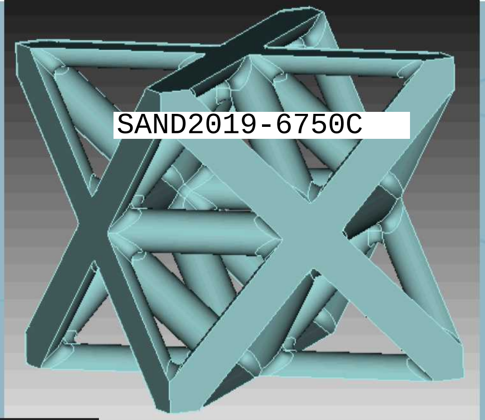
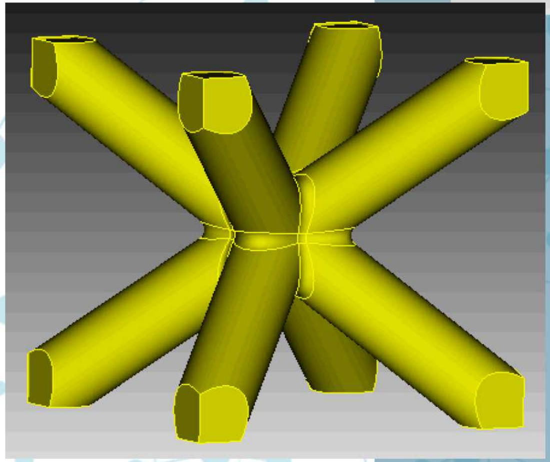
SAND2019-6750C

Nonlinear Topology Optimization with Microstructural Effects:

A Micromorphic Approach





PRESENTED BY

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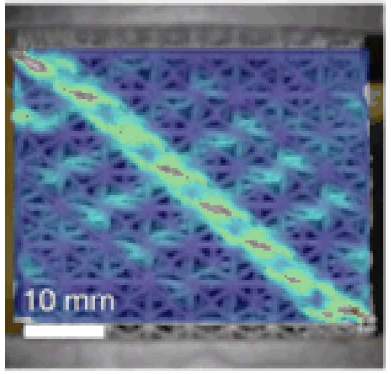

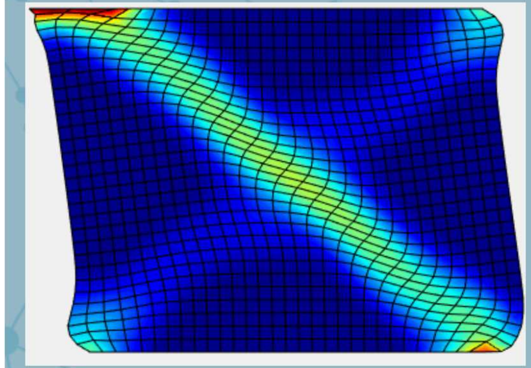

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Talking Points for Today

- Lattice Metamaterials for Novel Structural Designs

- Capturing Localization Effects through a Generalized Continuum Approach

- Topology Optimization for Structural Design

- Shear Band Localization and Dual-Lattice Metamaterials

Lattice Metamaterials for Novel Structural Designs

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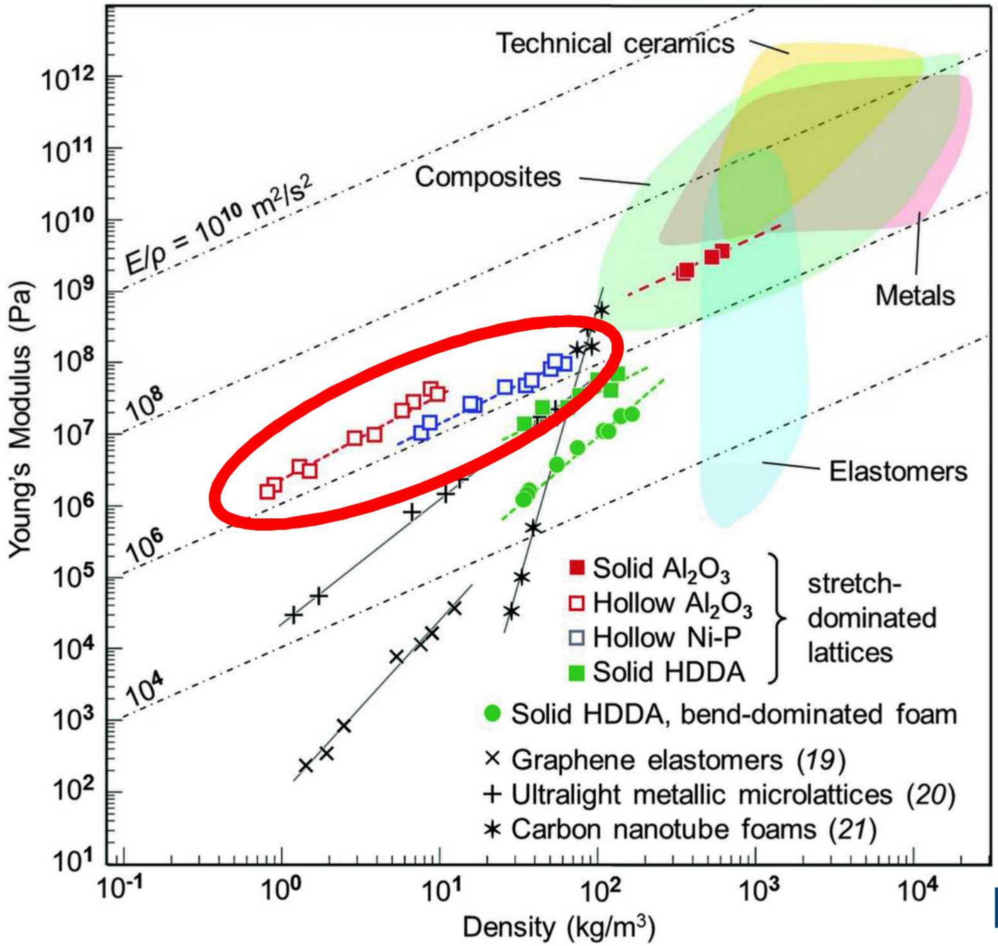
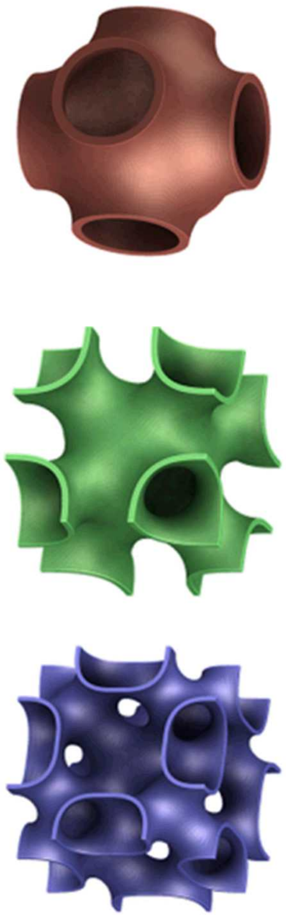
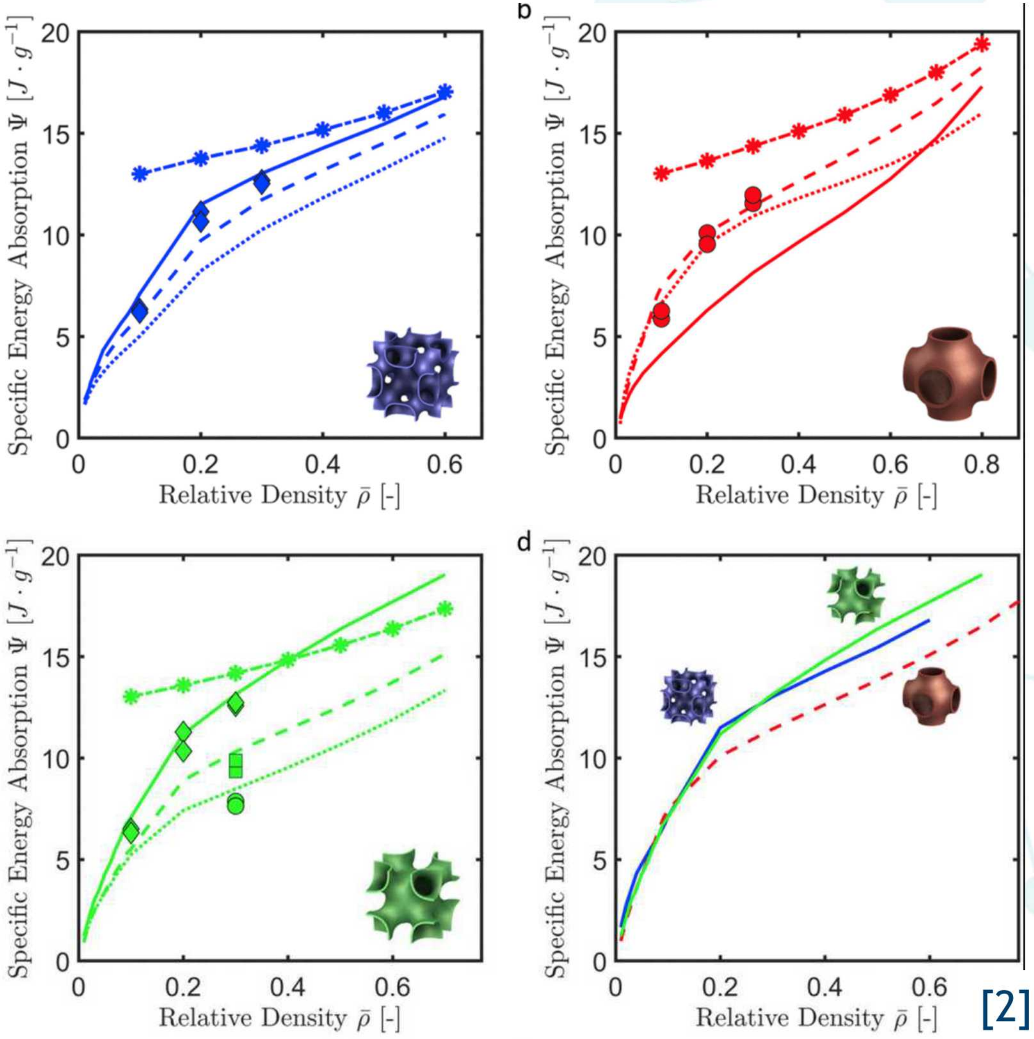
What Are Lattice Metamaterials?

