Variations in Damage from Same Shock Response Spectra

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Design Issue:

Shock Specifications do not capture durations or decay rate adequately

Study Objectives:

For same SRS, how is Damage affected by varying the decay rate of different component frequencies?

What causes differences in SRS and 'Damage' as decay rate changes?

How does a change in 'Damage' affect Probability of Failure (P99 etc)?

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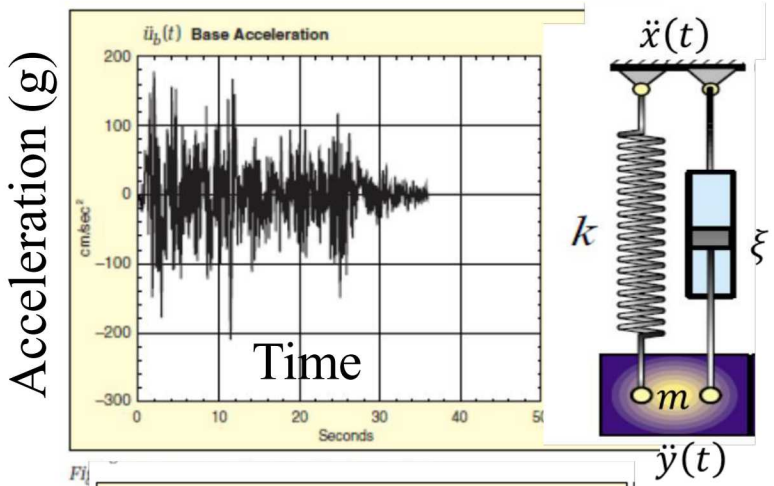
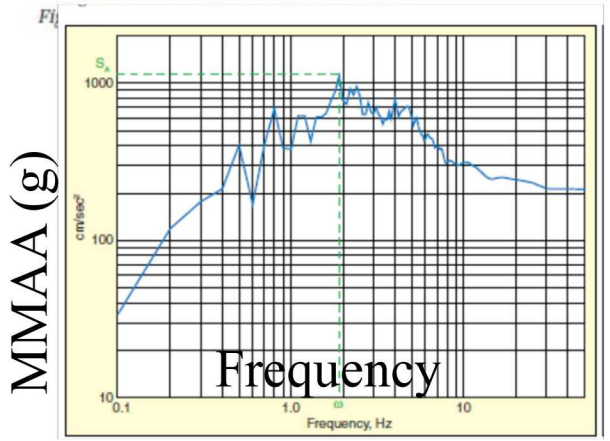
Theoretical Background

Shock Response Spectra (Biot M, 1932)



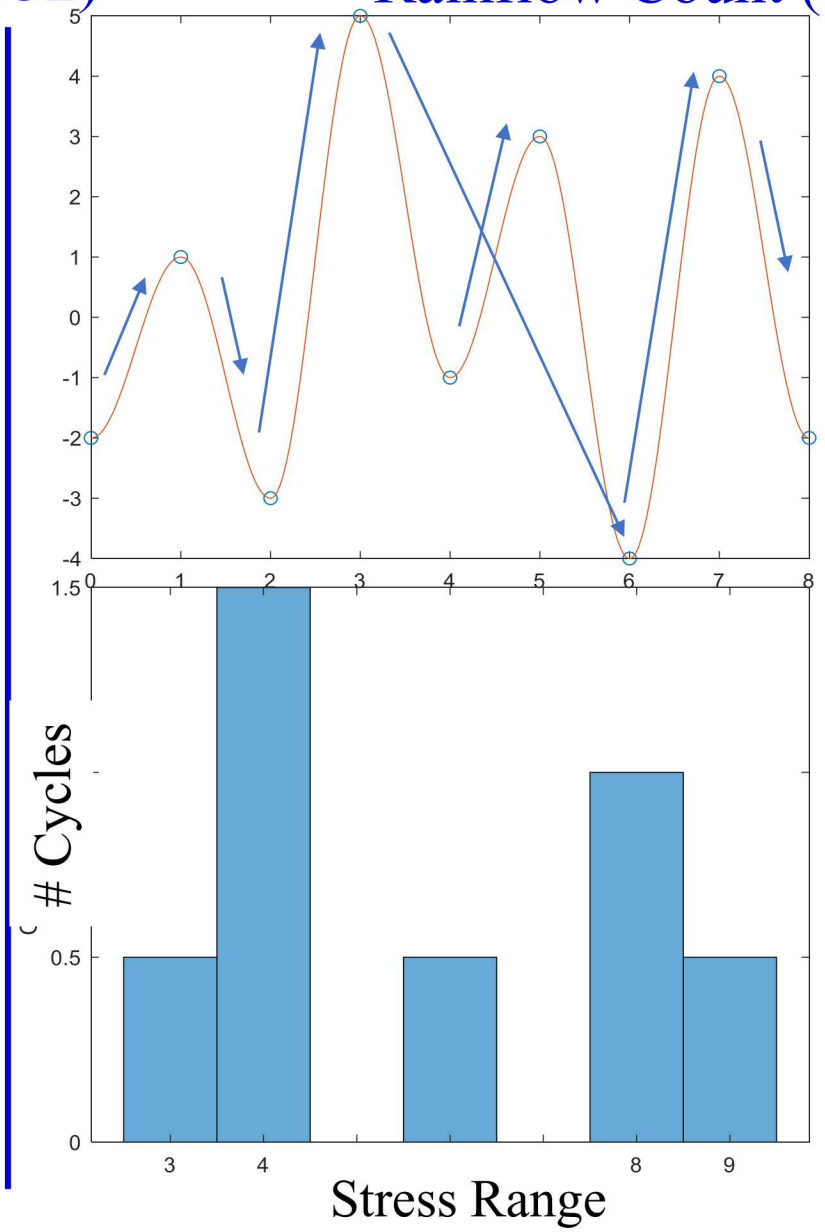


$\ddot{y}(t)$ Decay rate of Input?

$$= \frac{\omega}{\sqrt{1-\xi^2}} \int_0^t \ddot{x}(\tau) e^{-\xi\omega(t-\tau)} [(1-2\xi^2)\sin\omega\sqrt{1-\xi^2}\,(t-\tau)$$

$$+ 2\xi\sqrt{1-\xi^2}\cos\omega\sqrt{1-\xi^2}\,(t-\tau)]d\tau$$

Rainflow Count (©Matlab)

