

Digital quantum simulation of quantum dynamics and control



Alicia Magann^{1,2}, Mohan Sarovar¹

¹ Sandia National Laboratories, California

² Princeton University



Motivation

Can we use light to control chemistry?

Idea:

Use shaped fields to control molecular systems

Challenge:

Simulations of the dynamics and control of many quantum systems are infeasible

Problem for:

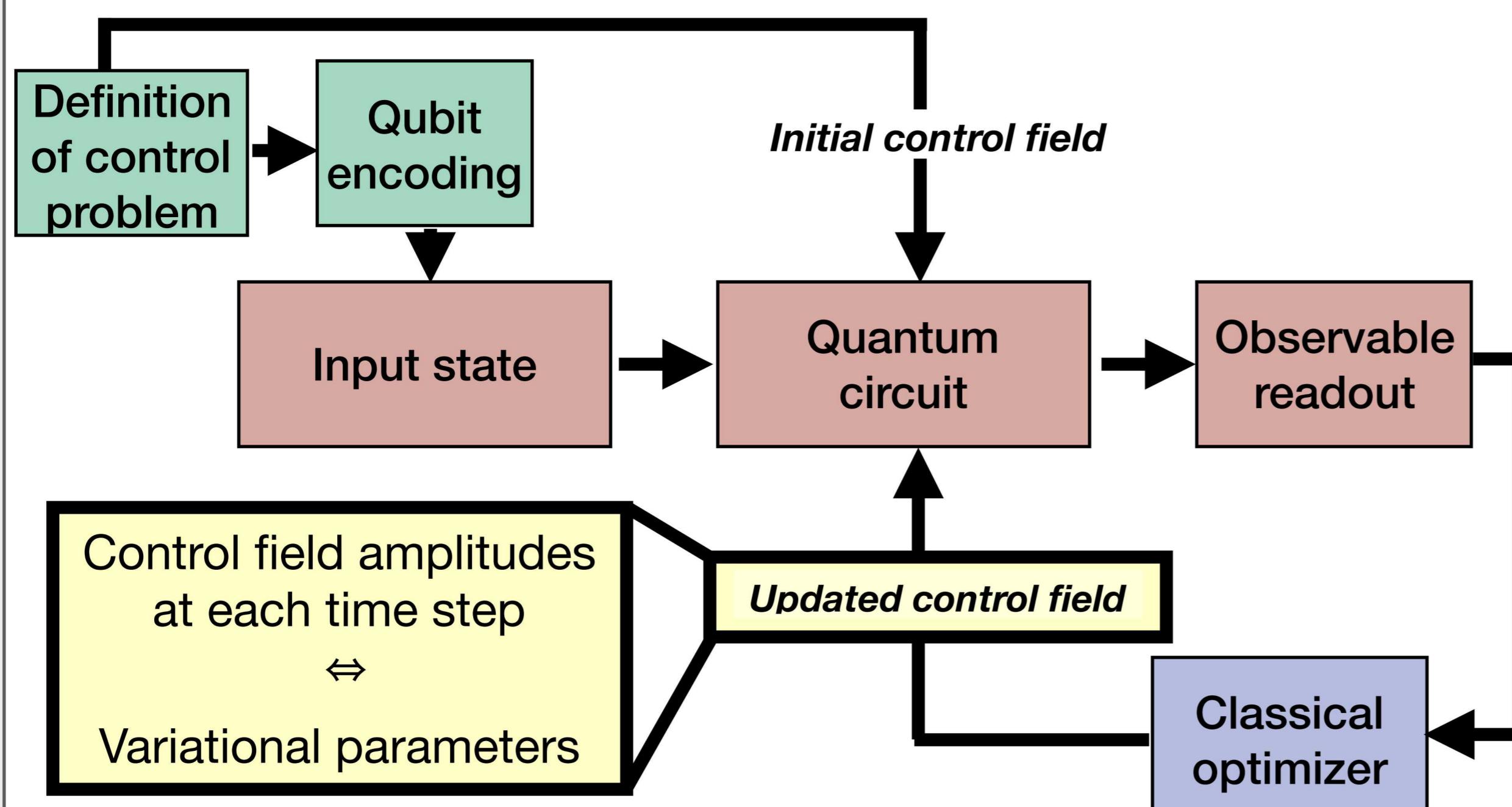
- feasibility studies
- experimental guidance
- control mechanism analysis

“Curse of dimensionality”

