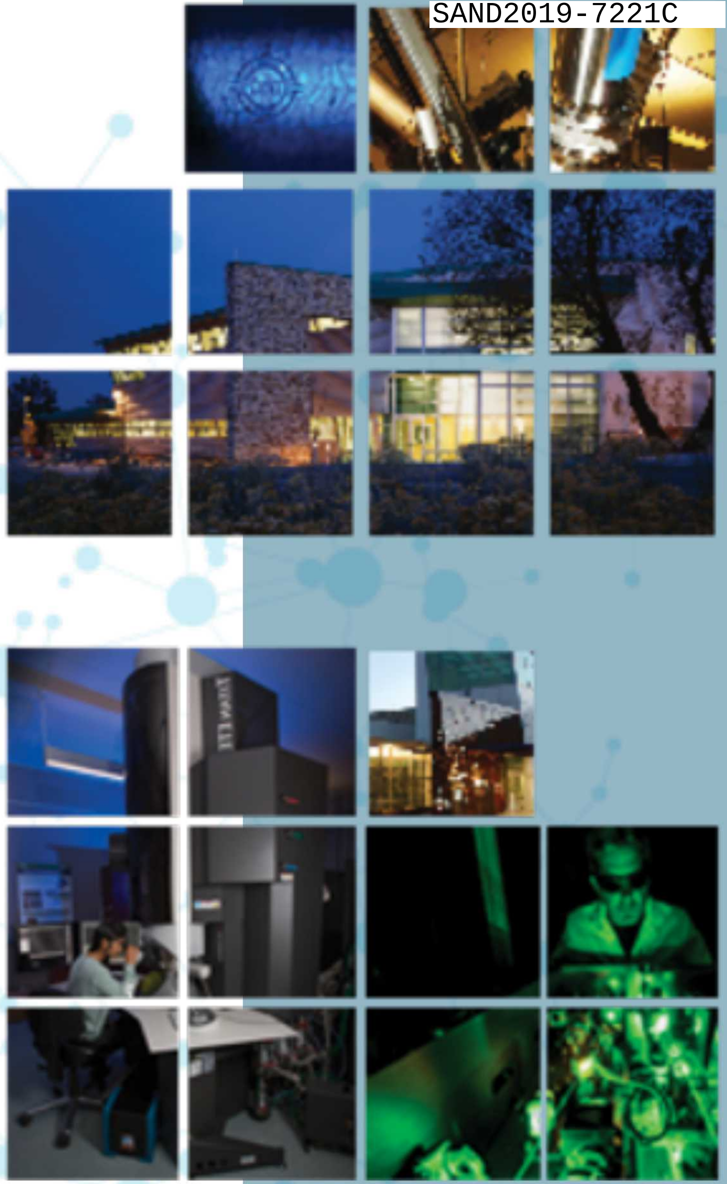
SAND2019-7221C

Statistical and Continuum Modeling of Dense Granular Flows at the Flow-Arrest Transition

Ishan Srivastava

Collaborators: Jeremy Lechman (*Sandia*), Leonardo Silbert (*CNM*), Gary Grest (*Sandia*)

Sandia National Laboratories



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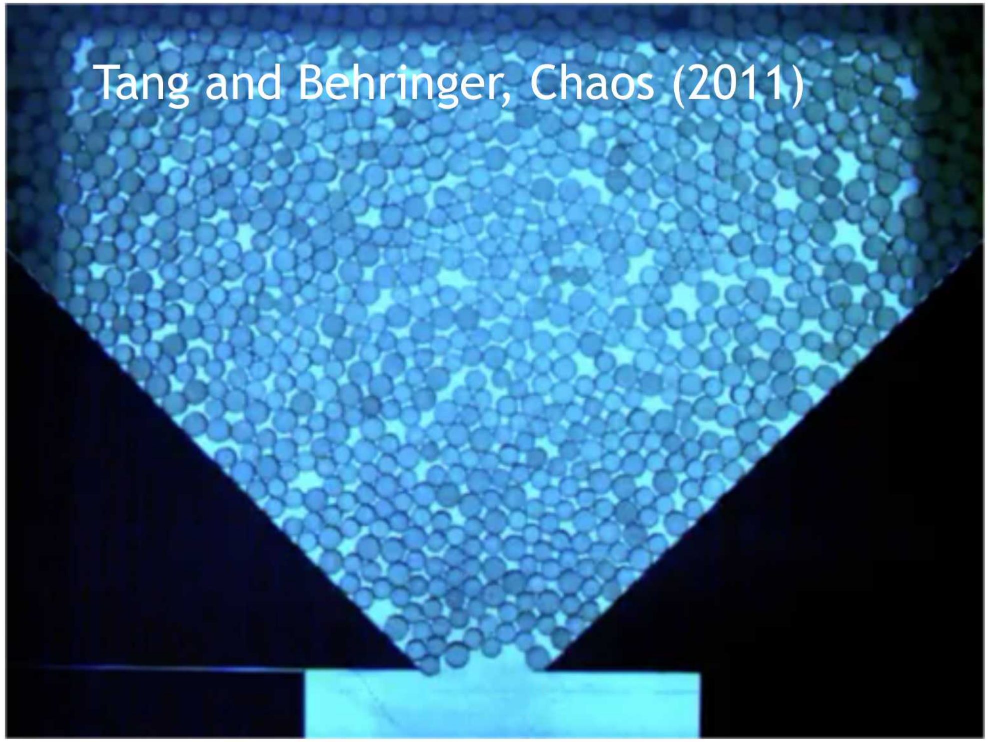
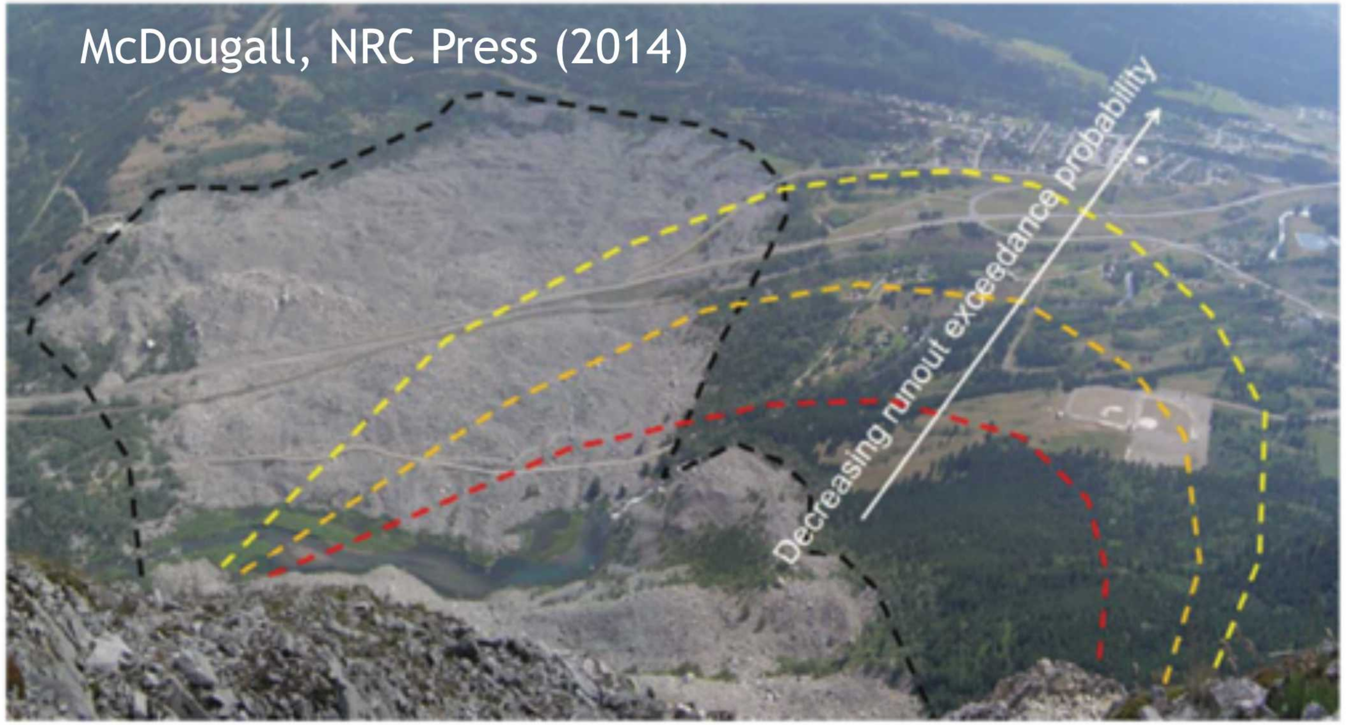
This work was performed, in part, at the Center for Integrated Nanotechnologies, an Office of Science User Facility operated for the U.S. Department of Energy (DOE) Office of Science.

Flow-Arrest Transition in Granular Materials

clogging in hopper

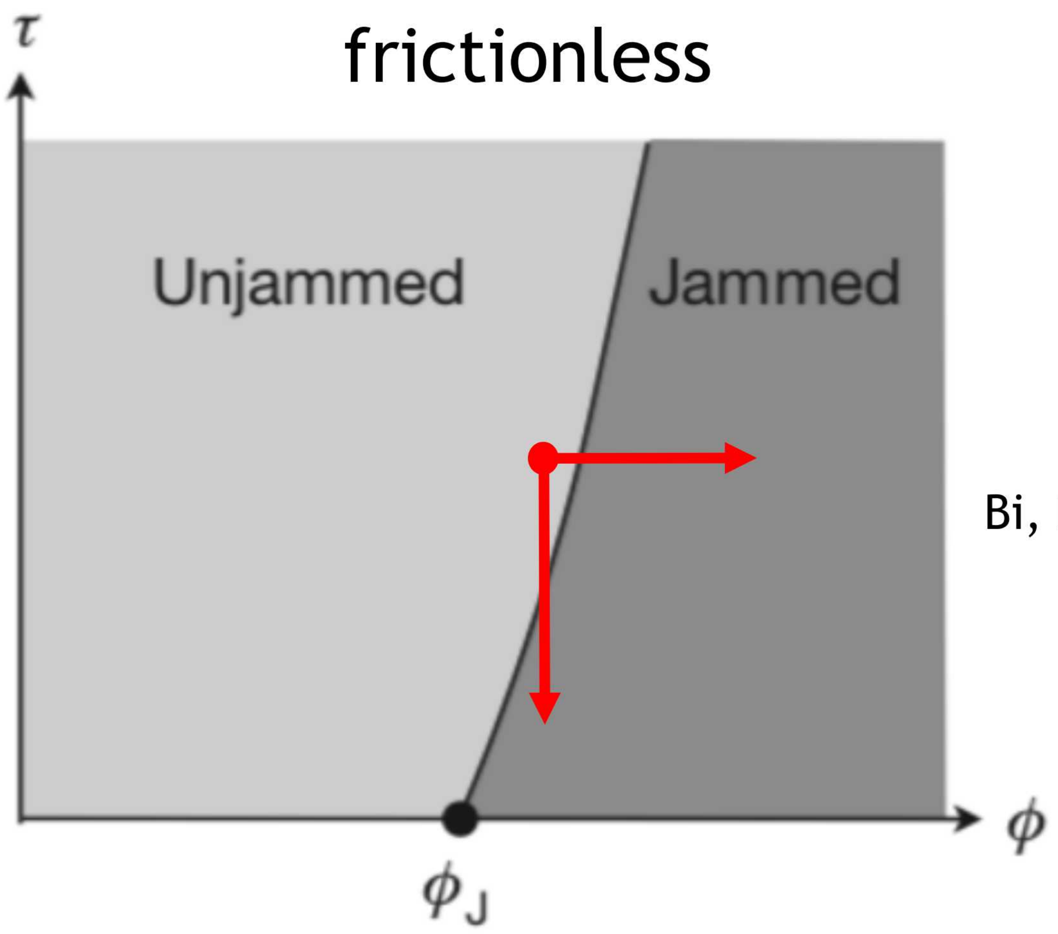


landslide runout probability

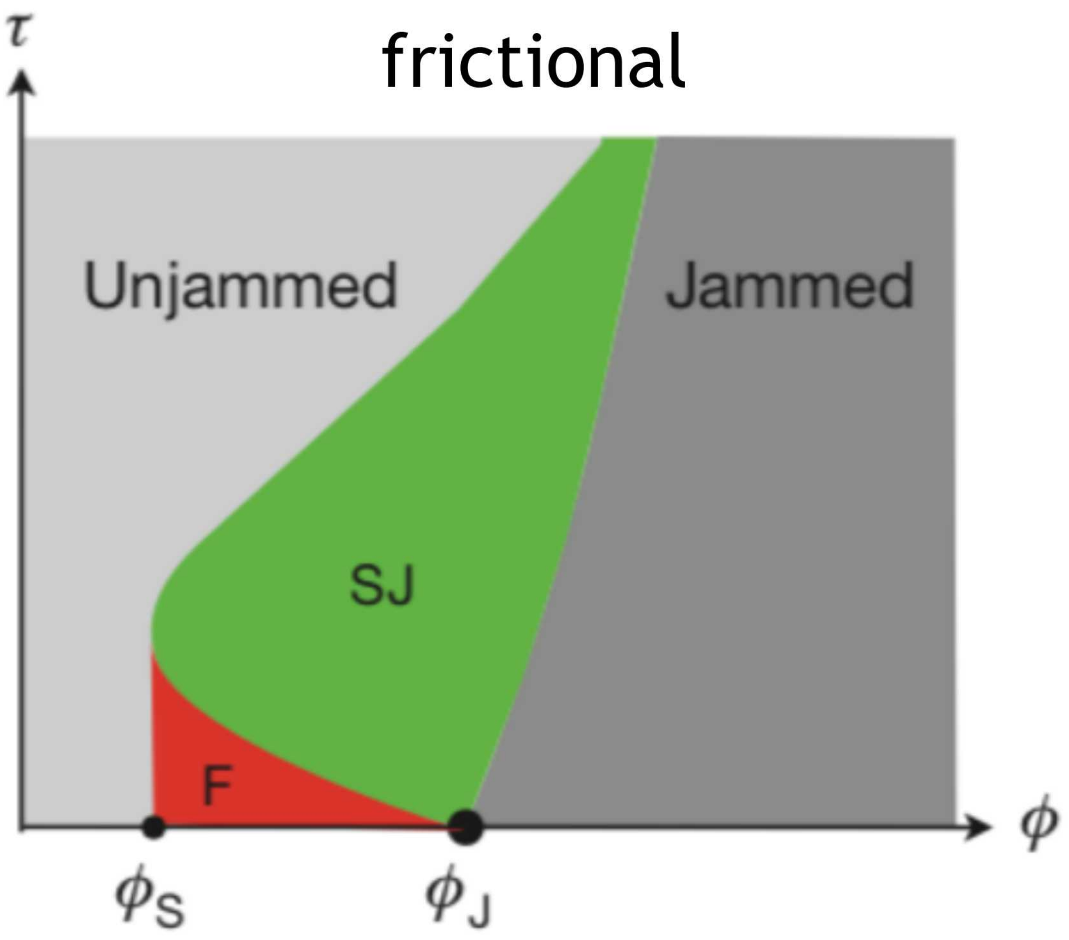


1. Will the flow arrest?
2. When will the flow arrest?
3. If it flows, how does it flow?

Will Flowing Granular Material Arrest (Jam)?