- **Exceptional weight-specific stiffness/strength[1]**



[1]





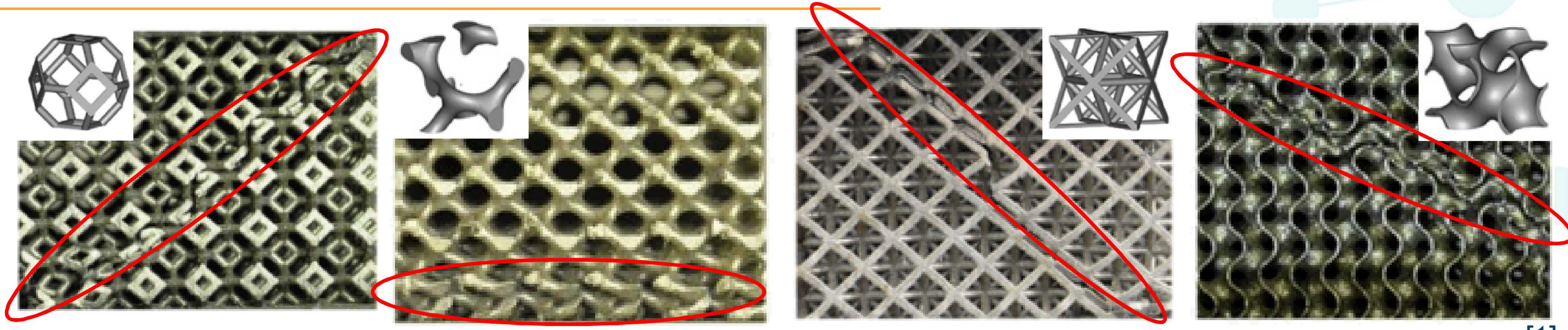
[2]

- **High specific energy absorption – Ultralight energy absorbers[2]**

[1] Zhang, et al. *Ultralight, ultrastiff mechanical metamaterials.* Science, 2014. **344**: p. 1373-1377.

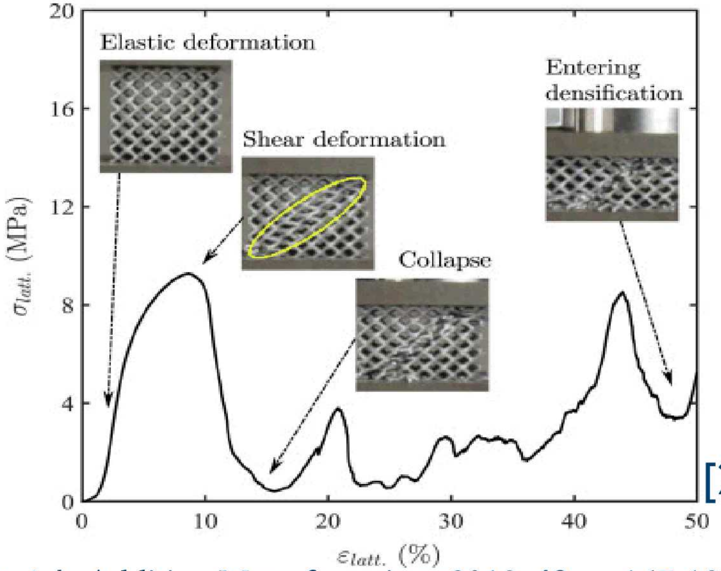
[2] Colin Bonatti and Mohr, D. *Smooth-shell metamaterials of cubic symmetry: Anisotropic elasticity, yield strength and specific energy absorption.* Acta Materialia, 2019. **164**: p. 301-321.

Localization in Lattice Structures due to Fabrication Defects and Geometrical Effects



[1]

- Lattice metamaterials in compression fail predominantly through localization
- Shear bands cause sharp drop in load capacity – limiting energy dissipation
- Compression bands allow for progressive collapse of lattice
- Ideal energy dissipating behavior involves homogeneous plastic deformation before progressive collapse at a high plateau stress

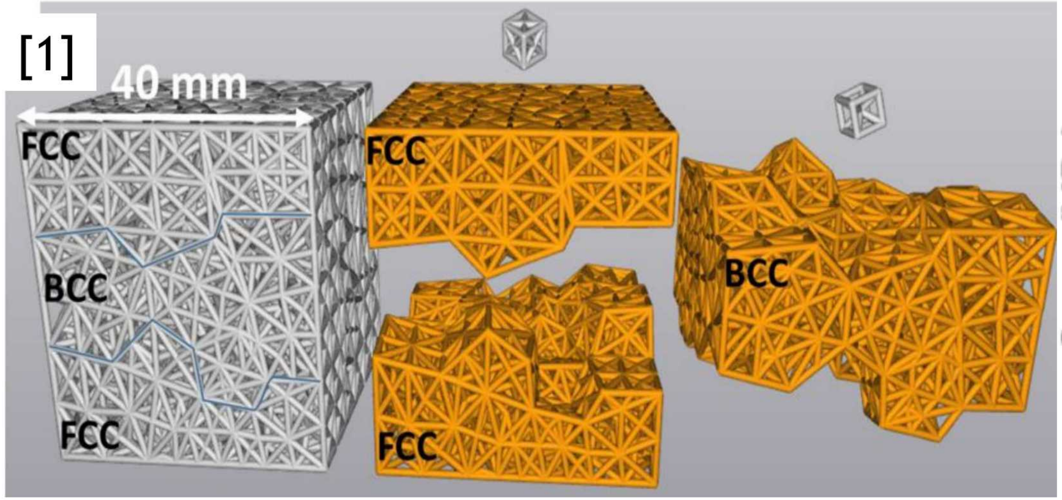


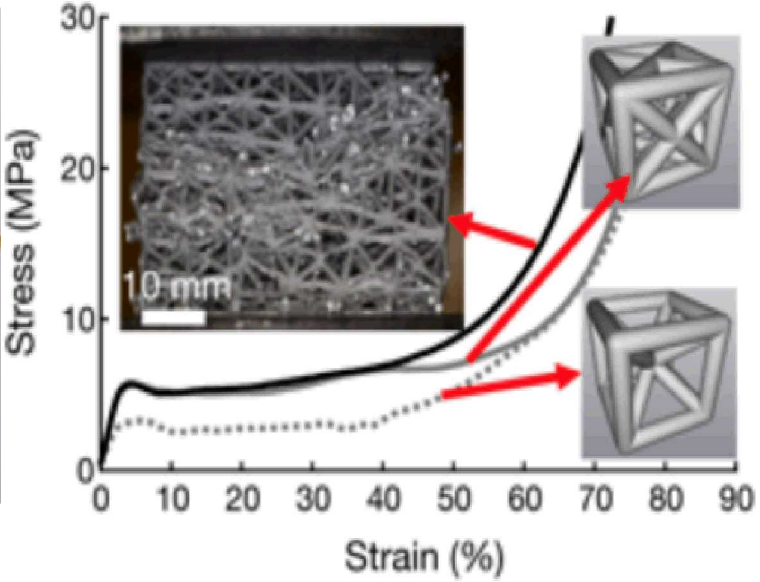
[2]

[1] Al-Ketan, et al. *Topology-mechanical property relationship of 3D printed strut, skeletal, and sheet based periodic metallic cellular materials.* Additive Manufacturing, 2018. **19**: p. 167-183.
[2] Maskery, et al. *A mechanical property evaluation of graded density Al-Si10-Mg lattice structures manufactured by selective laser melting.* Materials Science and Engineering A, 2016. **670**: p. 264-274.

Design Considerations with Lattice Metamaterials is non-intuitive

- Design approaches with traditional materials don't apply to lattice metamaterials – non-intuitive
- Combining different lattice topologies can allow for tailoring of properties and control of localization[1]





- Topology optimization provides rigorous way to explore design space
- Multiscale topology optimization approaches can account for microstructure but are based on homogenization
  - Becomes computationally very expensive for nonlinear problems
  - Homogeneous deformation of RVE cannot capture localization

[1] Pham, et al. *Damage-tolerant architected materials inspired by crystal microstructure.* Nature, 2019. **565**: p. 305-311.

