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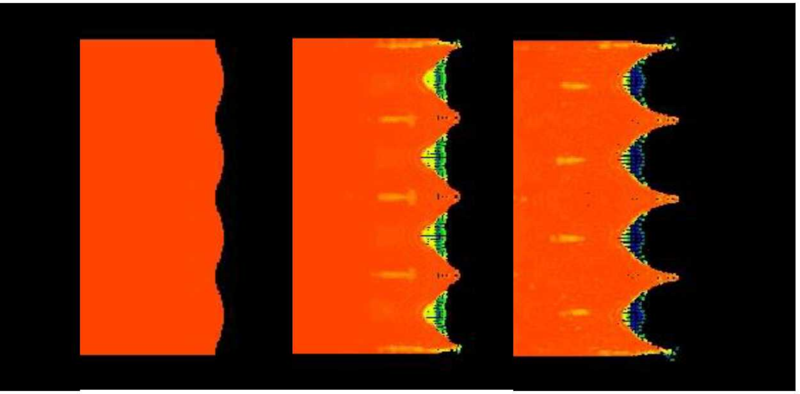
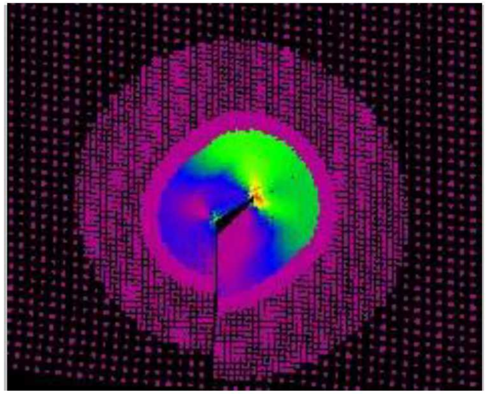
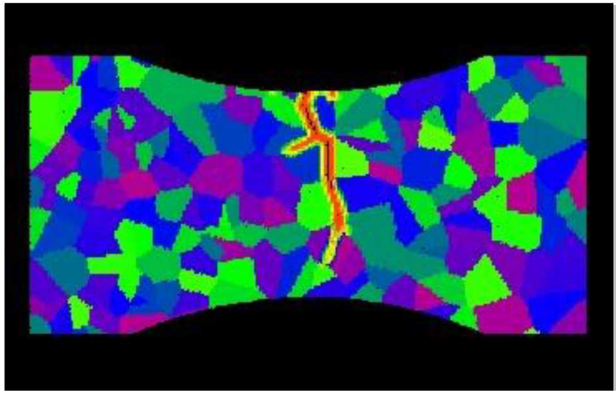

Local-peridynamic coupling with the splice method

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EMI Conference, Pasadena, CA, June 19, 2019

Outline

- Peridynamics basics
- Continuum splice
- Discretized splice
- Examples

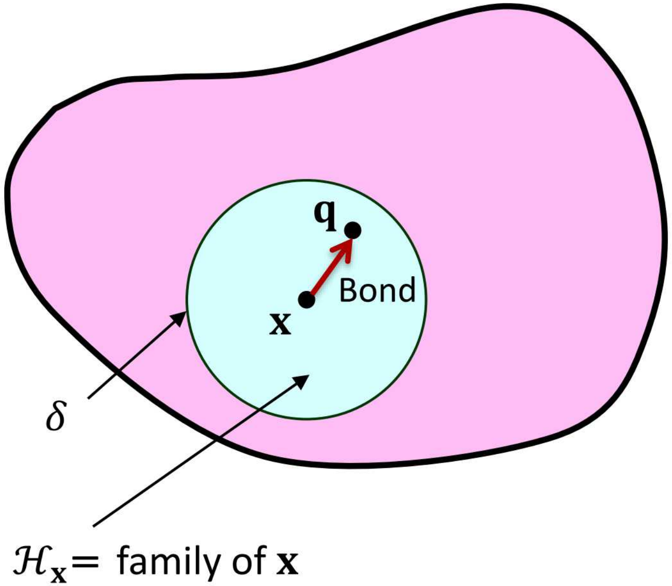
Peridynamics concepts: Horizon and family

- Any point $\mathbf{x}$ interacts directly with other points within a distance δ called the "horizon."
- The material within a distance δ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.

Peridynamic equilibrium equation

$$\int_{\mathcal{H}_{\mathbf{x}}} \mathbf{f}(\mathbf{q}, \mathbf{x})\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x}) = 0$$

$\mathbf{f}$ = bond force density



- The peridynamic field equations don't use spatial derivatives
  - so they are compatible with cracks.

General references
- SS, Journal of the Mechanics and Physics of Solids (2000)
- SS and R. Lehoucq, Advances in Applied Mechanics (2010)