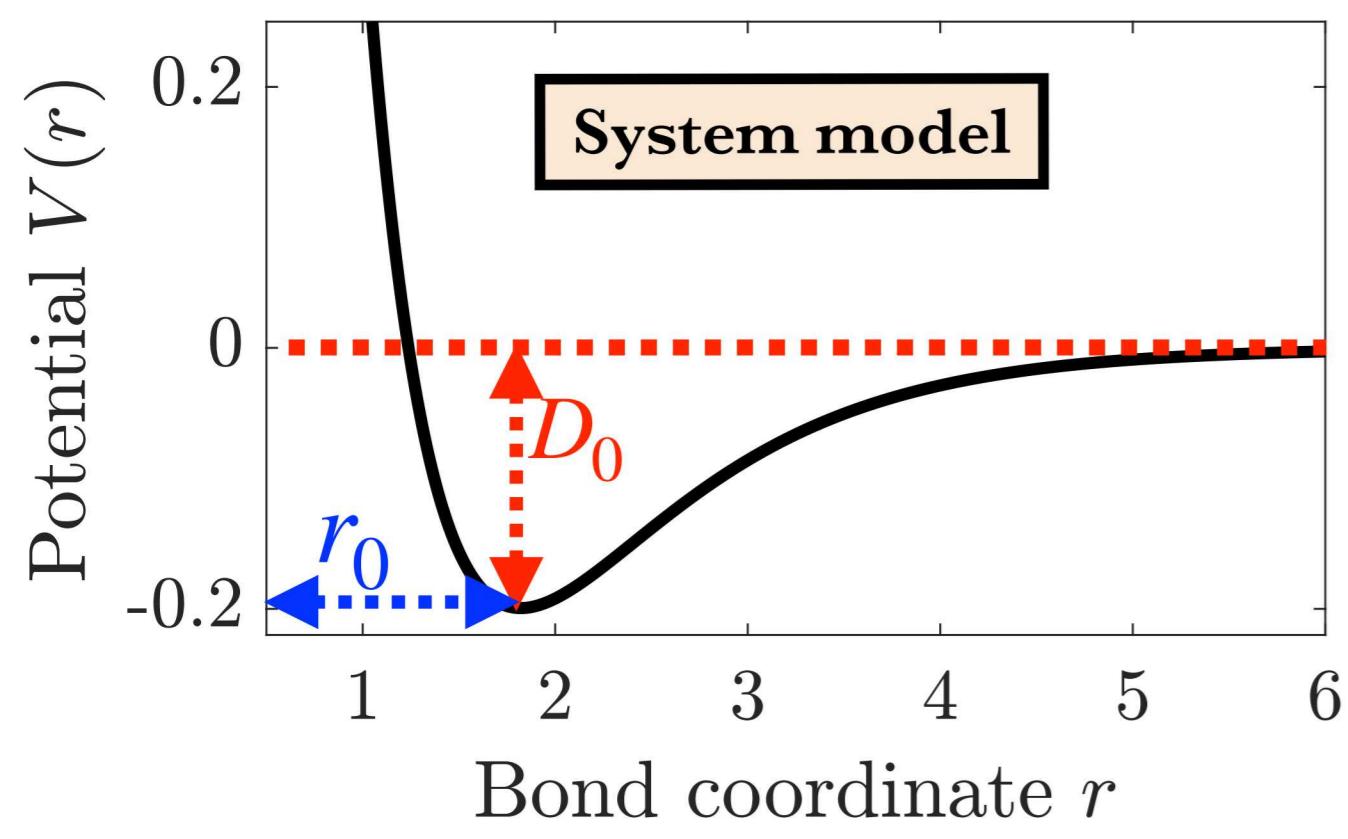
Goal:

Explore how a quantum computer could be used to address this challenge by serving as a digital quantum simulator in quantum control simulations