Capturing Localization Effects Through a Generalized Continuum Approach

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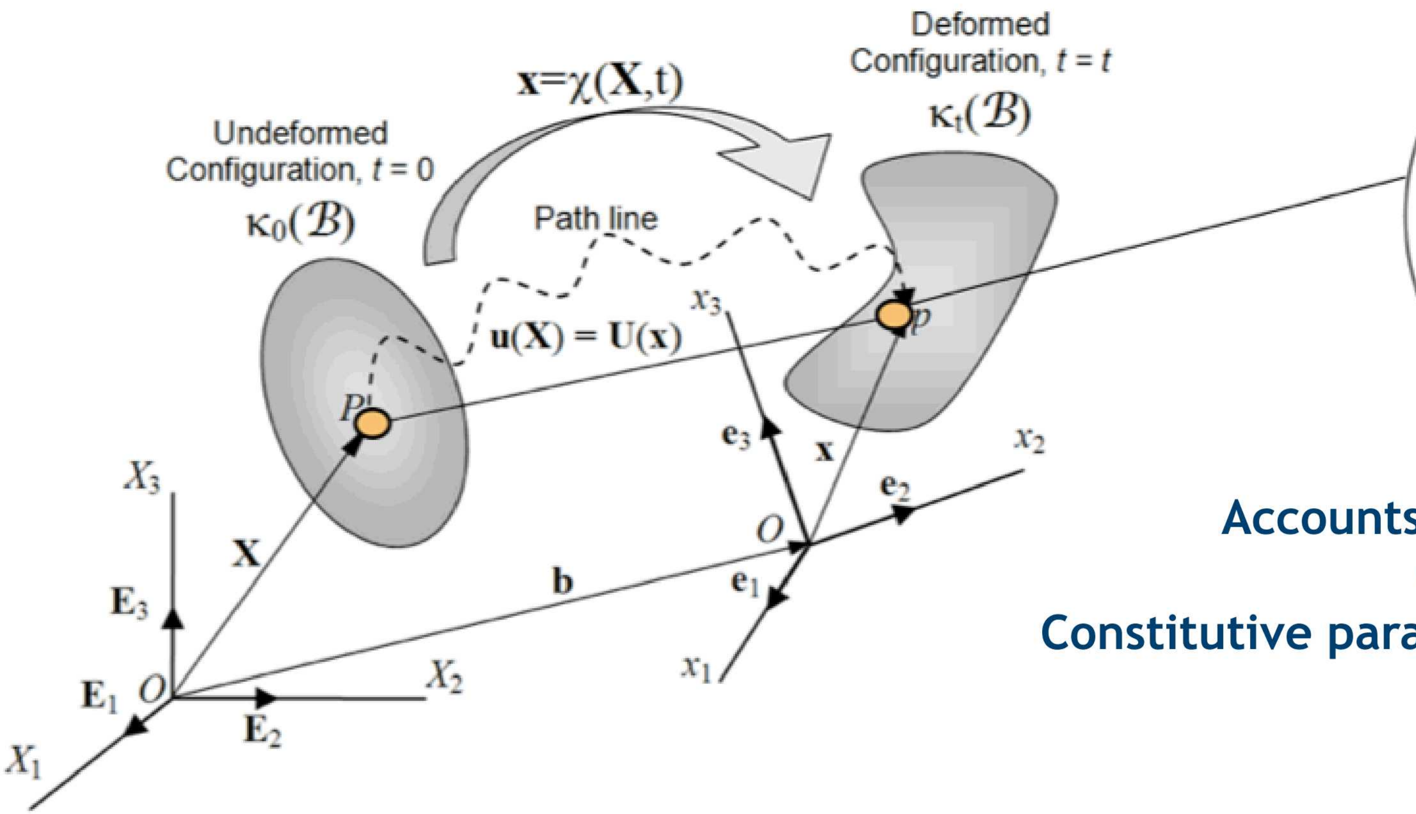
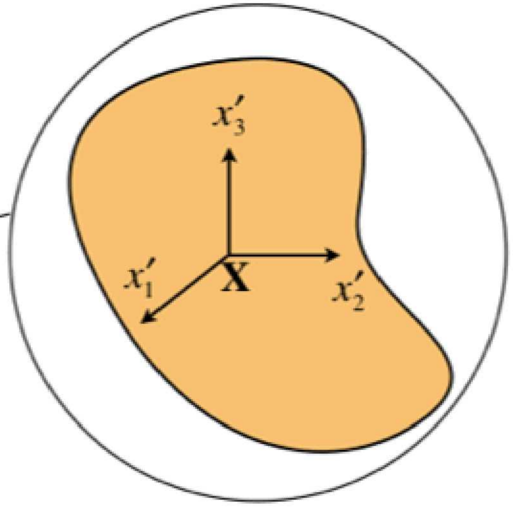
Accounting for Underlying "Microstructure" With Additional DOFs: Micromorphic Approach



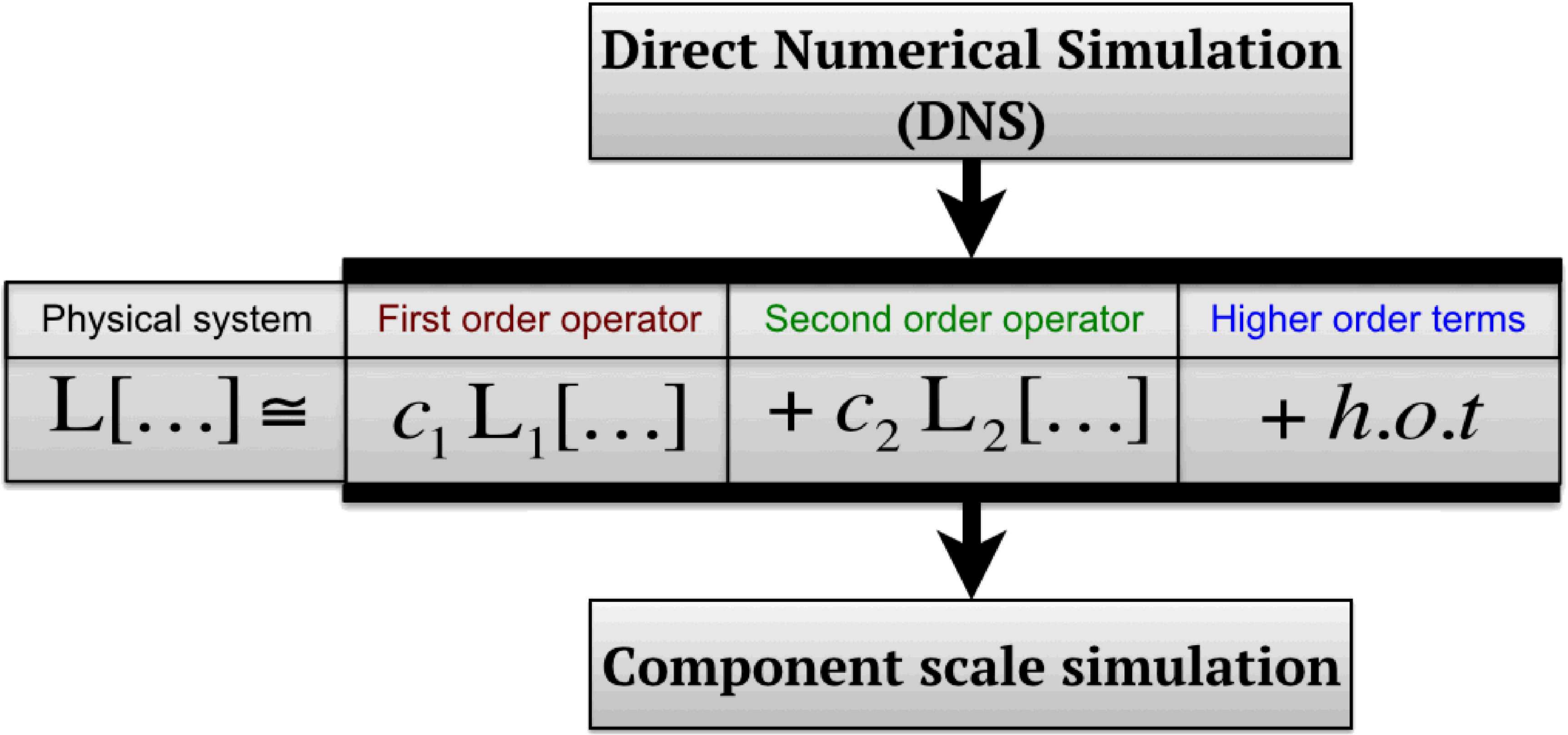


Accounts for rotation and stretching
Geometry agnostic
Constitutive parameters contain microstructure info

