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A Chance-Constrained model for Stochastic Unit Commitment

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Stochastic unit commitment

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Standard unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations?

Stochastic unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations, under uncertainty (of demand, prices, renewables...)?

But...

- Thermal generator operational limits are based on engineering judgments
- Can be exceeded in practice, for short periods
- System operators do run thermal generators beyond these limits

Stochastic unit commitment

Proposed model

- Allow thermal generators to "occasionally" violate operational limits
- Violations should be few (else, increased maintenance costs)
- Violations should not be large (there are absolute ratings of generators)
- 1% savings in energy production is worth $\approx$ \$1 billion per year in the U.S. alone

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Stochastic unit commitment

Proposed model

- Let $y_t^{g,\omega}$ denote a "non-nominal" operation in hour t for generator g in scenario ω
- During non-nominal operations, generator's operating region expands from $[\underline{P}^g, \overline{P}^g]$ to $[\underline{\underline{P}}^g, \overline{\overline{P}}^g]$
- Non-nominal mode of generation is more expensive
- Number of non-nominalities is few: $\frac{1}{|\Omega||\mathcal{T}||\mathcal{G}|} \sum_{g \in \mathcal{G}} \sum_{t \in T} \sum_{\omega \in \Omega} y_t^{g,\omega} \leq \varepsilon \leftarrow$ almost a chance-constraint!

What is a Chance Constraint?

This is a linear Joint Chance Constraint:

$$P(x_t \leq y_t^{\omega} + w_t^{\omega}, \forall t \in T) \geq 1 - \varepsilon$$

Background:

- Two-stage stochastic program with recourse
- First stage decision, x_t, second-stage decision, y_t^{ω}
- Possibly integer restrictions on x and/or y
- i.i.d. samples of uncertainty w_t^{ω}

Challenges of chance-constraint (CC) models

- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

Again...

Proposed model

- Let $y_t^{g,\omega}$ denote a "non-nominal" operation in hour t for generator g in scenario ω
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Setup

We use:

$$
\begin{aligned}
\overline{\overline{P}}^{g} &= (1+\beta)\overline{P}^{g} \\
\underline{\underline{P}}^{g} &= (1-\beta)\underline{P}^{g} \\
\overline{C}^{g} &= (1+\gamma)C^{L^{g},g} \\
\underline{C}^{g} &= (1+\gamma)C^{L^{g},g}.
\end{aligned}
$$

WECC240++ system with 85 thermal generators, 50 scenarios and RTS-GMLC system with 73 thermal generators, 16 scenarios

Computational results for the RTS-GMLC 16 scenario case for 10 July 2020.

Accepted: *Computational Management Science*

Table: MIP gap = 0.1%

ε	β	γ	Cost (M$)	Savings (%)	Time (sec)	MIP gap (%)
0			3.89	0.00%	33	-
0.01	0.05	0.1	3.84	1.21%	46	-
		0.2	3.84	1.20%	48	-
	0.1	0.1	3.83	1.51%	82	-
		0.2	3.83	1.50%	106	-
0.05	0.05	0.1	3.83	1.53%	65	-
		0.2	3.83	1.45%	100	-
	0.1	0.1	3.81	2.08%	1800	0.22%
		0.2	3.82	1.82%	1800	0.15%

- Increase $\varepsilon \Rightarrow$ increase savings
- Increase $\beta \Rightarrow$ increase savings
- Increase $\gamma \Rightarrow$ decrease savings

Cost savings for the RTS-GMLC 16 scenario case for 10 July 2020.

Accepted: *Computational Management Science*

Table: MIP gap = 0.1%

ε	β	γ	Optimal	Limited	No nuclear
0.01	0.05	0.1	1.21%	0.71%	1.06%
		0.2	1.20%	0.69%	1.04%
	0.1	0.1	1.51%	1.14%	1.15%
		0.2	1.50%	1.10%	1.11%
0.05	0.05	0.1	1.53%	0.70%	1.22%
		0.2	1.45%	0.69%	1.15%
	0.1	0.1	2.08%	1.14%	1.41%
		0.2	1.82%	1.10%	1.28%

Limited = at most one non-nominal operation per generator per day
No nuclear = no non-nominal operation for the nuclear unit in this system

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Stochastic unit commitment model

Indices and Sets:

$g \in \mathcal{G}$	Thermal generators.
$t \in \mathcal{T}$	Hourly time steps: $1, \ldots, T$; i.e., $[a, b) \in \mathcal{T} \times \mathcal{T}$ such that $b \geq a + UT^g$.
$l \in \mathcal{L}^g$	Piecewise production cost intervals for generator g: $1, \ldots, L_g$.
$s \in \mathcal{S}^g$	Start-up categories for generator g, from hottest (1) to coldest (S_g).
$\omega \in \Omega$	Scenarios: $\omega_1, \ldots, \omega_N$.