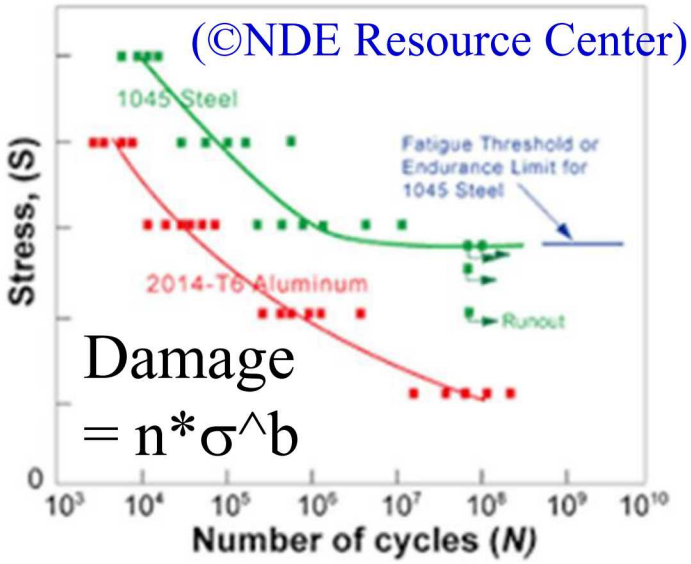


T=7×5 table (Histrogram c values)

Count	Range	Mean	Start	End
0.5	3	-0.5	0	1
0.5	4	-1	1	2
1	4	1	4	5
0.5	8	1	2	3
0.5	9	0.5	3	6
0.5	8	0	6	7
0.5	6	1	7	8

Fatigue Life (statistical)

(©NDE Resource Center)



Damage = n*σ^b

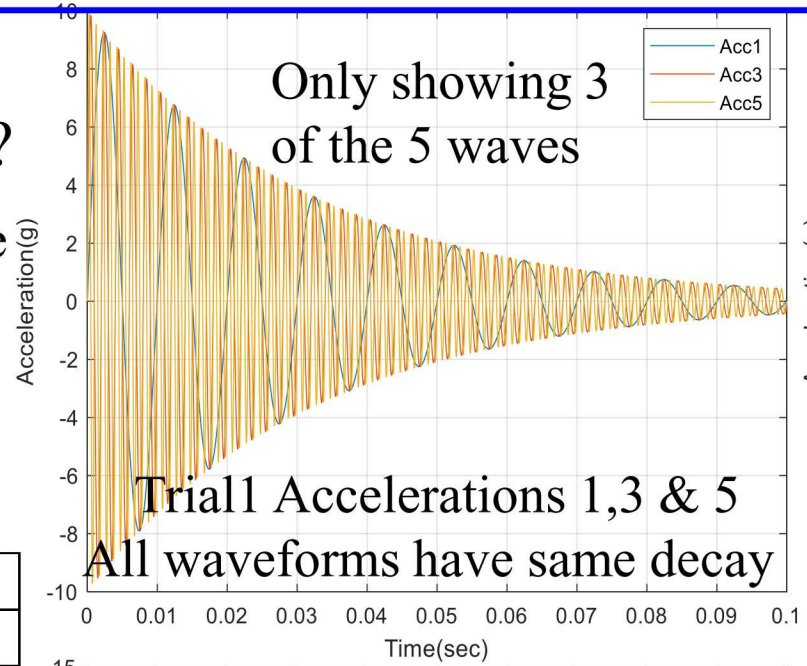
Comparing 2 Waveforms with 5 Frequencies: Trials 1 & 2

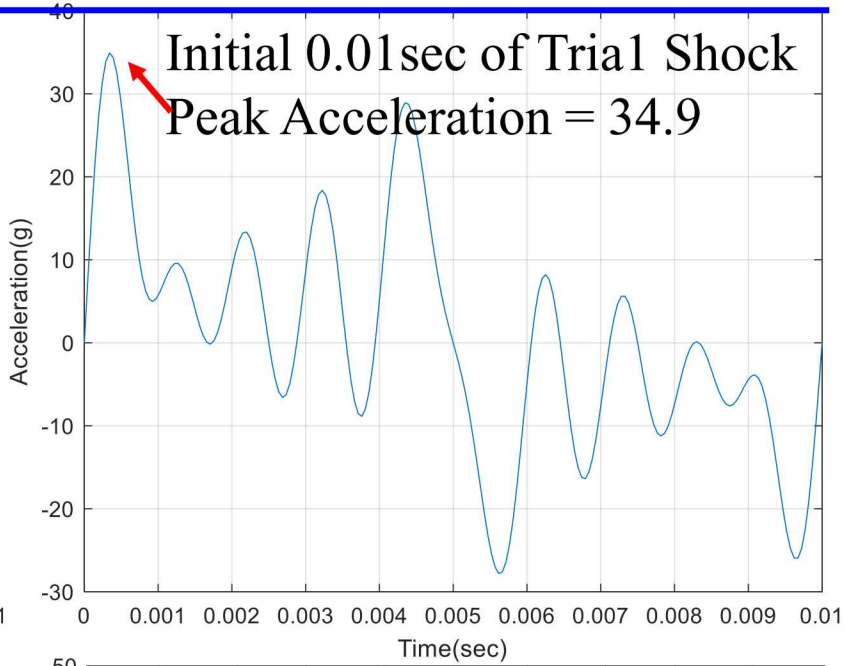
For same SRS, how is Damage affected by varying the decay rate?