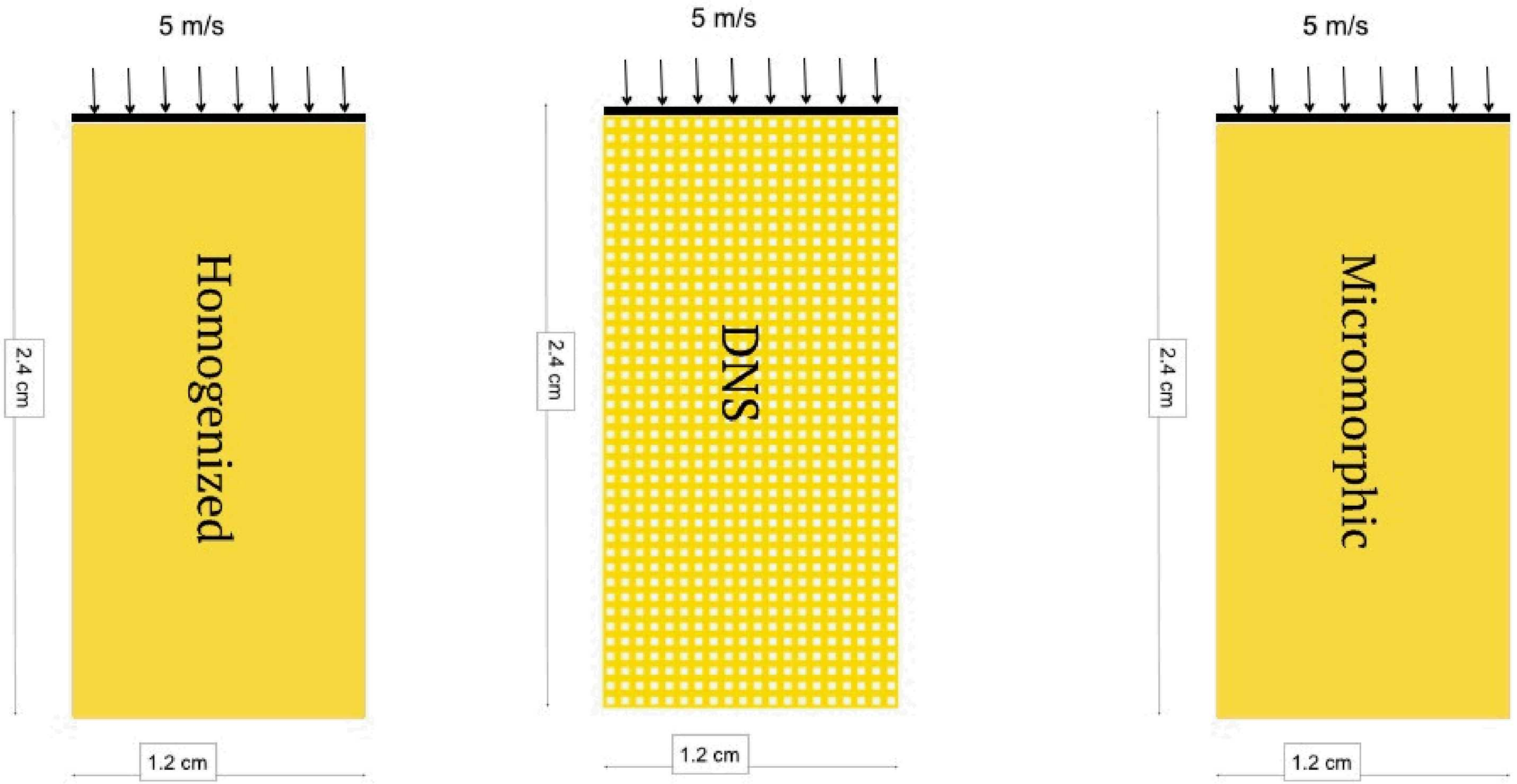
- Can capture significant gradients in macroscopic loading over microstructural features which cause
  - **Size dependent mechanical properties**
  - **Dispersion effects in wave propagation**
  - **Accumulation of plastic deformation in microstructure during localization**

[1] Eringen, A. Cemal. *Mechanics of micromorphic continua.* Springer, Berlin, Heidelberg, 1968.

Calibrating Micromorphic Models to Specific Microstructures



Example: Low Velocity Impact in Foam

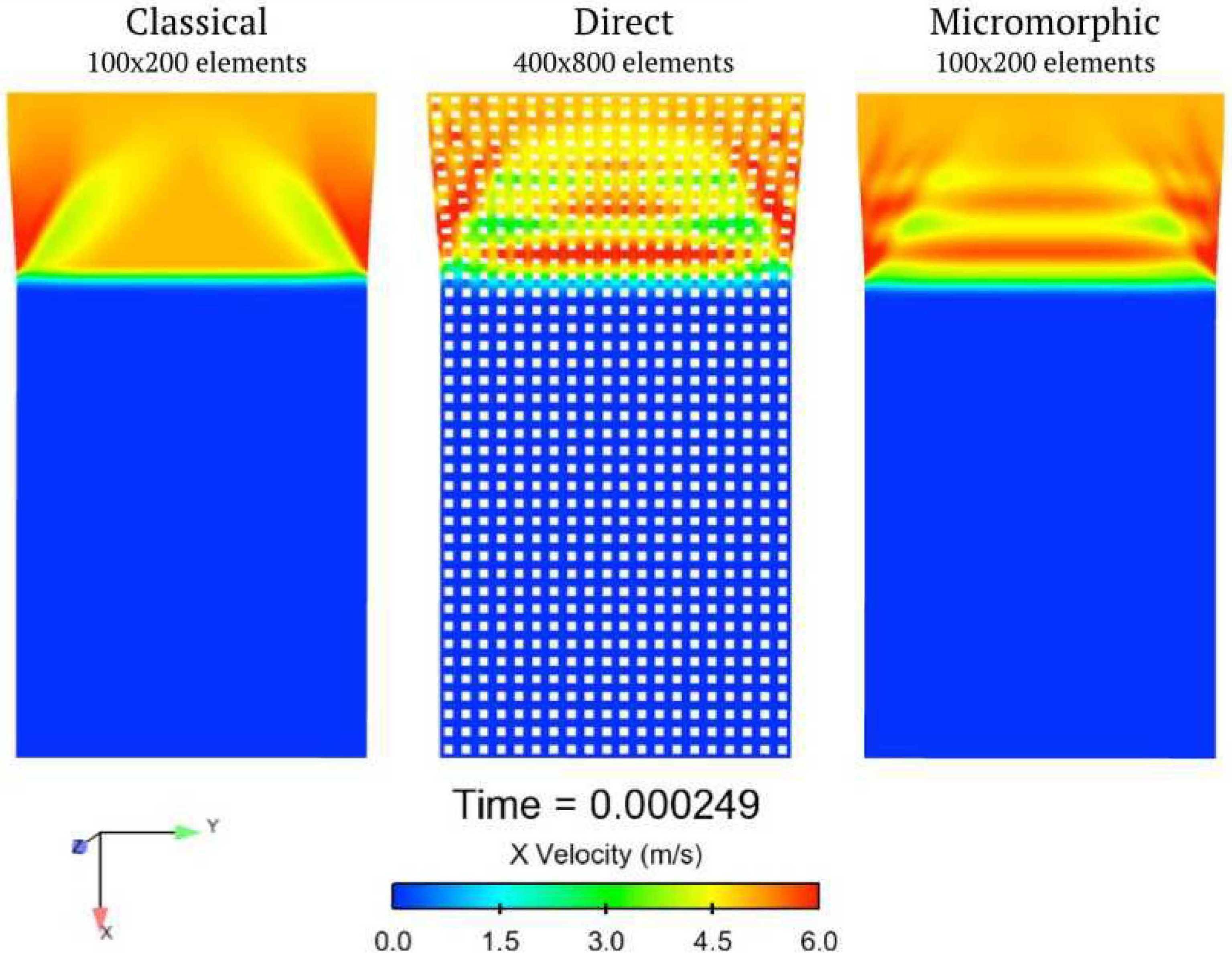


Classical (200 x 100 elements):