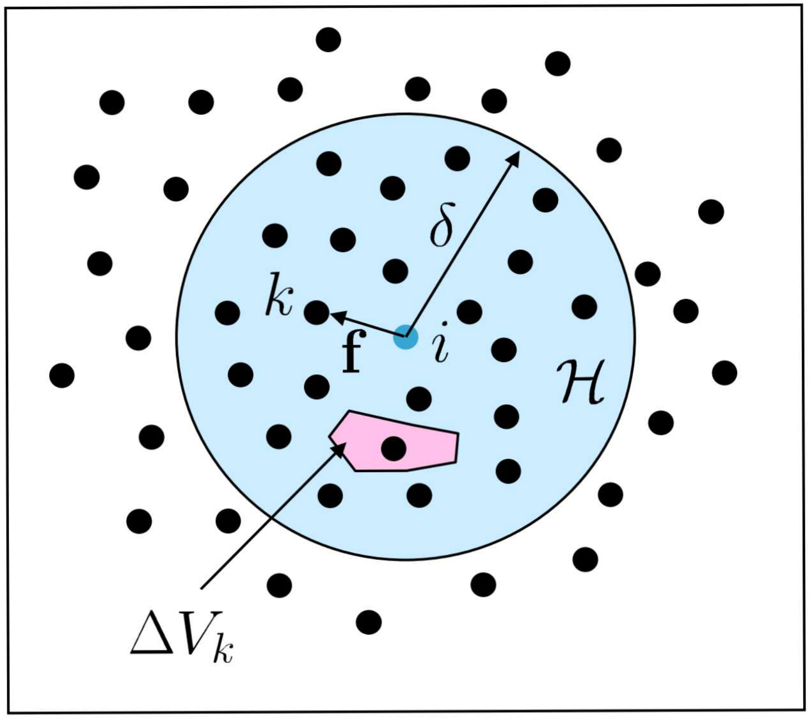
Simple particle discretization

- Integral is replaced by a finite sum: resulting method is meshless and Lagrangian.

$$\rho \ddot{\mathbf{y}}(\mathbf{x}, t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{x}', \mathbf{x}, t)\, dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}, t)$$

$$\downarrow$$

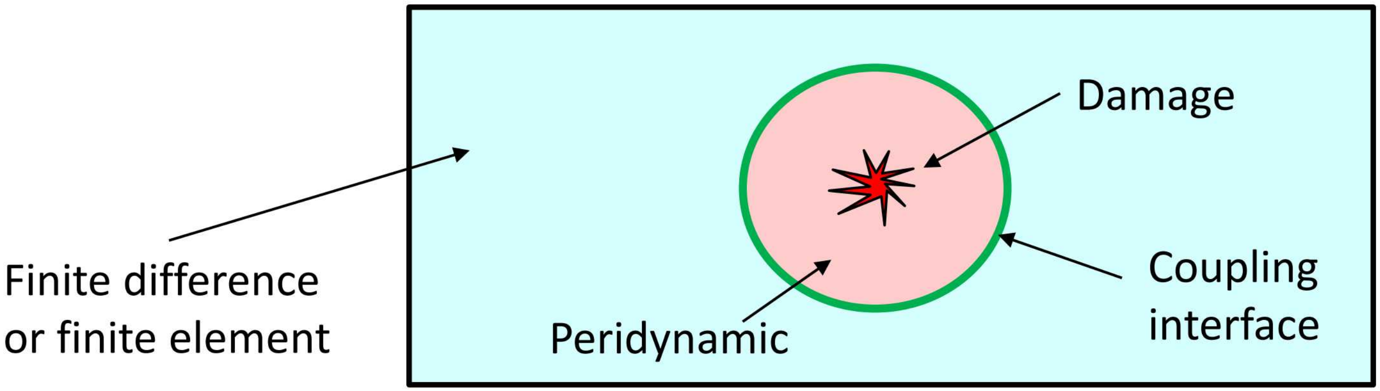
$$\rho \ddot{\mathbf{y}}_i^n = \sum_{k \in \mathcal{H}} \mathbf{f}(\mathbf{x}_k, \mathbf{x}_i, t)\, \Delta V_k + \mathbf{b}_i^n$$



- Discontinuous Galerkin is another viable method (Gunzburger, LS-DYNA).

Local-nonlocal coupling

- Reduce cost by using a nonlocal method only near actively growing cracks.
- Single grid with variable spacing.
  - Coarse grid: local (conventional) equations.
  - Fine grid: peridynamic.

