

A Modified Johnson-Cook Model for Dynamic Response of Metals with an Explicit Strain- and Strain-rate-dependent Thermosoftening Effect



PRESENTED BY

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A Classic Johnson-Cook Model

Based on scaling

$$\sigma = \left(A + B \cdot \varepsilon^n \right) \cdot \left(1 + C' \cdot \ln \dot{\varepsilon} \right) \cdot \left[1 - T^{*m} \right]$$

Work Hardening

Strain-rate
Strengthening

Temperature
Effect

“Temperature Rise” due to
“adiabatic heating”

“Adiabatic Thermo-softening”

$$T^* = \frac{T - T_0}{T_m - T_0}$$

T_0 Room temperature

T_m Melting temperature

A general practice:

- Use very low rate data as a reference strain rate (isothermal condition)
- Room-temperature tests at various strain rates
- Quasi-static tests at various temperatures

Johnson GR, Cook WH (1983) A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures. In: Proceedings of the 7th International Symposium on Ballistics, The Hague, The Netherlands, pp.541-7.

Revisit of Classic Johnson-Cook Model – Strain Rate Effect

$$\sigma = (A + B \cdot \varepsilon^n) (1 + C' \cdot \ln \dot{\varepsilon}) \left[1 - \left(\frac{T - T_0}{T_m - T_0} \right)^m \right]$$

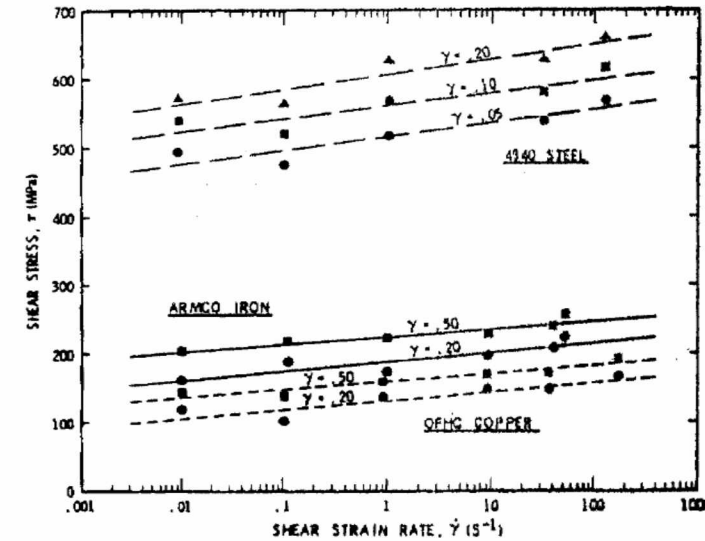
At $\varepsilon = 0$ ε is plastic strain

A represents yield strength

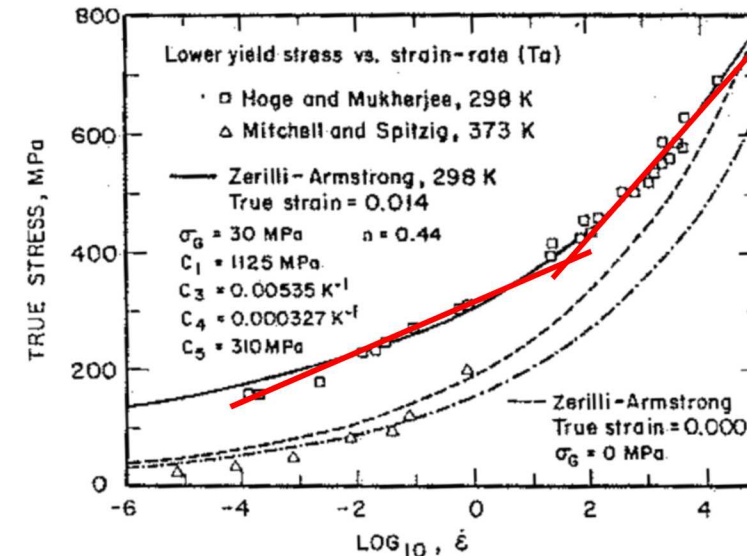
$T = T_0$ No temperature rise

$$\sigma = A \cdot (1 + C' \cdot \ln \dot{\varepsilon}) = A \cdot \left(1 + C \cdot \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)$$

Strain rate effect – linear to logarithm of strain rate, which may not be true for many materials.

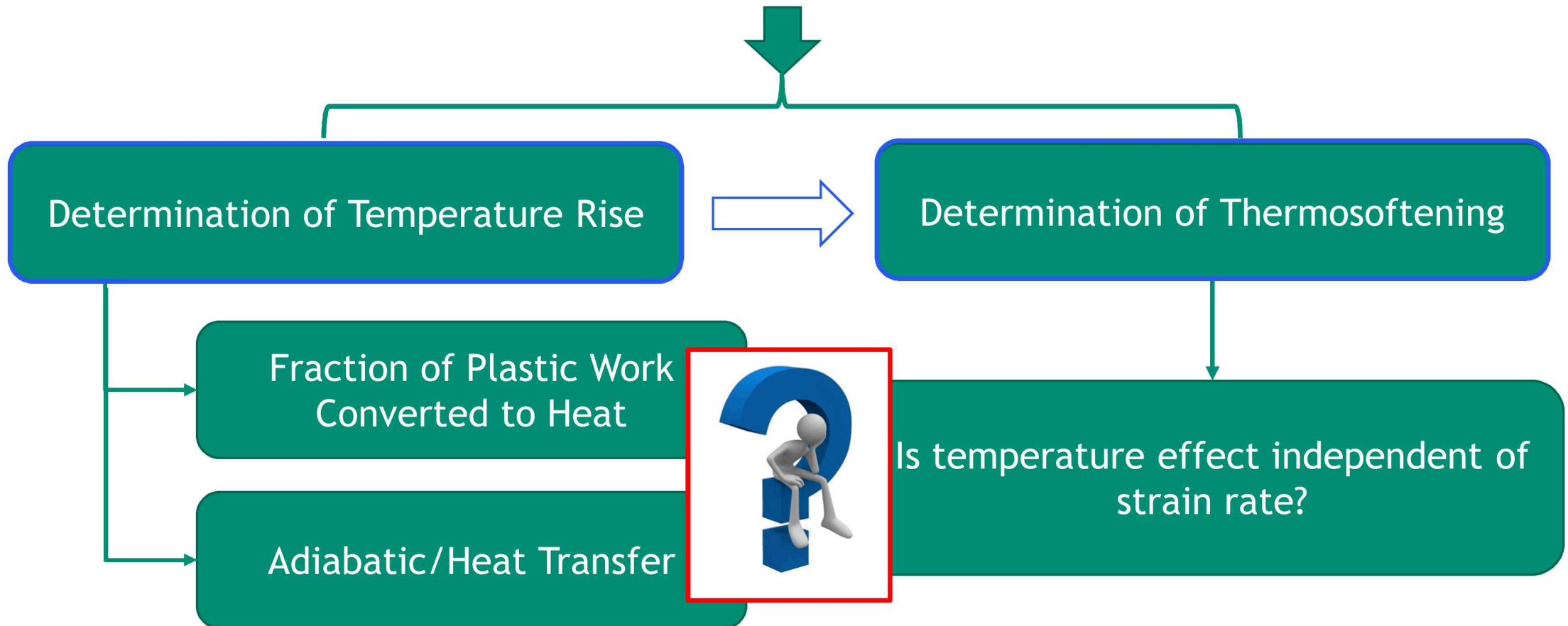


Johnson and Cook, 1983.