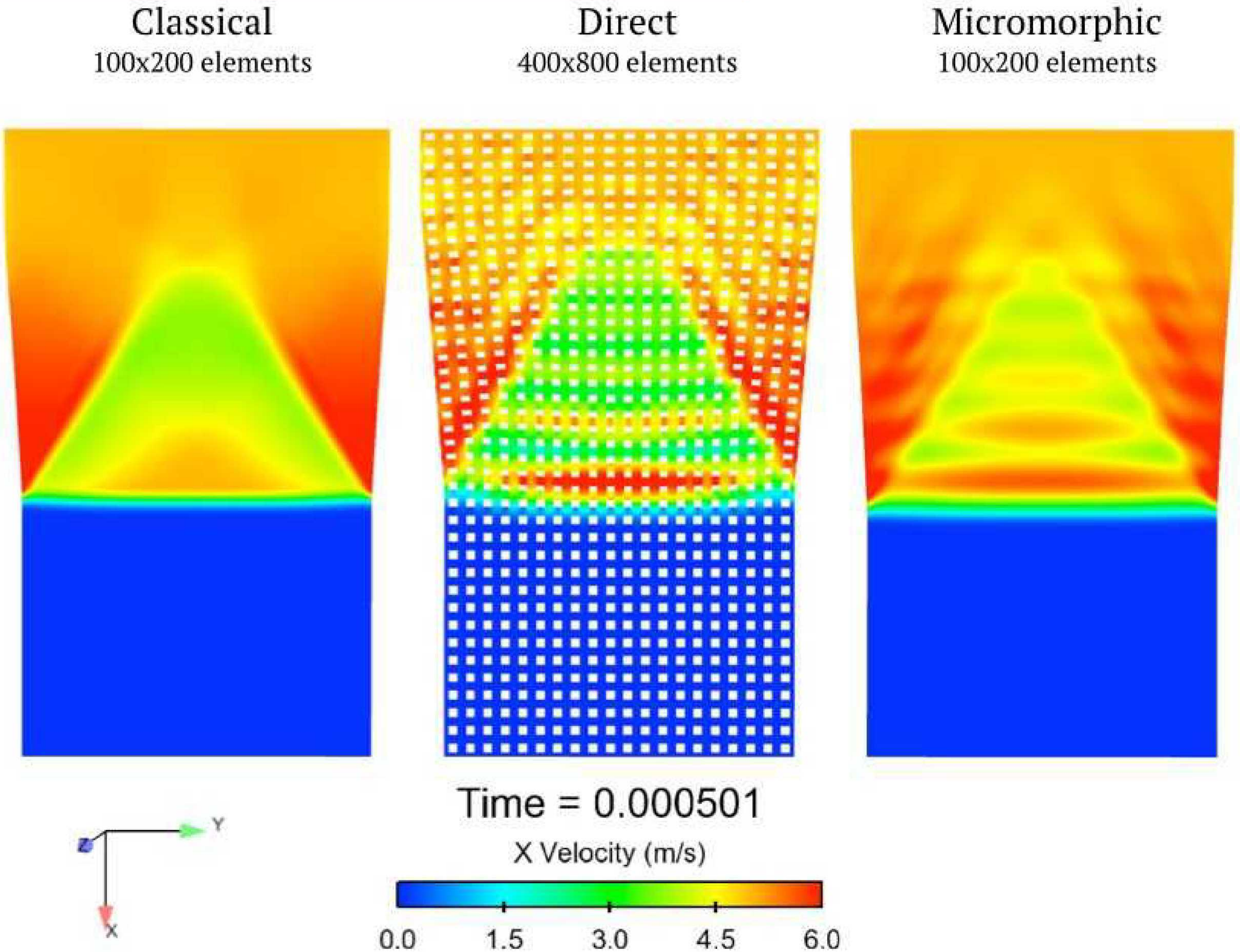
Direct (800 x 400 elements):
- 0.3 mm "pore" size (0.6 mm RVE)
- 25% porosity
- Solid density: 1100 kg/m^3 (Matweb.com)
- Young's modulus: 0.851 MPa (Fan, 2011)
- Poisson's ratio: 0.4

Micromorphic (200 x 100):
- microscale length: 0.6 mm
- Mindlin parameter (a.k.a. coupling constant): 0.3

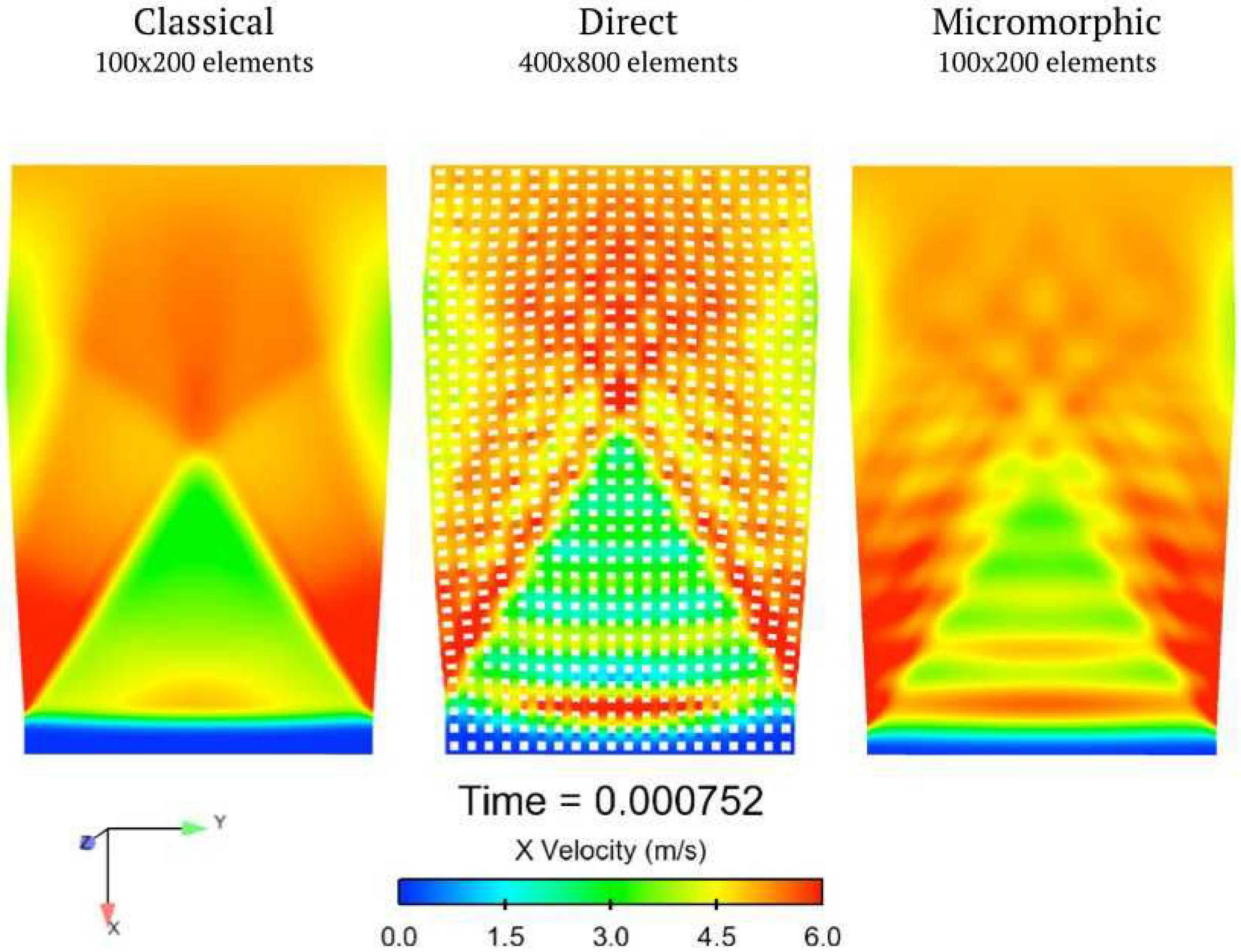
Example: Low Velocity Impact in Foam



Example: Low Velocity Impact in Foam



Example: Low Velocity Impact in Foam



Capturing Strain Softening and Localization: Regularization Through a Micromorphic Model

- Localization phenomena can lead to a loss of ellipticity in governing PDEs for classical continua
- Micromorphic continua involve length scale which regularizes this effect – suitable for simulating localization phenomena in lattice metamaterials
- Regularizing effect can be gained by replacing the microstrain tensor with a scalar plastic microstrain variable – only 1 additional DOF per continuum point
- Finite deformation theory considering elastoplastic softening and scalar plastic microstrain variable[1]:

Free Energy

$$\psi(\boldsymbol{C}^e, \chi, \boldsymbol{\nabla}\chi) = \psi_{ref}(\boldsymbol{C}^e, \alpha) + \frac{1}{2}H(\alpha - \chi)^2 + \frac{1}{2}\boldsymbol{K}.\boldsymbol{A}.\boldsymbol{K}$$

$\boldsymbol{K} = \boldsymbol{\nabla}\chi \quad \boldsymbol{A} = A\boldsymbol{I} \qquad \psi_{ref}(\boldsymbol{C}^e, \alpha)$ ⟶ Macroscale free energy

Micro Momentum Balance

$$\boldsymbol{\nabla}.\boldsymbol{b}_0 - a_0 = A\boldsymbol{\Delta}\chi - H\chi + H\alpha = \alpha - \chi + l^2\boldsymbol{\Delta}\chi = 0$$

$$l = \sqrt{A/H}$$

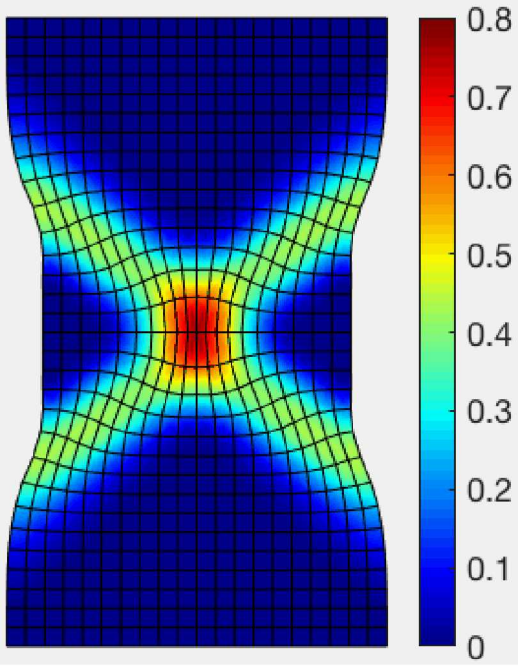
[1] Anand, et al. *A large-deformation gradient theory for elastic-plastic materials: Strain softening and regularization of shear bands*. International Journal of Plasticity, 2012. **30-31**: p. 116-143.