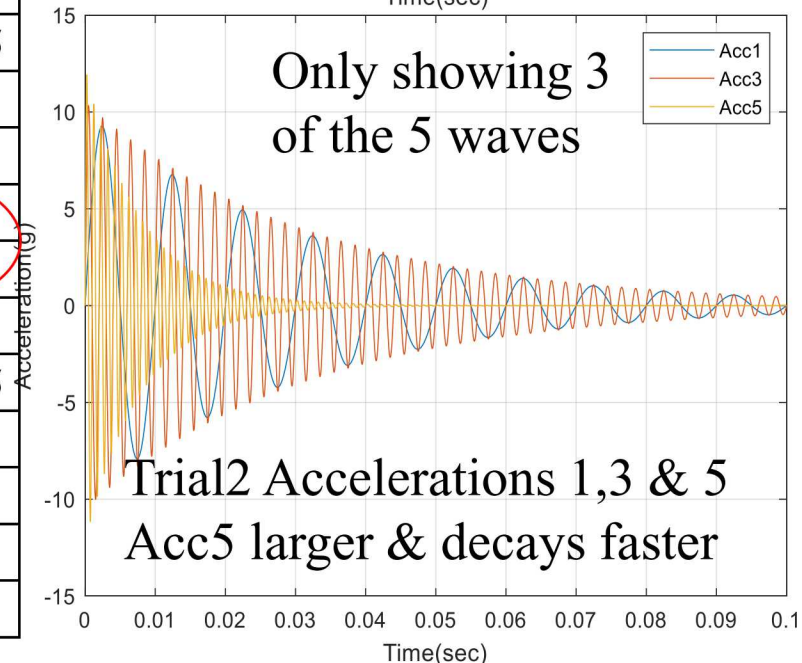
Trial 1: 5 Frequencies of same amplitude (100, 300, 500, 700 & 1000 Hz)
Trial 2: Damping of 700Hz & 1kHz increased by X4

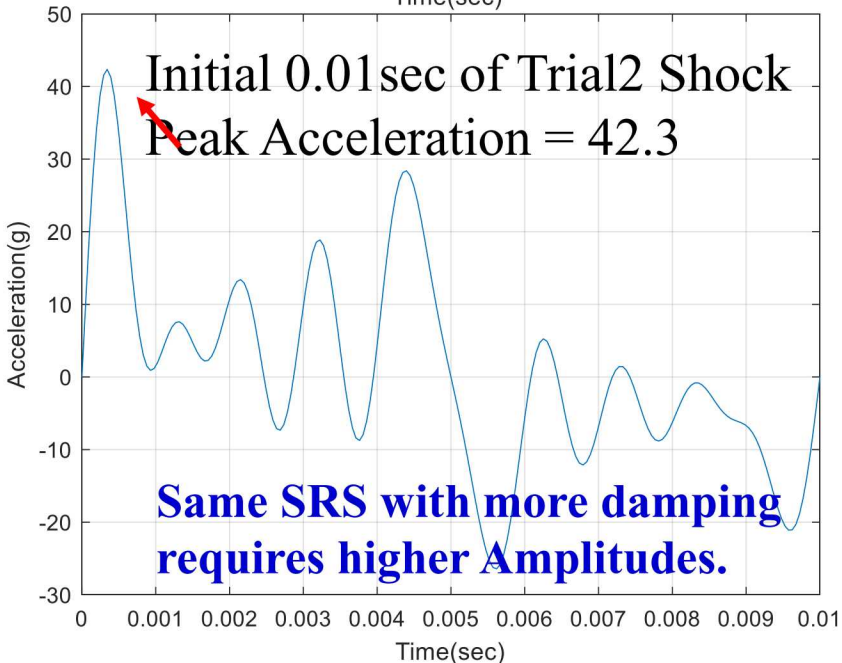
ξ x freq=$\alpha/2\pi$= Constant Exp Decay

Freq (Hz)	100	300	500	700	1000
Trial 1: Proportional damping to get same decay					
Damping	0.05	0.01667	0.01	0.0071	0.005
Amplitude	10	10	10	10	10
Trial 2: X4 damping of 700 & 1000 Hz					
Damping	0.05	0.01667	0.01	0.0286	0.02
Amplitude	10	10	10.5	16	12.3
Trial 3: X4 dampling of 100 & 300 Hz					
Damping	0.2	0.06668	0.01	0.0071	0.005
Amplitude	22.5	16	10.5	10	10
Trial 4: Adjust Damping & Amplitude to Trial 2					
Damping	0.05	0.01667	0.007	0.0286	0.02
Amplitude	10	10	10	16	12.3



