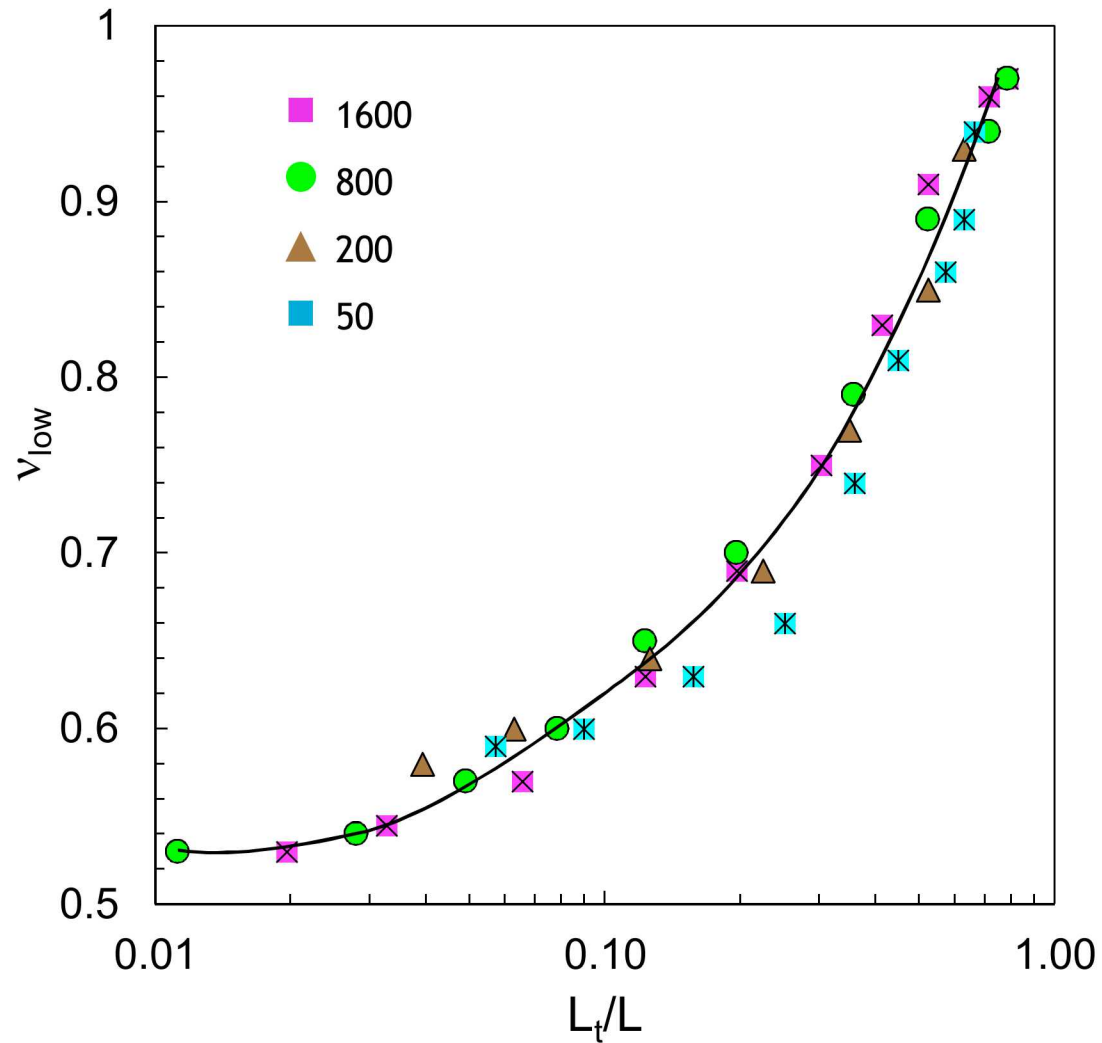
$\nu_{\text{low}}$  for low  $q$