Simulation framework



Example problem: Controlling the vibrational state of a molecular bond

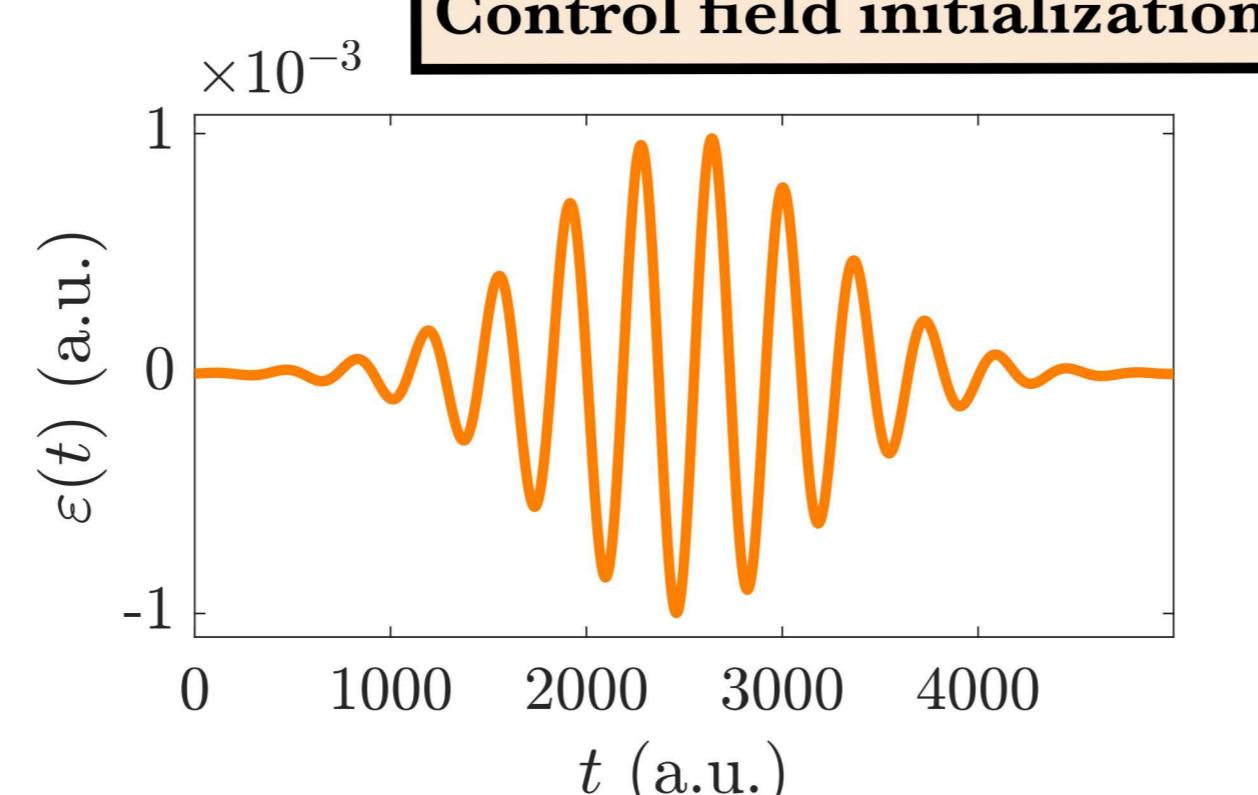


Control field specification

Set pulse duration as
 $T = 5,000 \text{ a.u.} \approx 121 \text{ fs}$

Set time step as
 $\Delta t = 2 \text{ a.u.}$

Identify field $\epsilon(t)$ to prepare particular vibrational eigenstate $|\psi\rangle$ at terminal time T



Illustration

The k th vibrational state is encoded as:

$$|k\rangle = |0\rangle_0 |0\rangle_1 \cdots |0\rangle_{k-1} |1\rangle_k |0\rangle_{k+1} \cdots |0\rangle_{d-1}$$

The operators are encoded as:

