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Fundamental limits of time-resolved velocimetry

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Time-resolved velocimetry is a core diagnostic in dynamic compression experiments. These diagnostics map > 100 THz optical frequencies, which cannot be measured directly, to a recordable signal phase, frequency, or amplitude. All three mappings have benefits and shortcomings, but the fundamental limits of each share several features. Velocity uncertainty is always proportional to wavelength and noise fraction, while the role of characteristic time scale depends on the encoding mechanism. This relationship determines the suitability for a particular diagnostic at short and long time scales.

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Optical velocimetry relies on the Doppler shift of reflecting light from a moving target. Since optical frequencies (> 100 THz) cannot be measured directly, that information must be encoded in a more tractable form. VISAR (Velocity Interferometry System for Any Reflector)[1] encodes velocity into signal phase, using multiple signals to unwrap phase from any quadrant. PDV (Photonic Doppler Velocimetry)[2] encodes velocity into signal frequency, while other techniques[3] map this information into signal amplitude.

This discussion focuses on the relationship between velocity uncertainty and characteristic time scale. The origin of this time scale and its impact depends on the encoding method. Sample rate, signal noise, and optical wavelength also play a role in velocity uncertainty.

I. DISCRETE SAMPLING

Consider a velocimetry signal sampled at a frequency $f_s \equiv 1/T$, where T is the interval between samples. Over $N \equiv 2M+1$ samples, the signal spans a duration $\tau = 2MT$ centered about a reference time t_R.

$$s_k = s(t_R + kT) \qquad k = -M..M \tag{1}$$

Suppose that the signal exactly matches a model function $g_k = g(t_R + kT)$ in the absence of random noise, but that the presence of noise causes model parameter p to vary slightly around its ideal value. The resulting signal variance:

$$\sigma_s^2 \approx \sum_{k=-M}^{M} \left(\frac{\partial g_k}{\partial p} \sigma_p \right)^2 \tag{2}$$

can be inverted to estimate model parameter uncertainty.

$$\sigma_p = \sigma_s \left[\sum_{k=-M}^{M} \left(\frac{\partial g_k}{\partial p} \right)^2 \right]^{-2} \tag{3}$$

A discretely sampled sinusoid has the form:

$$g_k = A \sin\left[\phi + 2\pi k f T\right] \tag{4}$$

where A is the amplitude, ϕ is the phase, and f is the frequency. Combining this model with Equation 3 leads to the following parameter uncertainties.[4]

$$\sigma_\phi = \sqrt{\frac{2}{f_S \tau}}\,\frac{\sigma}{A} \tag{5}$$

$$\sigma_f = \sqrt{\frac{6}{f_S \tau^3}}\,\frac{1}{\pi}\frac{\sigma}{A} \tag{6}$$

$$\sigma_A = \sqrt{\frac{2}{f_S \tau}}\,\sigma \tag{7}$$

Real velocity measurements can approach these limits in certain circumstances. For example, steady-state PDV uncertainties are comparable to Equation 6 for non-overlapping spectral features.[5]

II. VISAR PERFORMANCE

Phase mapping allows VISAR to measure arbitrarily large velocities with limited measurement bandwidth;[1] this type of encoding has difficulty with multiple velocities and rapid velocity changes. VISAR analysis is usually formulated in terms of fringe shift ($F \equiv \phi/2\pi$) instead of phase.

$$\sigma_F = \frac{\sigma}{A}\sqrt{\frac{1}{2\pi^2 f_S \tau}} \tag{8}$$

Equating τ with the interferometer delay provides a convenient conceptual test. Recording one sample per delay results in 2.3% fringe uncertainty, while 10 samples per delay yields 0.7% fringe uncertainty. These value are consistent with the 1–2% fringe uncertainty indicated by Barker.[6]

Converting fringe shift to velocity (scaling factor $\lambda/2\tau$) leads to a velocity uncertainty:

$$\sigma_v = \lambda\,\frac{\sigma}{A}\sqrt{\frac{1}{8\pi^2 f_S \tau^3}} \tag{9}$$

where λ is the optical wavelength (often 532 nm). The noise fraction σ/A is the inverse signal-to-noise ratio.

III. PDV PERFORMANCE

Frequency mapping allows PDV to operate under conditions that VISAR cannot, such as extremely low light return, multiple velocities, and measurement nonlinearity (e.g., clipping).[7] Instead of an interferometer delay, τ represents the local analysis duration, such as the Fourier transform size, used to calculate frequency. The penalty for this type of encoding is that velocity range is limited by recording bandwidth (scaling factor $\lambda/2$).