$$\sigma = \left(A + B \cdot \varepsilon^n \right) \cdot \left(1 + C' \cdot \ln \dot{\varepsilon} \right) \cdot \left[1 - \left(\frac{T - T_0}{T_m - T_0} \right)^m \right]$$

Implicit to Strain and Strain Rate



Proposed Modification of J-C Model with Explicit Thermo-softening

$$\sigma = \left(A + B \cdot \varepsilon^n \right) \cdot \underbrace{f(\dot{\varepsilon})}_{\text{A better description of strain-rate effect}} \cdot \underbrace{g(\varepsilon, \dot{\varepsilon})}_{\text{An explicit description of strain and strain-rate-dependent thermo-softening}}$$

A better description of strain-rate effect

An explicit description of strain and strain-rate-dependent thermo-softening

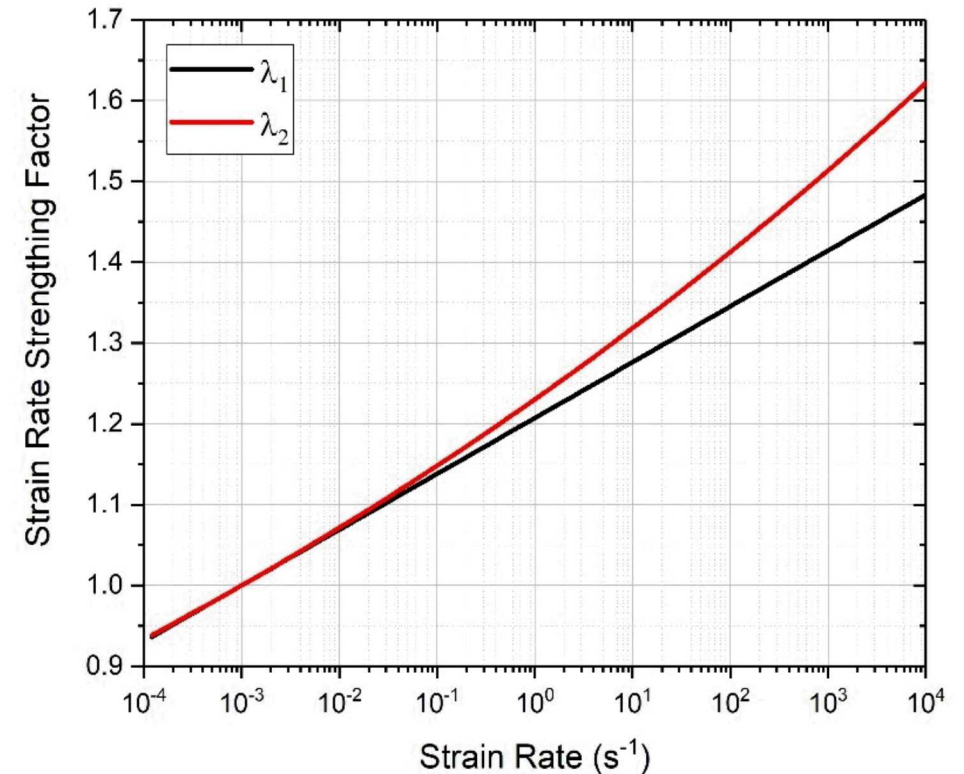
❖ A power-law form

- ✓ Zerilli and Armstrong, 1987
- ✓ Litonski 1977
- ✓ Vinh et al. 1979
- ✓ Klopp et al. 1985

$$f(\dot{\varepsilon}) = \lambda_2 = \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^\alpha$$

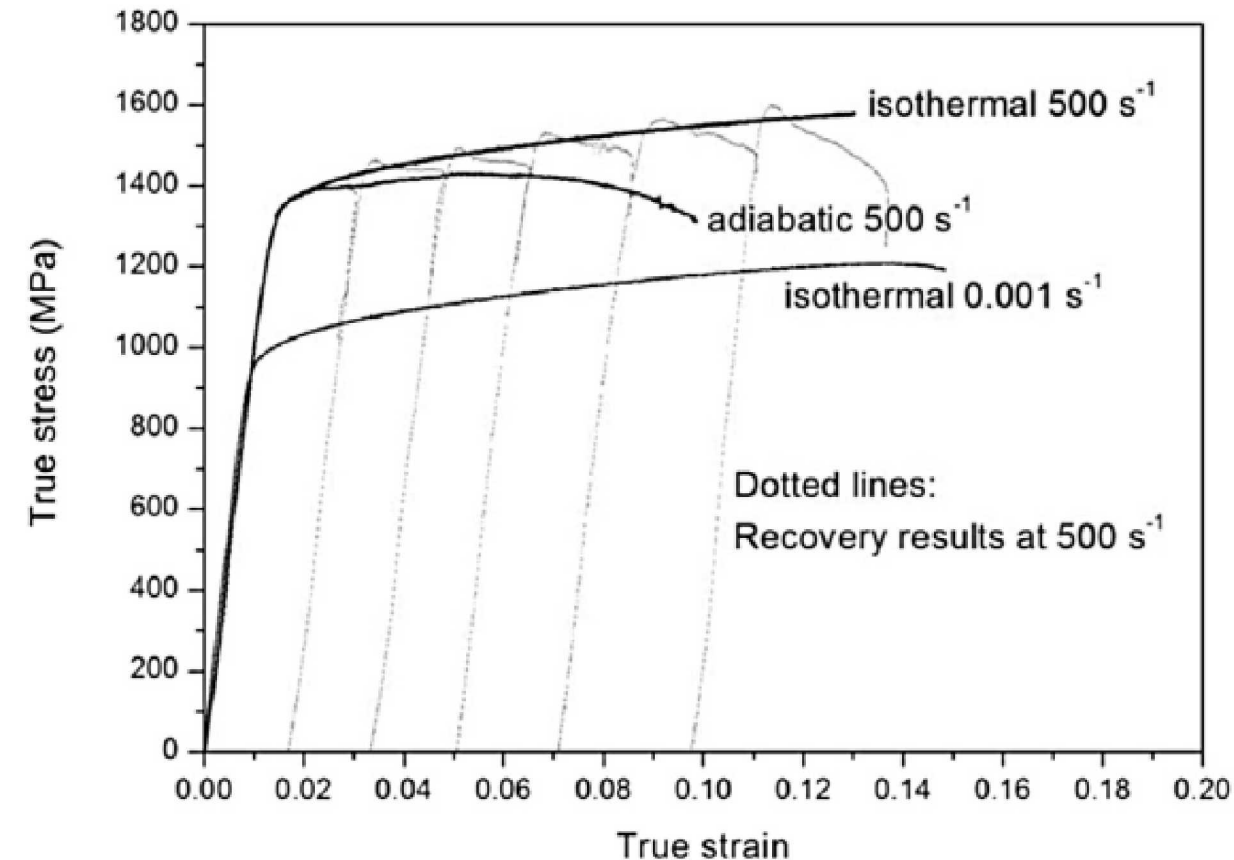


$$f(\dot{\varepsilon}) = \lambda_1 = 1 + C \cdot \ln \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)$$



6 Proposed Modification of J-C Model – Thermo-softening

Zhang et al. (2015) The constitutive responses of Ti-6.6Al-3.3Mo-1.8Zr-0.29Si alloy at high strain rates and elevated temperatures, Journal of Alloys and Compounds, 647:97-104.