Shear Band Localization

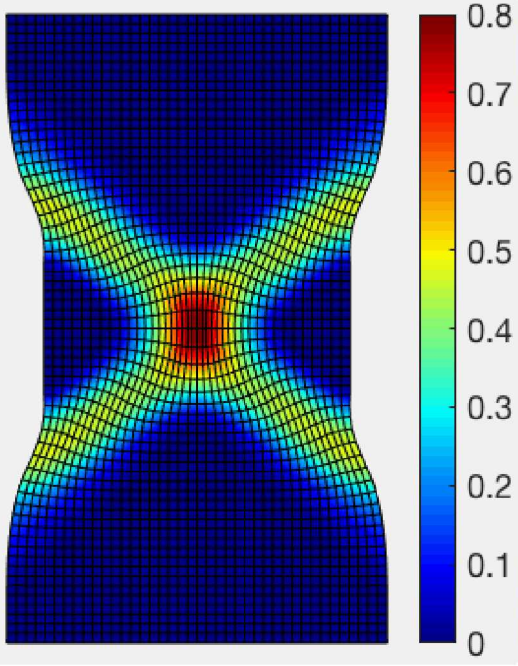
- Localization bands have finite width dictated by micromorphic parameters A,H,Z and hardening/softening modulus

- Inhomogeneous deformation induced by finite deformation kinematics (necking/bulging)

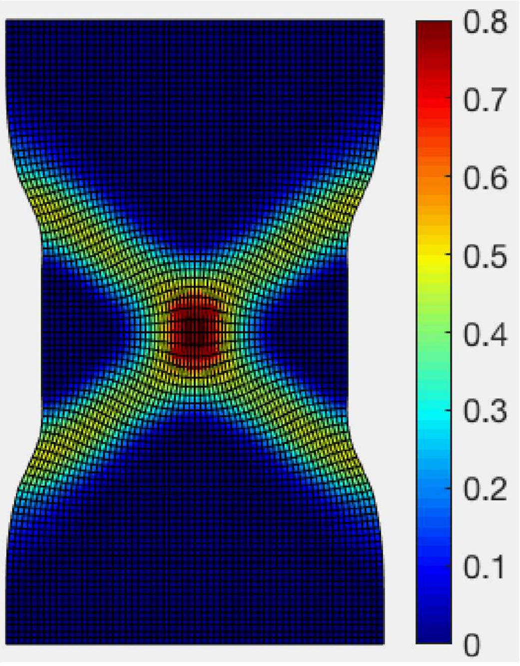
20x30 mesh

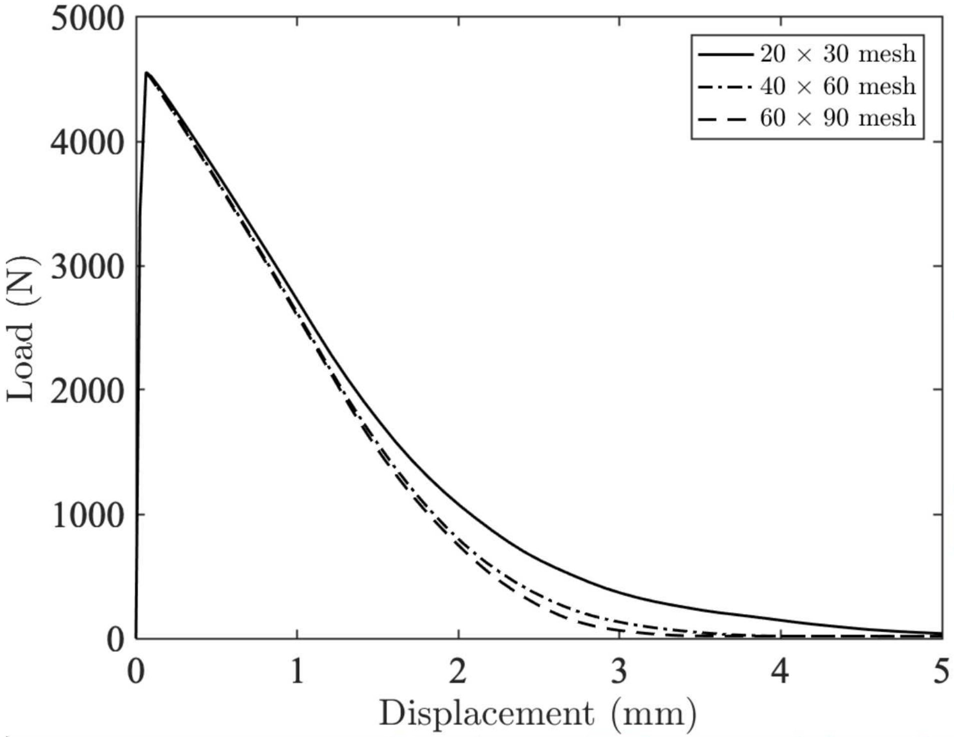


40x60 mesh



60x90 mesh





Topology Optimization for Structural Designs

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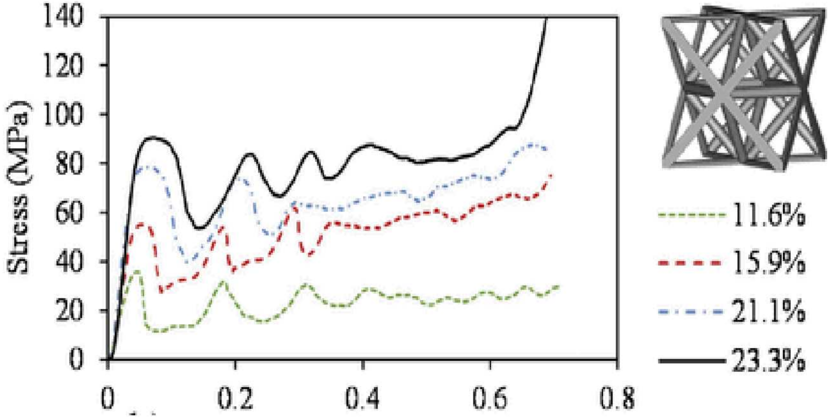
Combining Functionalities of FCC (Energy Dissipation) & BCC (No Softening) Lattices



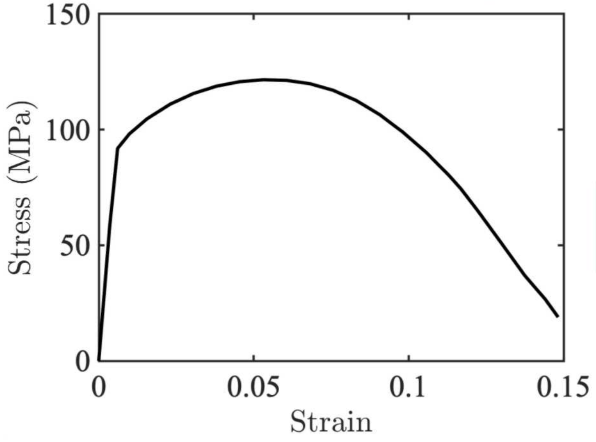
[1]

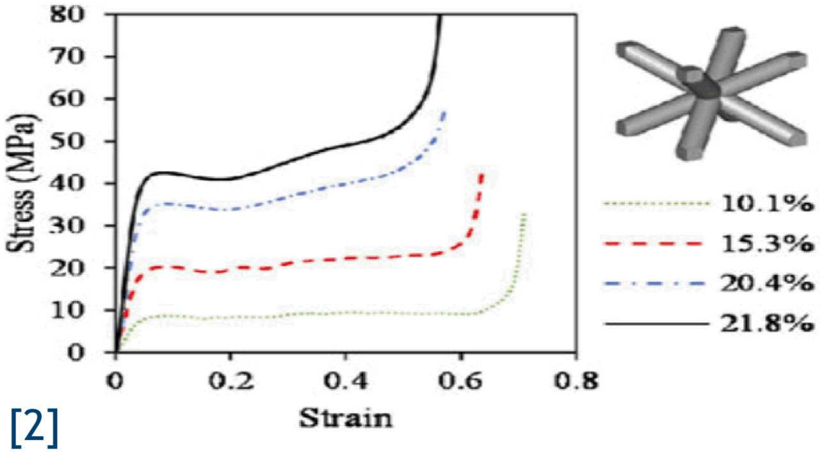
FCC Lattice

- Higher stiffness and yield stress
- Peak stress reached ~5% strain
- Severe softening after peak stress

$E \rightarrow$ 16.55 GPa $\sigma_{max} \rightarrow$ 37.07 MPa

$\sigma_y \rightarrow$ 23.17 MPa $K^h \rightarrow$ -220 MPa





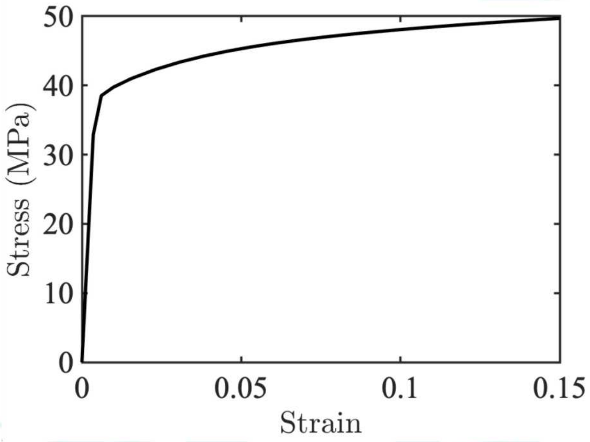
[2]

BCC Lattice

- Lower stiffness and yield stress
- Little to no softening
- Relatively level hardening until densification

$E \rightarrow$ 9.31 GPa $\sigma_{max} \rightarrow$ 11.47 MPa

$\sigma_y \rightarrow$ 9.57 MPa $K^h \rightarrow$ -20 MPa



[1] Al-Ketan, et al. *Topology-mechanical property relationship of 3D printed strut, skeletal, and sheet based periodic metallic cellular materials.* Additive Manufacturing, 2018. **19**: p. 167-183.
[2] Al-Ketan, et al. *The effect of architecture on the mechanical properties of cellular structures based on the IWP minimal surface.* Journal of Materials Research, 2018. **33**: p. 343-359.

