SAND2019-10063C

Sandia National Laboratories

Reformulating Friedman's Implicit Integrator as a Single Step Method for Maxwell's Equations as a First Order System

D. McGregor, E. Phillips, and E. Love

Introduction

We have reformulated Friedman's integrator for wave equations to serve as a single step method for the first order system.

- Allows simpler inclusion of complex closure models
- Naturally provides an accurate **B** for particle push
- Does not require differentiating **J**.
- Requires two fewer edge fields ($\mathbf{J}^{n-1/2}$, $\overline{\mathbf{E}}^{n-1}$)

Friedman's Original Integrator (for Electric Fields)

$$\frac{\mathbf{E}^{n+1}-2\mathbf{E}^{n}+\mathbf{E}^{n-1}}{\Delta t^2}+c_0^2\mathbf{curl}\ \mathbf{curl}\ \frac{\mathbf{E}^{n+1}+\frac{\theta}{2}\mathbf{E}^{n}+(1-\frac{\theta}{2})\overline{\mathbf{E}}^{n-1}}{2}+\frac{1}{\epsilon_0}\frac{\mathbf{J}^{n+1/2}-\mathbf{J}^{n-1/2}}{\Delta t}=0$$

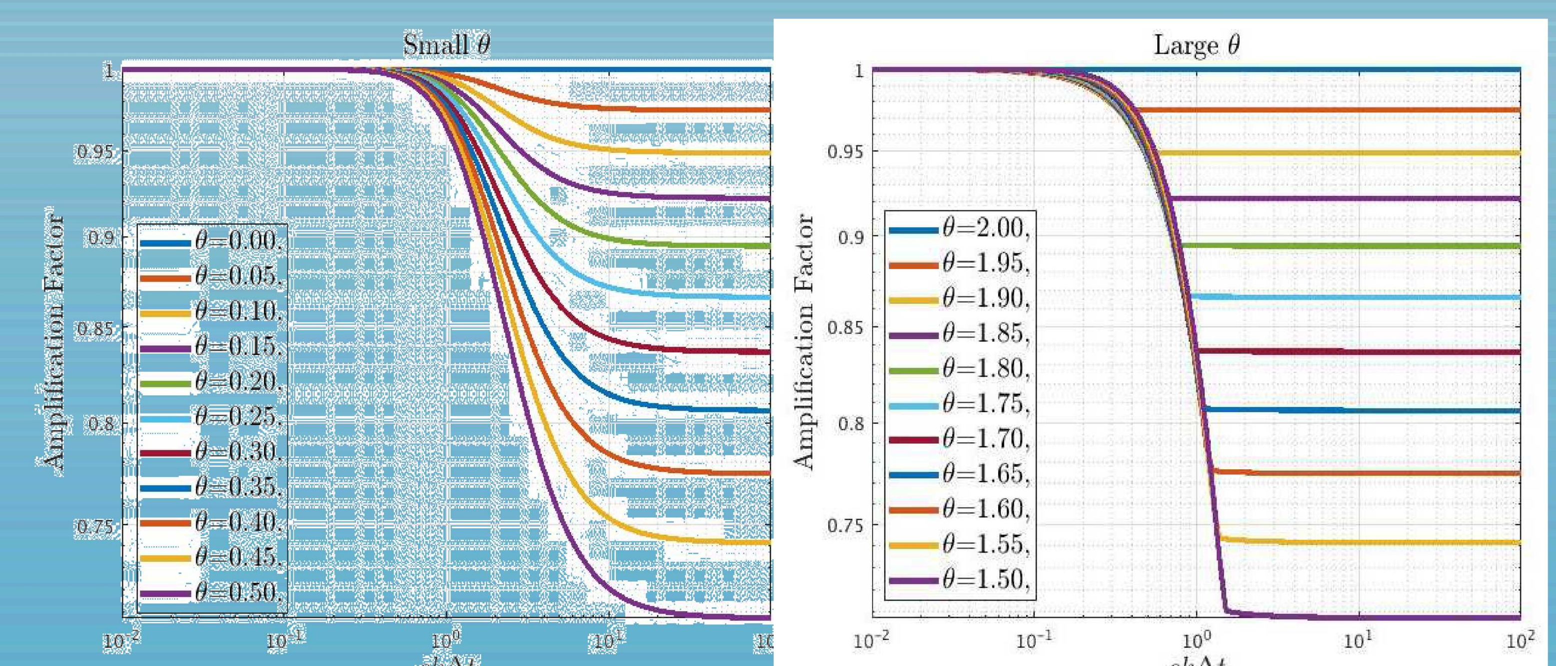
$$\overline{\mathbf{E}}^{n}=(1-\tfrac{\theta}{2})\mathbf{E}^{n}+\tfrac{\theta}{2}\overline{\mathbf{E}}^{n-1}$$