$$\sigma = \left(A + B \cdot \varepsilon^n \right) \cdot \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^\alpha \cdot \left[1 - \left(\frac{T - T_0}{T_m - T_0} \right)^m \right]$$

True Stress

True Plastic Strain

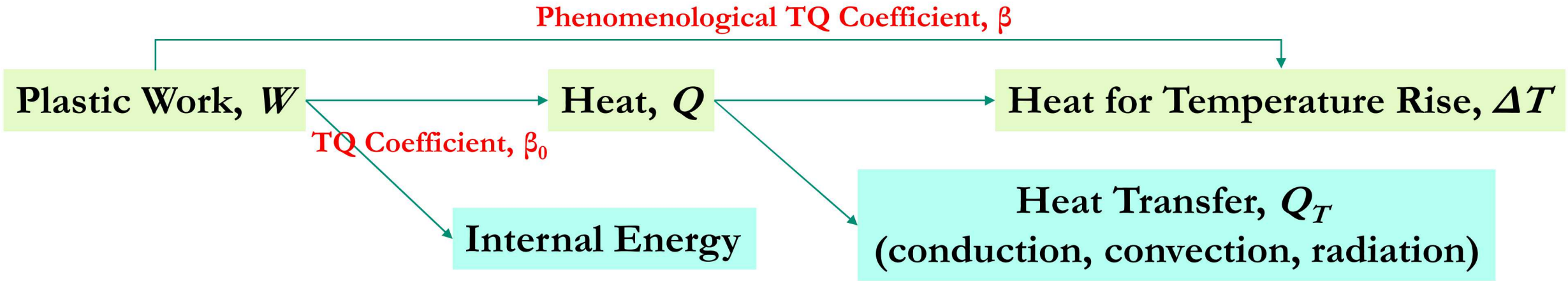
Isothermal

Adiabatic

Thermosoftening

$$\Delta T = T - T_0 = \frac{\beta \cdot \int \left(A + B \cdot \varepsilon^n \right) \cdot \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^\alpha d\varepsilon}{\rho_0 \cdot C_v}$$

7 Taylor-Quinney Coefficient

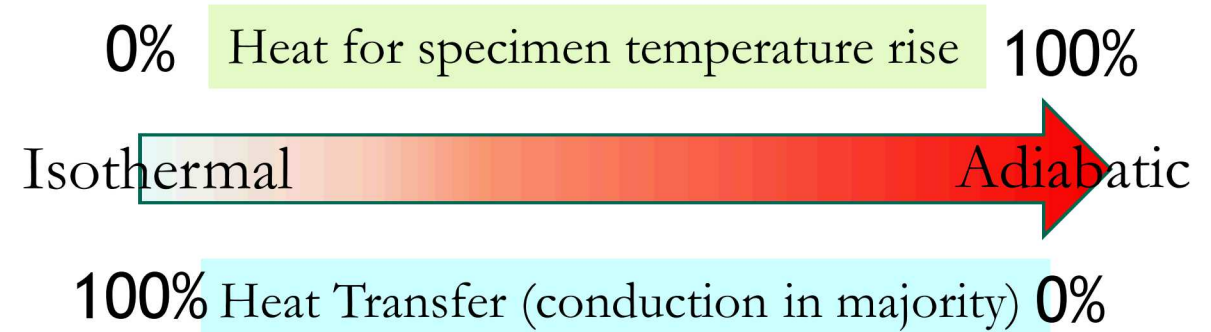


Taylor-Quinney coefficient $\beta_0 = \frac{d\dot{Q}}{d\dot{W}}$

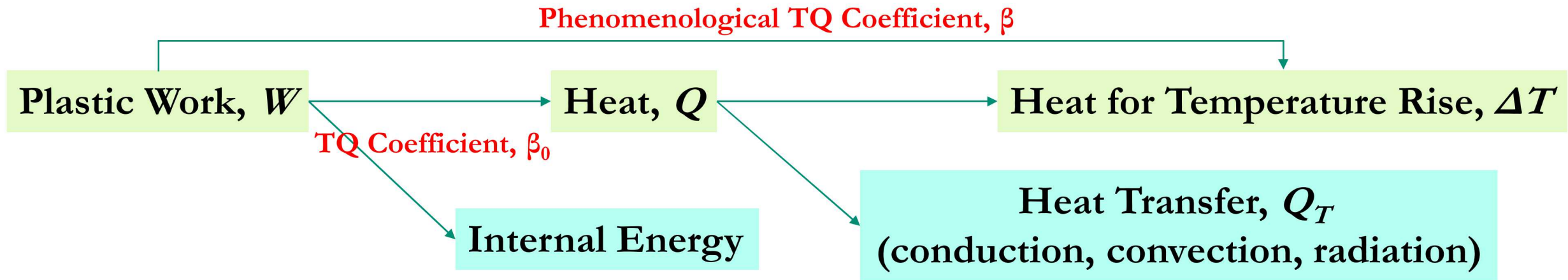
$$\Delta T = T - T_0 = \frac{Q - Q_T}{\rho_0 \cdot C_v} = \frac{\beta_0 \cdot W - Q_T}{\rho_0 \cdot C_v} = \frac{\beta_0 \cdot \int \sigma d\varepsilon - Q_T}{\rho_0 \cdot C_v}$$

Phenomenological Fraction of plastic work converted to heat

$$\Delta T = T - T_0 = \frac{\beta \cdot W}{\rho_0 \cdot C_v} = \frac{\beta \cdot \int \sigma d\varepsilon}{\rho_0 \cdot C_v}$$



Farren WS, Taylor GI (1925) The heat developed during plastic extension of metals, Proc. Royal Soc., pp423-451.
 Taylor GI, Quinney H (1934) The latent energy remaining in a metal after cold working, Proc. Royal Soc., pp307-326



$$\Delta T = T - T_0 = \frac{Q - Q_T}{\rho_0 \cdot C_v} = \frac{\beta_0 \cdot W - Q_T}{\rho_0 \cdot C_v} = \frac{\beta_0 \cdot \int \sigma d\varepsilon - Q_T}{\rho_0 \cdot C_v}$$



$$\Delta T = T - T_0 = \frac{\beta \cdot W}{\rho_0 \cdot C_v} = \frac{\beta \cdot \int \sigma d\varepsilon}{\rho_0 \cdot C_v}$$

$$\beta = \beta_0 - \frac{Q_T}{W} = \beta_0 - \frac{Q_T}{\int \sigma d\varepsilon}$$