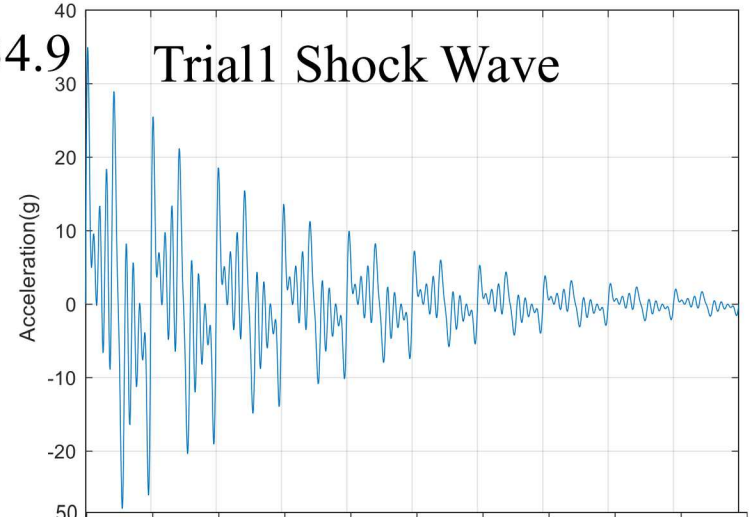






Same SRS but Different Damage: Trials 1 & 2

34.9



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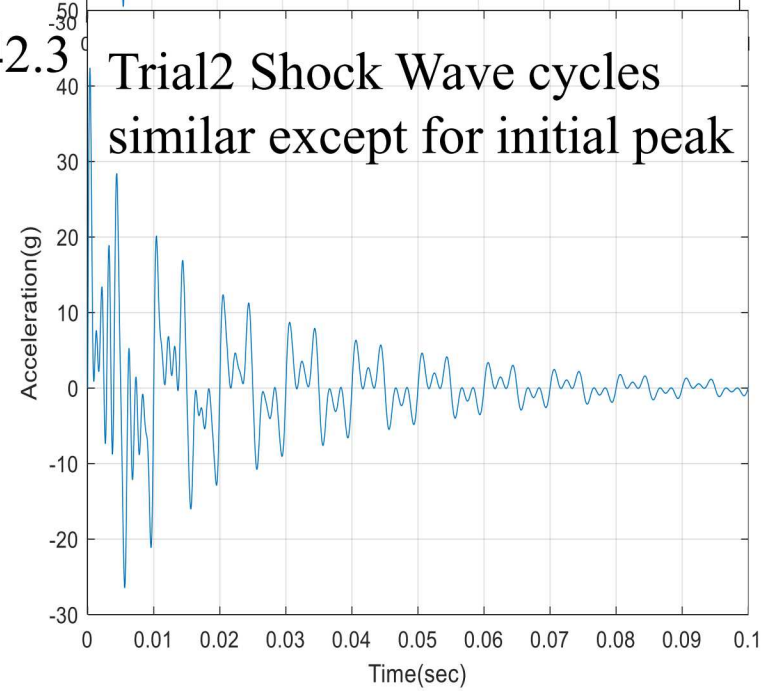
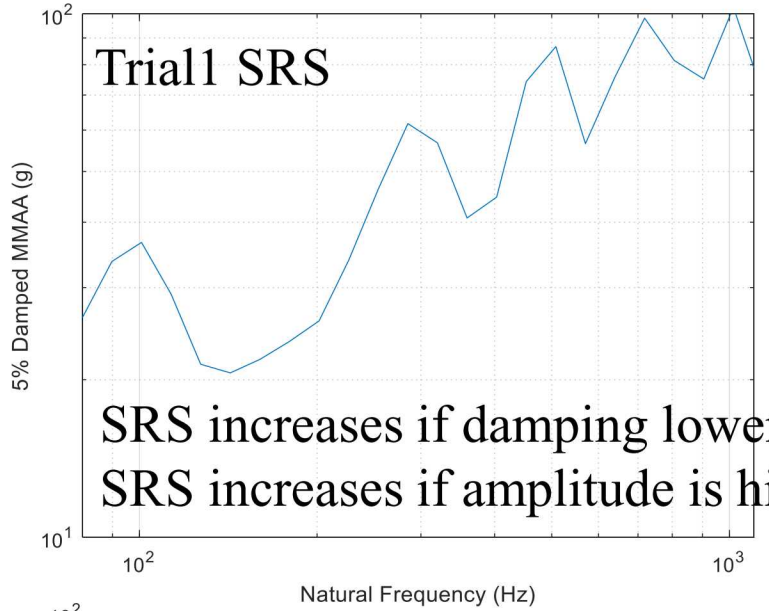
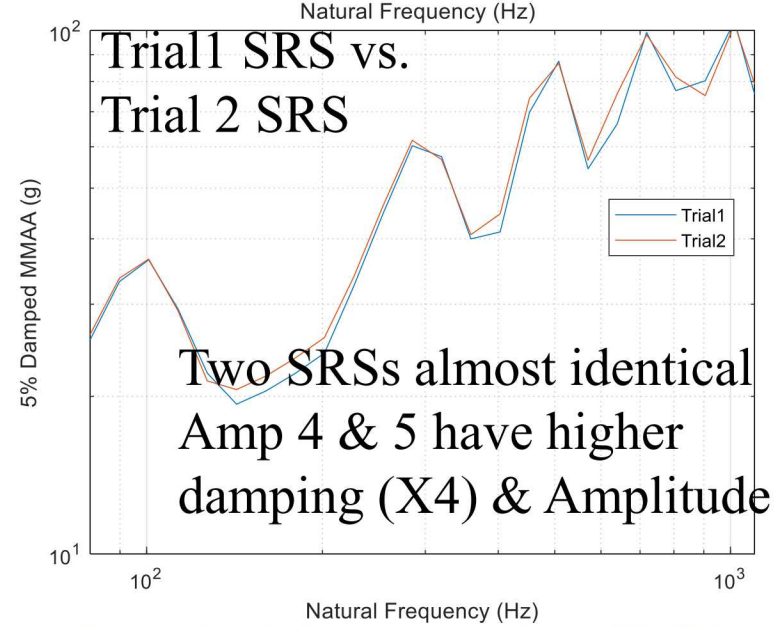
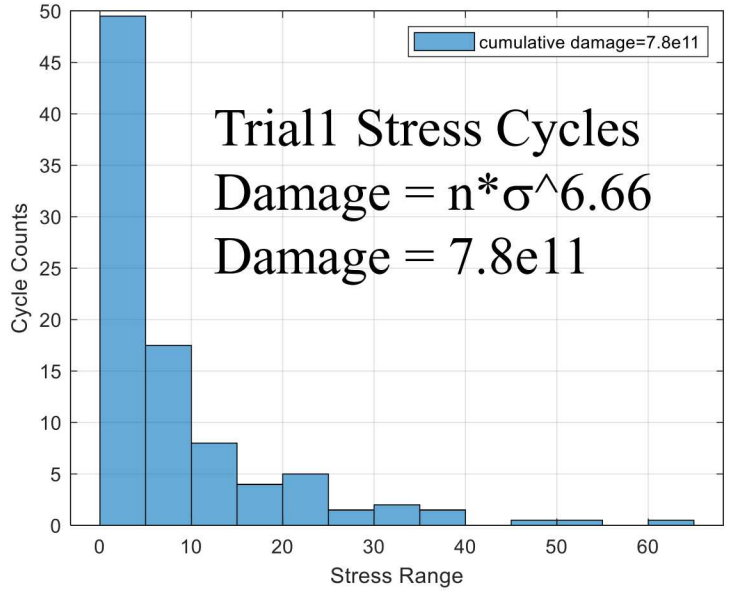
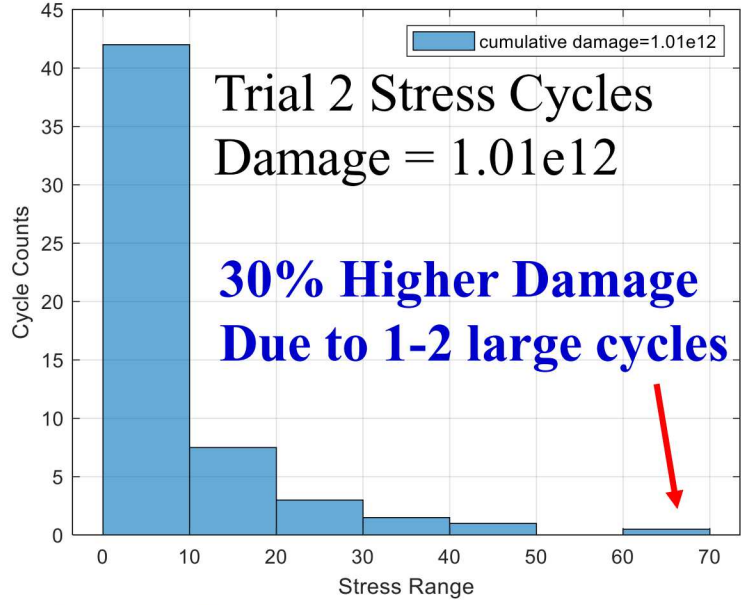