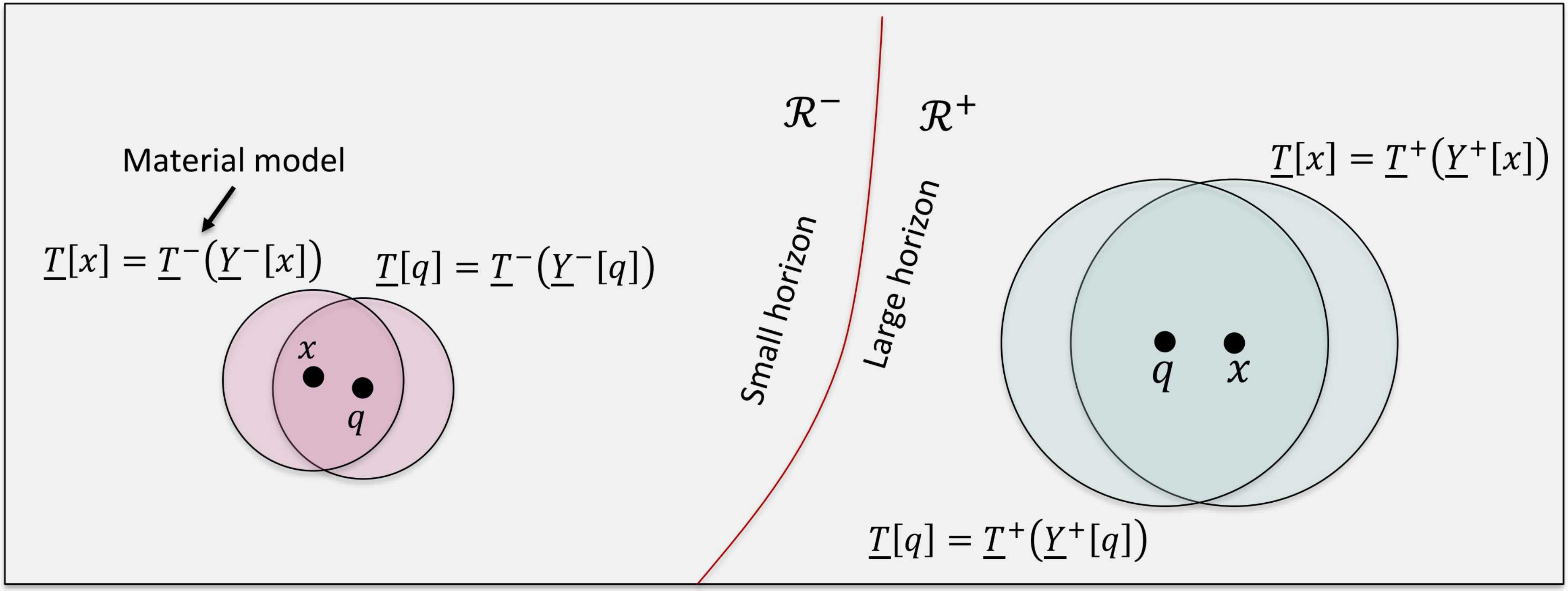


Continuum form of a splice connecting 2 PD regions with different horizons

- Far from the interface, everything is as usual.
- Force states come from whichever horizon applies.
- Force density at x:

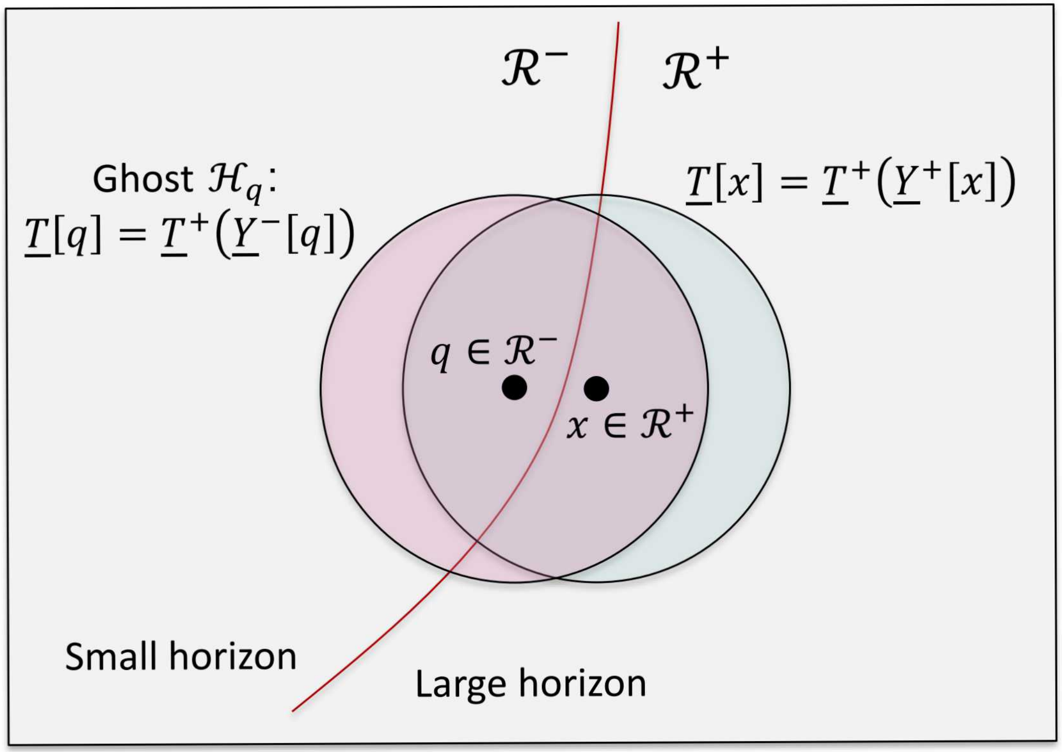
$$L(x) = \int_{\mathcal{H}_x} \{\underline{T}[x]\langle q - x\rangle - \underline{T}[q]\langle x - q\rangle\}\, dq$$

Force states



Continuum form of the splice, ctd.