Isothermal: $Q_T = Q = \beta_0 \cdot W$



$$\beta = 0$$



Adiabatic:



$$\beta = \beta_0$$

$$Q_T = 0$$

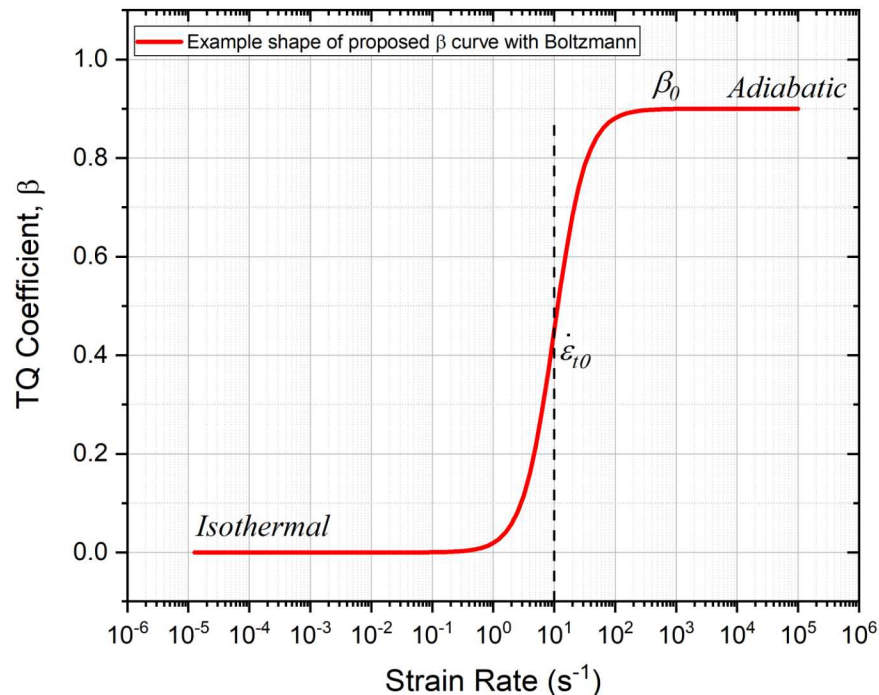
9 Determination of Temperature Rise with Taylor-Quinney Coefficient

$$\Delta T = T - T_0 = \frac{\beta_0 \cdot \int \sigma d\varepsilon - Q_T}{\rho_0 \cdot C_v} = \frac{\beta \cdot \int \sigma d\varepsilon}{\rho_0 \cdot C_v}$$

$$0 \leq \beta \leq \beta_0 \leq 1$$

β is easier than β_0 due to unknown heat transfer at lower strain rates.

Strain Rate Effect



Boltzmann Function

$$\beta(\dot{\varepsilon}) = \frac{\beta_0}{1 + \exp\left[-\frac{\ln \dot{\varepsilon} - \ln \dot{\varepsilon}_{t0}}{\delta}\right]}$$

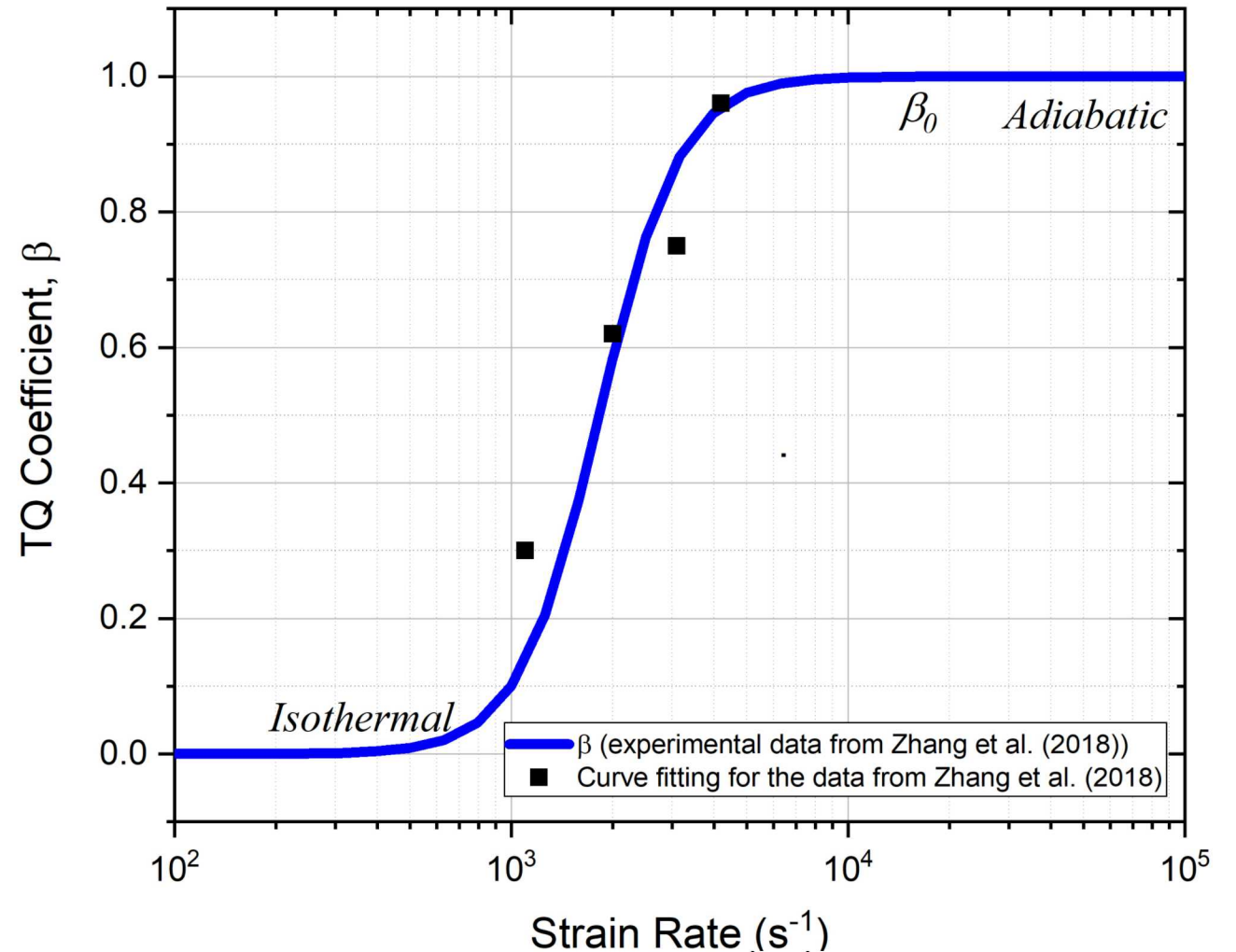
$$\beta(\dot{\varepsilon}) = \frac{\beta_0}{1 + \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{t0}}\right)^{-\frac{1}{\delta}}}$$

Boltzmann Description of Taylor-Quinney Coefficient

Zhang T, Guo Z-R, Yuan F-P, Zhang H-S (2018) Investigation on the plastic work-heat conversion coefficient of 7075-T651 aluminum alloy during an impact process based on infrared temperature measurement technology, Acta Mech Sinica, 34:327-333.

$$\beta(\dot{\epsilon}) = \frac{\beta_0}{1 + \exp\left[-\frac{\ln \dot{\epsilon} - \ln \dot{\epsilon}_{t0}}{\delta}\right]}$$

$$\beta(\dot{\epsilon}) = \frac{\beta_0}{1 + \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{t0}}\right)^{-\frac{1}{\delta}}}$$



Proposed Modification of J-C Model

$$\sigma = \left(A + B \cdot \varepsilon^n \right) \cdot f(\dot{\varepsilon}) \cdot g(\varepsilon, \dot{\varepsilon})$$