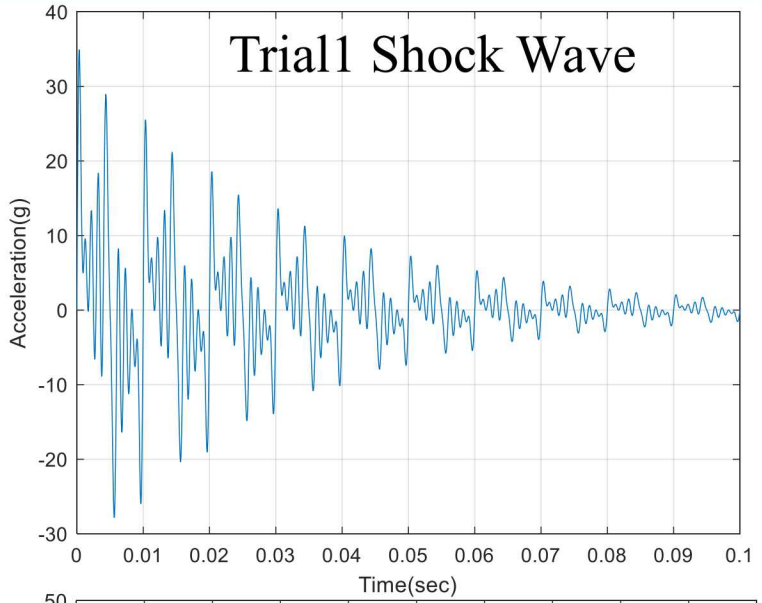




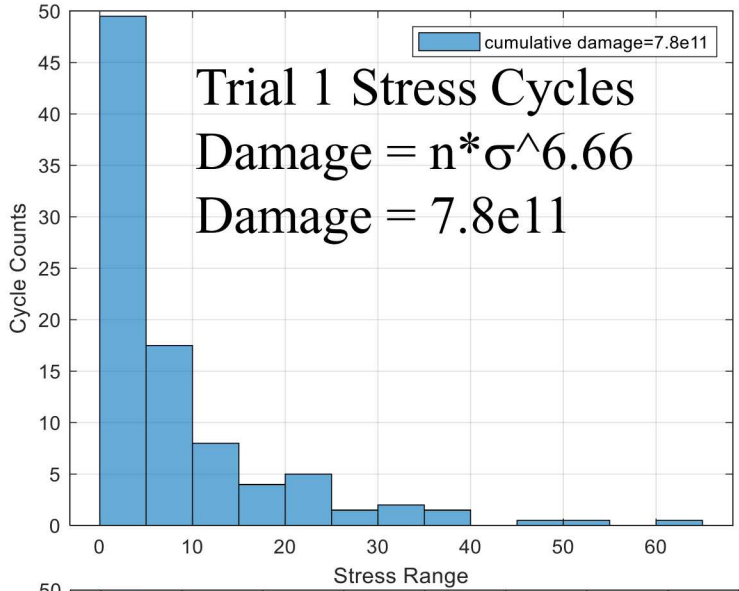


Assuming higher duration (=smaller damping) in recreating SRS leads to lower Damage - unconservative

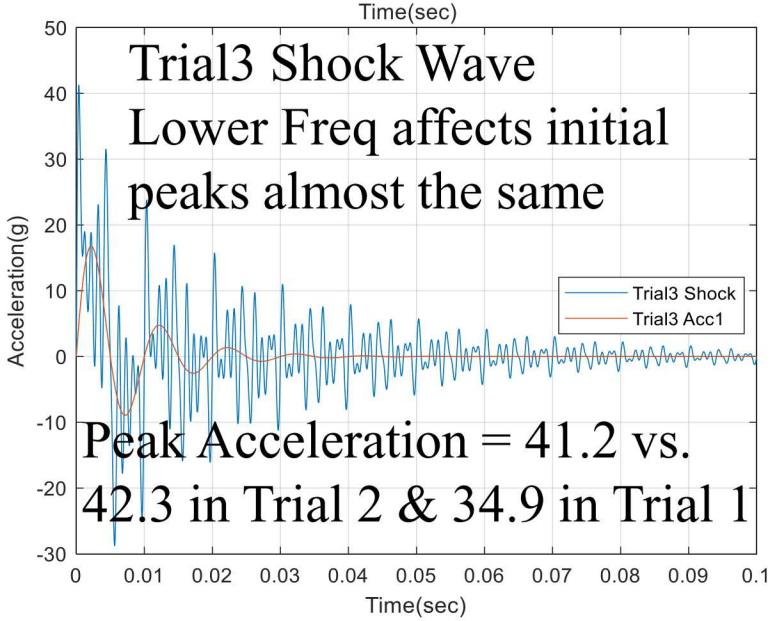
Same SRS but Different Damage: Trials 1 & 3