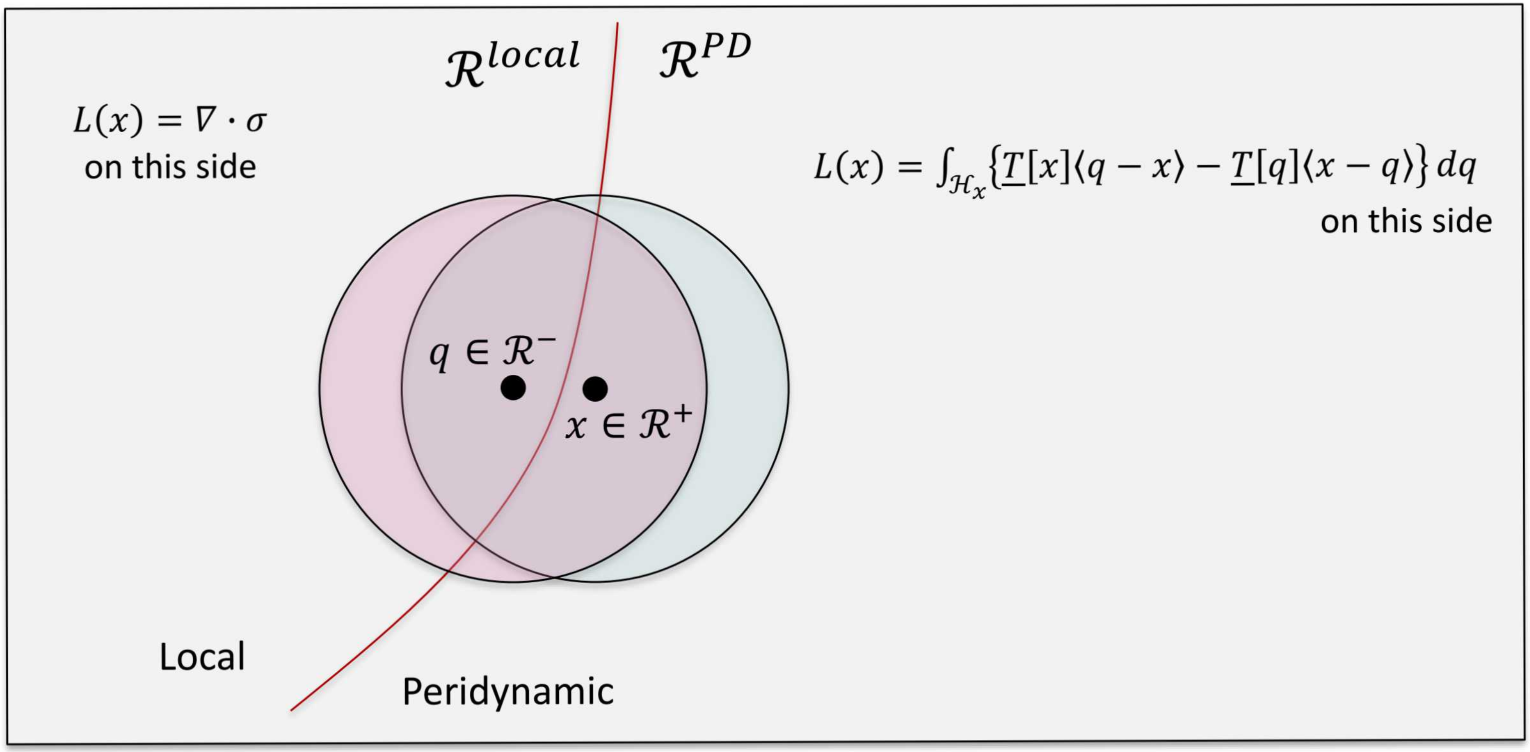
- At points whose horizon includes points on the other side:
  - Same expression for force density, but with material model "ghosted" from other side of the interface.
- Material models must "agree" for a homogeneous deformation.
  - i.e, the must produce the same partial stress tensor $\sigma = \int \underline{T} \times \underline{\xi}$ on both sides.



SS., Littlewood, D. and Seleson, P., 2015. Variable horizon in a peridynamic medium. *Journal of Mechanics of Materials and Structures*