Power-law Strain-rate Effect

*Explicit Strain- and Strain-rate- Dependent
Thermo-softening Effect*

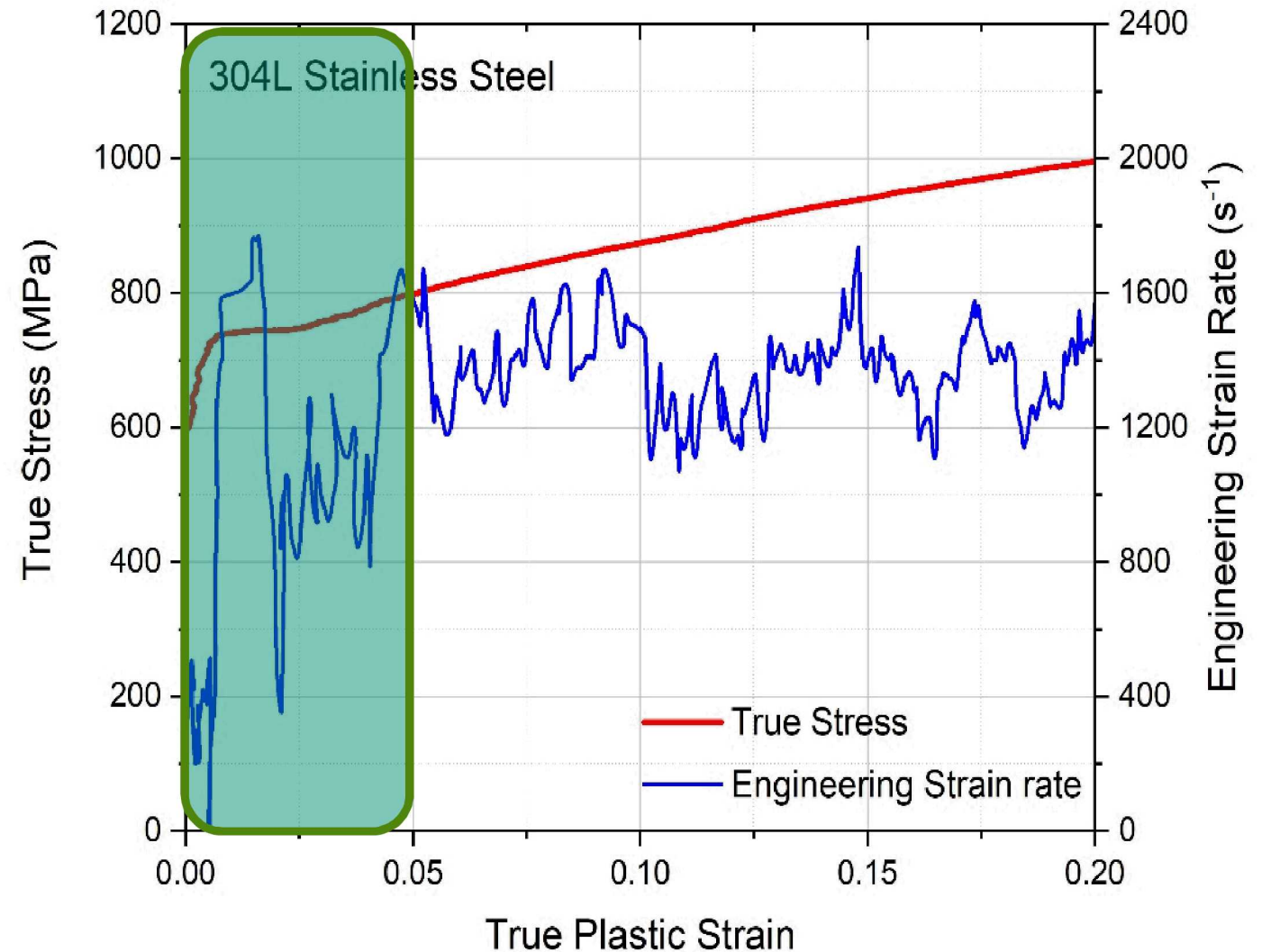
$$\sigma = \left(A + B \cdot \varepsilon^n \right) \cdot \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^\alpha \cdot \left\{ 1 - \left[\frac{D \cdot \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^\alpha \cdot Y(\varepsilon)}{1 + \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{t0}} \right)^{-\frac{1}{\delta}}} \right]^m \right\}$$

where

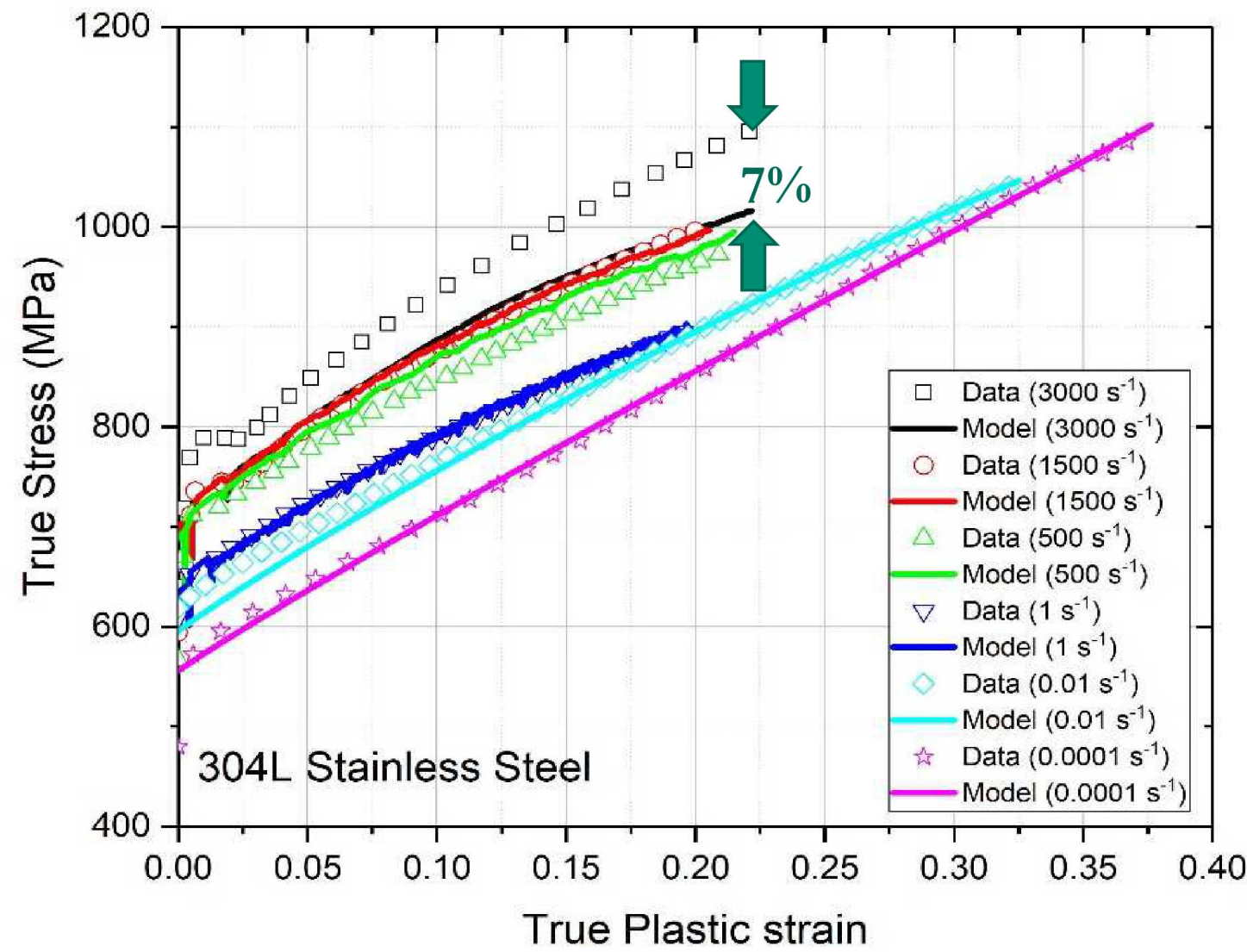
$$Y(\varepsilon) = \left(A + \frac{B}{n+1} \cdot \varepsilon^n \right) \cdot \varepsilon$$

Constant strain rates are preferred. However, in reality, it takes time/strain for strain rate to ramp up.

