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Chance-constrained Optimization: Approximations, Algorithms, and Applications

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Motivation

This is a linear Joint Chance Constraint (JCC):

$P(x_t \leq y_t^\omega + w_t^\omega, \forall t \in T) \geq 1 - \varepsilon$

Background:

- Two-stage stochastic program with recourse
- First stage decision, x_t, second-stage decision, y_t^ω
- Possibly integer restrictions on x and/or y
- i.i.d. samples of uncertainty w_t^ω

Challenges:

- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

Suggestions needed for:

Current interests:

- Deterministic approximations of CC
- Convergence of Lagrangian styled methods

Summary of results of Column 2

- Bonferroni lower bound and Dawson & Sankoff linearized upper bound consistently perform better than others
- Weaker correlation in uncertainty leads to easier-to-solve models
- MIQCP formulation of Dawson & Sankoff bound is challenging

Approximations

Key idea: Satisfying a JCC is an intersection of events. Failing a JCC is a union of events. Consider an optimization model with a JCC with a maximization objective.

$$\mathbb{P}(\textstyle\bigcup_{t \in T} F_t) \leq \varepsilon$$

$\mathbb{P}(\bigcup_{t \in T} F_t) = S_1 - S_2 + \cdots (-1)^{|T|-1} S_T$, where $S_k = \mathbb{P}(\sum_{1 \leq i_1 < \cdots < i_k \leq |T|} F_{i_1} \cap \cdots \cap F_{i_k})$.

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ORIGINAL PAPER

Check for updates

Approximating two-stage chance-constrained programs with classical probability bounds

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Abstract
We consider a joint-chance constraint (JCC) as a union of sets, and approximate this union using bounds from classical probability theory. When these bounds are used in an optimization model constrained by the JCC, we obtain corresponding upper and lower bounds on the optimal objective function value. We compare the strength of these bounds against each other under two different sampling schemes, and observe that a larger correlation between the uncertainties tends to result in more computationally challenging optimization models. We also observe the same set of inequalities to provide the tightest upper and lower bounds in our computational experiments.

Keywords Chance-constrained optimization · Bonferroni inequalities · Union bounds · Stochastic optimization · Approximations

Bonferroni bounds:

$$\mathbb{P}(\bigcup_{t \in T} F_t) \leq \quad S_1 \leftarrow LB \quad (1a)$$

$$\mathbb{P}(\bigcup_{t \in T} F_t) \geq \quad S_1 - S_2 \leftarrow UB. \quad (1b)$$

Tighter bounds from Sathe et al. [1980]:

$$\mathbb{P}(\bigcup_{t \in T} F_t) \leq \quad S_1 - \frac{2}{T} S_2 \leftarrow LB \quad (2a)$$

$$\mathbb{P}(\bigcup_{t \in T} F_t) \geq \quad \frac{S_1 + 2S_2}{T^2} \leftarrow UB. \quad (2b)$$

And more from Dawson and Sankoff [1967]:

$$\mathbb{P}(\bigcup_{t \in T} F_t) \geq \frac{{S_1}^2}{S_1 + 2S_2} \leftarrow \quad UB \quad (3a)$$

$$2\varepsilon S_2 \geq \alpha_n S_1 + \beta_n, n = 0, 1, \ldots |N| - 1 \leftarrow \quad UBlinearized \quad (3b)$$

Stochastic unit commitment

Stochastic unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations, under uncertainty (of demand, prices, renewables...)?

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Modeling Flexible Generator Operating Regions Via Chance-Constrained Stochastic Unit Commitment

Bismark Singh · Bernard Knueven · Jean-Paul Watson

Received: date / Accepted: date

Abstract We introduce a novel chance-constrained stochastic unit commitment model to address renewables production uncertainty in power systems operation. For most thermal generators, underlying technical constraints that are universally treated as “hard” by deterministic unit commitment models are in fact based on engineering judgments, such that system operators can periodically request operation outside these limits in non-nominal situations, e.g., to ensure reliability. We incorporate this practical consideration into a chance-constrained stochastic unit commitment model, specifically by at most occasionally allowing minor deviations from minimum and maximum thermal generator power output levels. We demonstrate that an extensive form of our model is computationally tractable for medium-sized power systems given modest numbers of renewables production scenarios. We show that the model is able to potentially save significant annual production costs by allowing infrequent and controlled violation of the traditionally hard bounds imposed on thermal generator production limits. Finally, we conduct a sensitivity analysis of optimal solutions to our model to various restrictions of thermal generator operational modes and analyze the structure of optimal solutions in terms of thermal generators selected for non-nominal operations.
under two restricted regimes and observe similar qualitative results.

Keywords Stochastic optimization · Unit commitment · Power systems operations · Chance constraints · Emergency operations

Proposal:

- Allow thermal generators to “occasionally” violate operational limits
- Violations should be **few** (else, increased maintenance costs)
- Violations should **not be large** (there are absolute ratings of generators)
- 1% savings in energy production is worth ≈ \$1 billion per year in the U.S. alone

Results demonstrated on US. RTS-GMLC and WECC240++ systems

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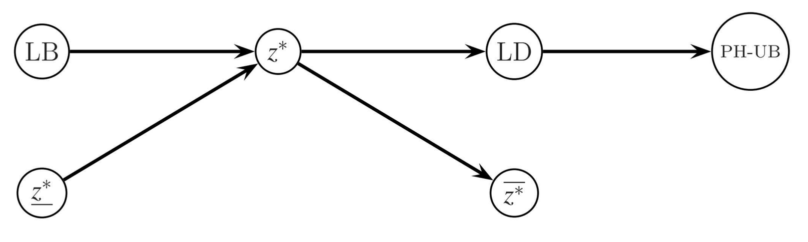
Lagrangian relaxation of CC

Proposal:

- Relax the “linking constraint” for a JCC, with a Lagrangian relaxation
- Solve using the standard subgradient algorithms
- Can instead approximately solve using Progressive Hedging algorithm

Consider a maximization objective. Arrow from source to sink indicates optimal value of source ≤ sink.



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A chance-constrained optimization model for day-ahead scheduling of a hybrid solar-battery storage system

Bismark Singh · Bernard Knueven

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Abstract We develop a novel chance-constrained optimization model for a hybrid solar-battery storage system. Solar power in excess of the promise is used to charge the battery, while power short of the promise is met by discharging the battery. We ensure reliable operations by using a joint chance constraint. Models with a few hundred scenarios are relatively tractable; for larger models, we demonstrate how a Lagrangian relaxation scheme provides improved results.

Keywords Chance constraints · Stochastic optimization · Solar power · Photovoltaic power station · Battery storage

1 Introduction

1.1 Motivation

$\lvert\Omega\rvert$	ε	LP		LR		LR-PH	
		Gap	Time	Gap	Time	Gap	Time
150	0.05	6.6%	1	1.4%	56	1.4%	68
	0.01	4.5%	1	-	8	-	9
300	0.05	6.8%	1	2.7%	272	3.3%	251
	0.01	4.4%	1	0.4%	210	1.1%	242
450	0.05	6.7%	2	2.5%	980	2.9%	487
	0.01	3.8%	2	-	660	0.5%	299
600	0.05	7.5%	3	3.3%	1936	3.7%	279
	0.01	4.2%	3	-	1581	0.6%	409
900	0.05	8.0%	9	3.8%	2369	4.7%	870
	0.01	4.4%	9	-	2362	1.5%	613
1200	0.05	8.2%	10	2.7%	2356	4.2%	896
	0.01	5.9%	9	0.9%	2351	2.4%	747

Bundling of scenarios offers advantages in PH algorithm.