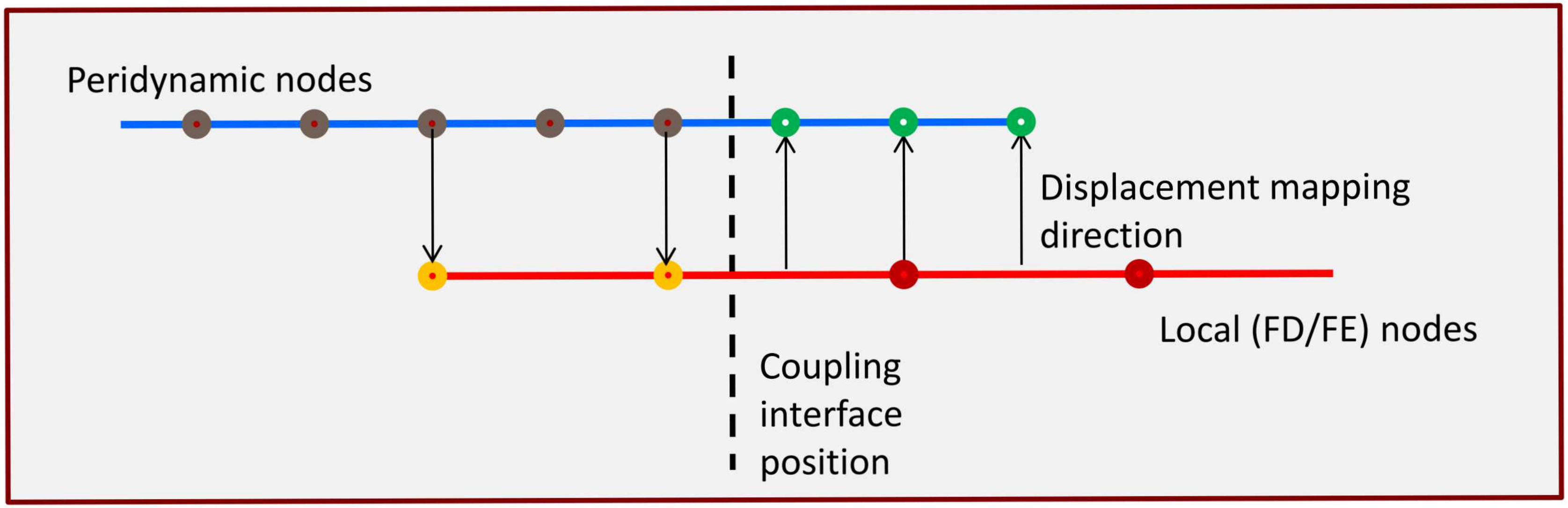
Continuum local-PD splice

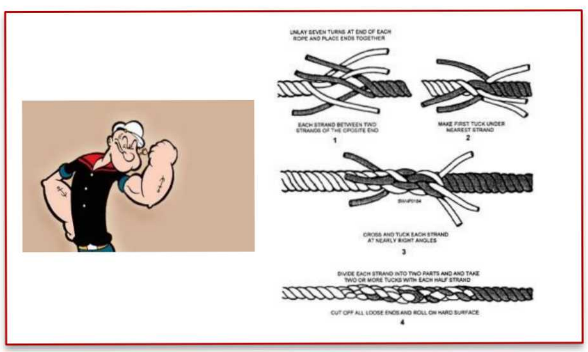
- Now let the small horizon approach zero.
- The local PDEs apply in $\mathcal{R}^-$ in the sense of a limit (assuming smooth deformation).
- The PD side still uses ghosting.

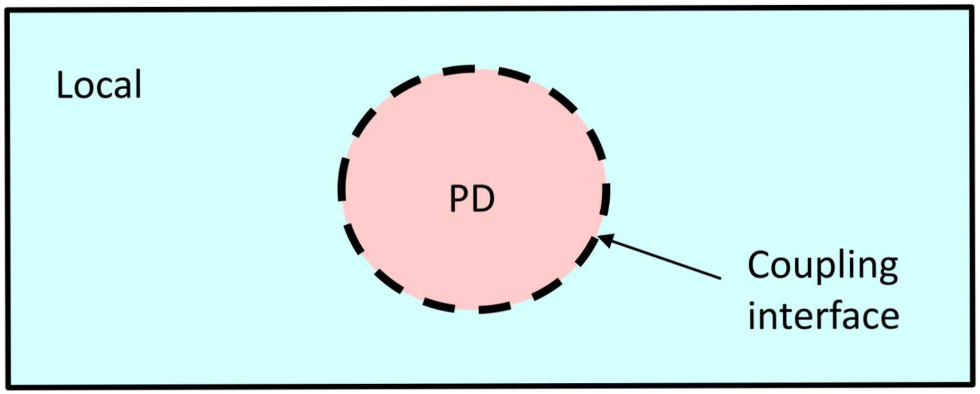


Discretized splice coupling method

- Regions exchange displacements and velocities only (not forces).
- Each region sees material just like itself on the other side of the interface.