Converting frequency to velocity leads to velocity uncertainty similar to Equation 9.

$$\sigma_v = \lambda \, \frac{\sigma}{A}\sqrt{\frac{3}{2\pi^2 f_S \tau^3}} \tag{10}$$

Both expressions vary with wavelength, noise fraction, and time scale in the same manner. If VISAR and PDV where operated at the same wavelength (e.g., 1550 nm) and time scale, the latter uncertainty would be $\sqrt{12}$ larger than the former uncertainty; this comparison ignores subtle differences in how τ relates to rise time for each diagnostic[5].

PDV is typically dominated by photon noise from the reference path, *i.e.* adding additional reference power does not reduce the noise fraction.[8] The limiting noise fraction for this condition is:

$$\frac{\sigma}{A} = \sqrt{\frac{e f_B}{2\rho P_T}} \tag{11}$$

where P_T is the Doppler-shifted (target) optical power at the detector. Combining this limit with Equation 10 leads to an expression that depends on wavelength and target power.

$$\sigma_v \geq \frac{1}{2\pi}\sqrt{\frac{\lambda h c}{P_T \tau^3}} \tag{12}$$

Figure 1 shows how uncertainty varies with time scale at several target power levels. For example, PDV uncertainty can be less than 10 m/s at 1 ns time scales and below 0.2 m/s at 10 ns for reasonable power levels (-20 dBm to -30 dBm).

The strong reduction of PDV uncertainty is a consequence of the $\tau^{-3/2}$ dependence in Equation 10. VISAR uncertainty (Equation 9) also scales with $\tau^{-3/2}$, but that benefit does not extent much beyond 10 ns for practical interferometers. PDV analysis is a software-controlled value that be increased up to about 3× the 10–90% rise time of interest.[5] For these reasons, PDV is the most reliable velocity diagnostic for time scales longer than 10 ns. Minute power returns, even -60 dBm (nanowatts) or less, can be overcome with a sufficiently large analysis duration.

Following the same logic, $\tau^{-3/2}$ scaling becomes problematic at short time scales. Practical experience indicates that analysis times below 1 ns yield poor results for anything but the most efficient probes, consistent with the trend in Figure 1. Although 10–100 m/s of uncertainty may not be a concern when measuring velocities above 10 km/s (<1% relative uncertainty), there is a domain of low-velocity transients poorly suited for PDV or VISAR.