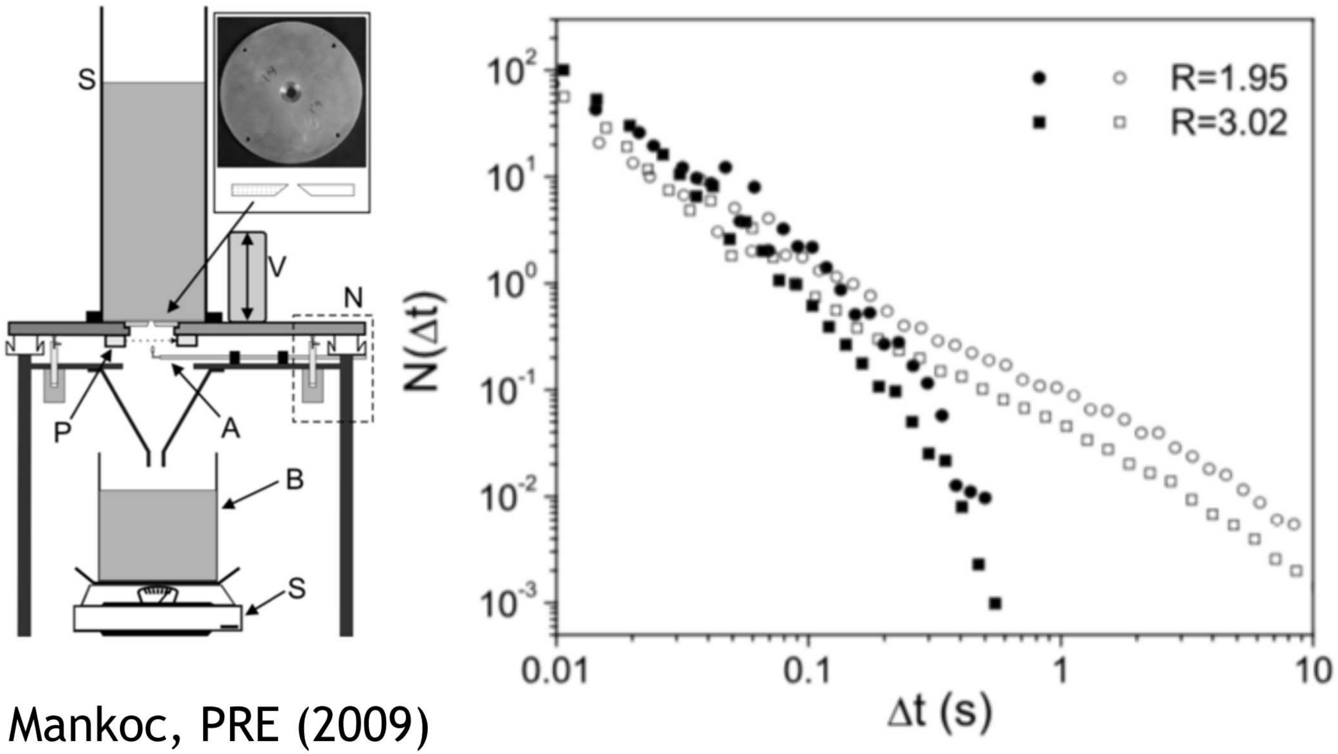
Bi, Nature (2011)



what is the phase diagram under controlled pressure conditions?

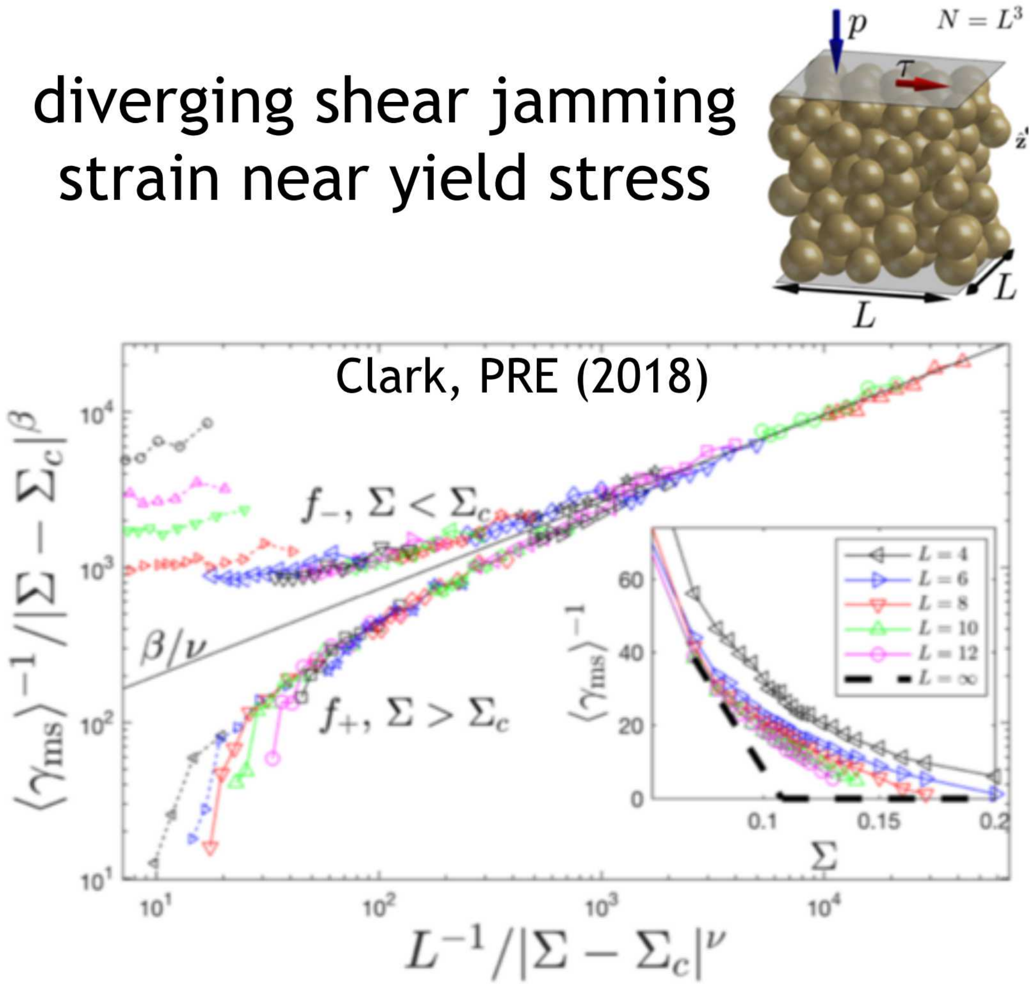
When will Flowing Granular Material Arrest?

distribution of clog times in silo discharge



Mankoc, PRE (2009)

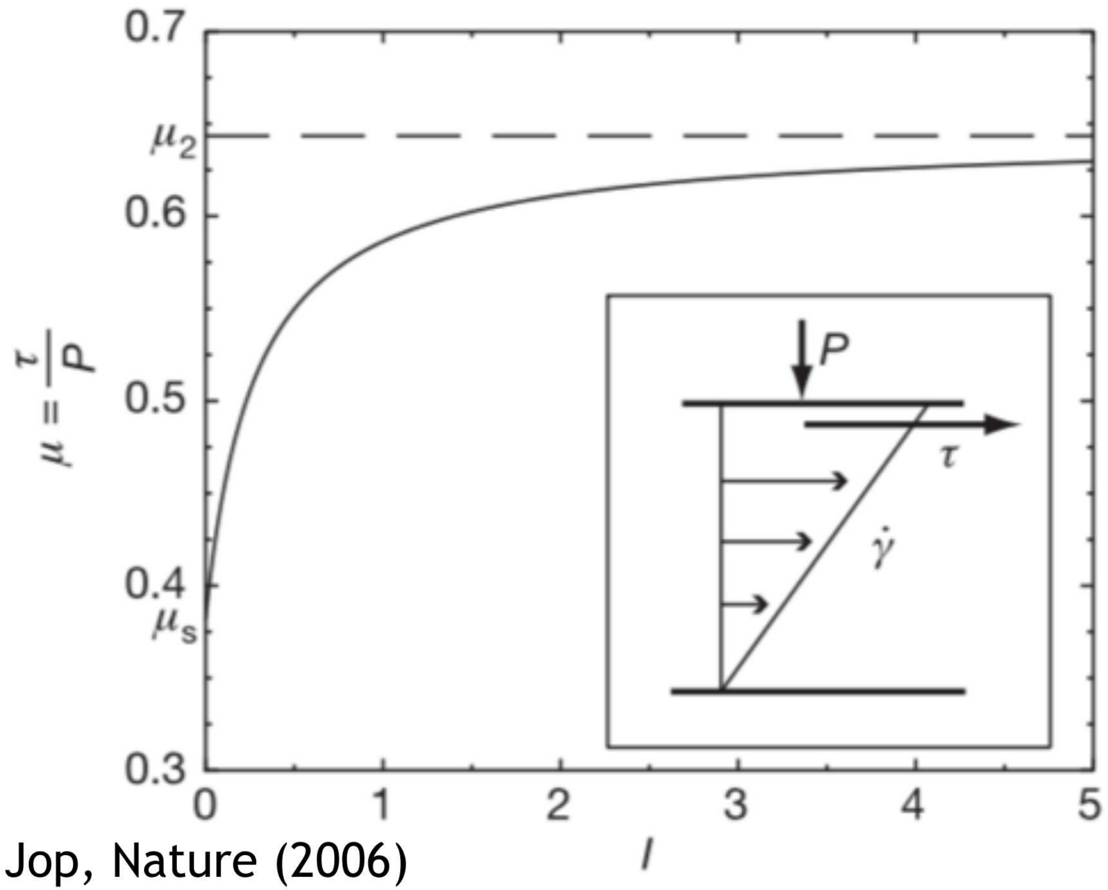
diverging shear jamming strain near yield stress



need a better characterization of transient flow before arrest

When it does Flow, how does it Flow? (Rheology)

inertial granular rheology



Jop, Nature (2006)

inertial Number: $I = \dot{\gamma} d \sqrt{\frac{\rho}{P}}$

(d : diameter
ρ: density)

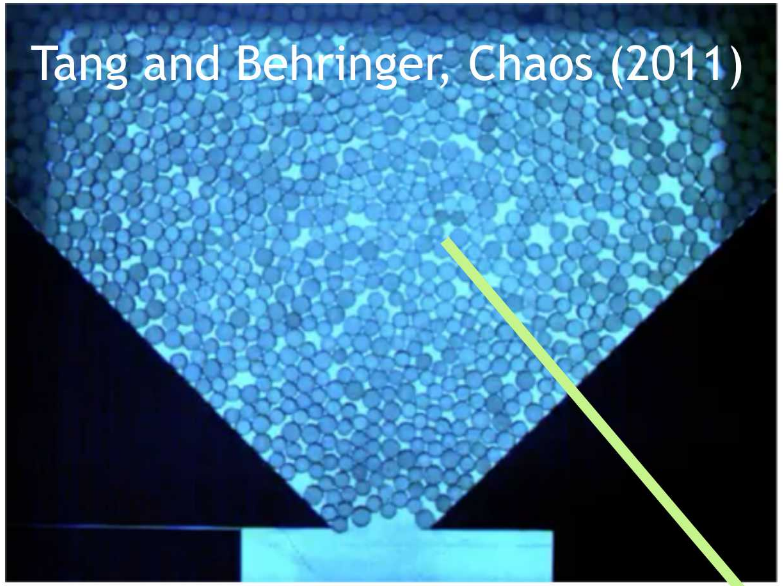
stress ratio: $\mu = \frac{\tau}{P}$

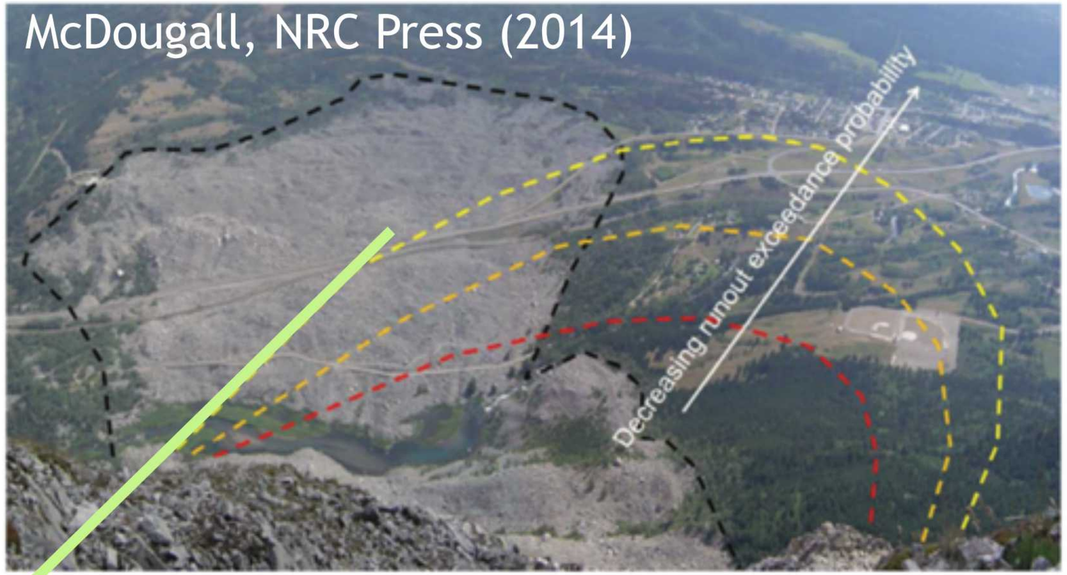
inertial rheology: $\mu(I) = \mu_s + \frac{\mu_s - \mu_s}{I_0/I + 1}$

(monotonic, yield criterion)

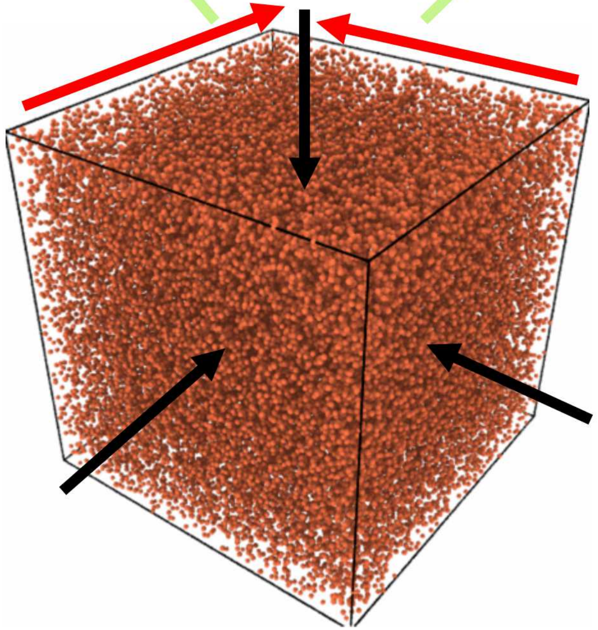
does this rheology sufficiently characterize 3D granular shear flows?