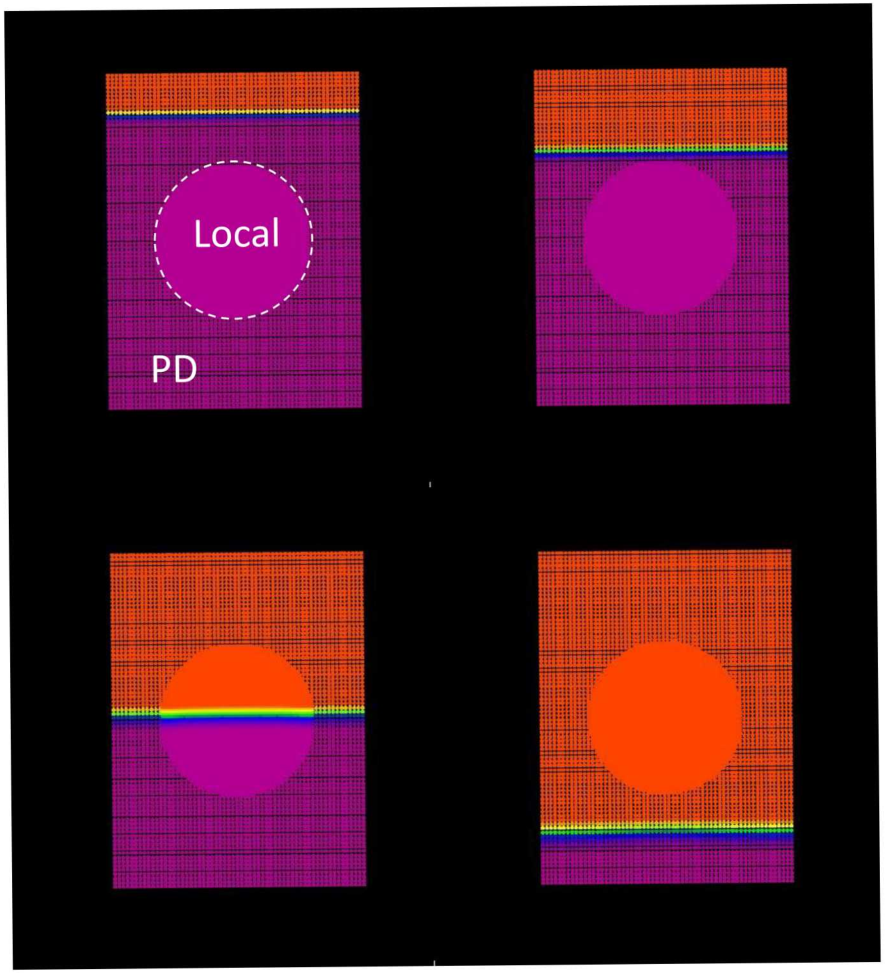






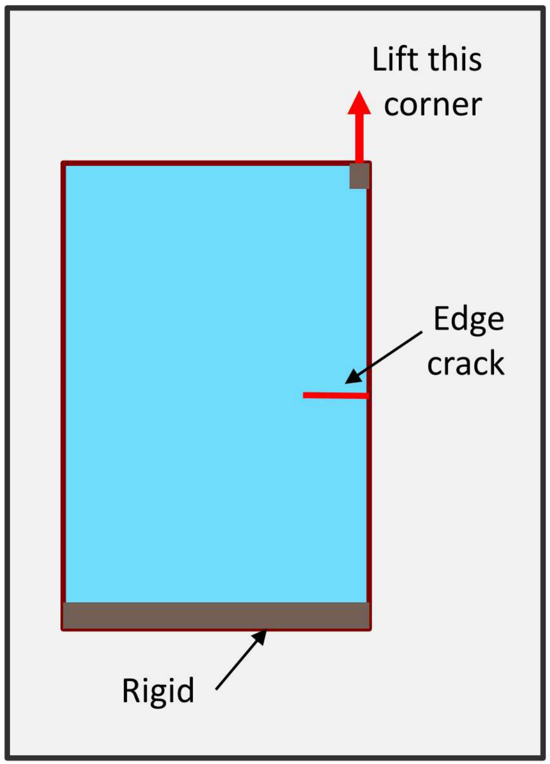
Effect on waves

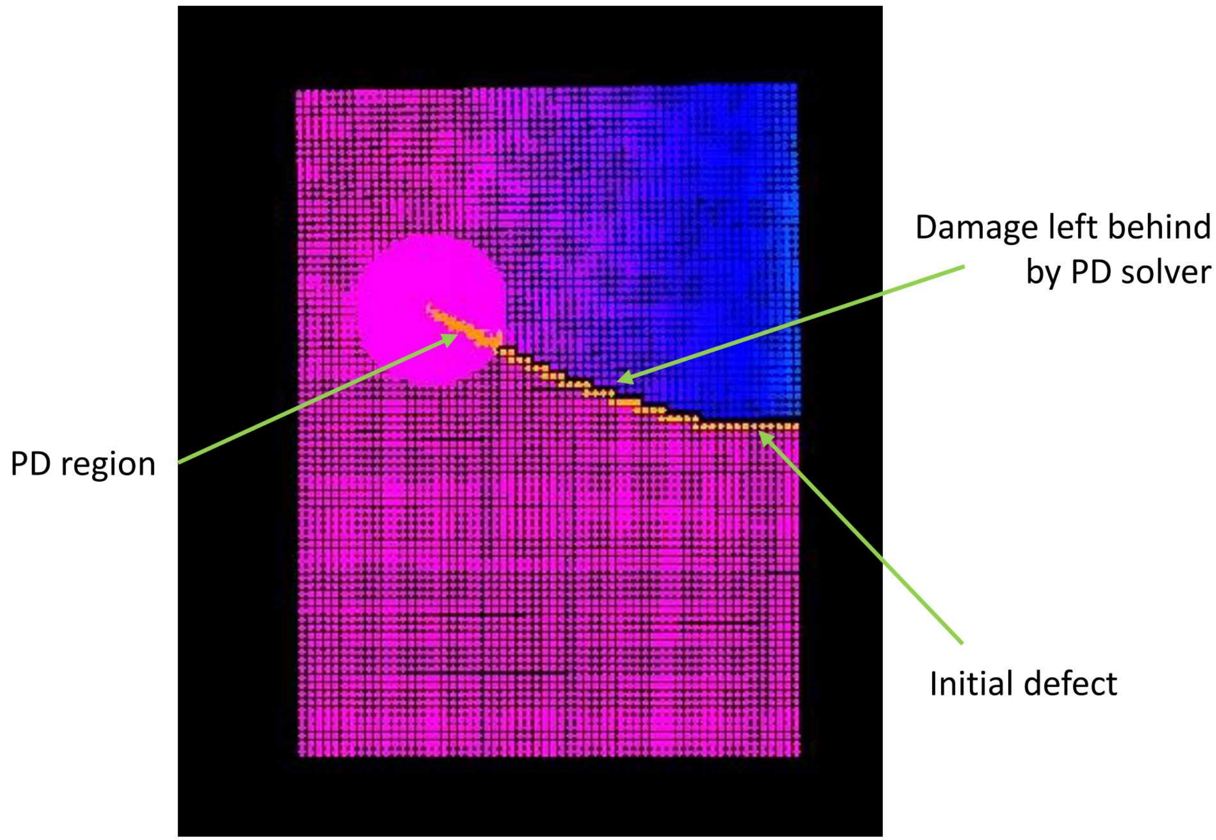
- Waves pass through between regions without obvious distortion.