Song B, Sanborn B, Heister JD, Everett RL, Martinez TL, Groves GE, Johnson EP, Kenney DJ, Knight ME, Spletzer MA, Haulenbeek KK, McConnell C (2019) An apparatus for tensile characterization of materials within the upper intermediate strain rate regime, Exp Mech, DOI:10.1007/s11340-019-00494-3.

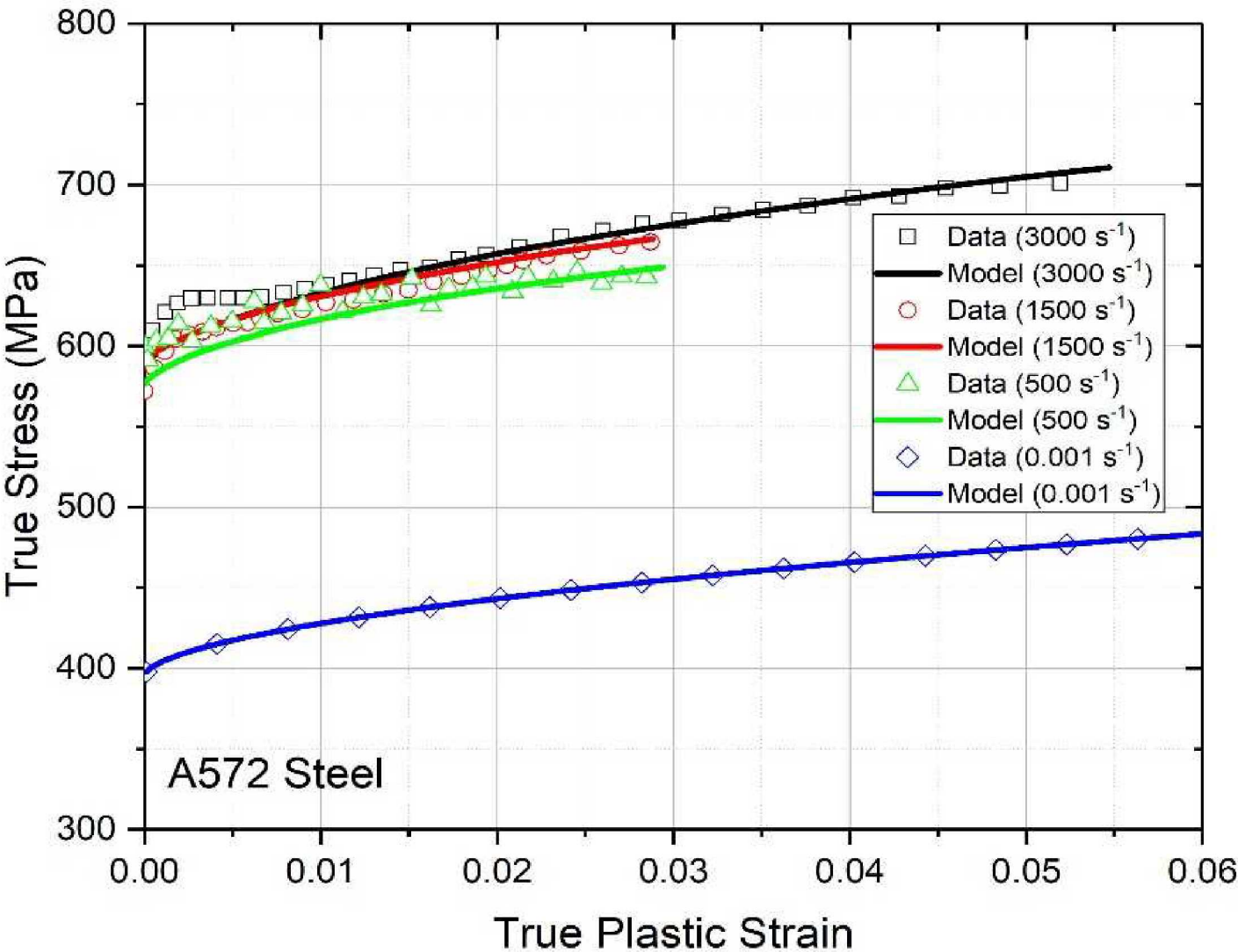


Modified J-C Model for 304L Stainless Steel



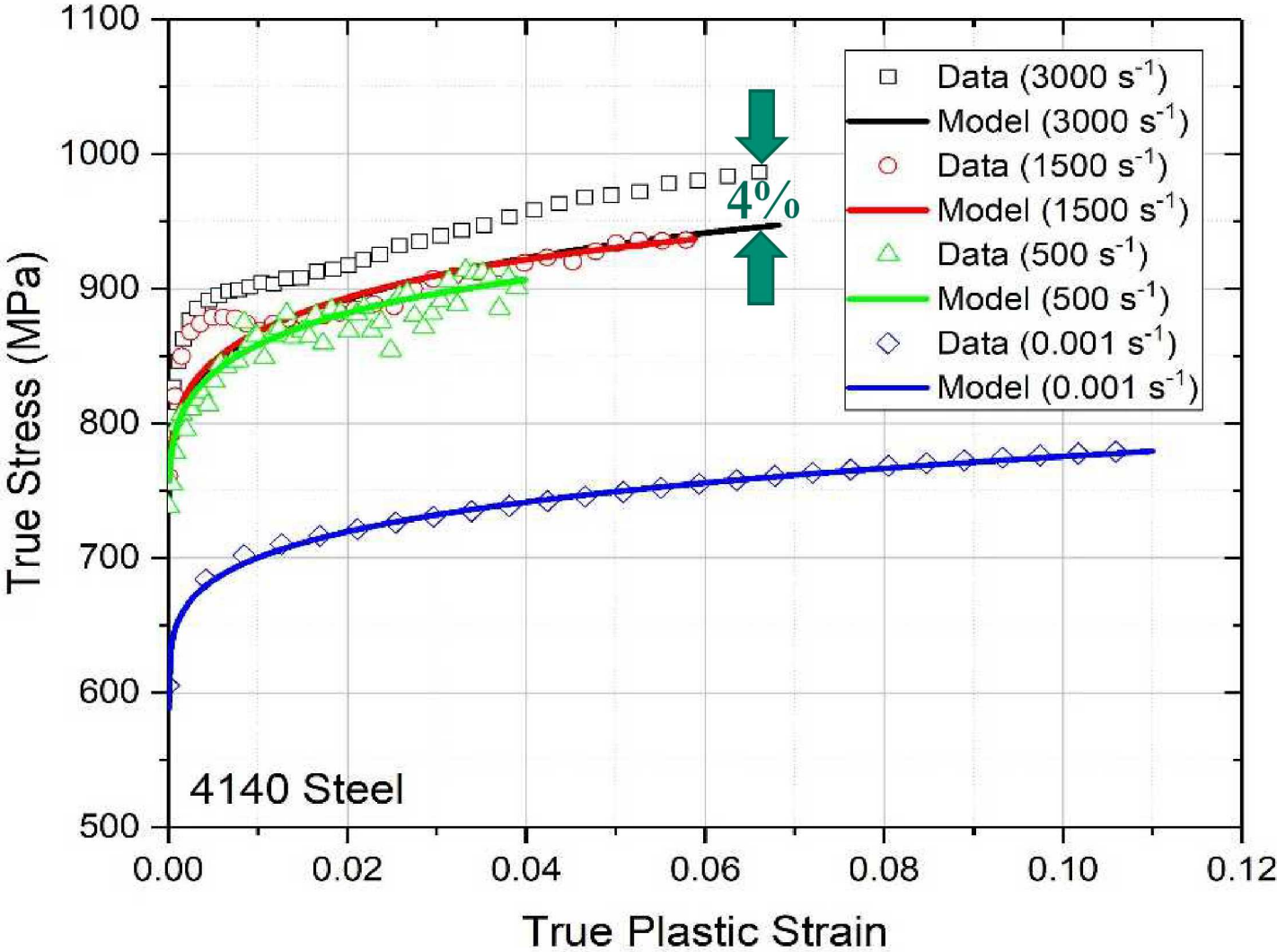
Constants	304L Stainless Steel
A	555.47 MPa
B	1378.87 MPa
n	0.9497
α	0.01503
D	9.4431×10^{-4}
δ	0.5327
m	1.3263
$\dot{\epsilon}_0$	$1 \times 10^{-4} s^{-1}$
$\dot{\epsilon}_{r0}$	$0.012 s^{-1}$