Flow-Arrest Transition: Problem Statement





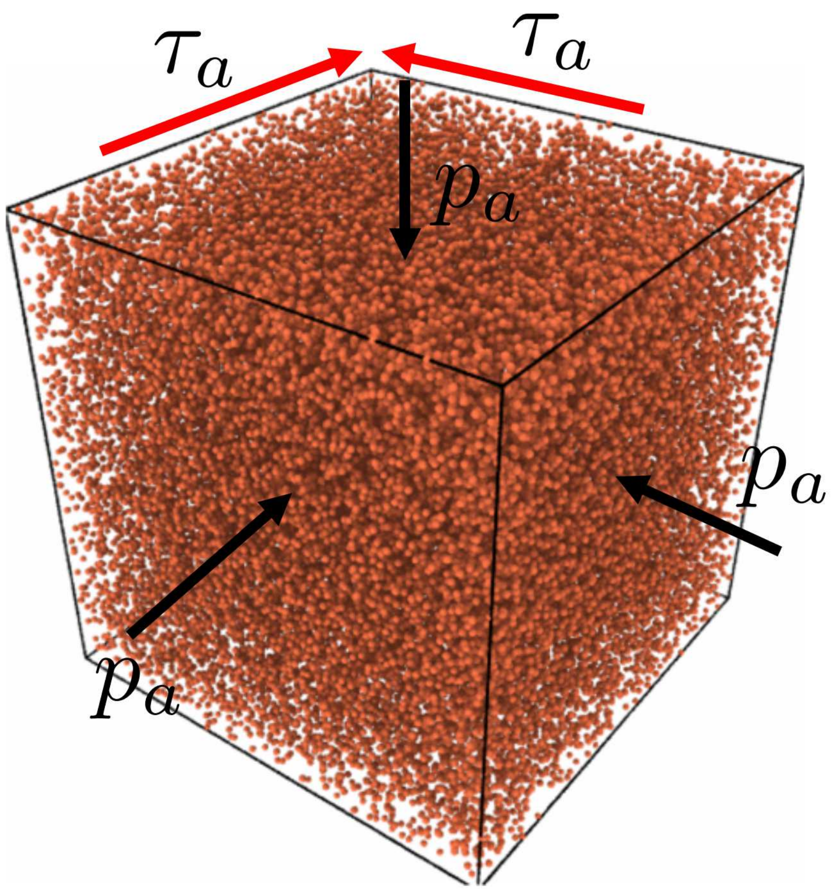
representative volume element under shear & pressure



under prescribed shear and pressure:

1. Will the flow arrest?
2. When will the flow arrest?
3. If it flows, how does it flow?

DEM Simulation Protocol



$$\boldsymbol{\sigma}_a = p_a \boldsymbol{I} + \begin{bmatrix} 0 & \tau_a & 0 \\ \tau_a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

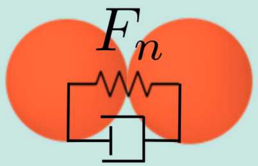
Parinello-Rahman dynamics
(isenthalpic-isotension ensemble in MD)

- fully periodic: **no boundary effects**
- uniform surface traction
- homogenous deformation/flow
- no shear bands

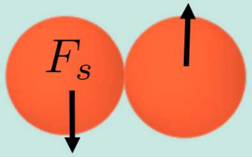
Srivastava, PRL (2019)

contact mechanics

Hookean contacts

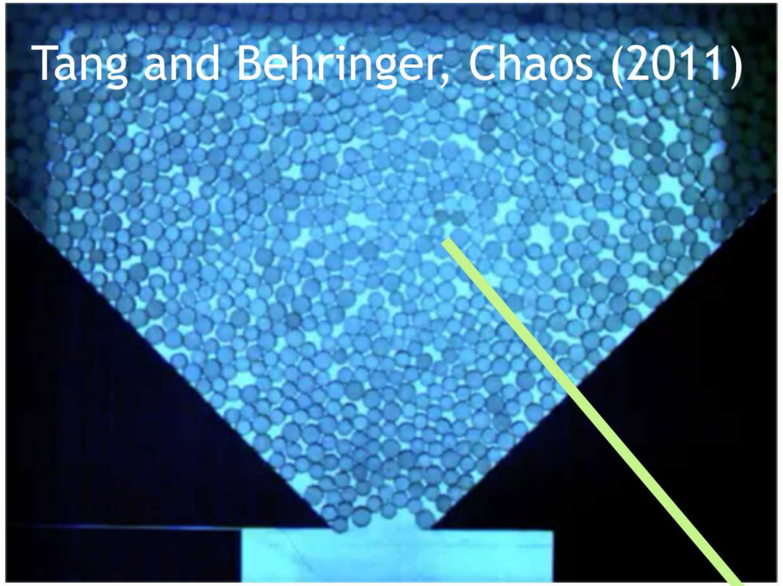


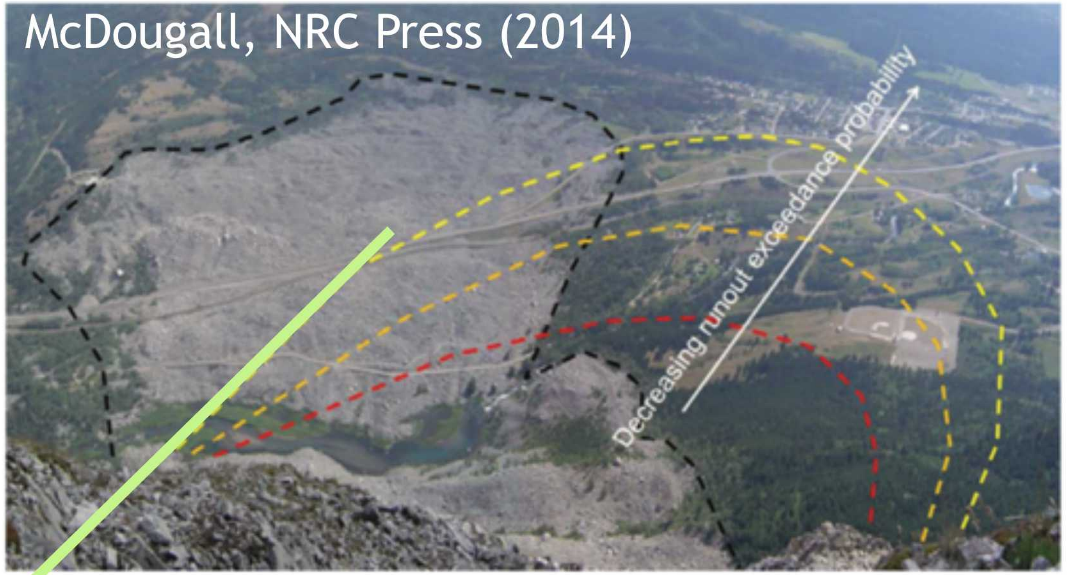
Coulomb surface friction



$$||F_s|| < \mu_s ||F_n||$$

Flow-Arrest Transition: Problem Statement





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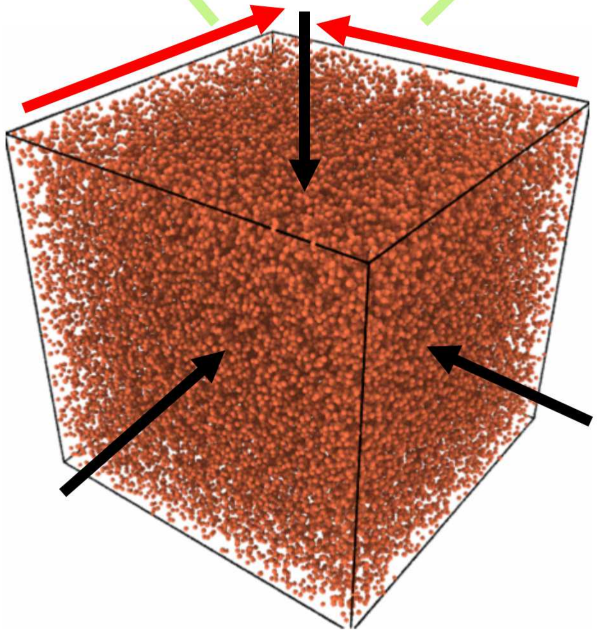
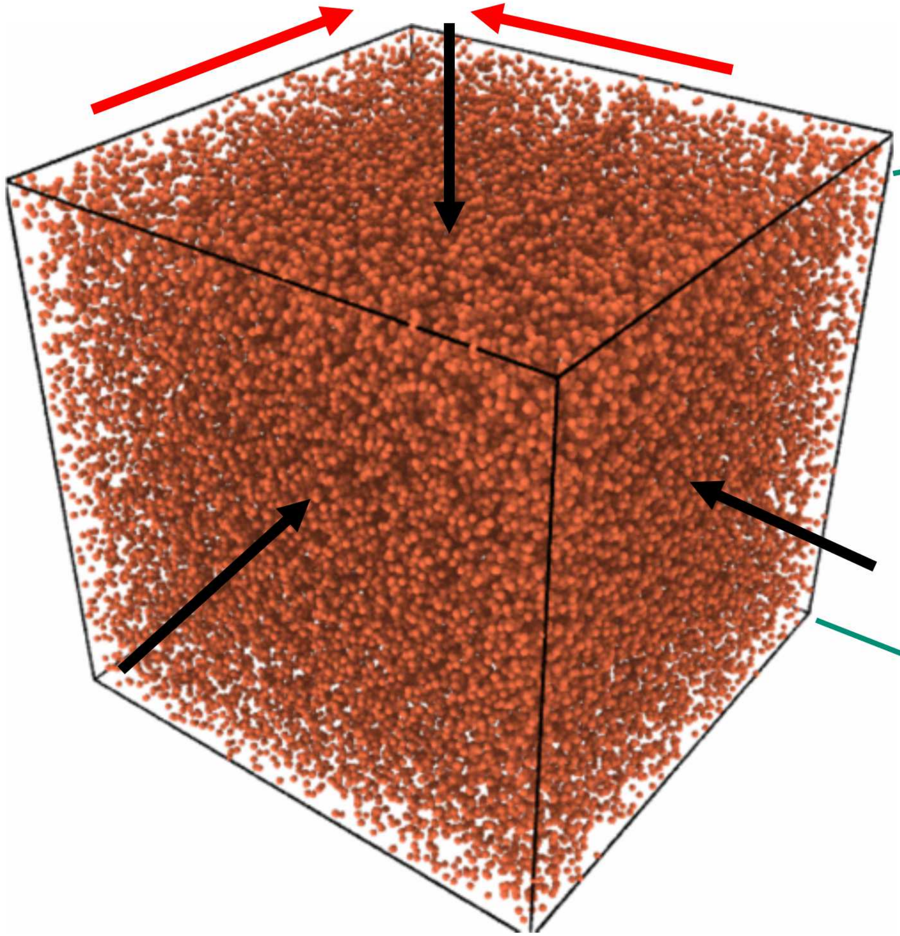


under prescribed shear and pressure:

1. Will the flow arrest?
2. When will the flow arrest?
3. If it flows, how does it flow?

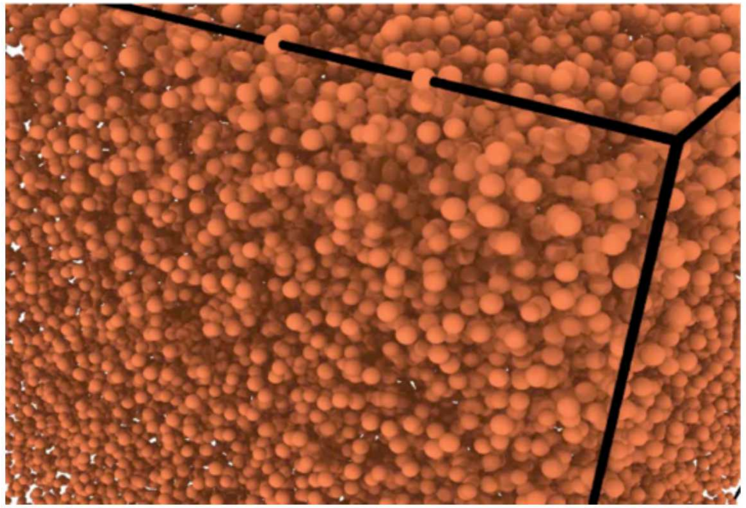
Example: Granular Flow vs. Arrest

initial dilute system: $\phi = 0.05$

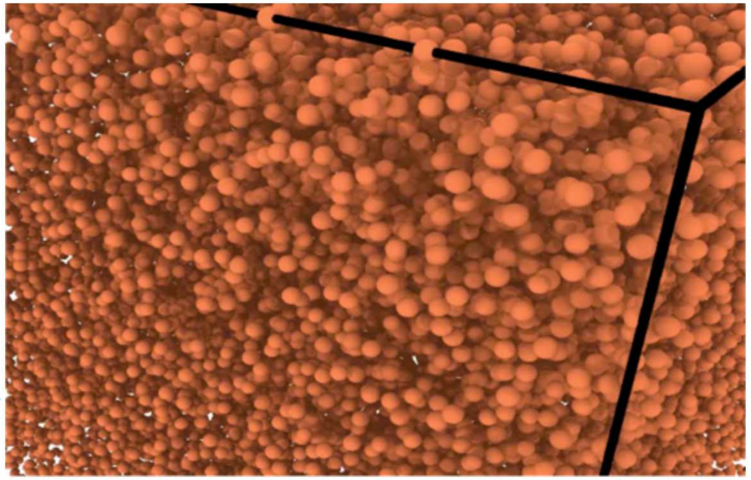


$$\boldsymbol{\sigma}_a = p_a \boldsymbol{I} + \begin{bmatrix} 0 & \tau_a & 0 \\ \tau_a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

steady flow

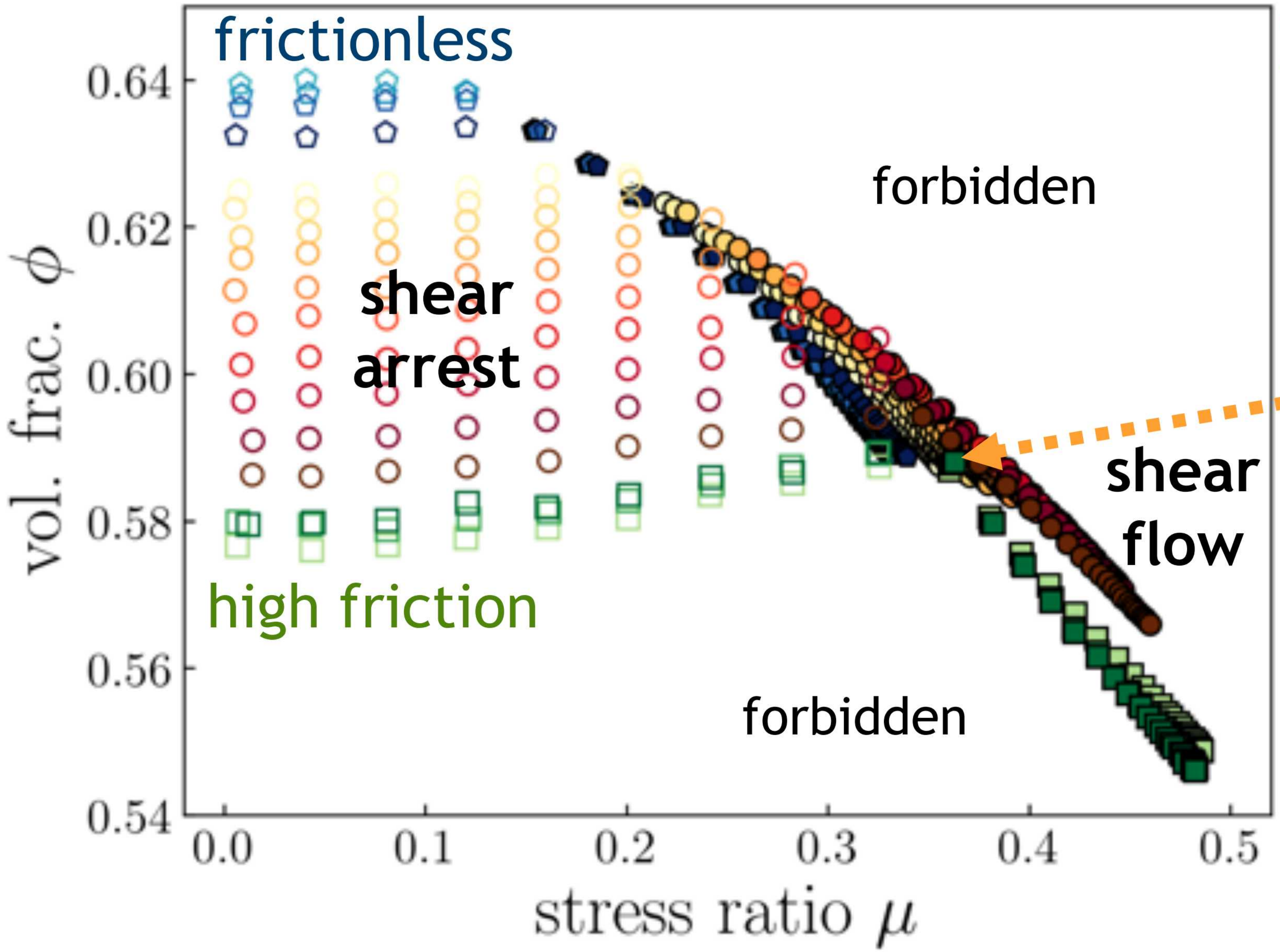


shear arrest



Srivastava, PRL (2019)