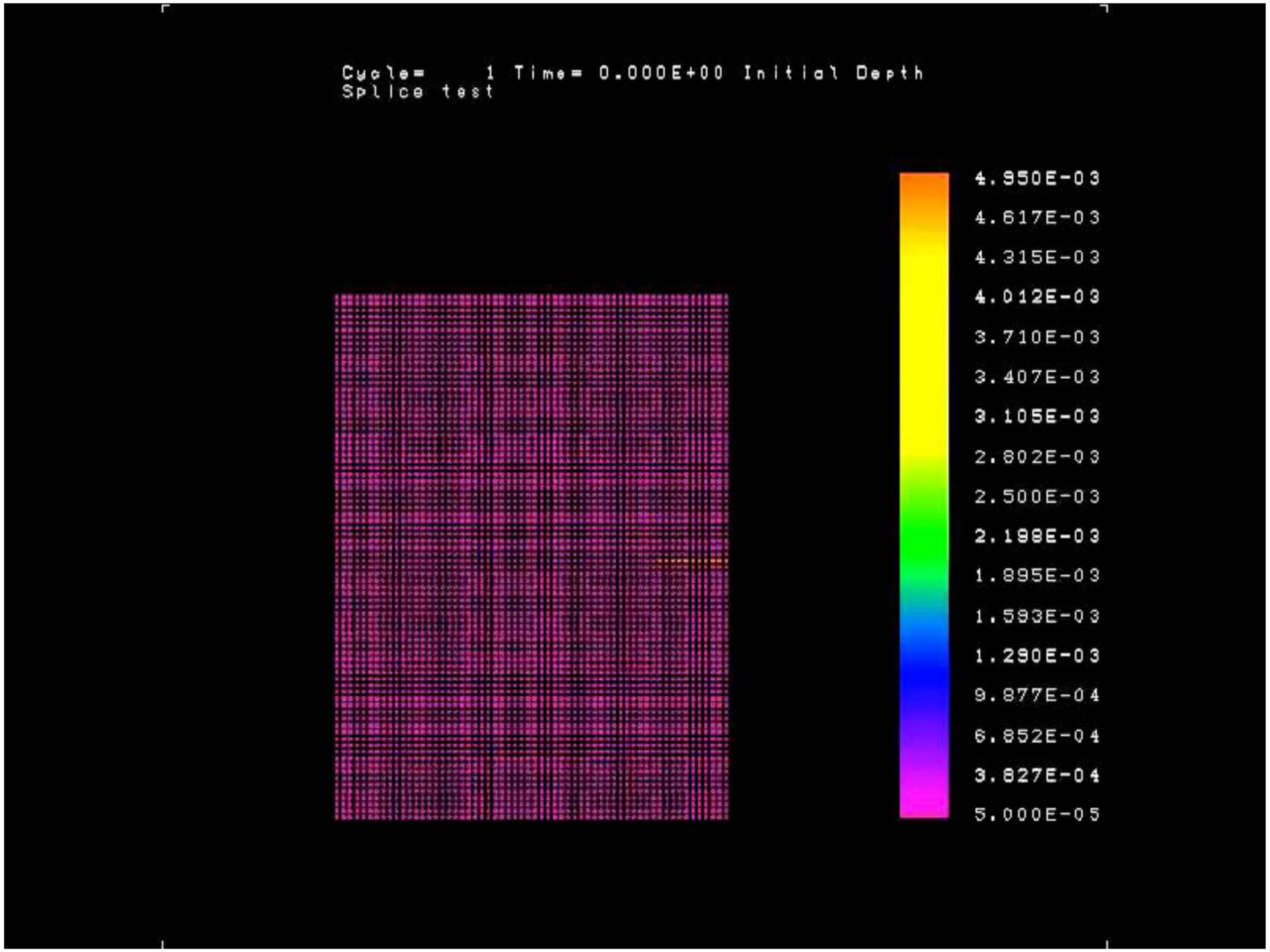
Adaptively moving the peridynamic region

- Peridynamic region follows the crack tip.
- Damage from PD is mapped back onto the local grid.