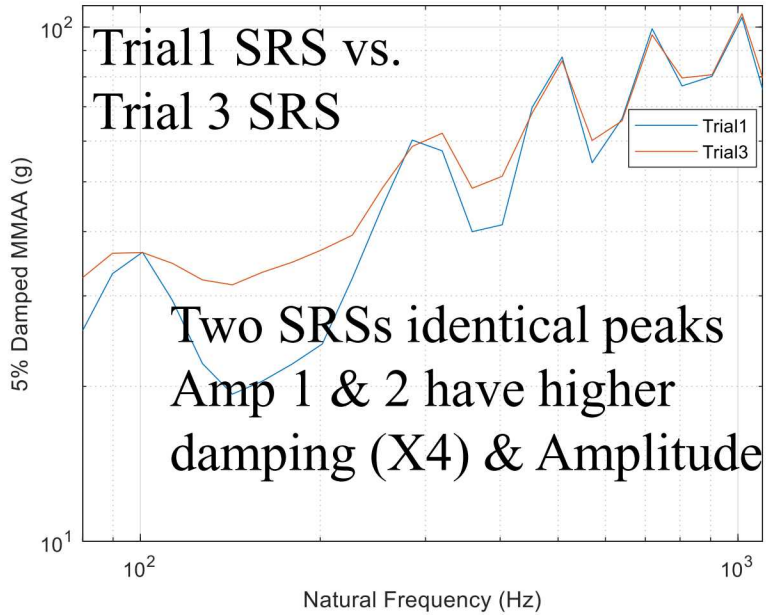


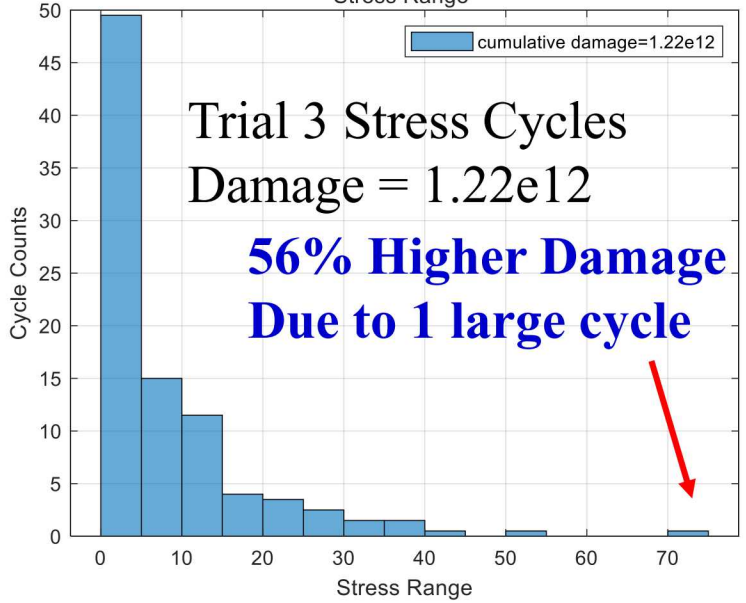
Trial 3: Damping of 100 & 300 Hz increased by X4



It is common practice to match just the peaks during reconstruction of acc. vs. time from an SRS.

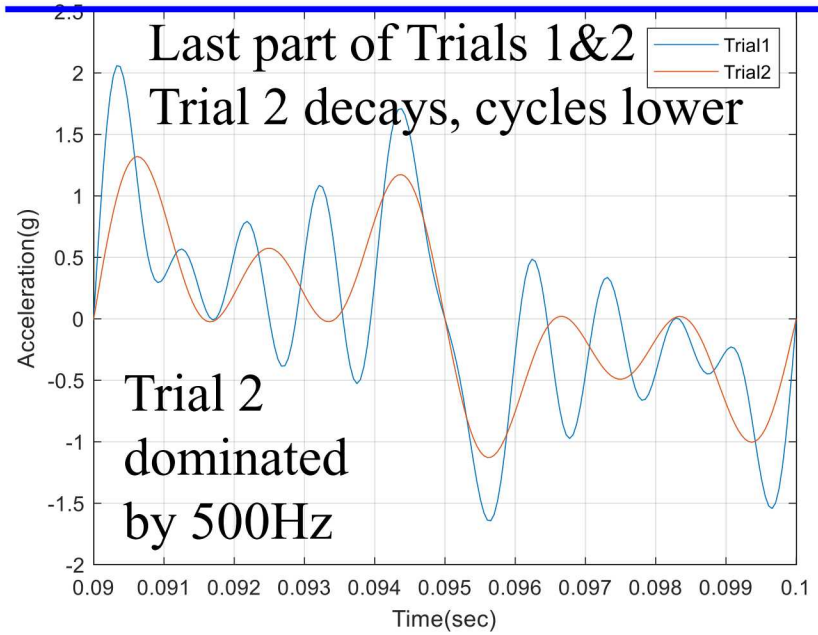




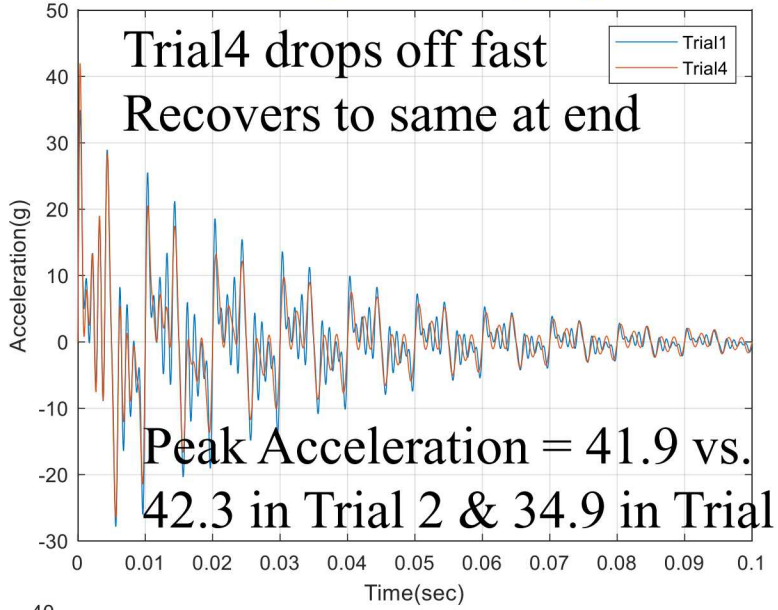


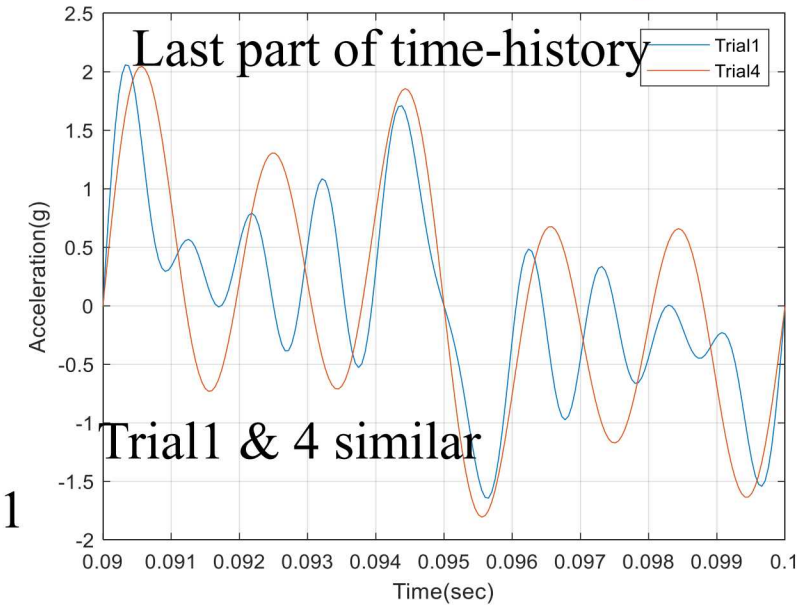
Increasing damping for 2 lower frequencies, Max Acc increase similar & Damage increase is higher