$\nu_{\text{high}}$  for high  $q$



# Large length scale behavior

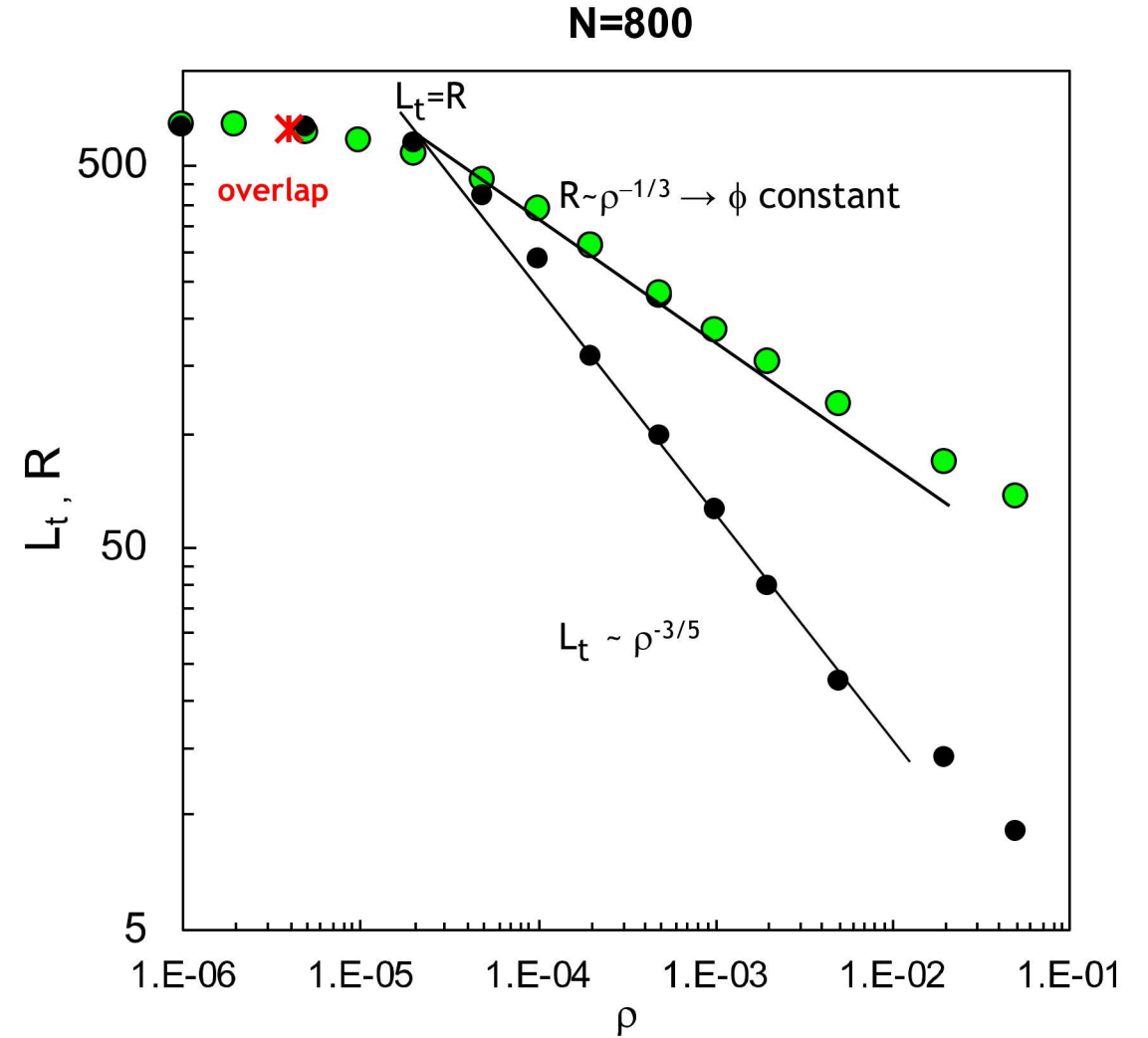
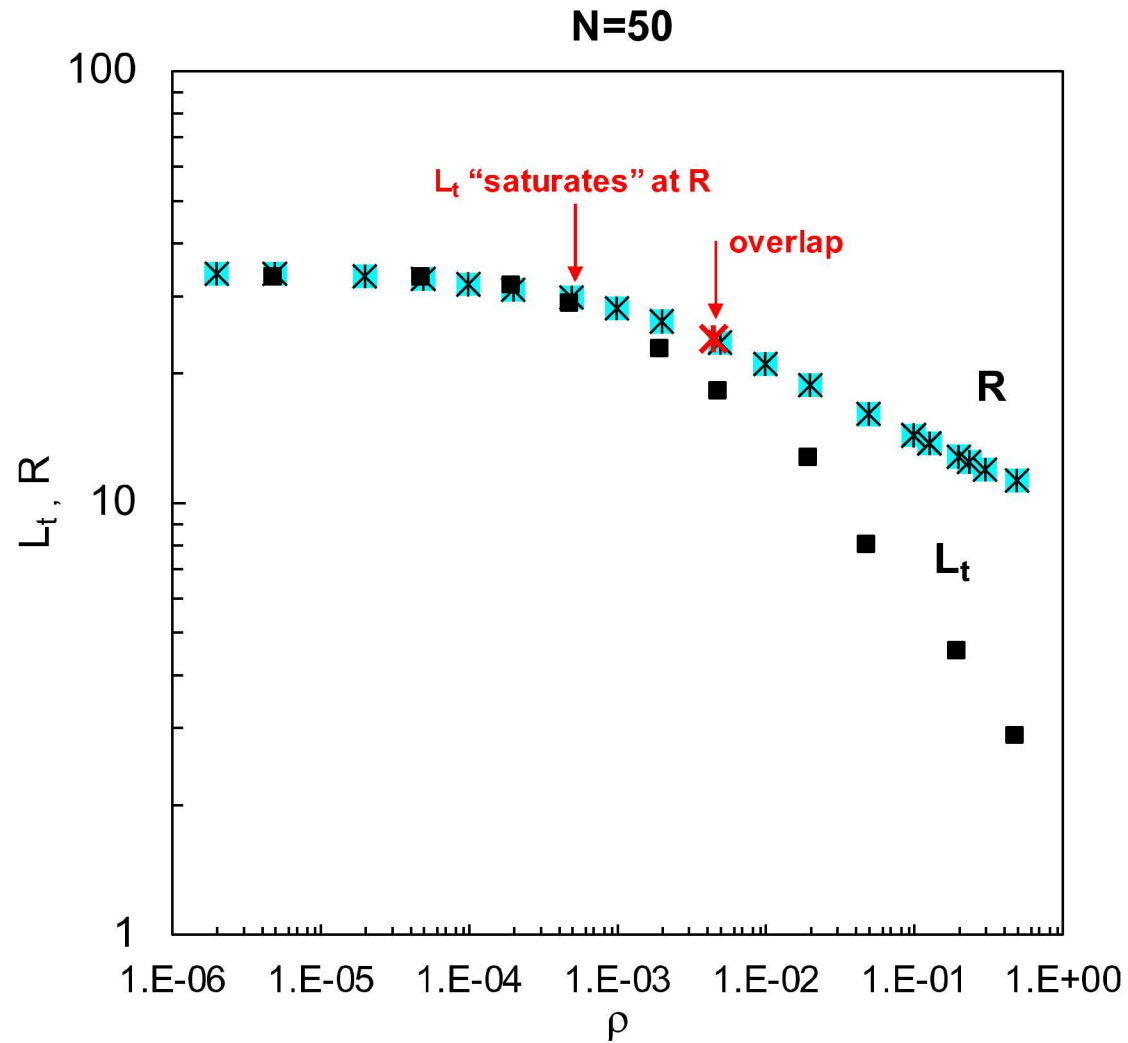