2nd order accurate for $\theta \in (0,2)$ and quiescent initial conditions.

$$\frac{\mathbf{E}^{n+1}+\frac{\theta}{2}\mathbf{E}^{n}+(1-\frac{\theta}{2})\overline{\mathbf{E}}^{n-1}}{2}=\frac{\mathbf{E}^{n+1}+\frac{\theta}{2}\mathbf{E}^{n}+(1-\frac{\theta}{2})^2\sum_{j=0}^{\infty}(\frac{\theta}{2})^j\mathbf{E}^{n-j-1}}{2}$$

$$\approx\frac{1+\frac{\theta}{2}+(1-\frac{\theta}{2})^2\sum_{j=0}^{\infty}(\frac{\theta}{2})^j}{2}\mathbf{E}(n\Delta t)+\frac{1-(1-\frac{\theta}{2})^2\sum_{j=0}^{\infty}(j+1)(\frac{\theta}{2})^j}{2}\Delta t\mathbf{E}'(n\Delta t)+\mathcal{O}(\Delta t^2)$$

For $\theta \in (0,2)$, the method is unconditionally stable with third order damping. Amplification factors can be determined by calculating the spectral radius of the amplification matrix.



Staggered Formulation for Electric & Magnetic Fields (I)

From Friedman's original paper: Apply Yee staggering to Ampere's Law and correct Faraday's Law to match the 2nd order stencil for **E**.

$$\begin{cases}\frac{\mathbf{E}^{n+1}-\mathbf{E}^{n}}{\Delta t}-c_0^2\mathbf{curl}\ \mathbf{B}^{n+1/2}+\frac{1}{\epsilon_0}\mathbf{J}^{n+1/2}=0\\ \frac{\mathbf{B}^{n+1/2}-\mathbf{B}^{n-1/2}}{\Delta t}+\mathbf{curl}\ \frac{\mathbf{E}^{n+1}+\frac{\theta}{2}\mathbf{E}^{n}+(1-\frac{\theta}{2})\overline{\mathbf{E}}^{n-1}}{2}=0\\ \overline{\mathbf{E}}^{n}=(1-\frac{\theta}{2})\mathbf{E}^{n}+\frac{\theta}{2}\overline{\mathbf{E}}^{n-1}\end{cases}$$

Eliminate **E** from Ampere's Law

$$\frac{\mathbf{B}^{n+1/2}-2\mathbf{B}^{n-1/2}+\mathbf{B}^{n-3/2}}{\Delta t^2}+c_0^2\mathbf{curl}\ \mathbf{curl}\ \frac{\mathbf{B}^{n+1/2}+\frac{\theta}{2}\mathbf{B}^{n-1/2}+(1-\frac{\theta}{2})^2\sum_{j=0}^{\infty}(\frac{\theta}{2})^j\mathbf{B}^{n-3/2-j}}{2}-\frac{1}{\epsilon_0}\mathbf{curl}\ \frac{\mathbf{J}^{n+1/2}+\frac{\theta}{2}\mathbf{J}^{n-1/2}+(1-\frac{\theta}{2})^2\sum_{j=0}^{\infty}(\frac{\theta}{2})^j\mathbf{J}^{n-3/2-j}}{2}=0$$

2nd order accurate at half steps!

Single Step Formulation for Electric & Magnetic Fields (II)

Advance Faraday's Law with Crank-Nicolson and correct Ampere's Law to match 2nd order stencil for **E**.

$$\begin{cases}\frac{\mathbf{E}^{n+1}-\mathbf{E}^{n}}{\Delta t}-c_0^2\mathbf{curl}\ \mathbf{B}^{n+1}-\frac{c_0^2\Delta t}{2}\mathbf{curl}\ \mathbf{curl}\ \overline{\mathbf{E}}^{n}+\frac{1}{\epsilon_0}\mathbf{J}^{n+1/2}=0\\ \frac{\mathbf{B}^{n+1}-\mathbf{B}^{n}}{\Delta t}+\mathbf{curl}\ \frac{\mathbf{E}^{n+1}+\mathbf{E}^{n}}{2}=0\\ \overline{\mathbf{E}}^{n+1}=(1-\frac{\theta}{2})\mathbf{E}^{n+1}+\frac{\theta}{2}\overline{\mathbf{E}}^{n}\end{cases}$$

Eliminate **E** from Ampere's Law

$$\frac{\mathbf{B}^{n+1}-2\mathbf{B}^{n}+\mathbf{B}^{n-1}}{\Delta t^2}+c_0^2\mathbf{curl}\ \mathbf{curl}\ \frac{\mathbf{B}^{n+1}+\frac{\theta}{2}\mathbf{B}^{n}+(1-\frac{\theta}{2})^2\sum_{j=0}^{\infty}(\frac{\theta}{2})^j\mathbf{B}^{n-j-1}}{2}-\frac{1}{\epsilon_0}\mathbf{curl}\ \frac{\mathbf{J}^{n+1/2}+\mathbf{J}^{n-1/2}}{2}=0$$

2nd order accurate on integer steps!