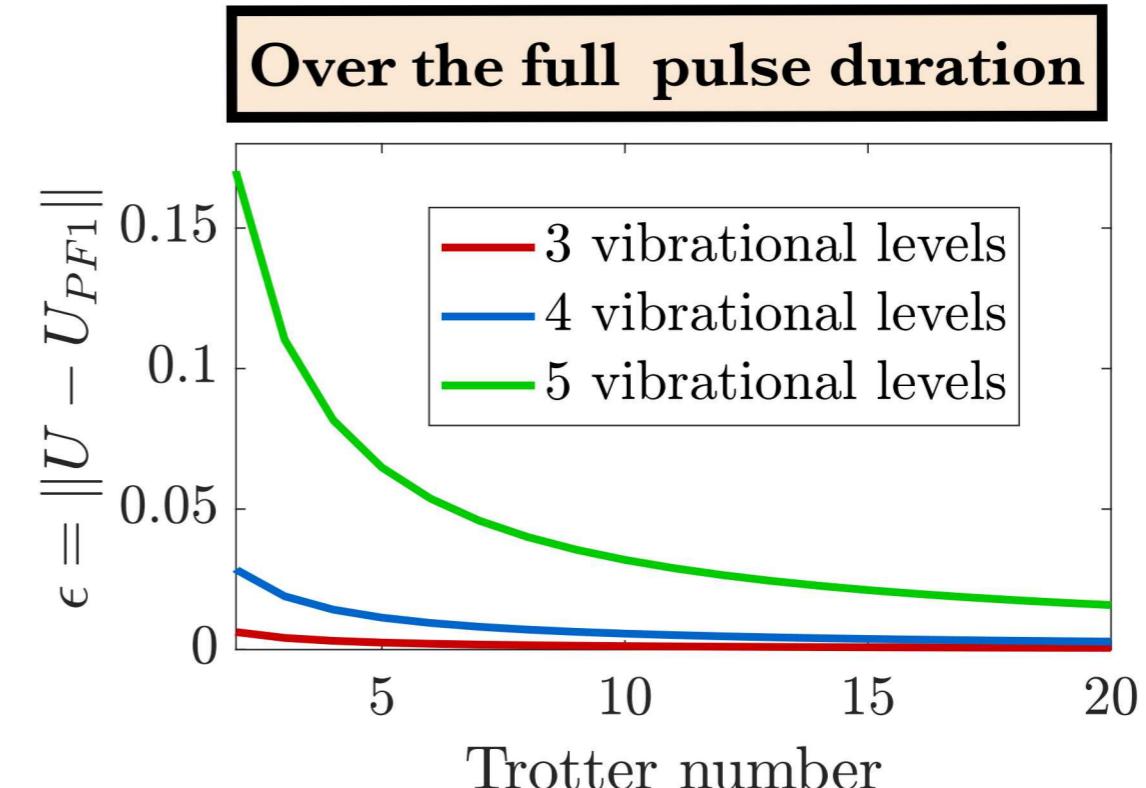
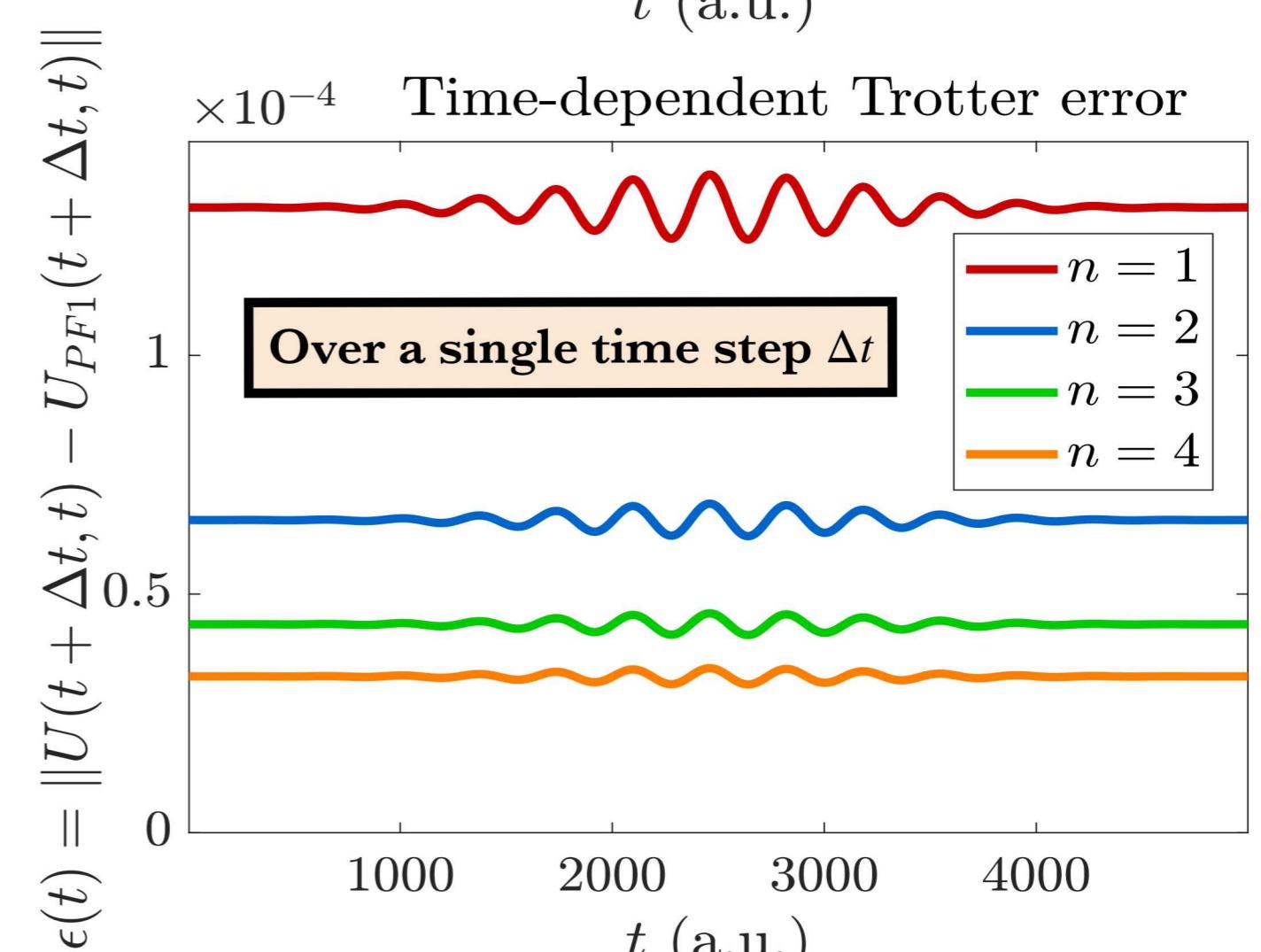
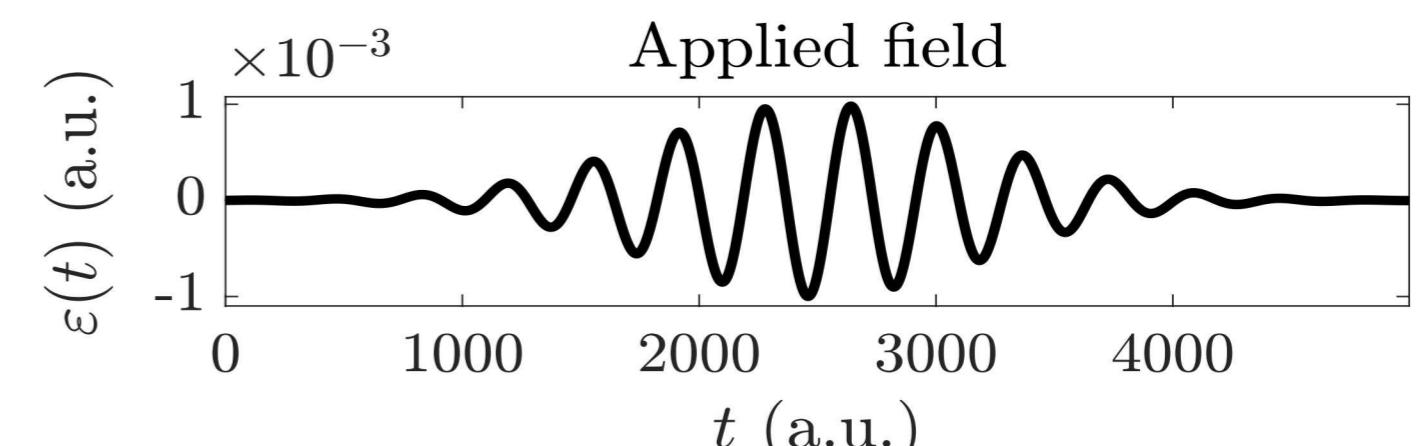
$$a^\dagger = \sum_{k=0}^{d-2} \sqrt{k+1} (\sigma_-^k \sigma_+^{k+1}) \quad a^\dagger a = \sum_{k=0}^{d-1} k \left(\frac{\sigma_z^k + 1}{2} \right)$$

Hamiltonian simulation algorithm used to evolve system under influence of control field

$$e^{-iH(t)\Delta t} \approx (e^{-iH_1 t/n} e^{-iH_2 t/n} \cdots e^{-iH_d t/n})^n \quad \text{“Trotter”}$$

To simulate time evolution over N time steps of length Δt , take the product

$$e^{-iH(t_N)\Delta t} \cdots e^{-iH(t_1)\Delta t} e^{-iH(t_0)\Delta t}$$



For simulation results, measure qubits in computational basis

Outlook

- Exploration and comparison of different quantum algorithms for Hamiltonian simulation
- Analysis of the tradeoff between **gradient** and **gradient-free** optimization algorithms for identifying control fields
- Estimation of required quantum resources
- Extensions to quantum control problems with greater complexity

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