The Math Slide: Formulating the Optimization Problem

Density-Based Parameterization

$$0 \leq \rho_e \leq 1$$

BCC Phase $\rho_e = 0$

FCC Phase $\rho_e = 1$

SIMP Interpolation

$$A_e = \rho_e^p A_{FCC} + (1 - \rho_e)^p A_{BCC}$$

$$A \in \{E, \sigma_y, \sigma_{max}, K^h\}$$

Design Problem

$$\min_{\boldsymbol{x}} f_0(\boldsymbol{x}) = -\int_t \int_{\Omega_0} \dot{w}^p \, dv \, dt$$ Maximize Plastic Work

$$\text{s.t.} \quad f_1(\boldsymbol{x}) = 1 - \frac{1}{V} \sum_e^{n_{ele}} \rho_e(\boldsymbol{x}) v_e - V_f \leq 0$$ Volume Fraction Constrain (BCC Phase)

$$f_2(\boldsymbol{x}) = \left[\sum_e^{n_{ele}} \sum_{r=1}^{n_{ipt}} \left(\alpha_{e_r}^N\right)^q \right]^{\frac{1}{q}} - \hat{\alpha} \leq 0$$ Maximum Accumulated Plastic Strain Constraint

$$\boldsymbol{R}^k\left(\hat{\boldsymbol{u}}^k, \hat{\boldsymbol{u}}^{k-1}, \boldsymbol{c}^k, \boldsymbol{c}^{k-1}, \boldsymbol{\rho}(\boldsymbol{x})\right) = \boldsymbol{0}, \qquad k = 1,2, \ldots, N$$ Implicit Global Constraint

$$\boldsymbol{H}^k\left(\hat{\boldsymbol{u}}^k, \hat{\boldsymbol{u}}^{k-1}, \boldsymbol{c}^k, \boldsymbol{c}^{k-1}, \boldsymbol{\rho}(\boldsymbol{x})\right) = \boldsymbol{0}, \qquad k = 1,2, \ldots, N$$ Explicit Global Constraint

$$\boldsymbol{0} \leq \boldsymbol{x} \leq \boldsymbol{1}$$ Box Constraint

Adjoint Sensitivity Analysis: Defining Local and Global Variables and Associated Constraints

Global Variables and Constraints

Scalar plastic microstrain, F-bar elements for incompressibility

$$\hat{\boldsymbol{u}}^k = \begin{bmatrix} \boldsymbol{u}^k \\ \boldsymbol{\chi}^k \end{bmatrix} \qquad \boldsymbol{R}^k = \begin{bmatrix} \boldsymbol{R}_1^k \\ \boldsymbol{R}_2^k \end{bmatrix} = \begin{bmatrix} \overset{n_{ele}}{\underset{e=1}{\mathcal{A}}} \boldsymbol{F}_{int}^{e,u^k} \\ \overset{n_{ele}}{\underset{e=1}{\mathcal{A}}} \boldsymbol{F}_{int}^{e,\chi^k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{F}_{int}^{e,u^k} = \sum_{r=1}^{n_{ipt}} w_r \boldsymbol{B}_{e_r}^{u\ T} {r_{e_r}^k}^{-a} \overline{\boldsymbol{P}}_{e_r}^k \qquad \boldsymbol{F}_{int}^{e,\chi^k} = \sum_{r=1}^{n_{ipt}} w_r \left(\boldsymbol{N}_{e_r}^{\chi\ T} \boldsymbol{N}_{e_r}^{\chi} \boldsymbol{\chi}_e^k - \boldsymbol{N}_{e_r}^{\chi\ T} \alpha_{e_r}^k + \frac{A}{H} \boldsymbol{B}_{e_r}^{\chi\ T} \boldsymbol{B}_{e_r}^{\chi} \boldsymbol{\chi}_e^k \right)$$

Local Variables and Constraints

Finite deformation isotropic elasto-viscoplasticity with micromorphic regularization

$$\boldsymbol{c}^k = \begin{bmatrix} \boldsymbol{c}_1^k & \dots & \boldsymbol{c}_{n_{ele}}^k \end{bmatrix}^T$$

$$\boldsymbol{c}_e^k = \begin{bmatrix} \boldsymbol{c}_{e_1}^k & \dots & \boldsymbol{c}_{e_{n_{ipt}}}^k \end{bmatrix}^T$$

$$\boldsymbol{c}_{e_r}^k = \begin{bmatrix} \boldsymbol{b}_{e_r}^{e^k} & \alpha_{e_r}^k & \Delta\gamma_{e_r}^k \end{bmatrix}^T$$

$$\boldsymbol{H}_{e_r}^k = \begin{cases} \boldsymbol{h}_{e_{r1}}^k = \boldsymbol{b}_{e_r}^{e^k} - \boldsymbol{b}_{e_r}^{e,tr}.\exp\left[-2\Delta t \boldsymbol{A}_{e_r}^k\right] = \boldsymbol{0} \\ h_{e_{r2}}^k = \alpha_{e_r}^k - \alpha_{e_r}^{k-1} - \Delta\gamma_{e_r}^k \\ h_{e_{r3}}^k = \sqrt{\frac{3}{2}} \left\| \boldsymbol{s}_{e_r}^k \right\| \left(\frac{\Delta t}{\mu \Delta\gamma_{e_r}^k + \Delta t} \right)^{\vartheta} - \zeta\left(\alpha_{e_r}^k, \boldsymbol{\chi}_e^k\right) \end{cases}$$

[1] Alberdi, R., et al., *A unified framework for nonlinear path-dependent sensitivity analysis in topology optimization.* International Journal for Numerical Methods in Engineering, 2018. **115**: p. 1-56.

Examples for Shear Band Localization and Dual Lattice Optimization

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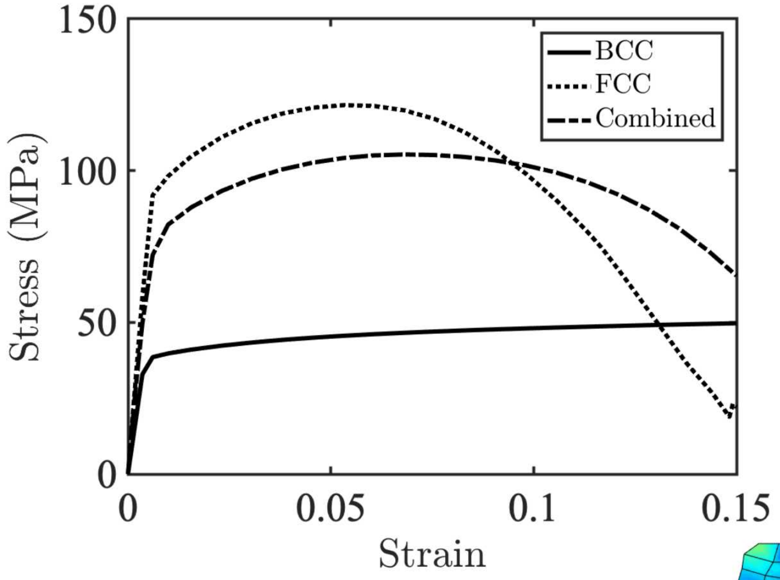
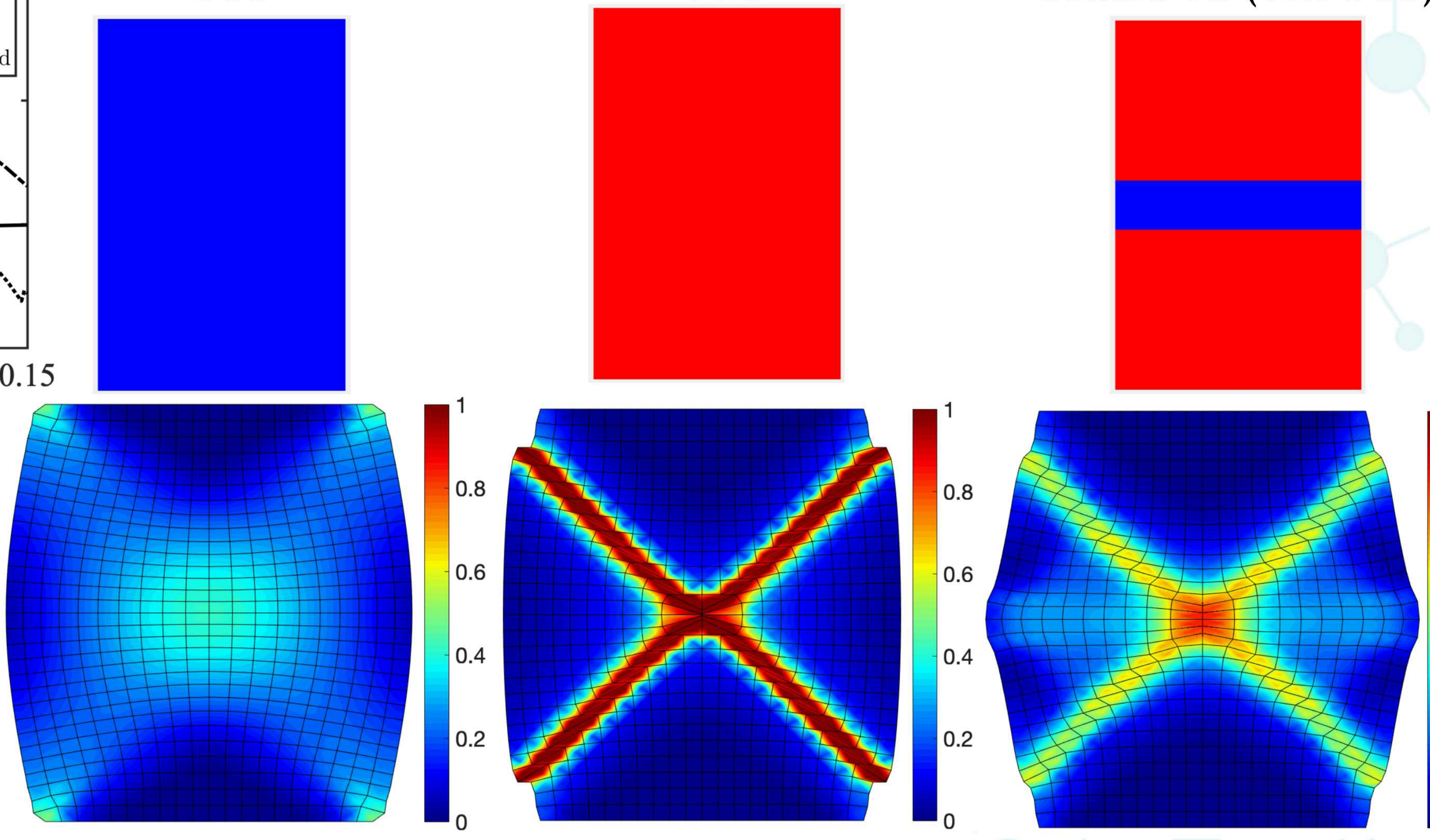
Naïve Combination of Lattices Leads to Improvement in Energy Dissipation