References

A. Friedman, "A second-order implicit particle mover with adjustable damping." J. Comput. Phys. 1990

Code Verification for the single step formulation

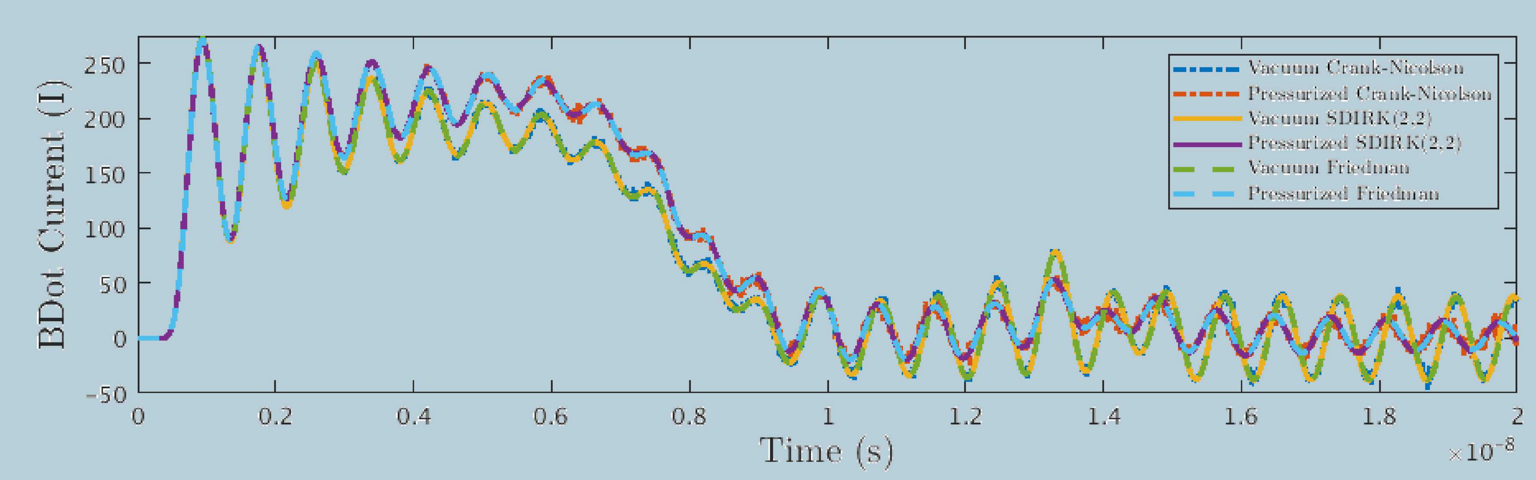
$$\mathbf{A_0}(x,y)=(0,0,\sin(k_x x)\sin(k_y))$$

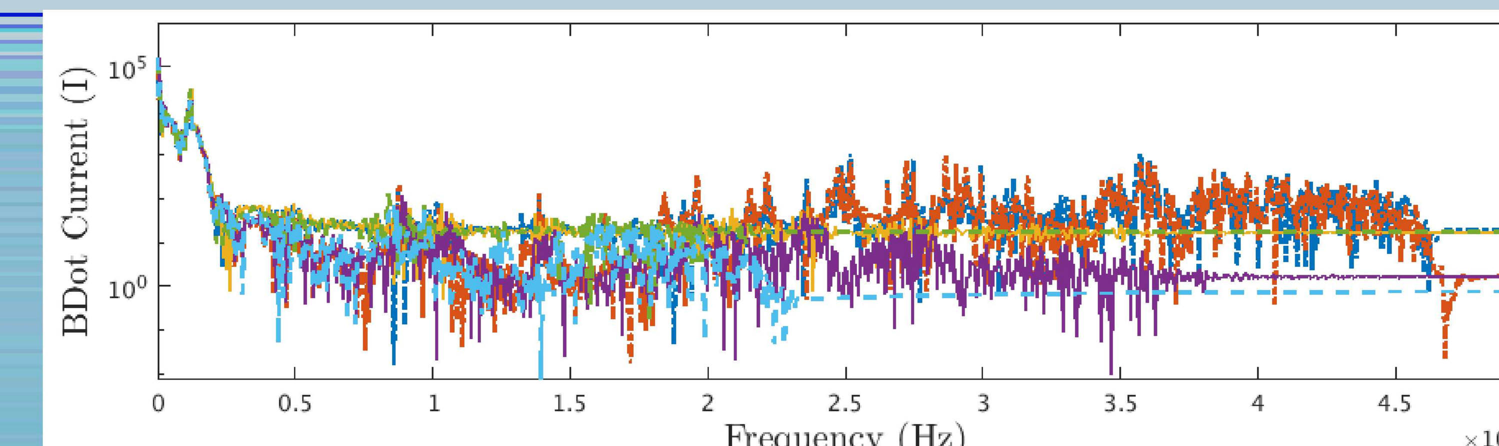
$$\begin{cases}\mathbf{J}=J_0\sin(\omega t)\mathbf{A_0}\\ \mathbf{E}=E_0(\cos(c_0 kt)-\cos(\omega t))\mathbf{A_0}\\ \mathbf{B}=E_0(\frac{1}{\omega}\sin(\omega t)-\frac{1}{ck}\sin(ckt))\mathbf{curl}\ \mathbf{A_0}\end{cases}\qquad \begin{cases}J_0=\frac{I}{|A|}\\ E_0=\frac{\omega J_0}{\epsilon_0((ck)^2-\omega^2)}\\ k_x=k_y\\ \omega=4ck\end{cases}$$