Modified J-C Model for A572 Steel



Constants	A572 Steel
A	394.98 MPa
B	418.33 MPa
n	0.5522
α	0.03157
D	2.6460×10^{-6}
δ	0.7173
m	0.3098
$\dot{\epsilon}_0$	$1 \times 10^{-3} \text{ s}^{-1}$
$\dot{\epsilon}_{r0}$	128 s^{-1}

Modified J-C Model for 4140 Steel



Constants	4140 Steel
A	562.41 <u>MPa</u>
B	330.62 <u>MPa</u>
n	0.1654
α	0.01968
D	1.589×10^{-7}
δ	0.7111
m	0.3388
$\dot{\epsilon}_0$	$1 \times 10^{-3} \text{ s}^{-1}$
$\dot{\epsilon}_{r0}$	73 s ⁻¹

Bridgman Correction for True Stress-Strain Response During Necking

- True strain

$$\varepsilon_T = \ln \frac{A_0}{A}$$

- Average true axial stress at the smallest cross section

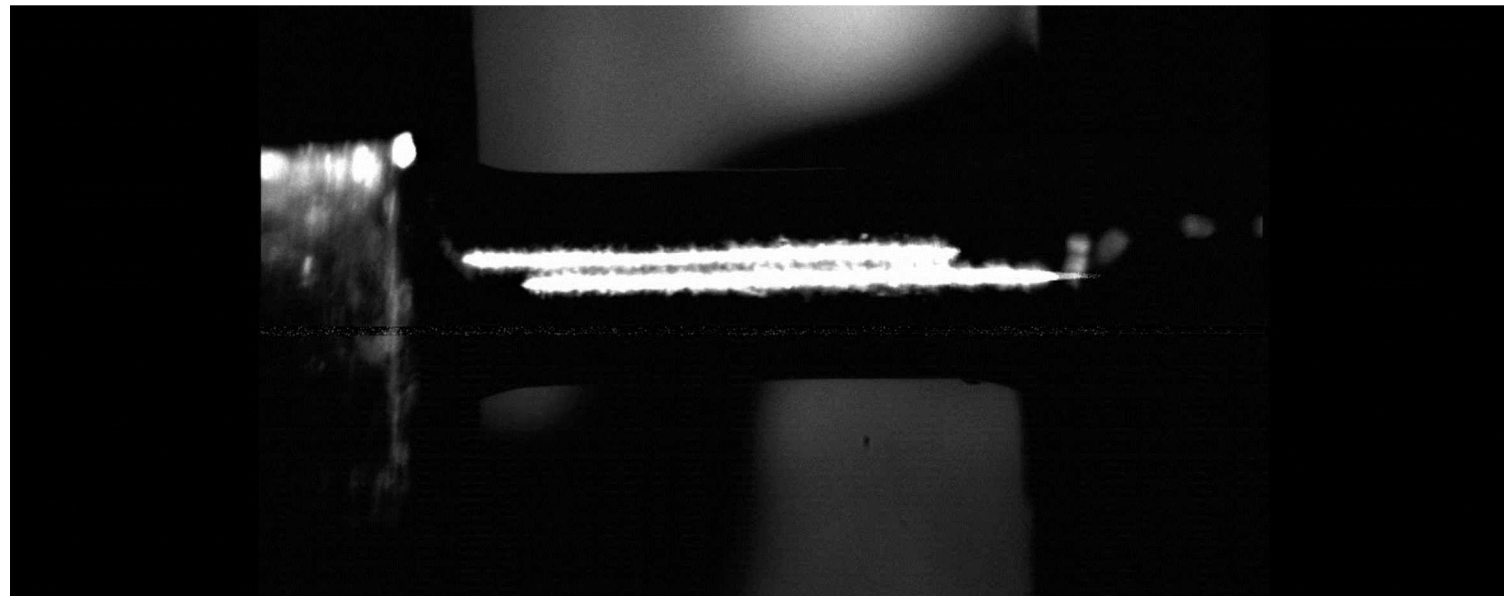
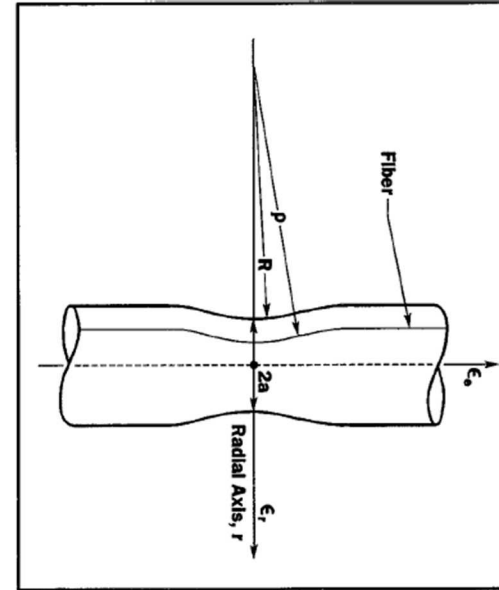
$$(\sigma_a)_{av} = \frac{F}{A} = \sigma_E \frac{A_0}{A}$$