Transition from random walk at high density  
to directed walk  
just straight segment at large enough  $N$

Maybe at large  $N$  ( $\gtrsim 800$ ) overlapping behavior?

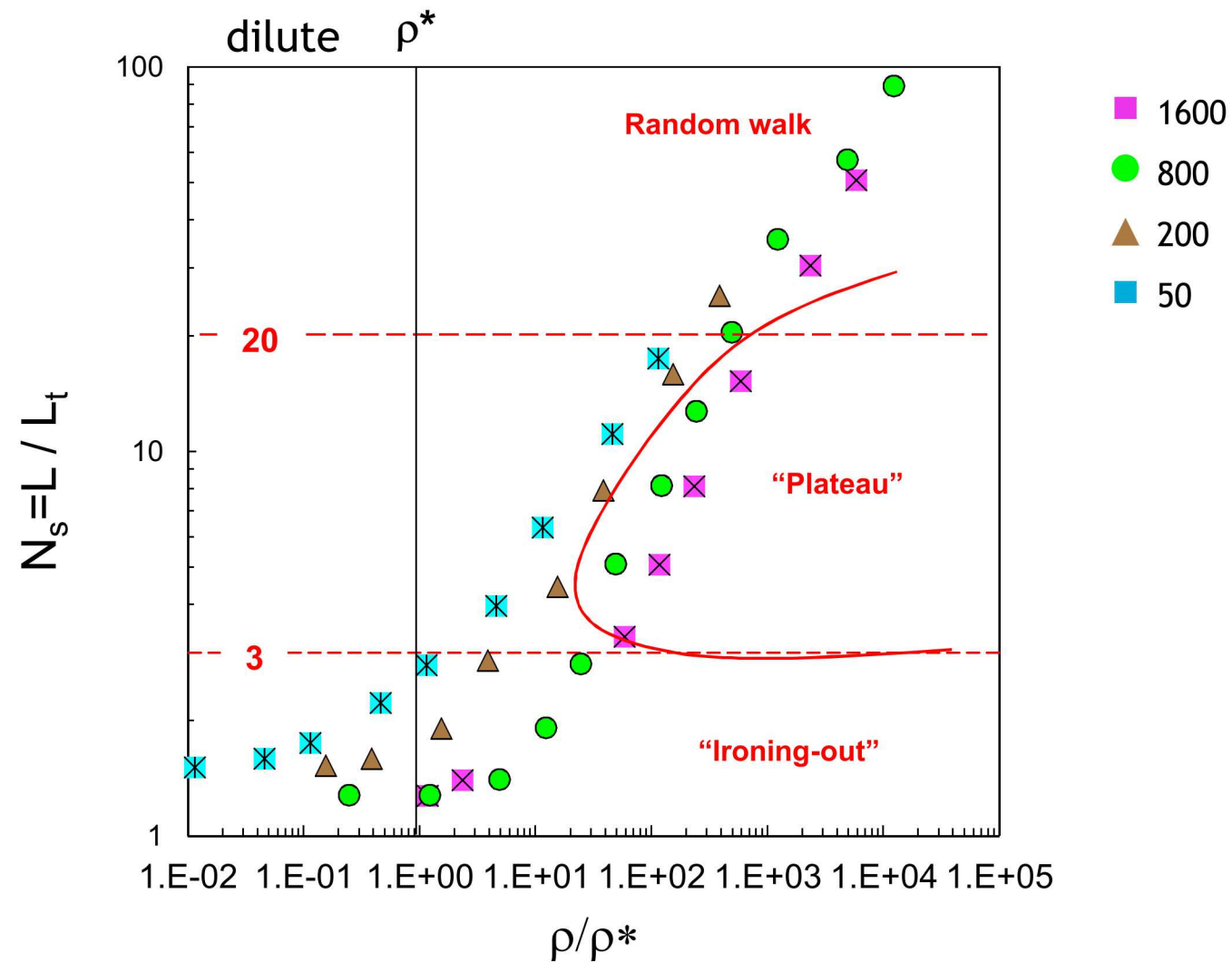
$v_{\text{high}}$  starts at 0.90 and goes to 0.97

# Segment length $L_t$ as a function of density





# Density Dependence: Number of Segments



# Summary



Extended simulations out to much larger  $N$  (1600) and very much lower  $\rho$

Find  $c^* \sim N^{-2.5}$  over the range of  $N$   
reaching the unscreened dilute limit  
at very large  $N$  ( $= 800$ )  
shouldn't have expected  $N^{-2}$

The chain structure at  $c^*$  varies significantly.  
For  $N \geq 800$ , chains are 'fully' stretched at  $c^*$   
 $S(q) \sim q^{-1/\nu}$  with  $\nu = 0.95+$

Density dependence of chain structure  
Two regimes: straight segments and directed walk  
Straight segment length increases with decreasing density  
Results imply simple description possible

Need better experimental data for  $c^*$   
As always would like larger  $N$  for simulations

# The Collaboration

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Jon Bollinger, SNL  
Gary Grest, SNL  
Michael Rubinstein, Duke