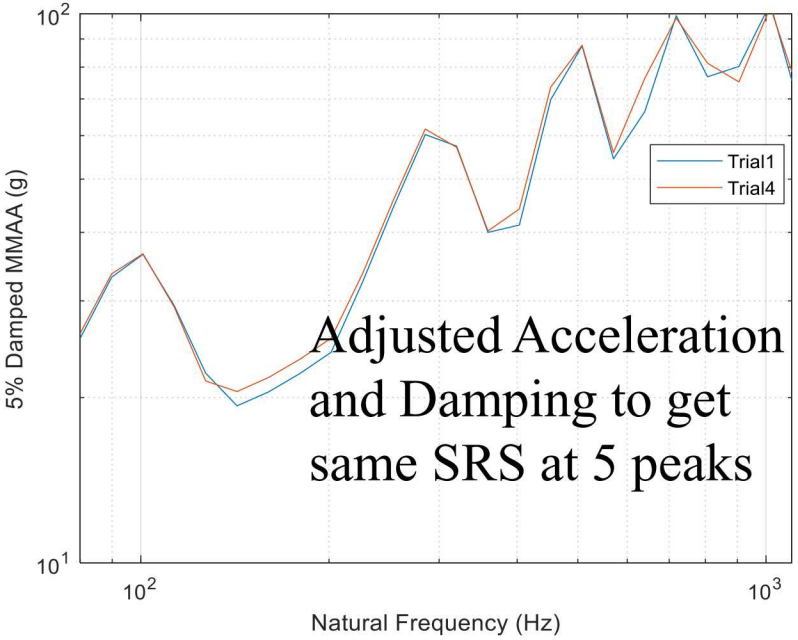
Same SRS & Overall Decay, Different Damage: Trials 1 & 4