Flow-Arrest Phase Diagram



open symbols: arrest
filled symbols: flow

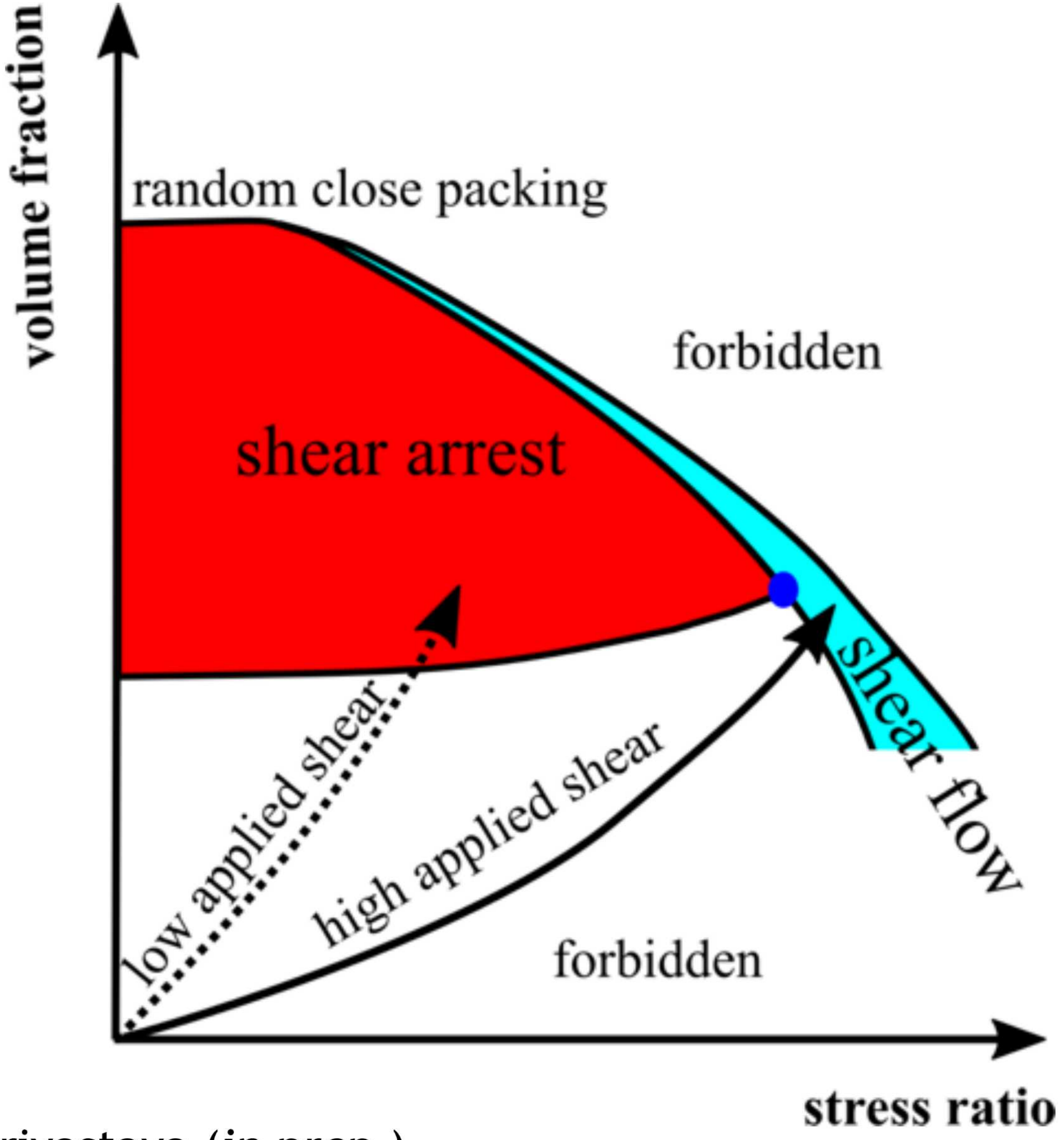
critical state

$$\mu = 0.36$$

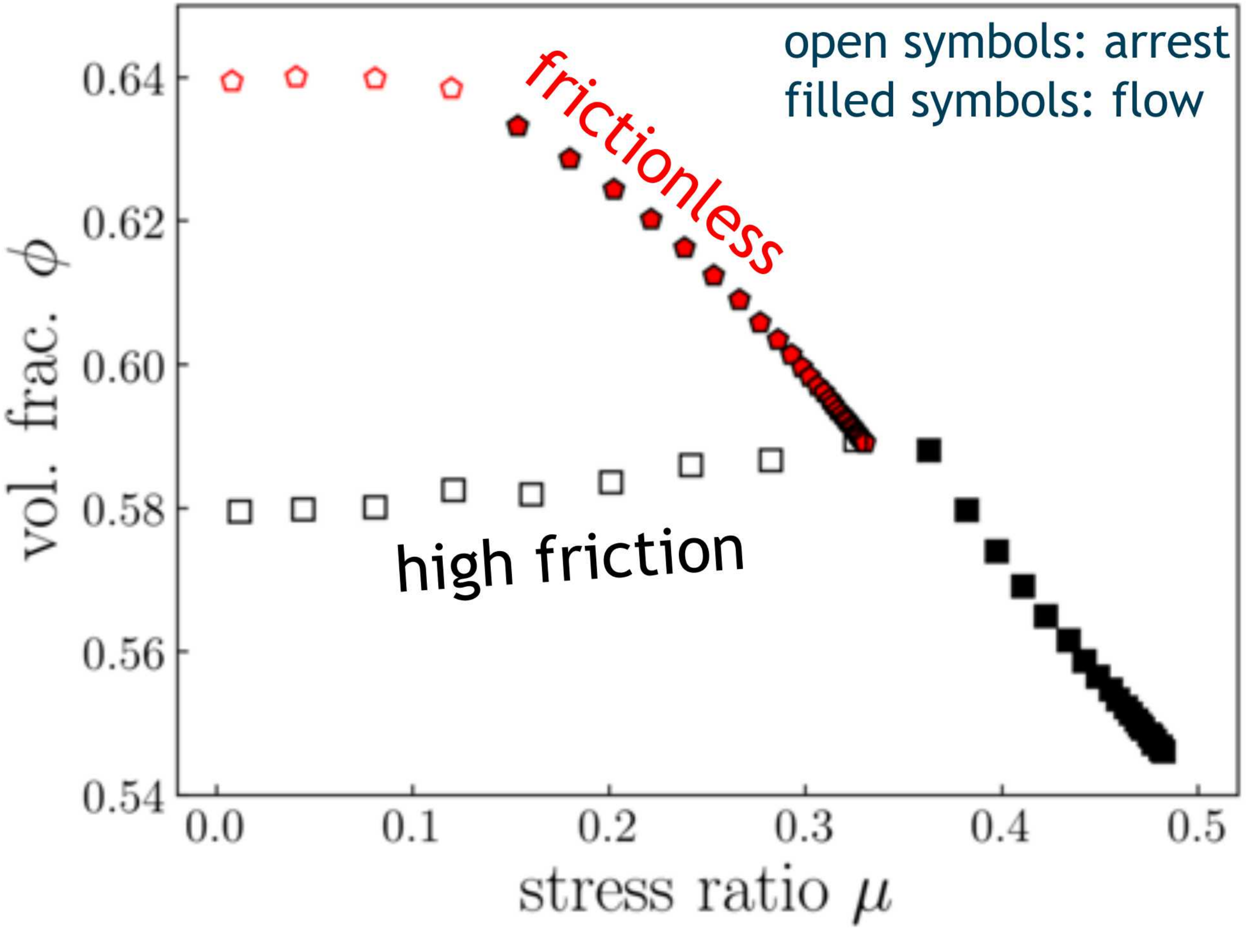
$$\phi = 0.59$$

Srivastava (*in prep.*)

Paths to Flow or Arrest

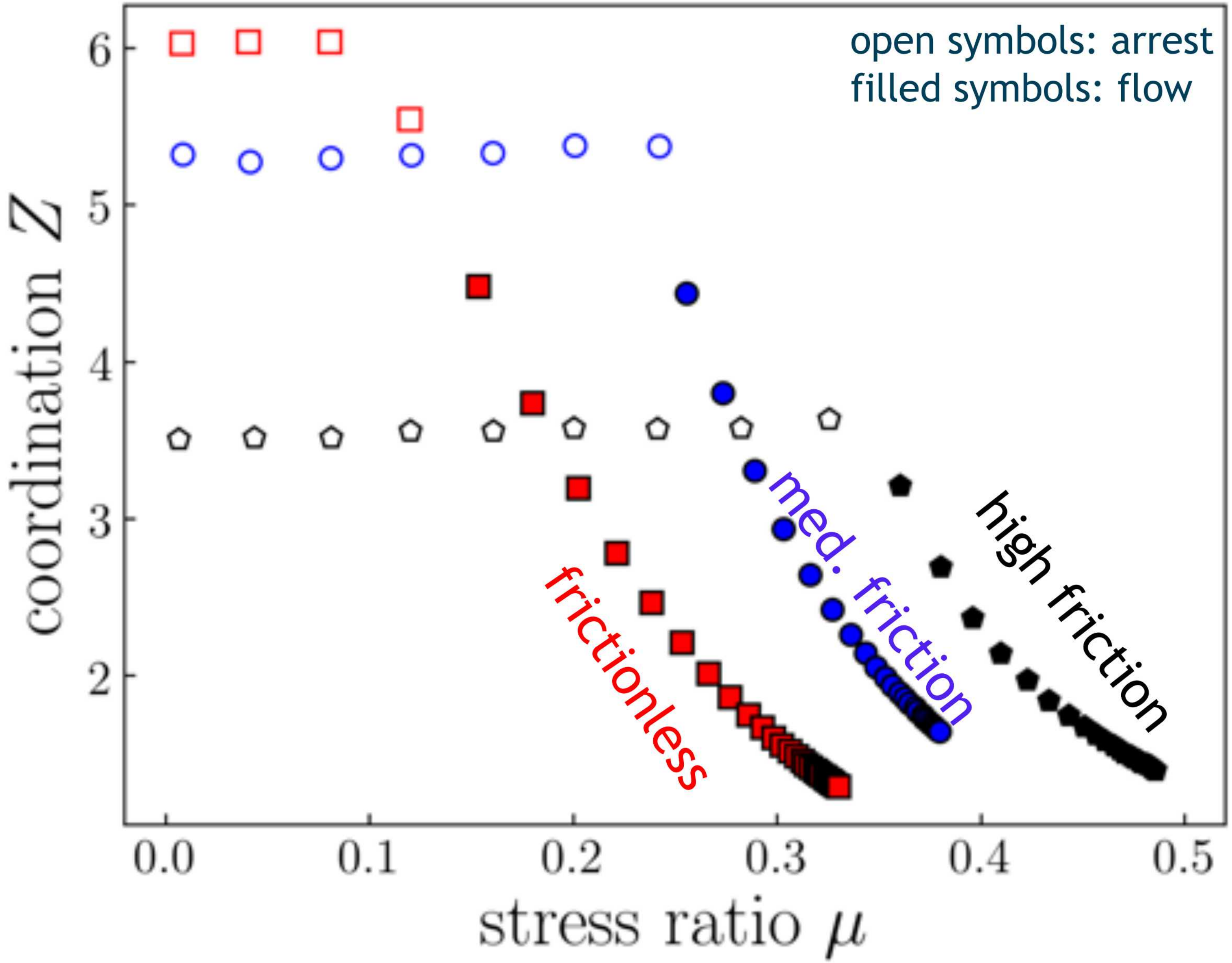


well-defined flow & arrest states for every friction



Srivastava (*in prep.*)

Coordination at Flow-Arrest Transition

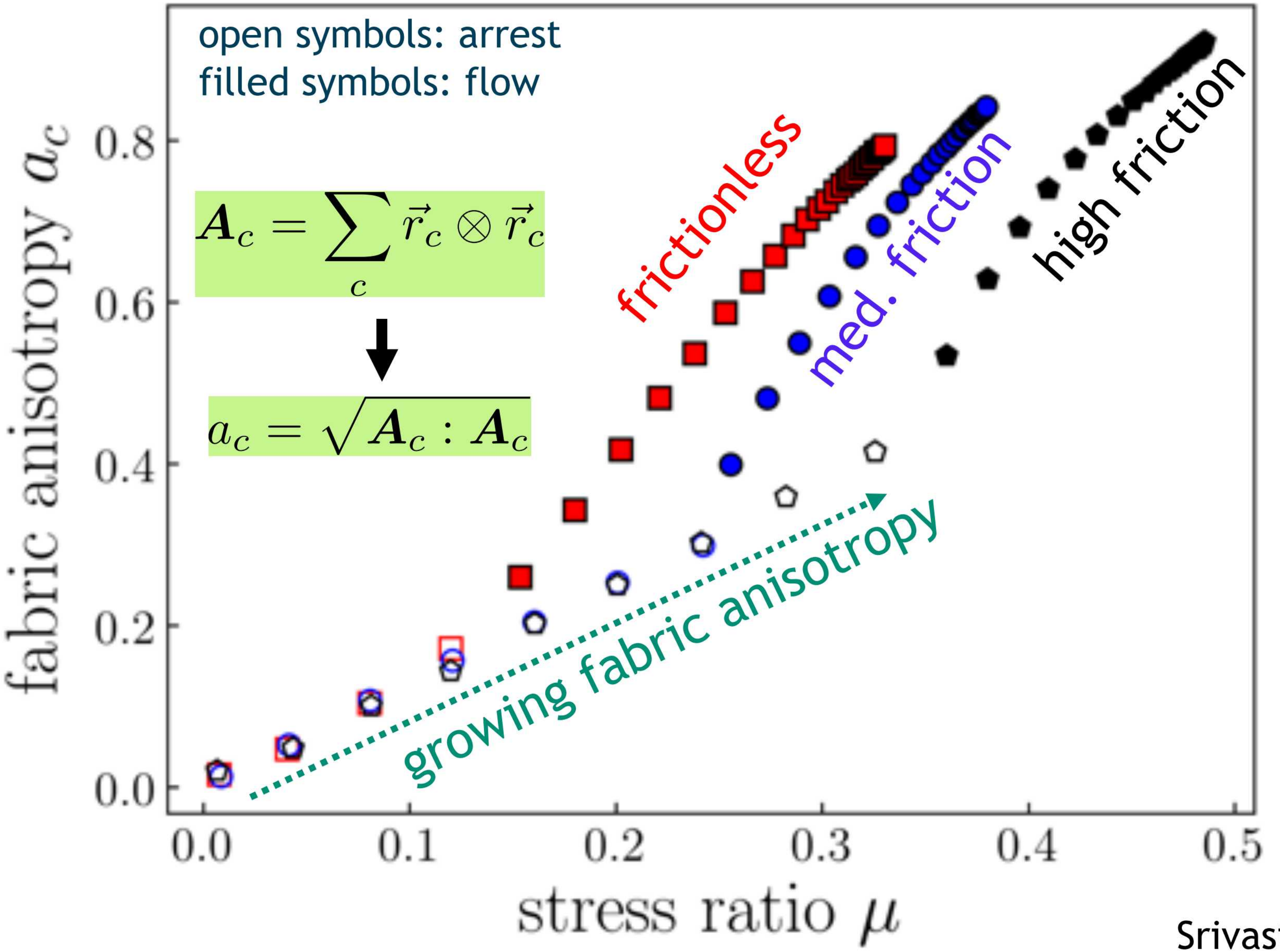


well-defined coordination at arrest for every friction

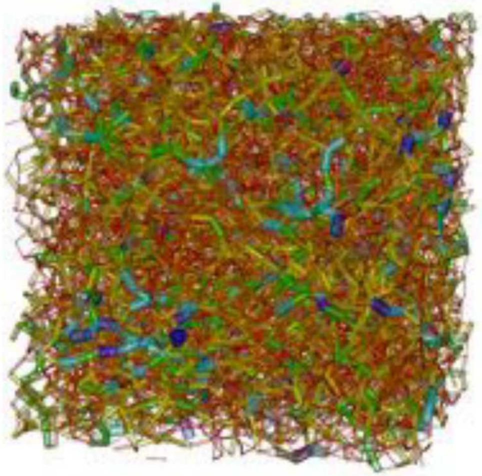
low coordination for high shear flows

Srivastava, (*in prep.*)