- Uniaxial stress correction

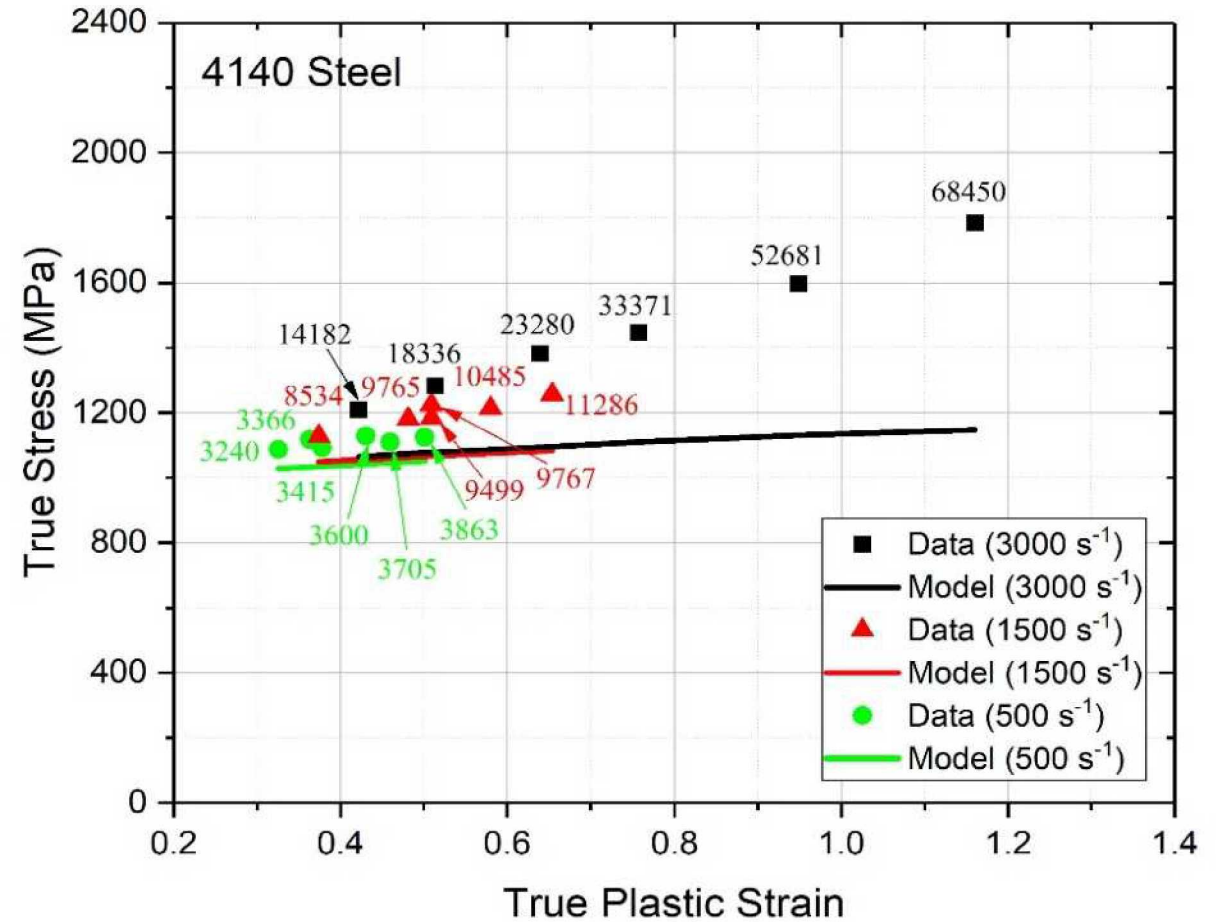
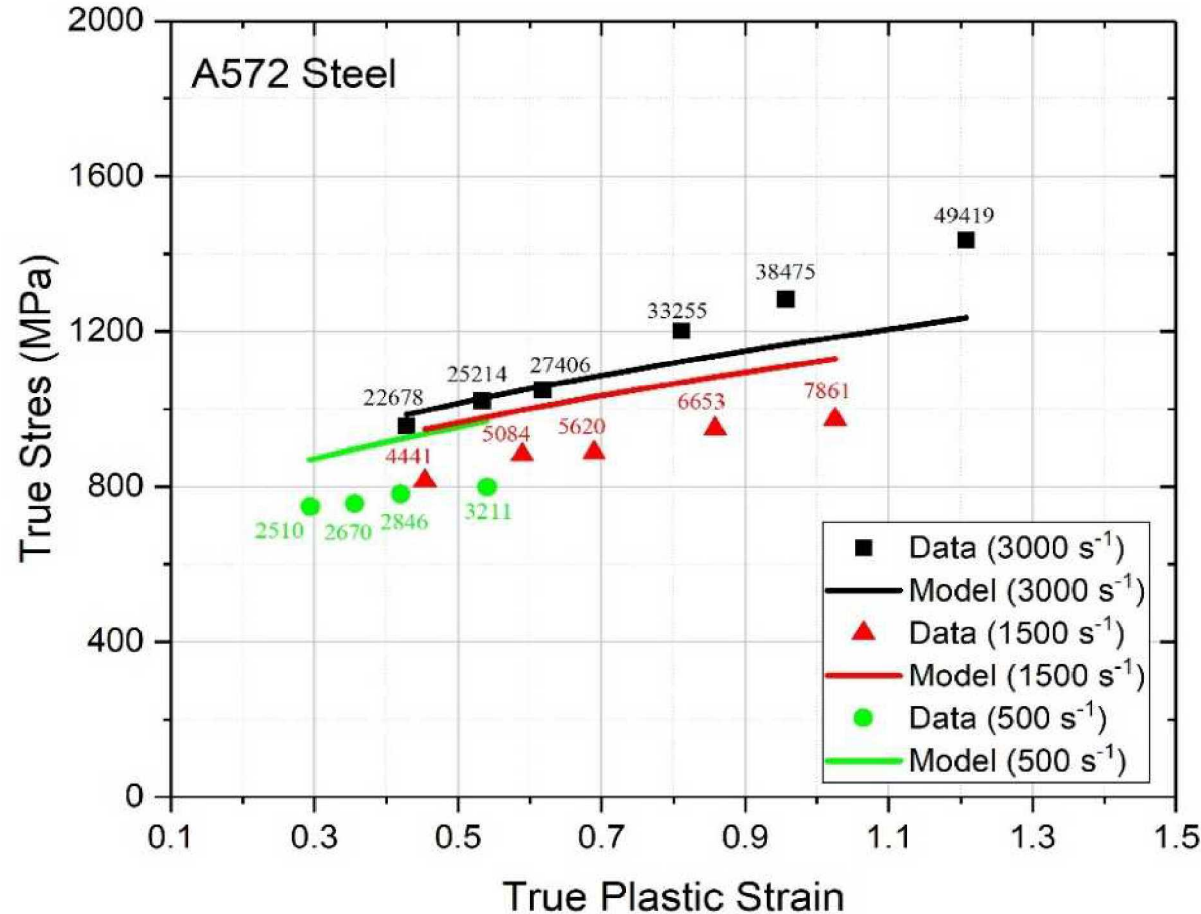
$$\sigma_T = k(\sigma_a)_{av}$$

$$k = \left[\left(1 + \frac{2R}{a} \right) \ln \left(1 + \frac{a}{2R} \right) \right]^{-1}$$

- Requires instantaneous measurements of minimum cross sectional area, A , and radius, R .



Modified J-C Model for True Stress-Strain Response During Necking



Uncertainties:

- Description of strain rate effect
- Experimental data corrected with Bridgman method

Song, B., Sanborn, B., Thompson, A., Reece, B., Attaway, S., and Teeter, R., High strain rate tensile response of A572 and 4140 steel, In: 2017 ASME International Mechanical Engineering Congress & Exposition (IMECE), November 3-9, 2017, Tampa, FL

Conclusions

- The Johnson-Cook model was modified with
 - power-law strain rate effect
 - explicit strain- and strain-rate-dependent thermosoftening using
 - a Boltzmann description of phenomenological Taylor-Quinney Coefficient

$$\sigma = \left(A + B \cdot \varepsilon^n \right) \cdot \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^\alpha \cdot \left\{ 1 - \left[\frac{D \cdot \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^\alpha \cdot Y(\varepsilon)}{1 + \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{t0}} \right)^{-\frac{1}{\delta}}} \right]^m \right\}$$

$$Y(\varepsilon) = \left(A + \frac{B}{n+1} \cdot \varepsilon^n \right) \cdot \varepsilon$$

- The modified Johnson-Cook model was tested for three different materials – 304L stainless steel, A572 and 4140 steels
 - the model described the tensile stress-strain response prior to necking better than that during necking
 - Description of strain rate effect
 - Uncertainties of Bridgman correction