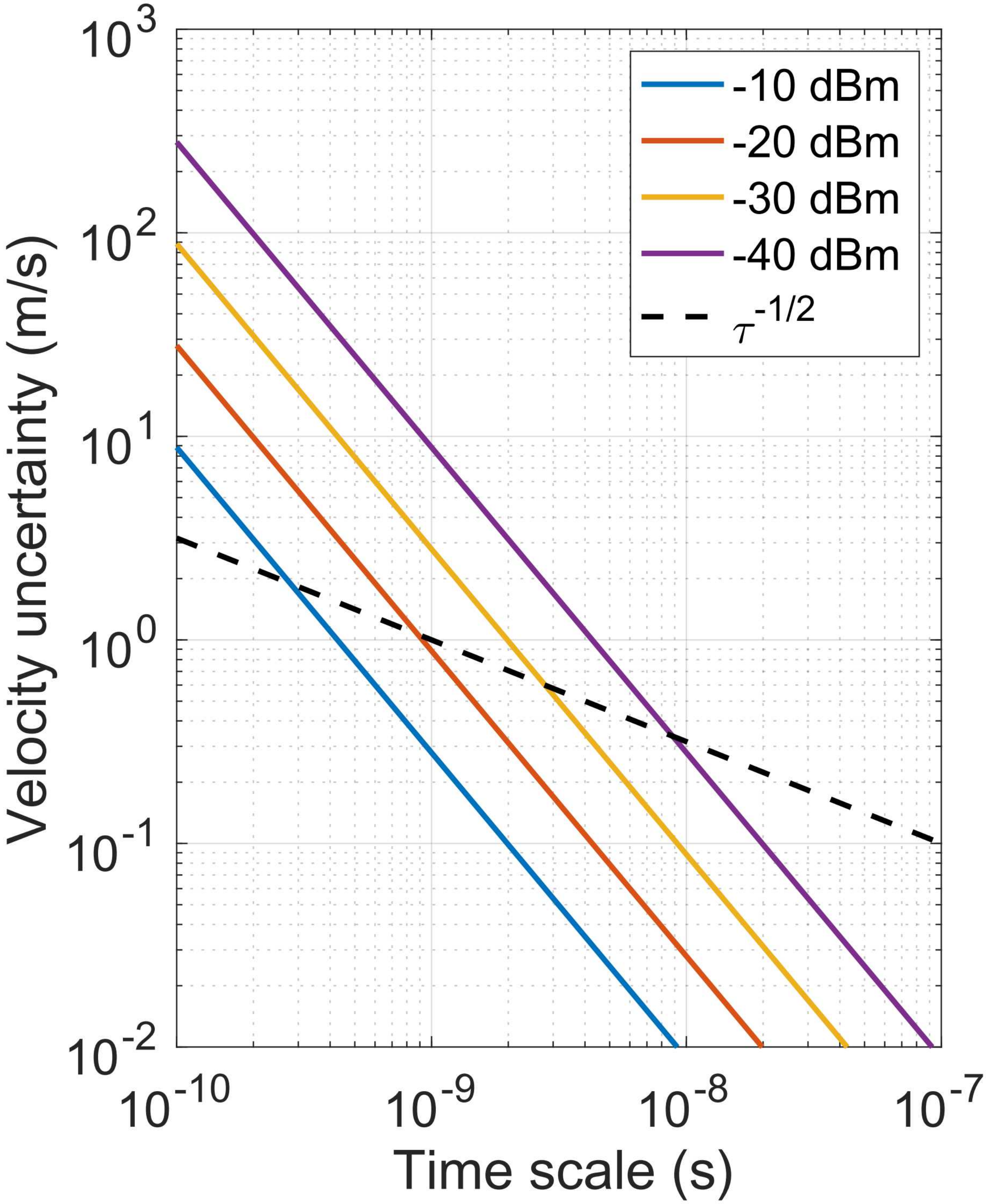

FIG. 1. Limiting velocity uncertainty in PDV at 1550 nm

IV. WEIRD PERFORMANCE

WEIRD (Wavelength Encoded Intensity Ratio Diagnostic) techniques utilize narrow absorptions to detect minute changes in optical wavelength. RALF (Rubidium Atomic Line Filtered)[3] velocimetry is one example of WEIRD measurement, but other implementations are possible. A key element WEIRD measurements is the ratio of Doppler-shifted light passing through a filter to a reference signal of unfiltered, Doppler-shifted light, which removes spurious light variations. Equation 7 must therefore be used twice, and for simplicity it is assumed that both signals have similar noise levels.

Amplitude sensitivity Q is often specified in logarithmic absorption per unit frequency, with characteristic values of 0.1–10 dB/GHz. The high end of the sensitivity range (10 dB/GHz) yields 13 dB amplitude change for 1 km/s (1550 nm operating range); lower sensitivities support a wider velocity range at coarser resolution. The velocity resolution of a WEIRD measurement is:

$$\sigma_v = \lambda \, \frac{\sigma}{A} \left(\frac{10}{Q \ln 10} \right) \sqrt{\frac{1}{f_S \tau}}. \tag{13}$$

Wavelength and noise fraction scale directly with uncertainty, as in the previous two sections, but the time variation is $\tau^{-1/2}$ instead $\tau^{-3/2}$. The dashed line Figure 1 illustrates the slower change of velocity uncertainty with time scale for $\tau^{-1/2}$ scaling. While WEIRD does not benefit from longer time scales as much as PDV, it also does not suffer as badly at short time scales. Minimum time scales in WEIRD analysis is dictated purely by the amount of signal smoothing—there is no need for a full oscillation cycle as in PDV analysis.

V. SUMMARY

Optical velocimetry is based on encoding Doppler shift onto the phase, frequency, or amplitude of recorded signal. Regardless diagnostic implementation, the limiting performance depends on the encoding method. Phase and frequency encodings have a velocity uncertainty that scales with $\tau^{-3/2}$; PDV can benefit from that scaling for $>$ 10 ns time scales while VISAR cannot. The disadvantage of $\tau^{-3/2}$ scaling becomes apparent at short time scales: 0.1 ns time scales have 32× the uncertainty of 1 ns time scales. WEIRD uncertainty scales with $\tau^{-1/2}$ and may be better suited to sub-nanosecond measurements than VISAR or PDV.

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