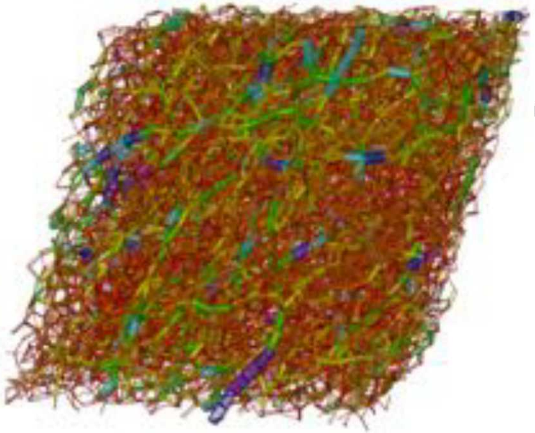
Fabric at Flow-Arrest



low-shear arrested



high-shear arrested

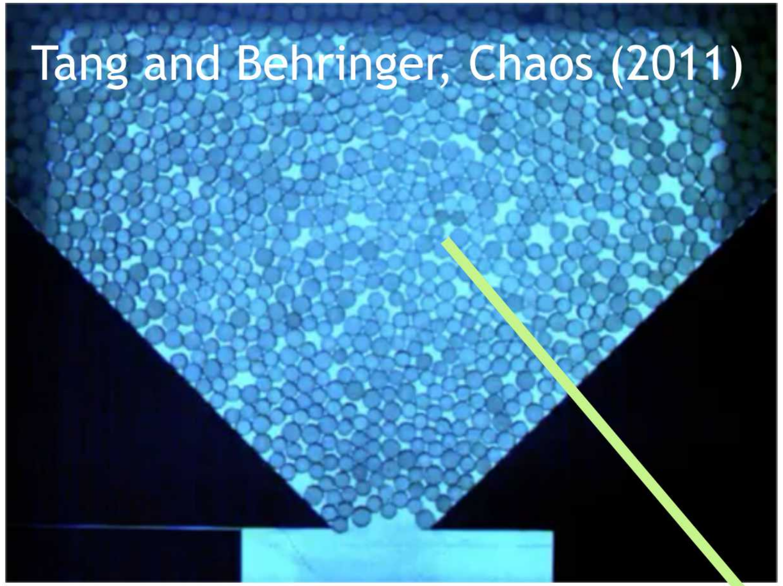


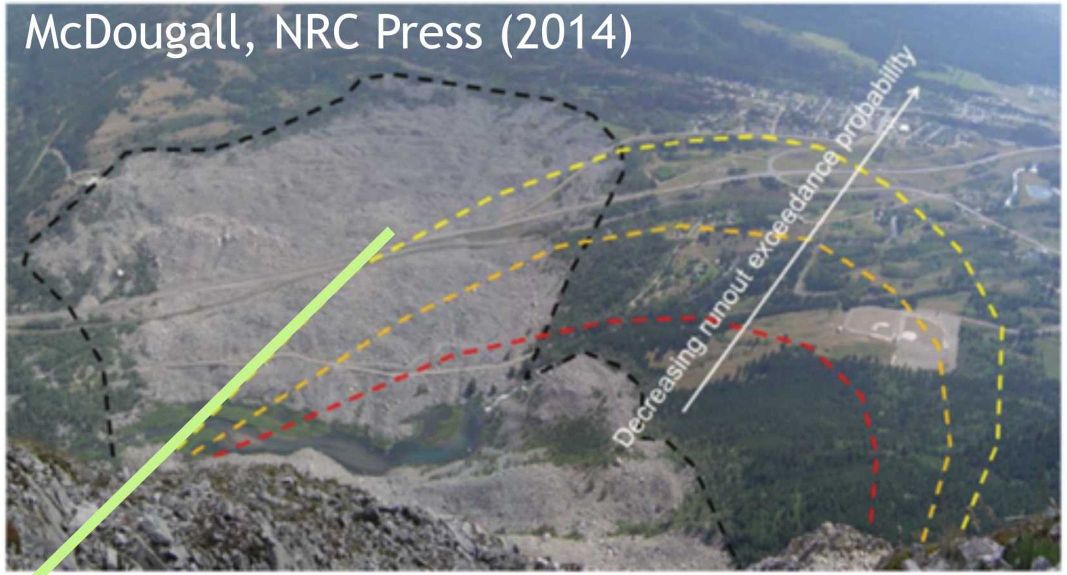
same ϕ, Z

different structure

Srivastava, (*in prep.*); Srivastava, (*in review*)

Flow-Arrest Transition: Problem Statement





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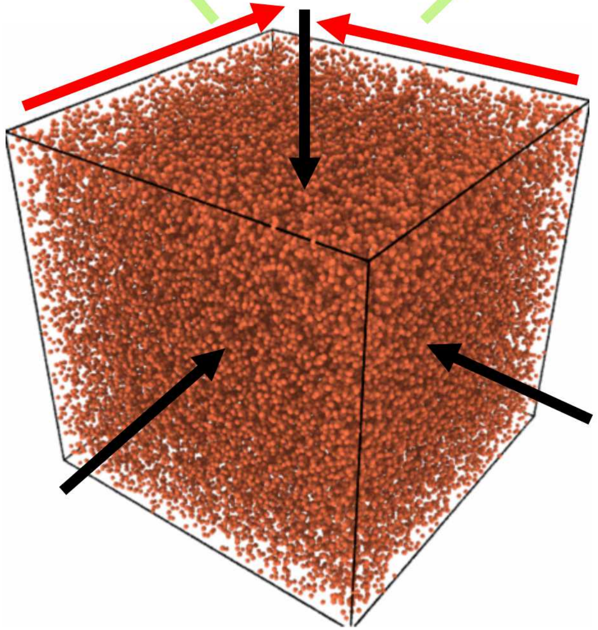
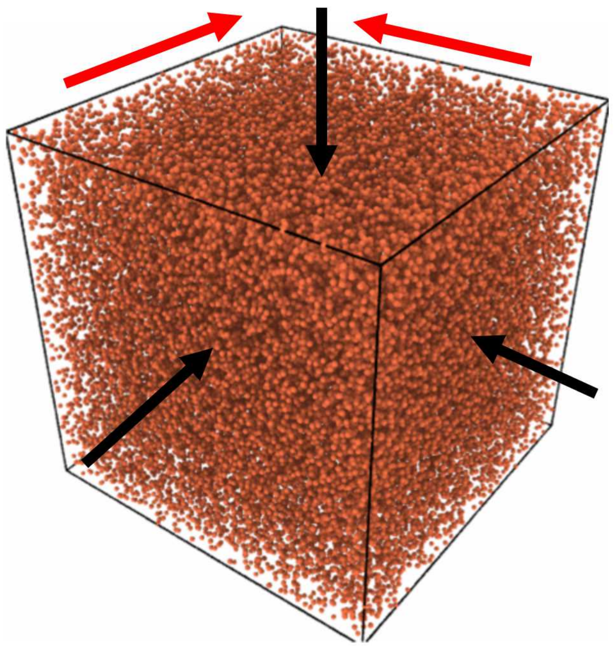


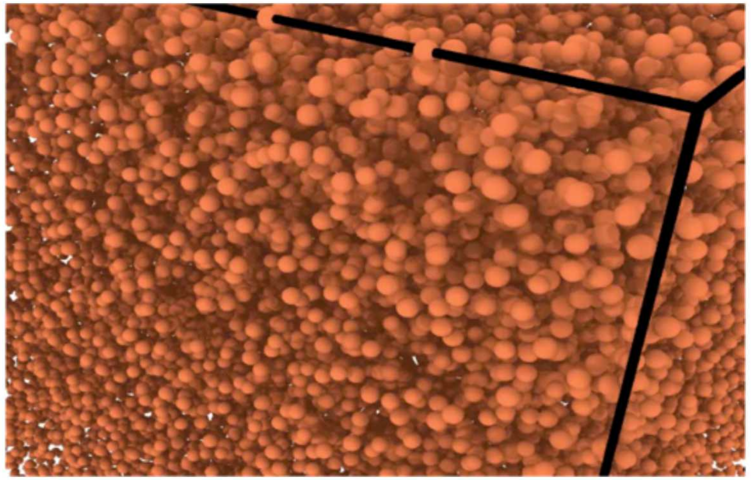
under prescribed shear and pressure:

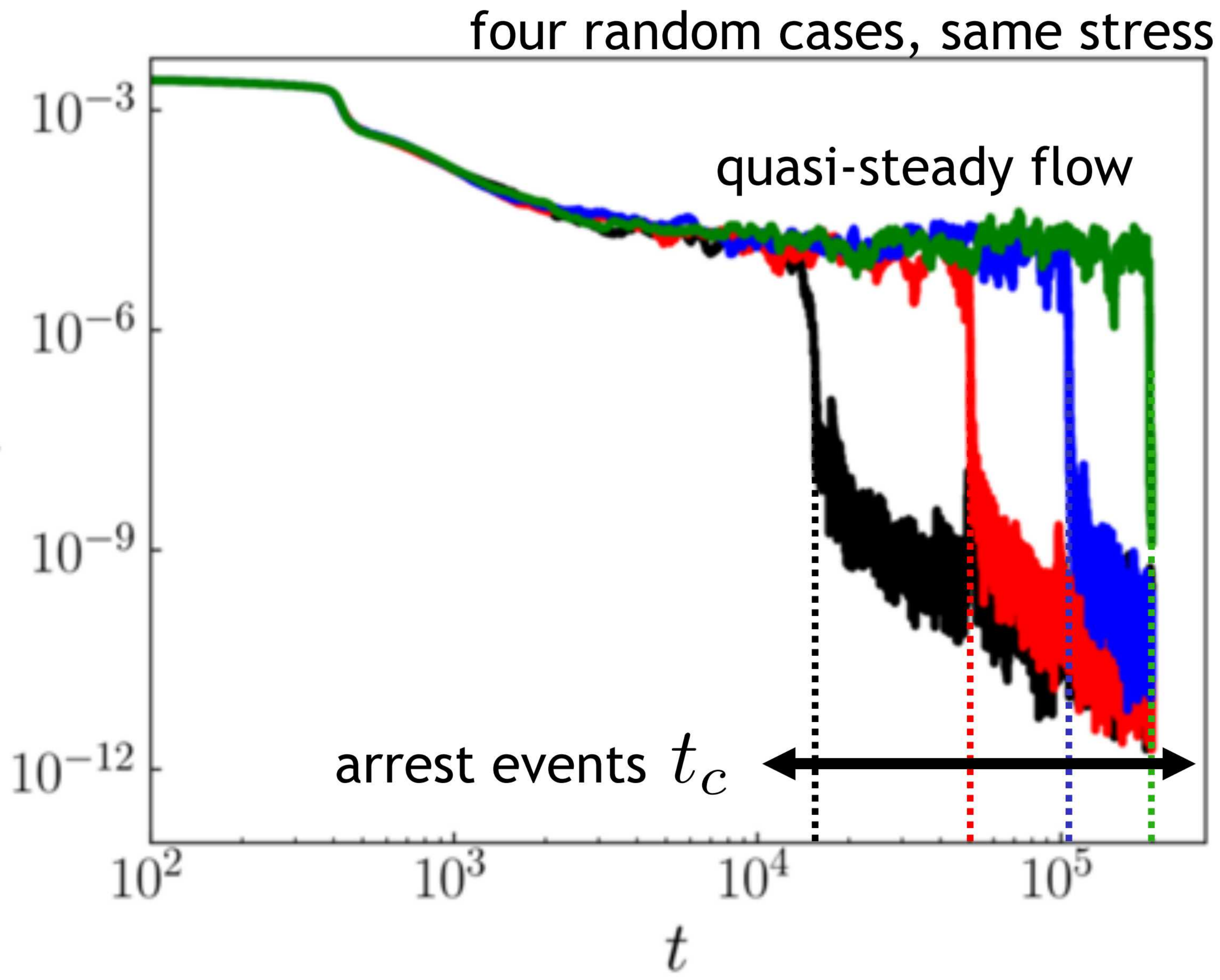
1. Will the flow arrest?
2. When will the flow arrest?
3. If it flows, how does it flow?

Time to Arrest is Stochastic



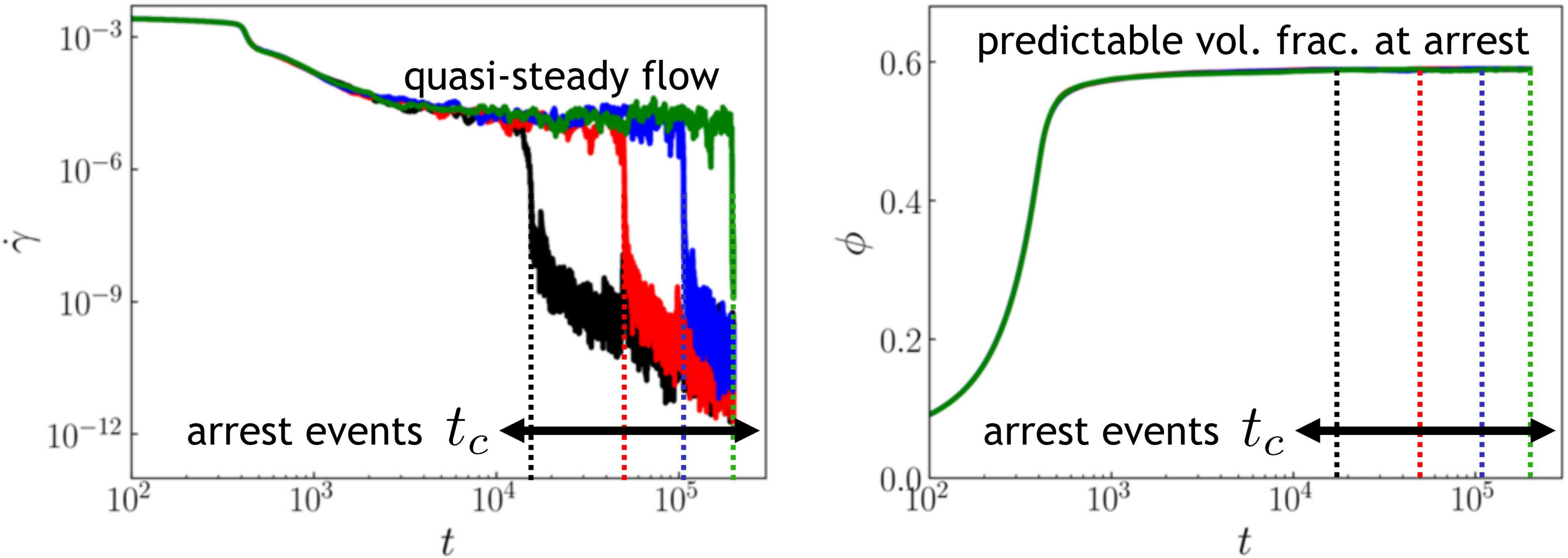
shear arrest





Srivastava, PRL (2019)

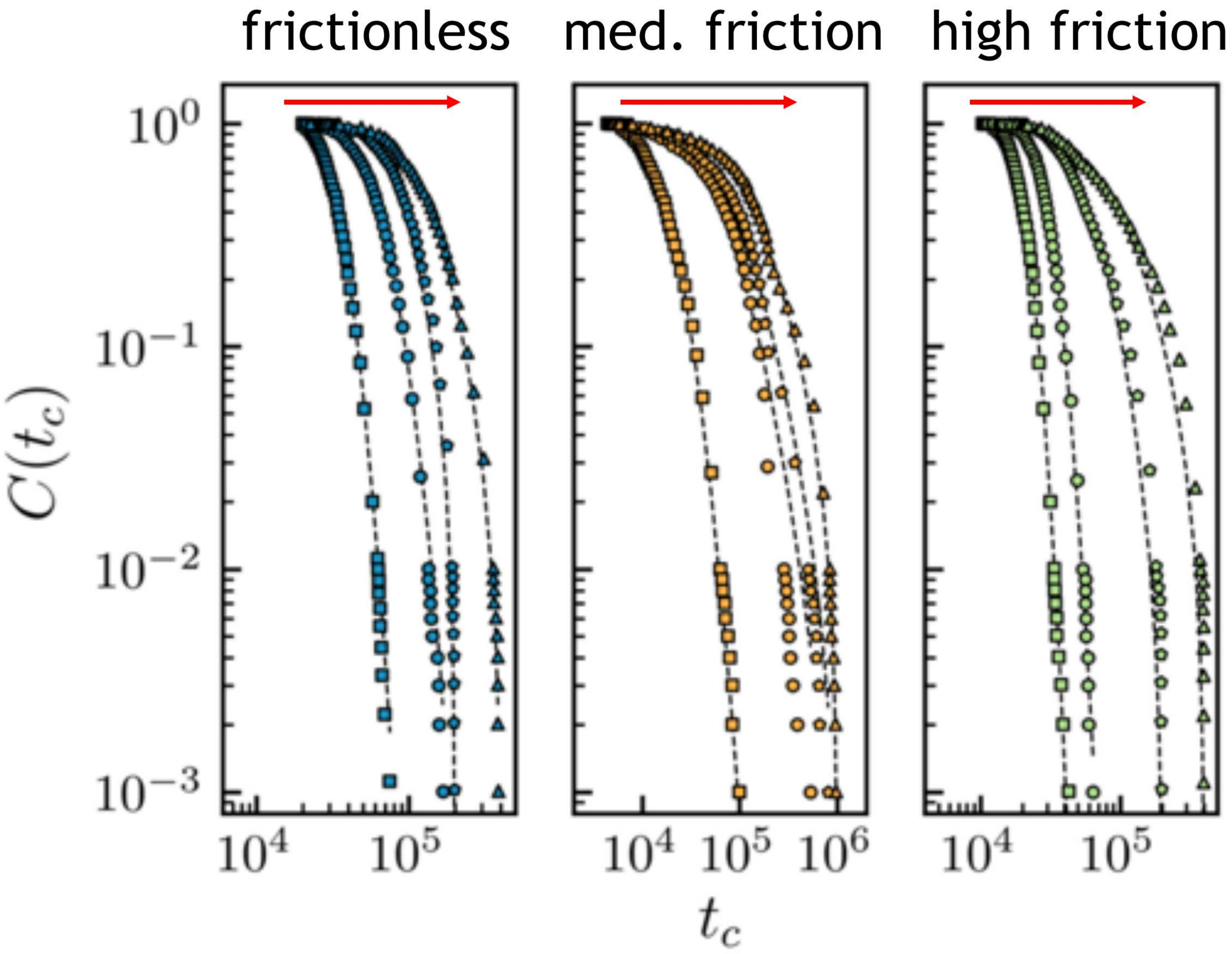
Volume Fraction at Arrest is Deterministic



difficult to predict when a flow will arrest,
but sure of volume fraction & coordination upon arrest