Parameters: First Stage

$C^{l,g}$	Marginal cost for piecewise segment l for generator g (\$/MWh).
$\overline{C}^g$	Marginal cost for production above $\overline{P}^g$ (\$/MWh).
$\underline{C}^g$	Marginal cost for production below $\underline{P}^g$ (\$/MWh).
$C^{R,g}$	Cost of generator g running and operating at minimum production $\underline{P}_g$ (\$/h).
$C^{s,g}$	Start-up cost of category s for generator g (\$).
DT^g	Minimum down time for generator g (h).
$\overline{P}^g$	Maximum power output for generator g under normal operations (MW).
$\overline{\overline{P}}^g$	Maximum power output for generator g under non-nominal operations (MW).
$\underline{P}^g$	Minimum power output for generator g under normal operations (MW).
$\underline{\underline{P}}^g$	Minimum power output for generator g under non-nominal operations (MW).
$\overline{\overline{P}}^{l,g}$	Maximum power available for piecewise segment l for generator g (MW) (with $\overline{P}^{0,g} = \underline{P}^g$).
RD^g	Ramp-down rate for generator g (MW/h).
RU^g	Ramp-up rate for generator g (MW/h).
SD^g	Shutdown ramp rate for generator g (MW/h).
SU^g	Start-up ramp rate for generator g (MW/h).
TC^g	Time down after which generator g goes cold (h).
$\underline{T}^{s,g}$	Time offline after which the start-up category s is available (h) (with $\underline{T}^{1,g} = DT^g$, $\underline{T}^{S^g,g} = TC^g$)
UT^g	Minimum up time for generator g (h).

Stochastic unit commitment model

Parameters: Second Stage

D_t^ω	Load (demand) at time t in scenario ω (MW).
$\overline{W}_t^\omega$	Maximum power from renewables at time t in scenario ω (MW).
$\underline{W}_t^\omega$	Minimum power from renewables at time t in scenario ω (MW).

Variables: First Stage

u_t^g	Commitment status of generator g at time t, $\in \{0, 1\}$.
v_t^g	Start-up status of generator g at time t, $\in \{0, 1\}$.
w_t^g	Shutdown status of generator g at time t, $\in \{0, 1\}$.
$x_{[t,t')}^g$	Indicator arc for shutdown at time t, start-up at time t', uncommitted for $i \in [t, t')$, for generator g, $\in \{0, 1\}$, $[t, t')$ such that $t + DT^g \leq t' \leq t + TC^g - 1$.

Variables: Second Stage

$p_t^{g,\omega}$	Power above minimum from generator g at time t in scenario ω (MW).
$\overline{P}_t^{g,\omega}$	Power above maximum from generator g at time t in scenario ω (MW).
$\underline{P}_t^{g,\omega}$	Power below minimum from generator g at time t in scenario ω (MW).
$p_t^{l,g,\omega}$	Power from piecewise interval l for generator g at time t in scenario ω (MW).
$r_t^{h,\omega}$	Power from renewables at time t in scenario ω (MW).
$y_t^{g,\omega}$	Non-nominal operation status of generator g at time t in scenario (MW).

Stochastic unit commitment

$$\min \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left(\sum_{l \in \mathcal{L}^g} \mathbb{E}[C^{l,g}\, p_t^{l,g,\omega} + \overline{C}^g\, \overline{P}_t^{g,\omega} + \underline{C}^g\, \underline{P}^{g,\omega}] + C^{R,g}\, u_t^g + c_t^{SU,g} \right) \tag{1}$$

subject to:

$$u_t^g - u_{t-1}^g = v_t^g - w_t^g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{2a}$$

$$\sum_{i=t-UT^g+1}^{t} v_i^g \leq u_t^g \qquad \forall t \in [UT^g, T],\ \forall g \in \mathcal{G} \tag{2b}$$

$$\sum_{i=t-DT^g+1}^{t} w_i^g \leq 1 - u_t^g \qquad \forall t \in [DT^g, T],\ \forall g \in \mathcal{G} \tag{2c}$$

$$\sum_{t'=t-TC^g+1}^{t-DT^g} x_{[t',t)}^g \leq v_t^g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{2d}$$

$$\sum_{t'=t+DT^g}^{t+TC^g-1} x_{[t,t')}^g \leq w_t^g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{2e}$$

$$c_t^{SU,g} = C^{S,g} v_t^g + \sum_{s=1}^{S^g-1} (C^{s,g} - C^{S,g}) \left(\sum_{t'=t-\underline{T}^{s+1,g}+1}^{t-\underline{T}^{s,g}} x_{[t',t)}^g \right) \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{2f}$$