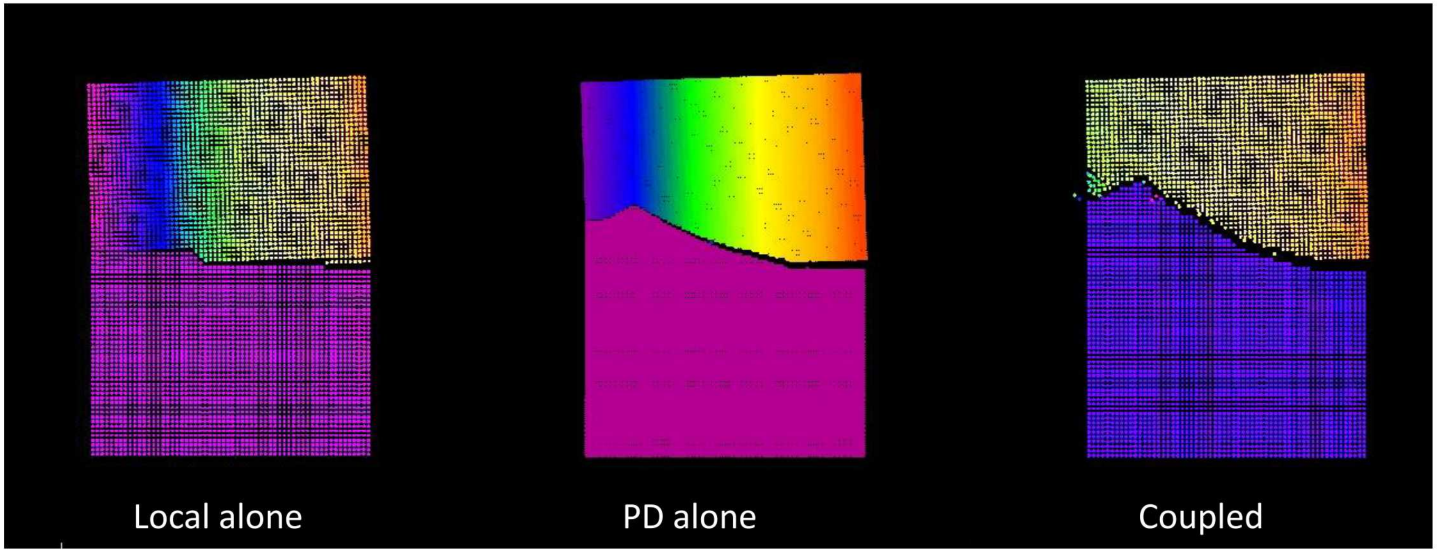


Adaptivity video

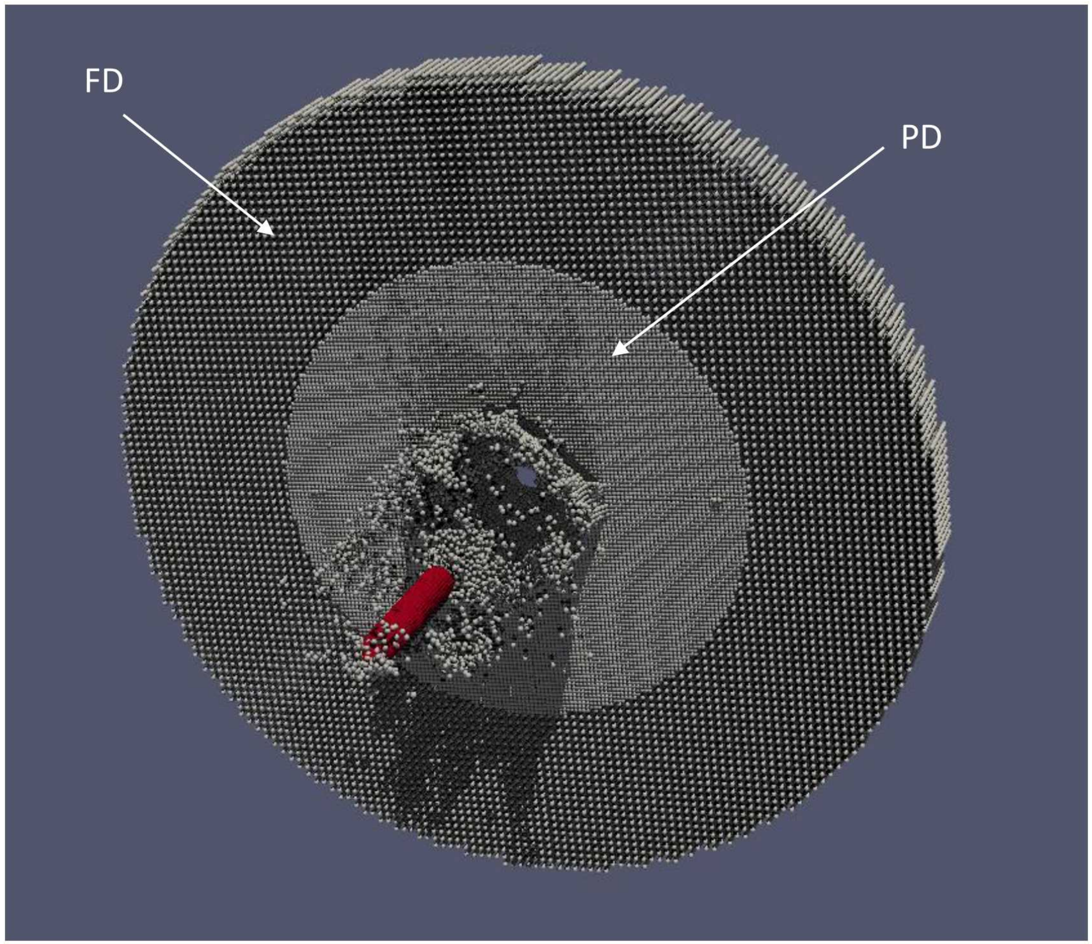


Compare coupled method vs. single methods individually

- Coupled local-PD results are close to PD alone.
- Crack path is more reasonable than with local alone (using element death).

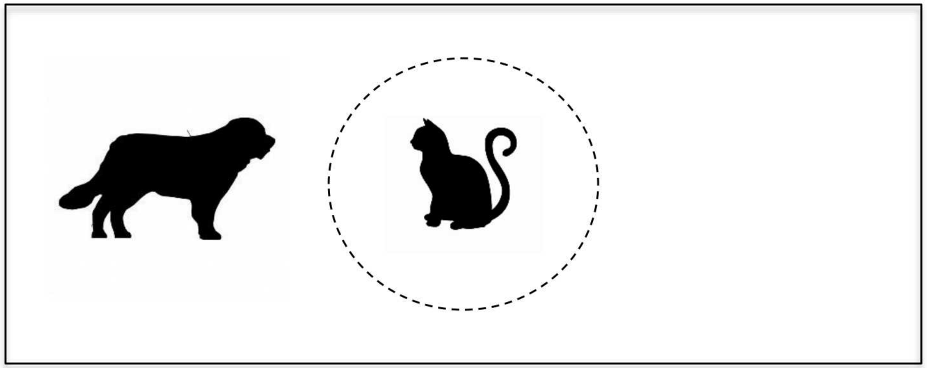


Coupling example in 3D



What does accuracy mean in coupling?

- The local and nonlocal regions are dogs and cats, respectively.
  - They don’t like to talk to each other.
  - There is no single right way to define the continuum coupling.
  - So it is meaningless ask which continuum coupling method is the most accurate.
- The best we can do seems to be:
  - Require the coupled solution to converge to the local solution as $\delta \to 0$.
  - For fixed δ, require the discretized model to converge as $\Delta x \to 0$.



Discussion

- Good:
  - All we do is interpolate displacements.
    - No need to define forces between the regions.
  - One region can be coarser than the other.
- Bad:
  - The local and PD regions need to have the “same” material model.
    - Otherwise the coupling fails to conserve momentum.
  - Trick to ensure this consistency:
    - Define the local material model by

$$\sigma(F) = \int \underline{T}(F\xi)\langle\xi\rangle \otimes \xi.$$