Srivastava, PRL (2019)

Time to Arrest: Long-tailed Distributions



increasing stress ratio μ

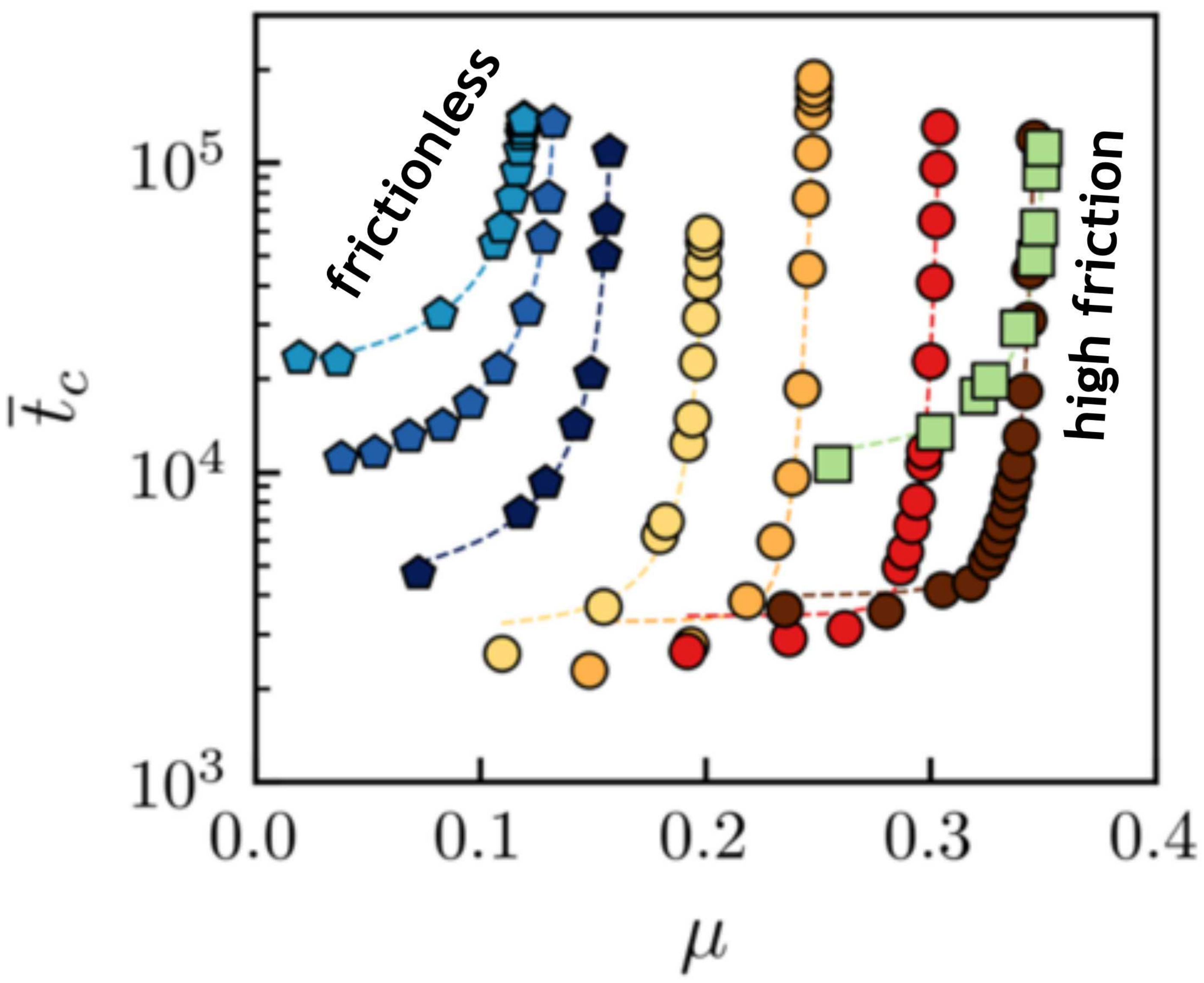
long-tailed *log-normal* distribution of arrest times

longer tails at higher stress ratio

arrest times: complementary cumulative distribution

Srivastava, PRL (2019)

Divergence of Mean Arrest Times

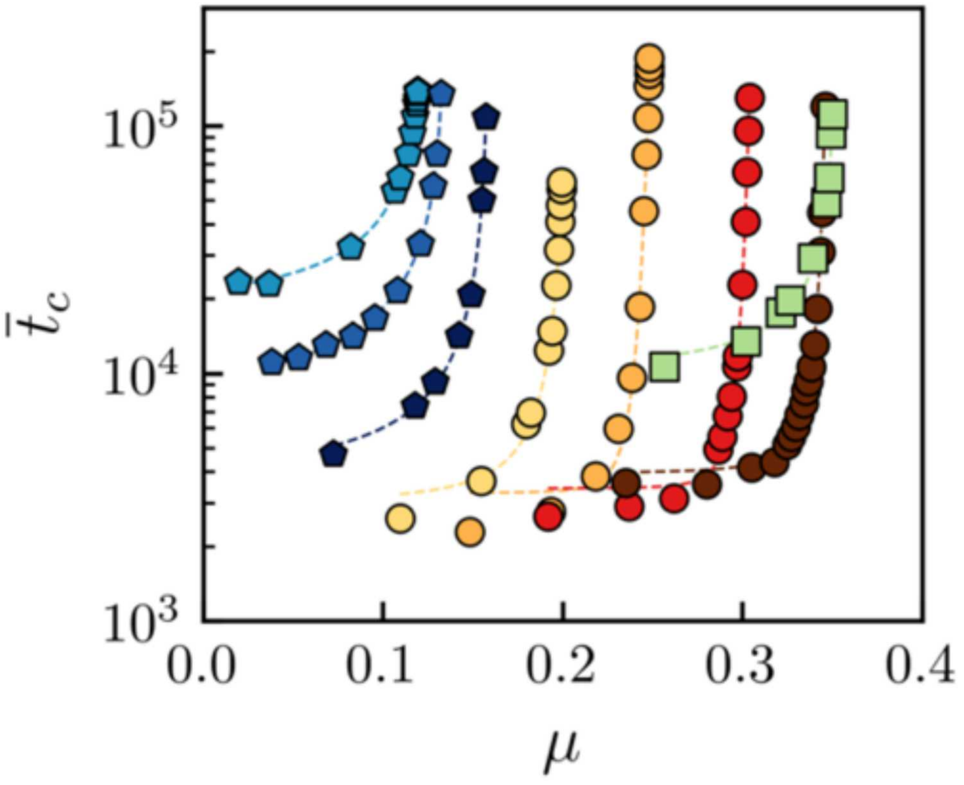


$$\bar{t}_c \sim (\mu_c - \mu)^{\alpha}$$

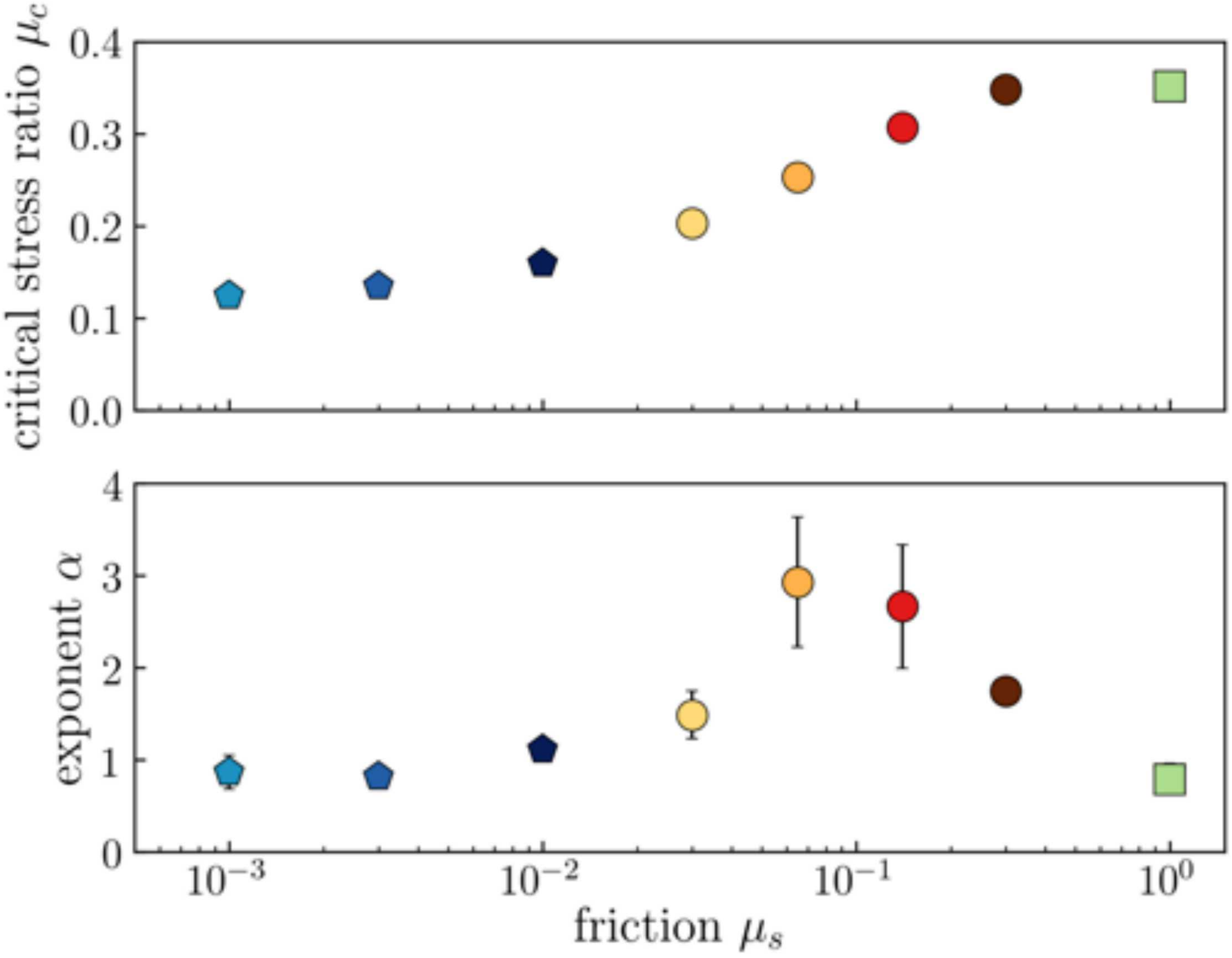
power-law divergence of mean arrest times at a critical stress ratio

Srivastava, PRL (2019)

Power-law Divergence: Critical Exponents

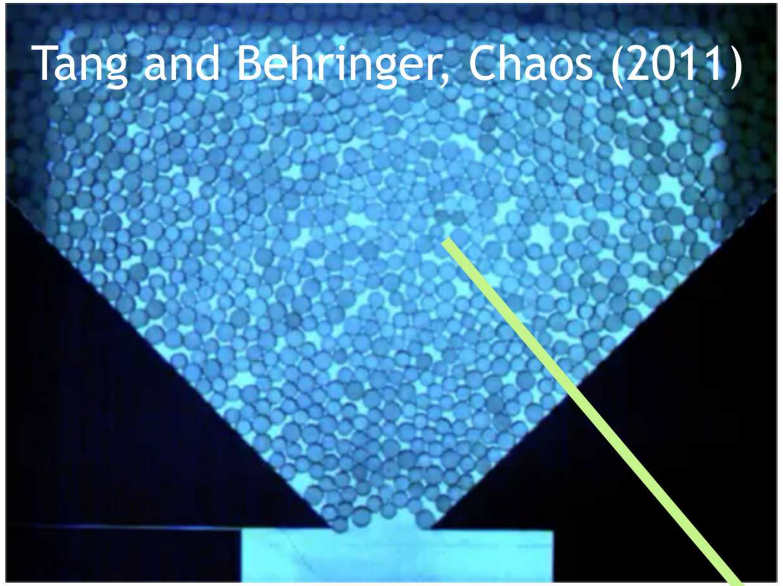


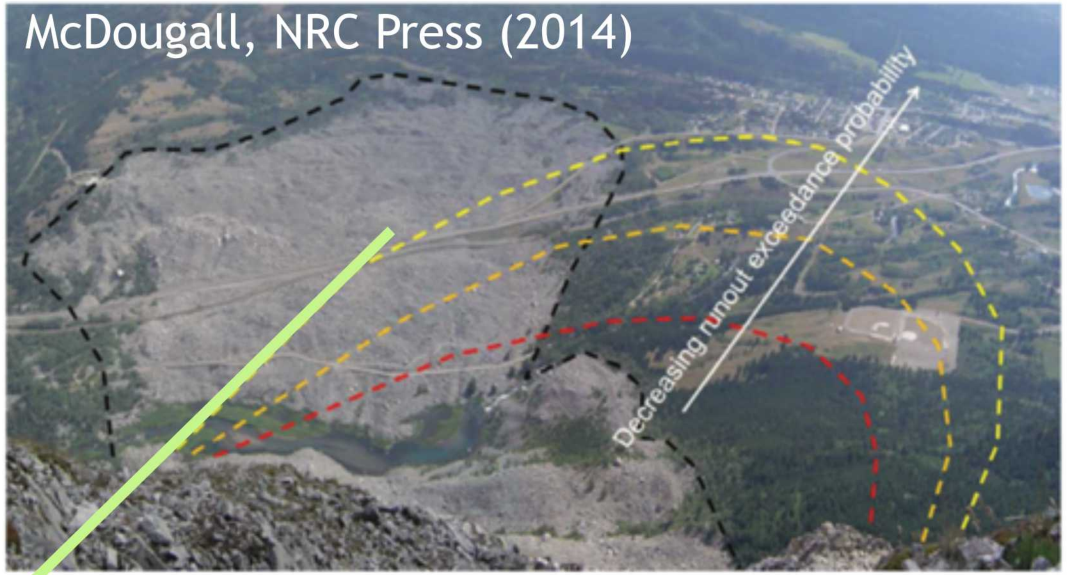
$$\bar{t}_c \sim (\mu_c - \mu)^{\alpha}$$



Srivastava, PRL (2019)

Flow-Arrest Transition: Problem Statement





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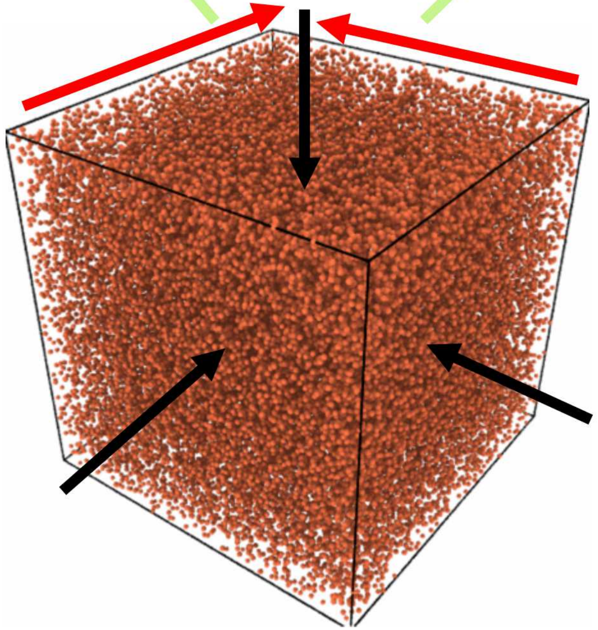

under prescribed shear and pressure:

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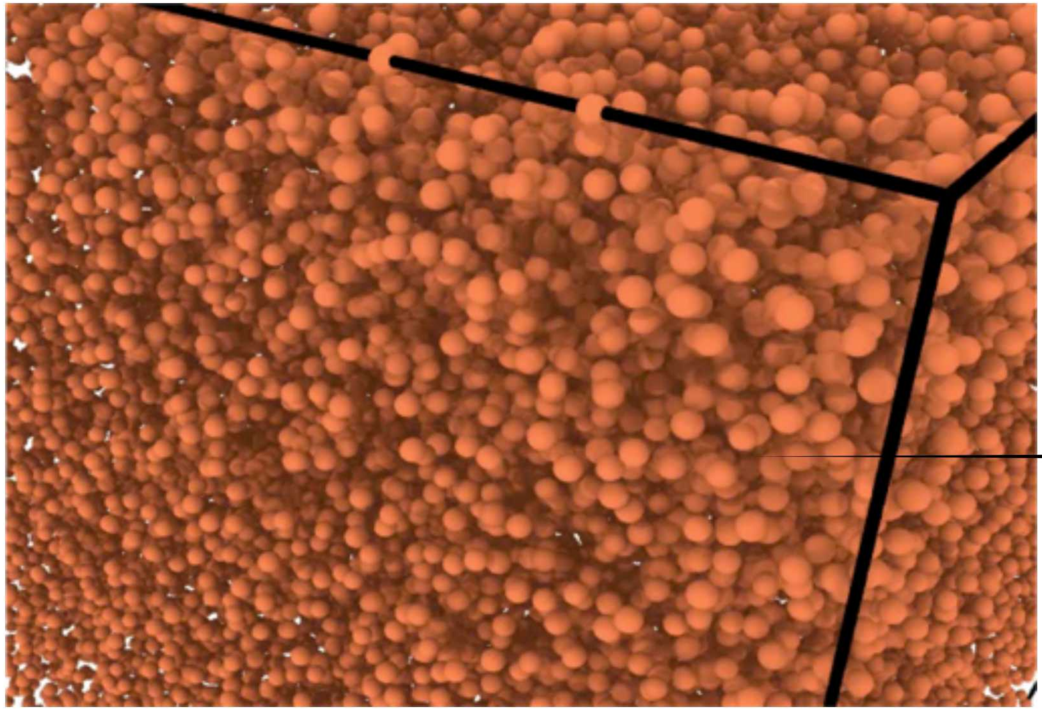
Planar Shear Rheology



after appropriate coordinate transformation

vel. gradient

$$\nabla u = \begin{bmatrix} 0 & \dot{\gamma} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

strain rate

$$\boldsymbol{D} = \begin{bmatrix} 0 & \dot{\gamma}/2 & 0 \\ \dot{\gamma}/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

spin

$$\boldsymbol{W} = \begin{bmatrix} 0 & \dot{\gamma}/2 & 0 \\ -\dot{\gamma}/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Cauchy stress

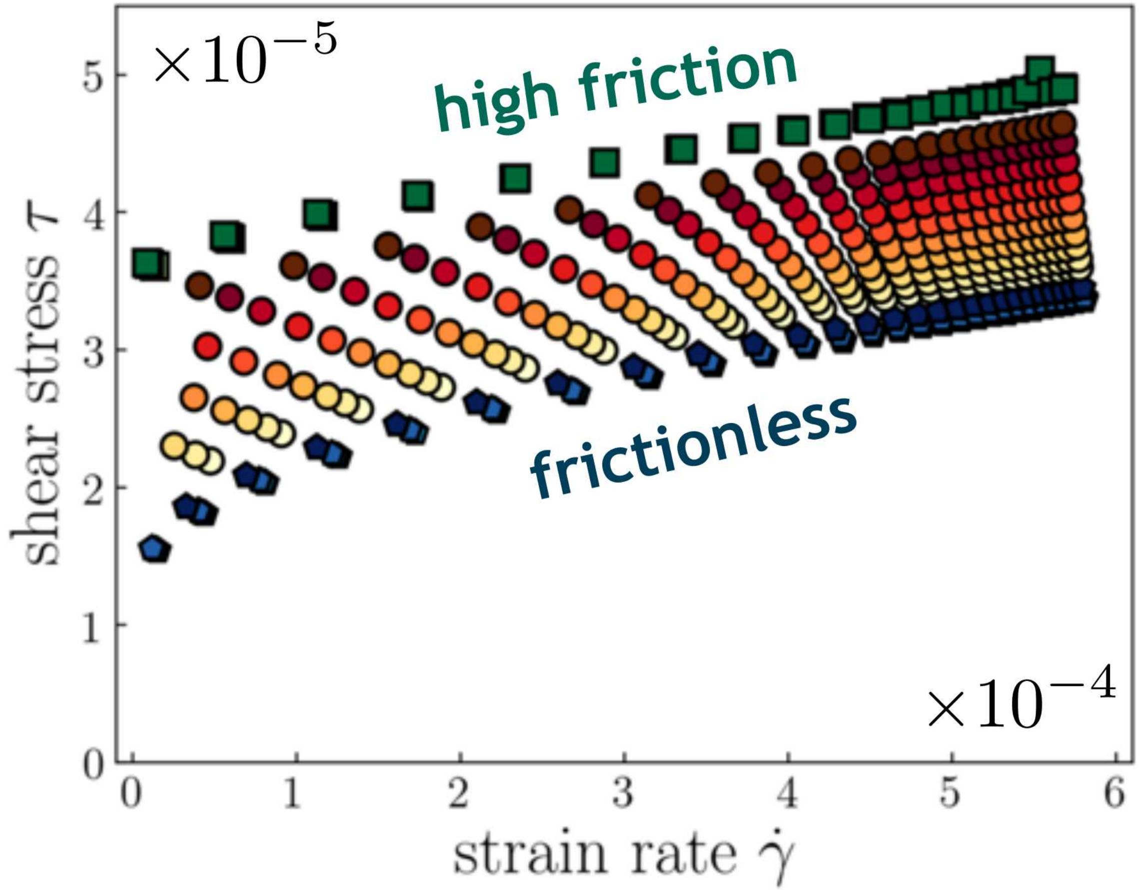
$$\boldsymbol{\sigma} = \sum_c \vec{r}_c \otimes \vec{F}_c$$

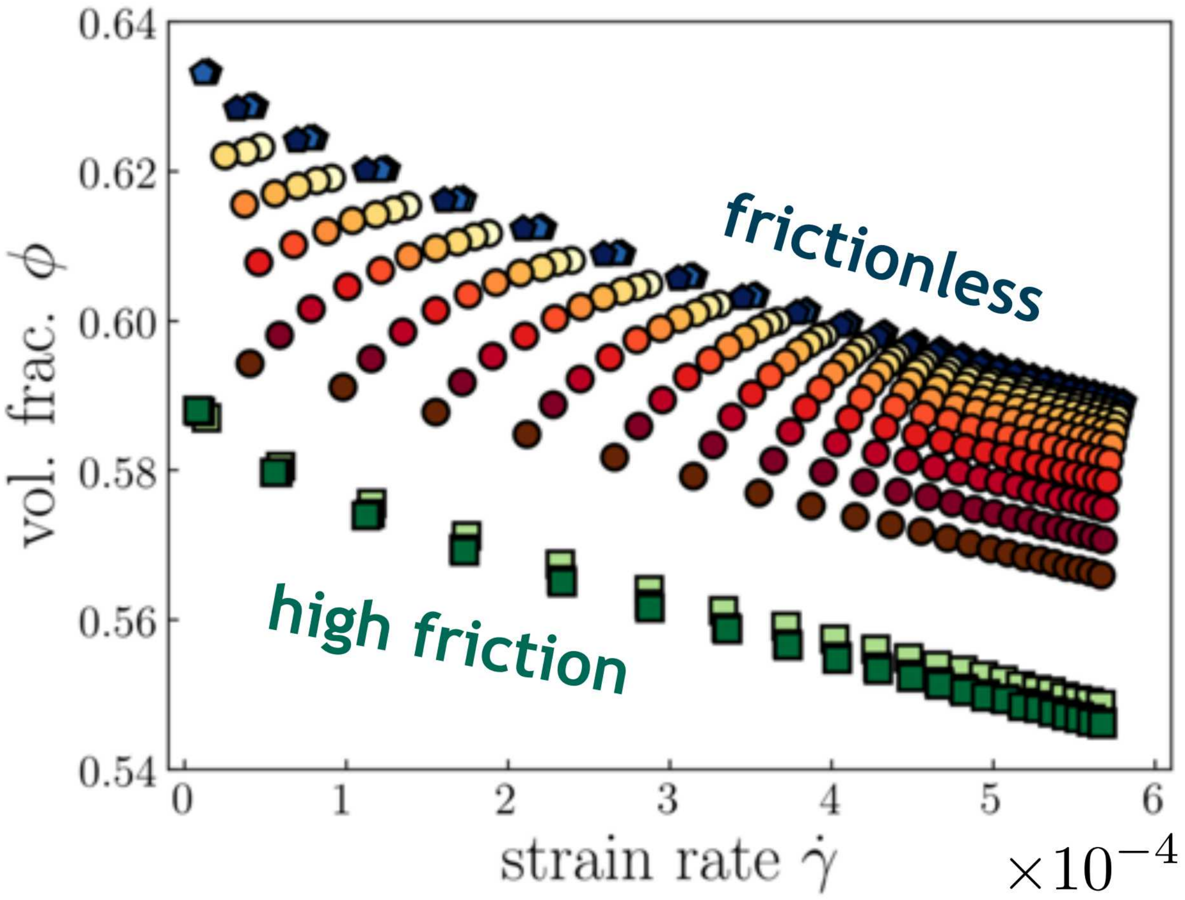
Rheology

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{D})?$$

Shear Stress and Dilation

Srivastava (*in prep.*)

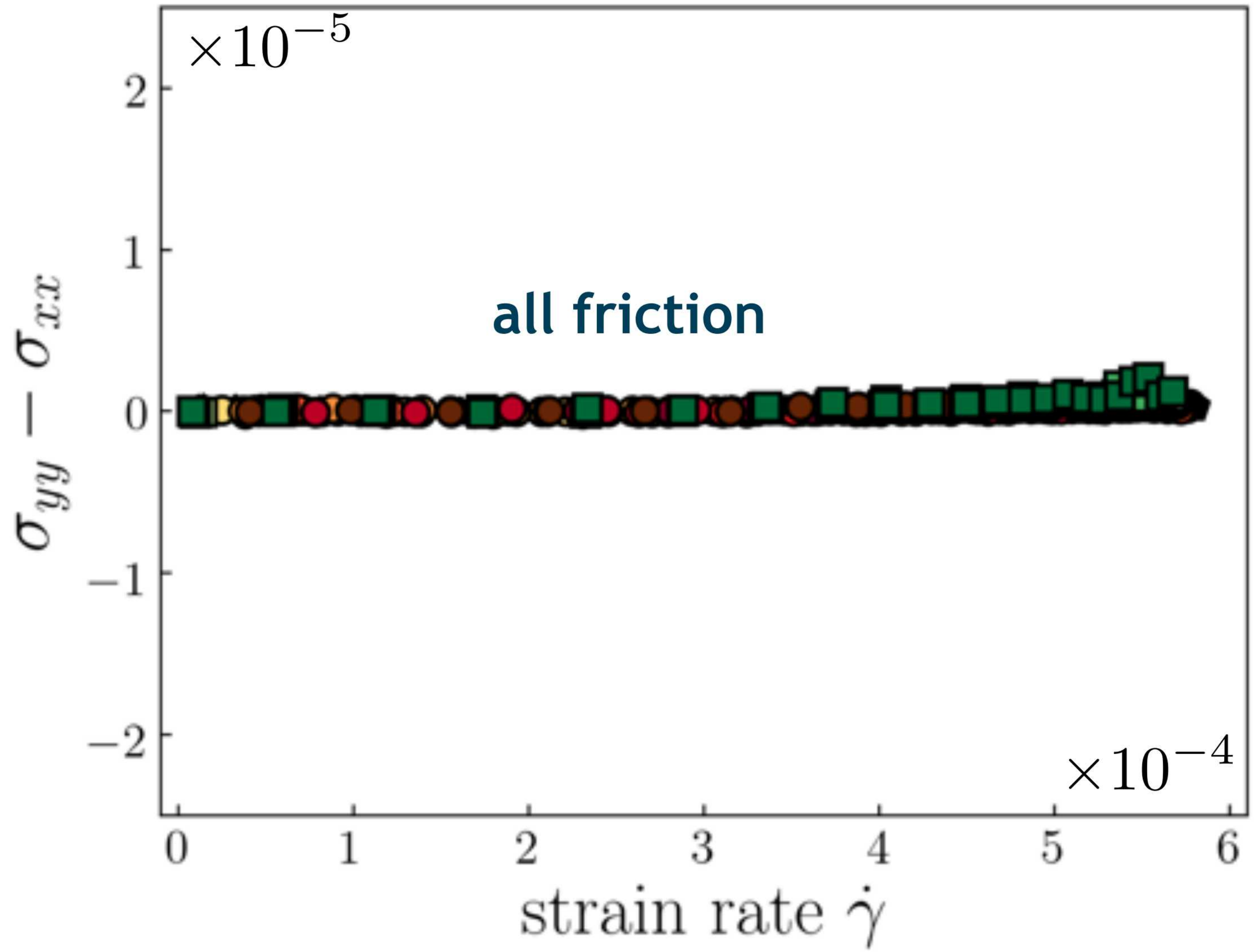




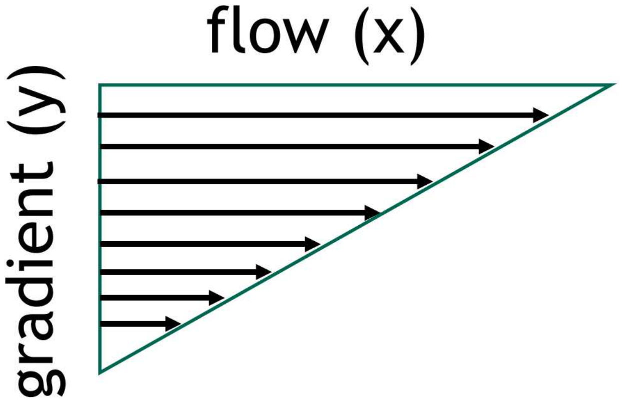
$$\boldsymbol{\sigma} = -P\boldsymbol{I} + \boldsymbol{\tau_D} \quad \rightarrow \quad \tau = \sqrt{\frac{1}{2}\boldsymbol{\tau_D} : \boldsymbol{\tau_D}}$$

$$\dot{\gamma} = \sqrt{\frac{1}{2}\boldsymbol{D} : \boldsymbol{D}}$$

First Normal Stress Difference?