Stochastic unit commitment

$$p_t^{g,\omega} \le (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SU^g)v_t^g - (\overline{P}^g - SD^g)w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \tag{3a}$$

$$p_t^{g,\omega} \le (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SU^g)v_t^g \quad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G}^1,\ \forall \omega \in \Omega \tag{3b}$$

$$p_t^{g,\omega} \le (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SD^g)w_{t+1}^g \quad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G}^1,\ \forall \omega \in \Omega \tag{3c}$$

$$p_t^{g,\omega} - p_{t-1}^{g,\omega} \le (SU^g - RU^g - \underline{P}^g)v_t^g + RU^g u_t^g \quad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{3d}$$

$$p_{t-1}^{g,\omega} - p_t^{g,\omega} \le (SD^g - RD^g - \underline{P}^g)w_t^g + RD^g u_{t-1}^g \quad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{3e}$$

$$p_t^{g,\omega} = \sum_{l \in \mathcal{L}^g} p_t^{l,g,\omega} \quad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{3f}$$

$$p_t^{l,g,\omega} \le (\overline{P}^{l,g} - \overline{P}^{l-1,g})u_t^g \quad \forall t \in \mathcal{T},\ \forall l \in \mathcal{L}^g,\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{3g}$$

Stochastic unit commitment

$$y_t^{g,\omega} \leq u_t^g - v_t^g - w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \tag{4a}$$

$$y_t^{g,\omega} \leq u_t^g - v_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \tag{4b}$$

$$y_t^{g,\omega} \leq u_t^g - w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \tag{4c}$$

$$\overline{P}_t^{g,\omega} \leq \left(\overline{\overline{P}} - \overline{P}\right) y_t^{g,\omega} \quad \forall t \in \mathcal{T}, \ \forall g \in \mathcal{G}, \ \forall \omega \in \Omega \tag{4d}$$

$$\underline{P}_t^{g,\omega} \leq \left(\underline{P} - \underline{\underline{P}}\right) y_t^{g,\omega} \quad \forall t \in \mathcal{T}, \ \forall g \in \mathcal{G}, \ \forall \omega \in \Omega \tag{4e}$$

$$\sum_{g \in \mathcal{G}} \left(p_t^{g,\omega} + \overline{P}_t^{g,\omega} - \underline{P}_t^{g,\omega} + \underline{P}^g u_t^g\right) + r_t^\omega = D_t^\omega \forall t \in \mathcal{T}, \ \forall \omega \in \Omega \tag{5}$$

$$\frac{1}{|\mathcal{G}||\mathcal{T}||\Omega|} \sum_{\omega \in \Omega} \sum_{g \in \mathcal{G}} \sum_{t \in T} y_t^{g,\omega} \leq \varepsilon \tag{6}$$

$$p_t^{l,g,\omega} \in \mathbb{R}_+ \quad \forall t \in \mathcal{T}, \ \forall l \in \mathcal{L}^g, \ \forall g \in \mathcal{G}, \ \forall \omega \in \Omega \tag{7a}$$

$$p_t^{g,\omega}, \overline{P}_t^{g,\omega}, \underline{P}_t^{g,\omega} \in \mathbb{R}_+ \quad \forall t \in \mathcal{T}, \ \forall g \in \mathcal{G}, \ \forall \omega \in \Omega \tag{7b}$$

$$r_t^{n,\omega} \in \left[\underline{W}_t^{n,\omega}, \overline{W}_t^{n,\omega}\right] \quad \forall t \in \mathcal{T}, \forall n \in \mathcal{N}, \forall \omega \in \Omega \tag{7c}$$

$$u_t^g, v_t^g, w_t^g \in \{0,1\} \quad \forall t \in \mathcal{T}, \ \forall g \in \mathcal{G} \tag{7d}$$

$$x_{[t,t')}^g \in \{0,1\} \quad \forall [t,t') \in \mathcal{X}^g, \ \forall g \in \mathcal{G} \tag{7e}$$

$$y_t^{g,\omega} \in \{0,1\} \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega. \tag{7f}$$

Computational results for the WECC240++ 50 scenario test case for 11 May 2013.

Table: MIP gap = 0.1%

ε	β	γ	Cost (K$)	Savings (%)	Time (sec)	MIP gap (%)
0			64.41	0.00%	183	-
0.01	0.05	0.1	64.20	0.33%	275	-
		0.2	64.21	0.31%	242	-
	0.1	0.1	64.03	0.59%	258	-
		0.2	64.04	0.58%	317	-
0.05	0.05	0.1	63.86	0.85%	275	-
		0.2	63.90	0.80%	343	-
	0.1	0.1	63.35	1.64%	378	-
		0.2	63.42	1.55%	371	-

- Increase $\varepsilon \Rightarrow$ increase savings
- Increase $\beta \Rightarrow$ increase savings
- Increase $\gamma \Rightarrow$ decrease savings

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