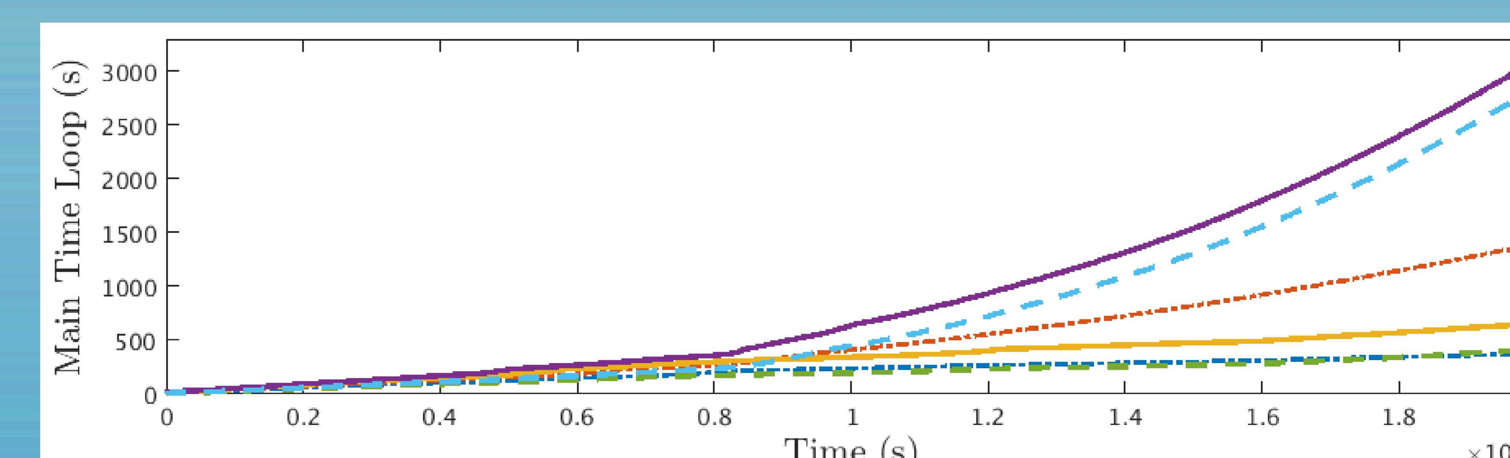
Compute Relative L2 Errors at $6\pi\omega^{-1}$

$\frac{2\pi}{kh}$	$\frac{2\pi}{\omega\Delta t}$	$\frac{\Vert\mathbf{E}-\mathbf{E}_h\Vert_{L^2}}{\Vert\mathbf{E}\Vert_{L^2}}$	rate	$\frac{\Vert\mathbf{B}-\mathbf{B}_h\Vert_{L^2}}{\Vert\mathbf{B}\Vert_{L^2}}$	rate
20	10	3.53e-2	–	1.80e-2	–
40	20	8.93e-3	1.99	4.49e-3	2.00
80	40	2.24e-3	2.00	1.12e-3	2.00
160	80	5.60e-4	2.00	2.80e-4	2.00

Radiation Driven BDot Cavity at 0.0 and 0.01 Tor







Sandia National Laboratories is a multimission laboratory managed and operated by National Technology &Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.
SAND No. 2019-DRAFT

Sandia National Laboratories