flow plane

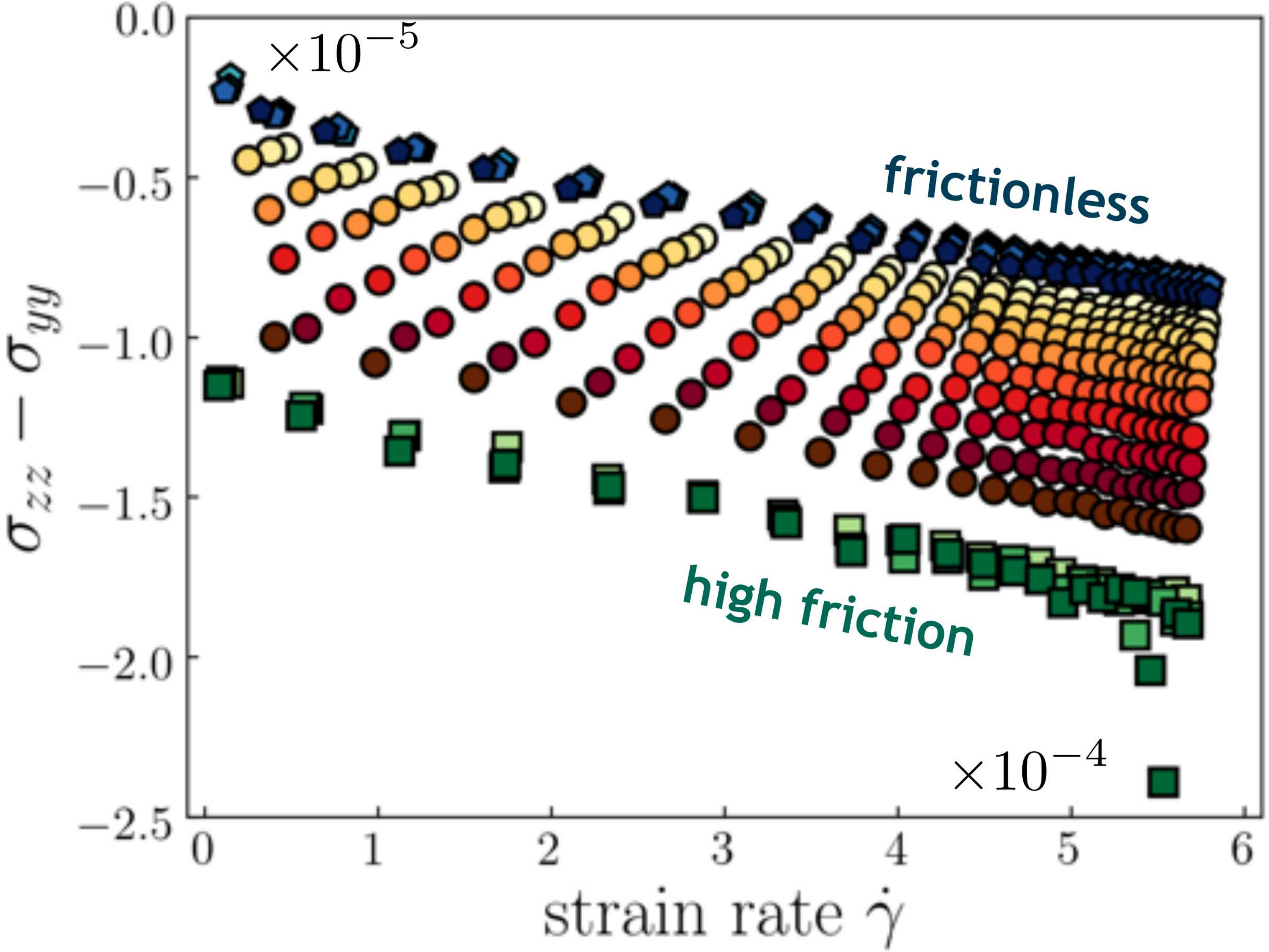


no first normal stress difference: co-axial flow

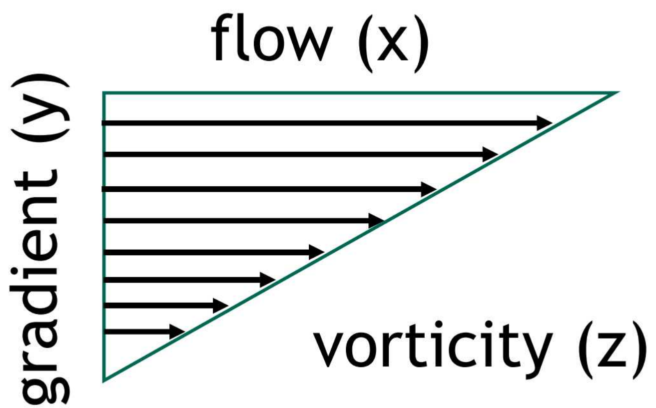
in the flow plane, stress and strain rate vectors are aligned

Srivastava (*in prep.*)

Second Normal Stress Difference?



flow plane

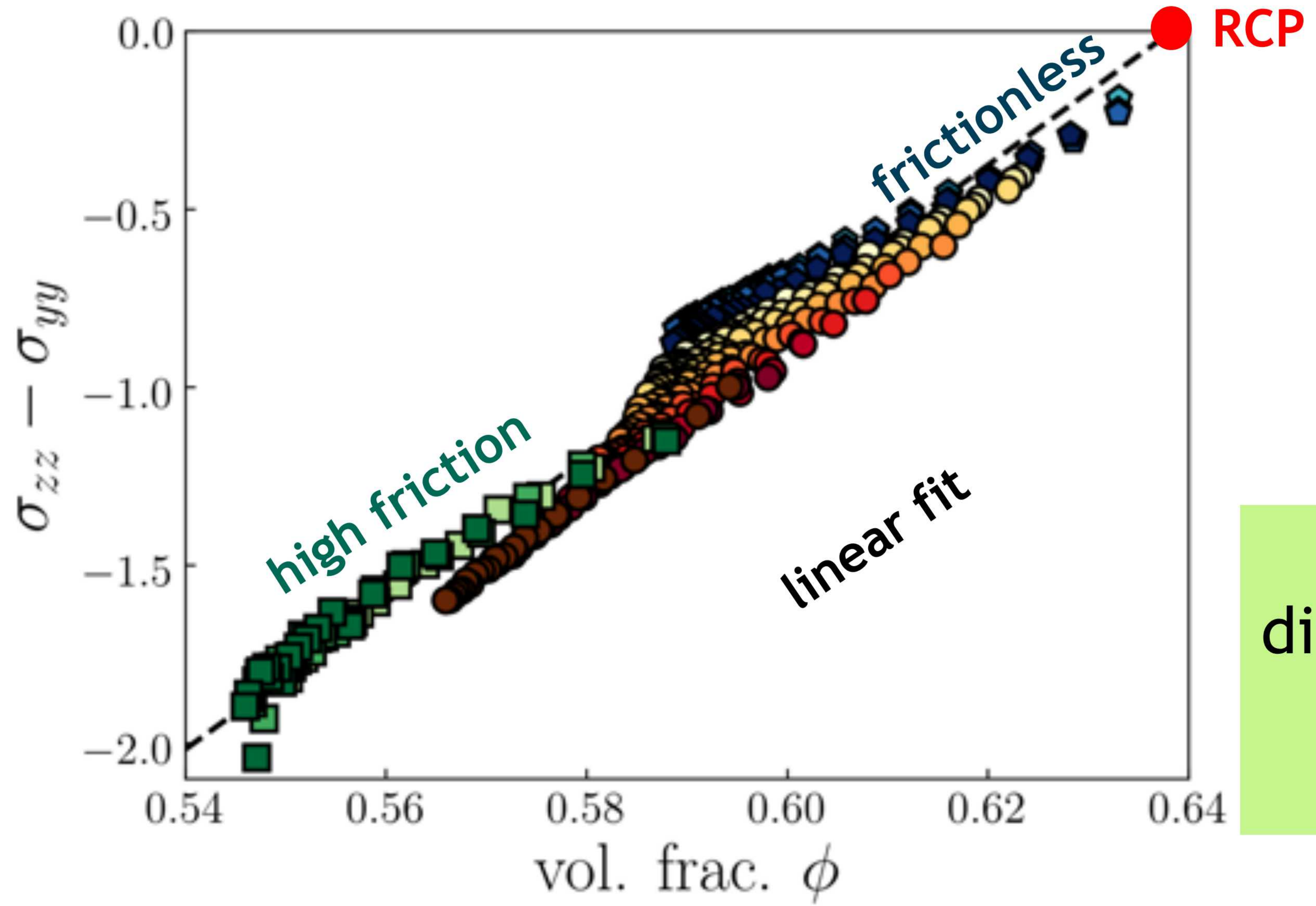


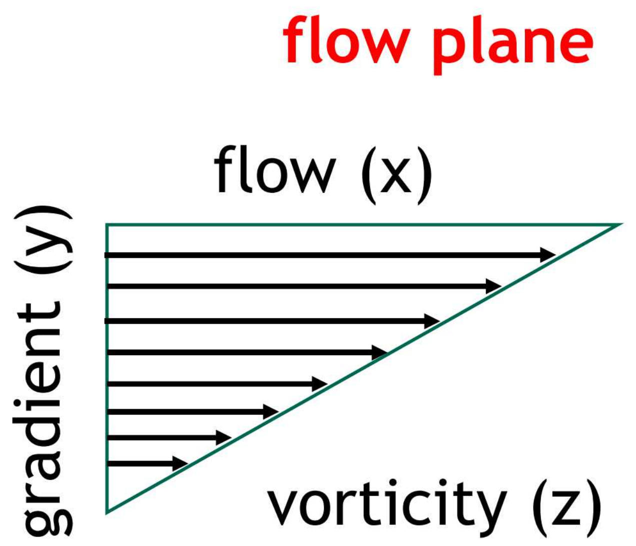
significant second normal stress difference

pressure loss in vorticity linked to dilation?

Srivastava (*in prep.*)

2nd Normal Stress Difference Vanishes at RCP



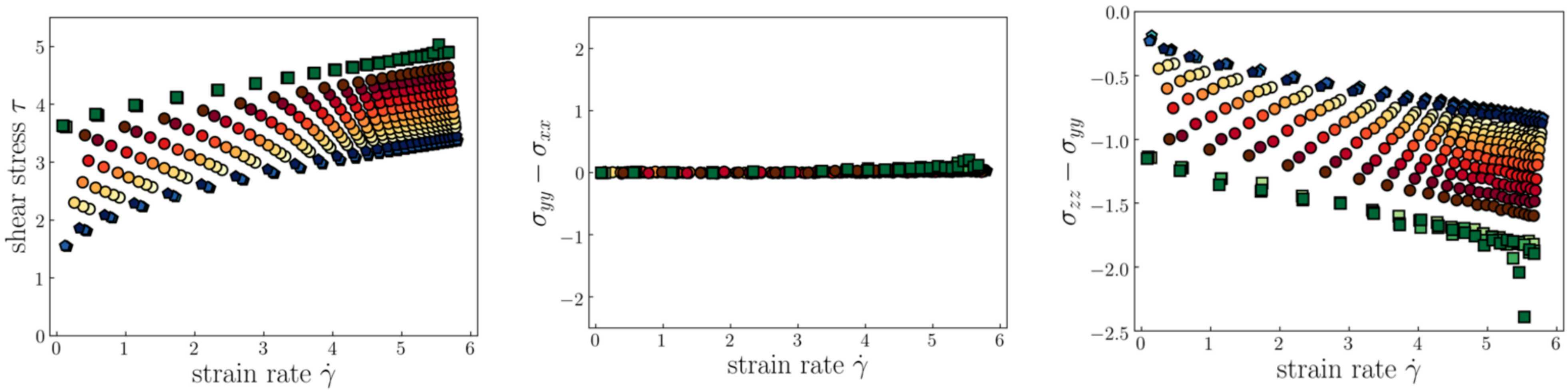


no 2nd normal stress difference & flow-induced dilation at RCP for frictionless

Peyneau, PRE (2008)

Srivastava (*in prep.*)

Granular Rheology: Planar Shear Flows



stress tensor (viscometric flow)

$$\boldsymbol{\sigma} = -P\boldsymbol{I} + \eta\left(\dot{\gamma}\right)\boldsymbol{D} + \alpha\left(\dot{\gamma}\right)\boldsymbol{D}^2$$

Goddard (1986); Rajagopal (1994); Massoudi (2001)

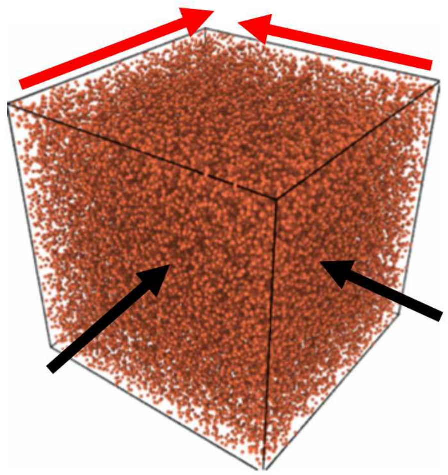
stress tensor for steady planar granular shear flow resembles a Reiner-Rivlin viscometric fluid

Srivastava (*in prep.*)

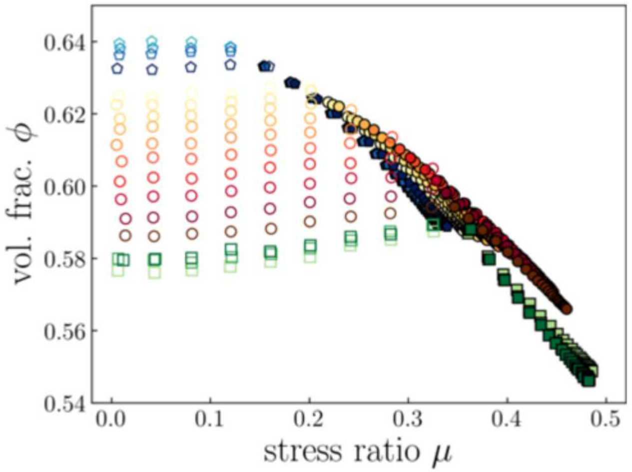
Conclusions & Outlook

Email: isriva@sandia.gov

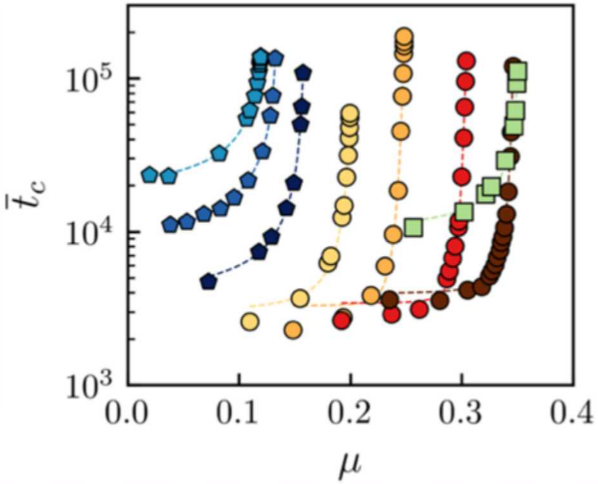
CONCLUSIONS



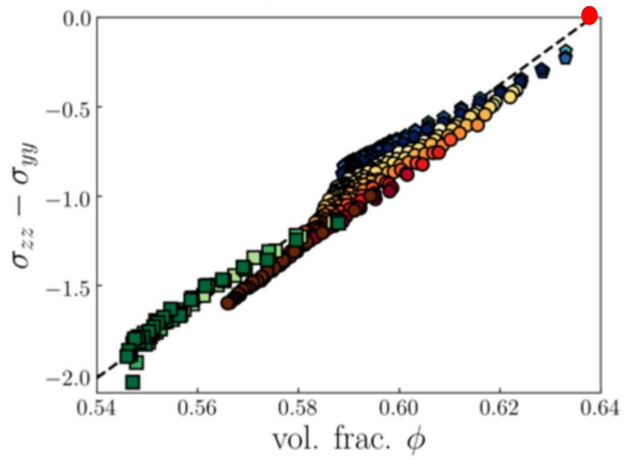
IF: flow-arrest diagram



WHEN: diverging arrest times



HOW: beyond $\mu(I)$ rheology



OUTLOOK:

- are shear-arrested states fragile/stable? à la [Bi, Behringer, Nature (2011)]
- is there a diverging length scale at shear-arrest near critical yield?
- how do normal stresses manifest in flow fields? dilation-driven vortex flows? [Krishanraj and Nott, Nature (2016)]

Stress States at Yielding and Flow

$$\underbrace{\Delta E(t)}_{\text{change in kinetic energy}} = \underbrace{\int_\Gamma f_i \Delta u_i \mathrm{d}S}_{\text{boundary traction work}} - \underbrace{\int_V \sigma_{ij} \frac{\partial\left(\Delta u_i\right)}{\partial x_j} \mathrm{dV}}_{\text{second order work (constitutive)}}$$

Arrest (Jammed): $\int_\Gamma f_i \Delta u_i \mathrm{d}S = \int_V \sigma_{ij} \frac{\partial\left(\Delta u_i\right)}{\partial x_j} \mathrm{dV}$

equilibrium: balance of internal and external stress

Yielding: $\int_\Gamma f_i \Delta u_i \mathrm{d}S > \int_V \sigma_{ij} \frac{\partial\left(\Delta u_i\right)}{\partial x_j} \mathrm{dV}$

rapid increase in kinetic energy: imbalance of internal and external stresses

Hill, J. Mech. Phys. Sol., 6, 1958