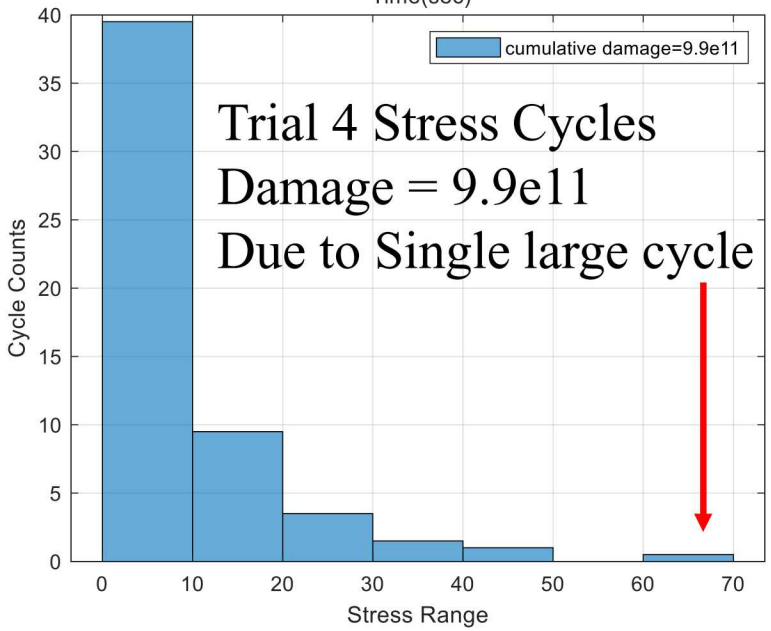


Adjust damping of 500Hz to match SRS & Final Decay





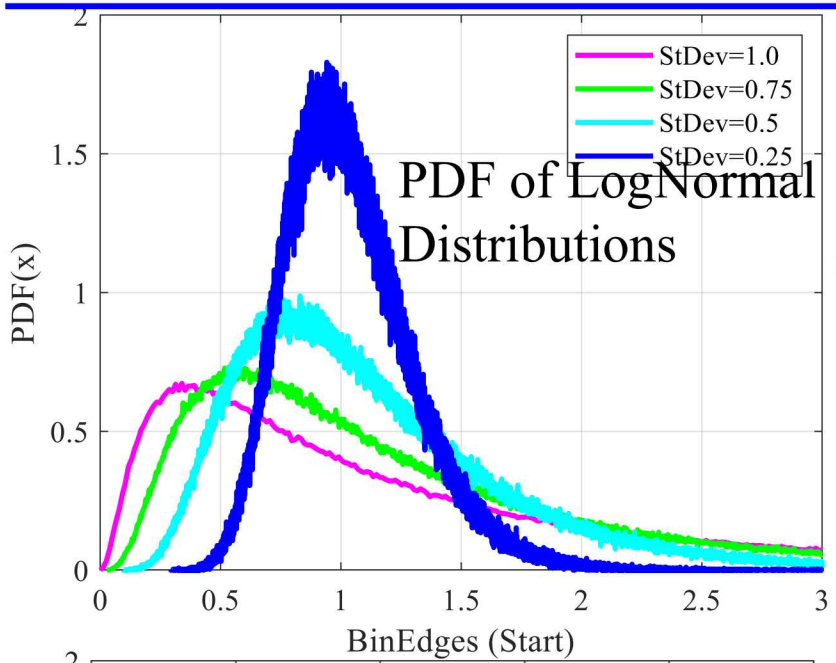


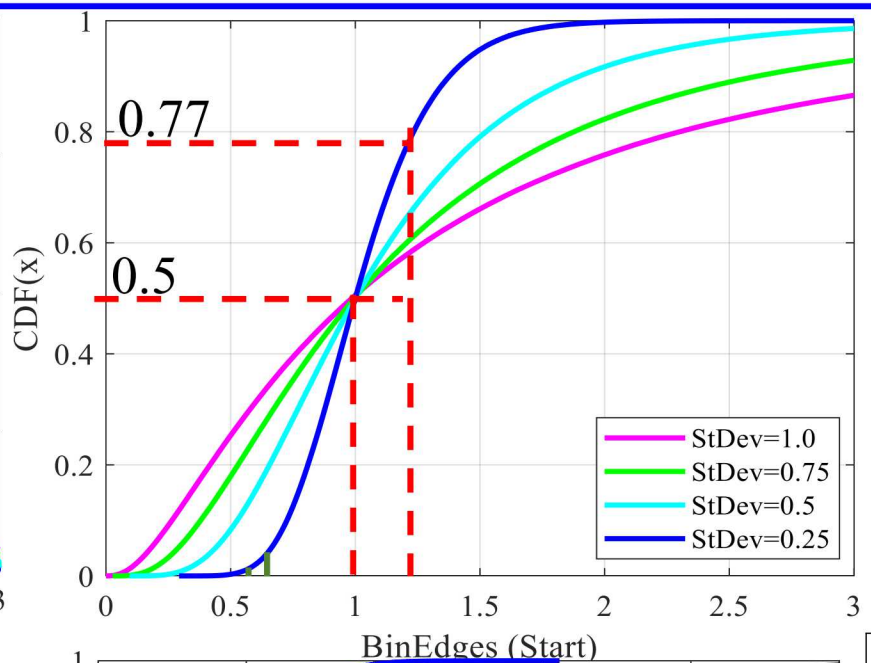


Conclusions

- Same SRS can have significantly more damage due to variation in damping.
- Assuming longer duration is unconservative (leads to greater damage)
- The increase in damage is due to 1 or 2 large cycles.

Effect of 20% Change in Damage on Probability of Failure





How does 20% change affect P(??)

Damage Failure data is log-normal

x is lognormal if y=log(y) is normal

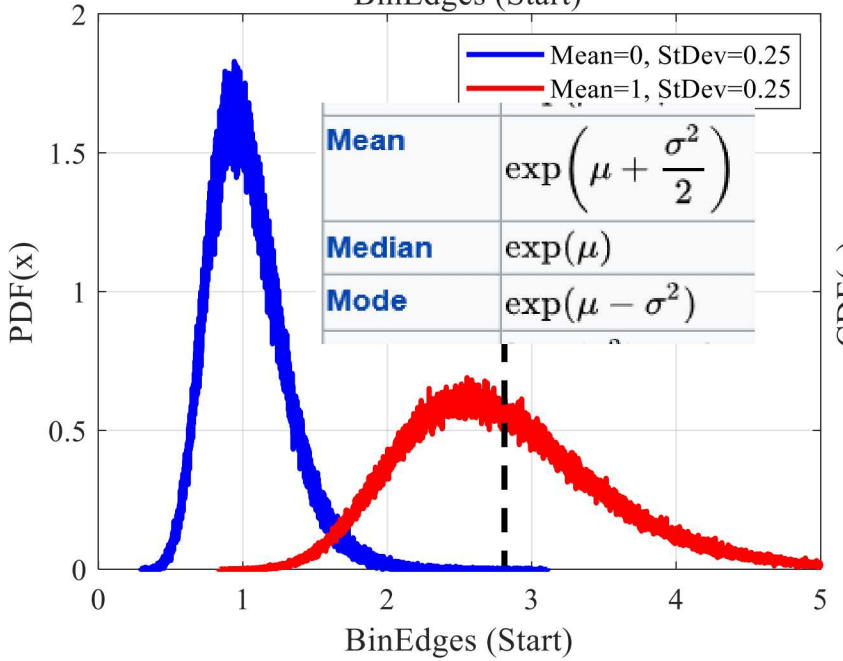
Each simulation has 1e6 data points

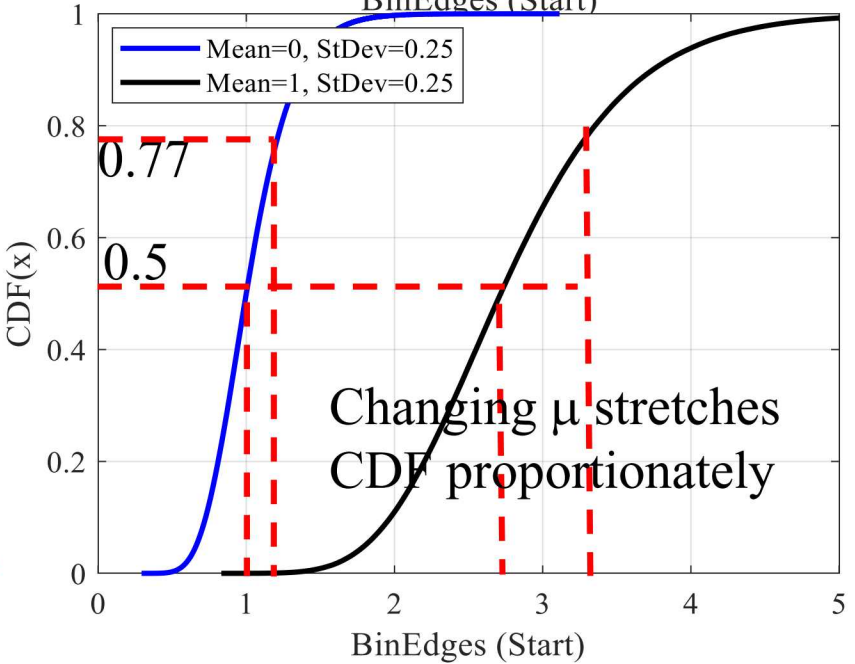
Determine Damage for P(1% Failure)

Change by 20% and determine P(??)

PDF	$\frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right)$
CDF	$\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left[\frac{\ln x-\mu}{\sqrt{2}\sigma}\right]$
Quantile	$\exp(\mu+\sqrt{2\sigma^2}\operatorname{erf}^{-1}(2F-1))$

Used Matlab erfinv





Mean	St. Dev.	P-Ratio (Simulation)	P-Ratio (Equations)
0	1	1.75	1.61
0	0.75	1.84	1.86
0	0.5	2.48	2.49
0	0.25	5.45	5.51
1	0.25	5.5	5.51

P99 can become P95 depending on σ (not μ) of Normal Distribution.

For x1.3 Damage P99 becomes P90

For x1.56 Damage P99 becomes P70