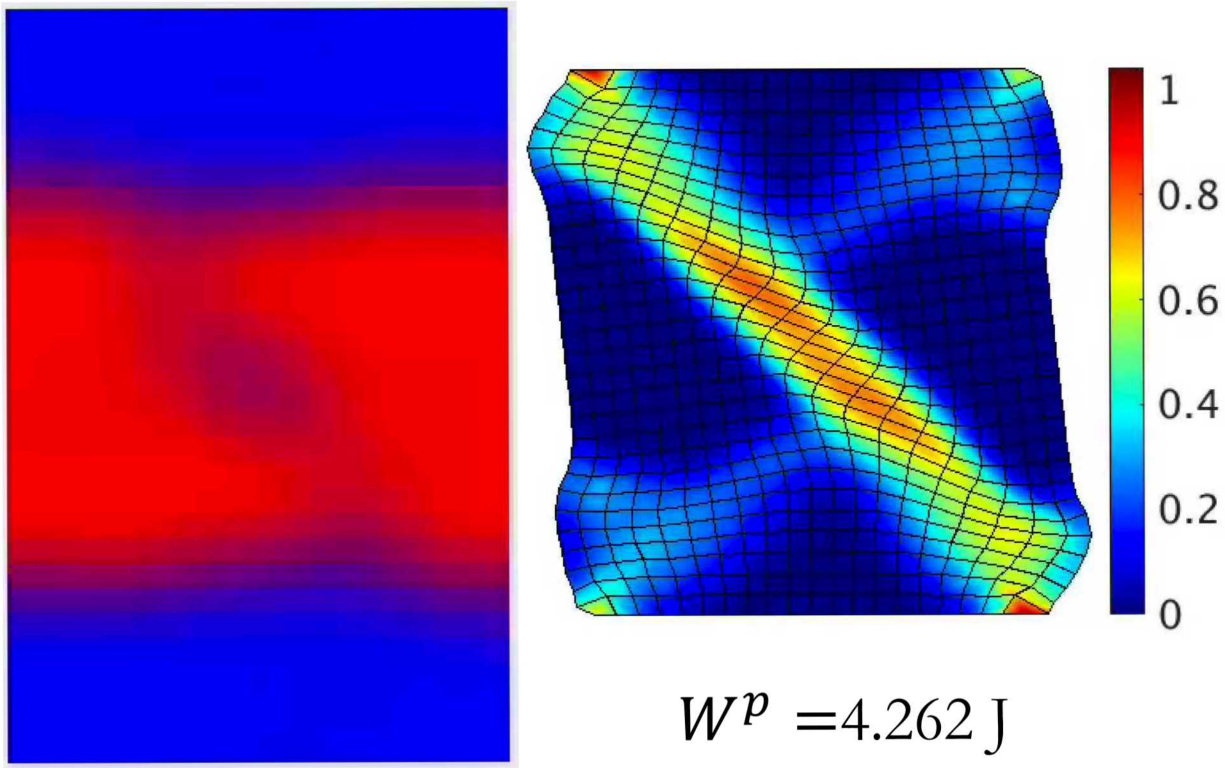
BCC FCC Combined (13% BCC)



Dissipated energy: 1.043 J 8.109 J 8.218 J

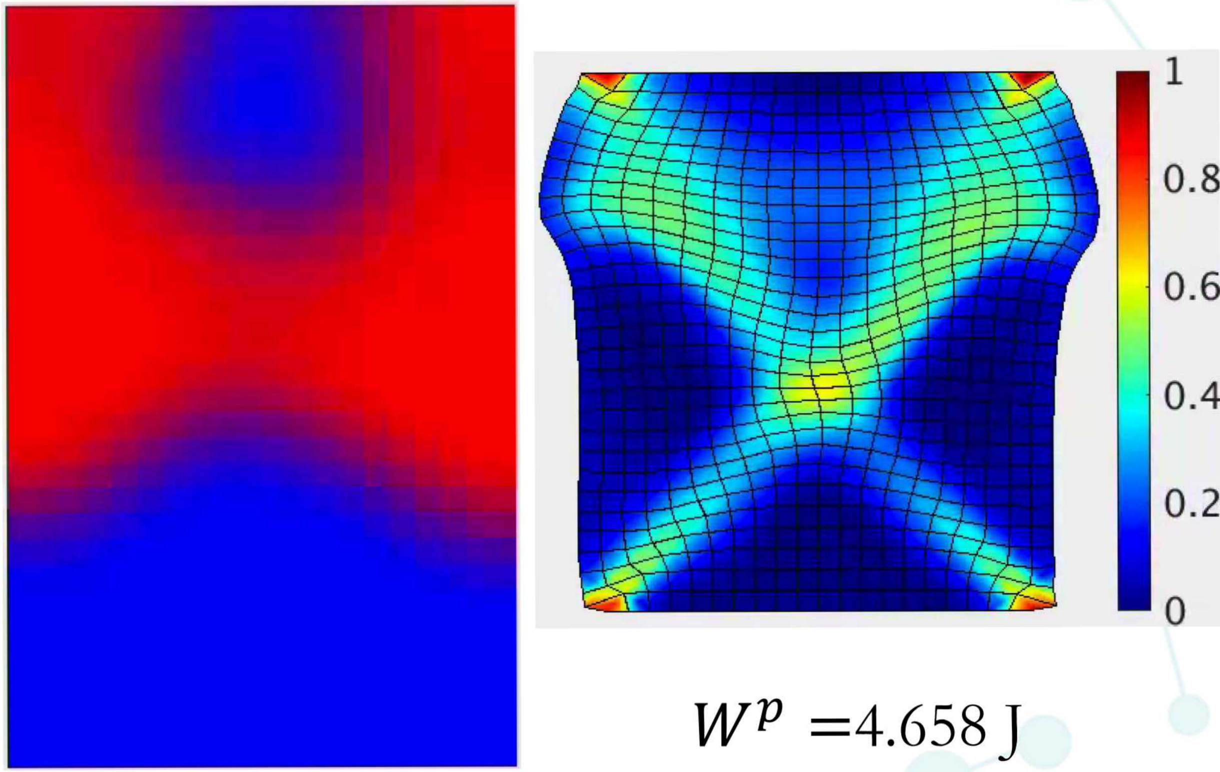
Preliminary Optimization Results

$V_f = 0.5$



W^p =4.262 J

$V_f = 0.9$



W^p =4.658 J

- Both designs experience less severe localization than FCC alone
- Problem is highly sensitive to changes in topology

Conclusions and Future Perspectives

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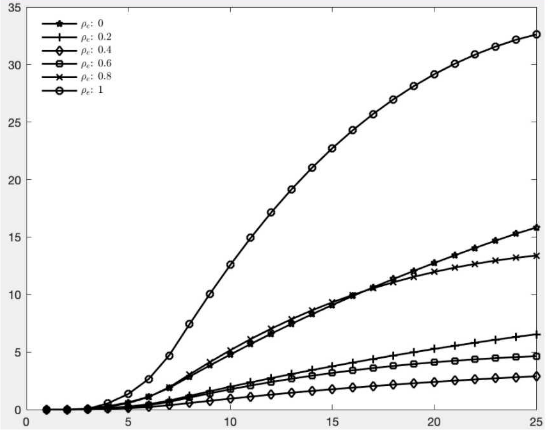
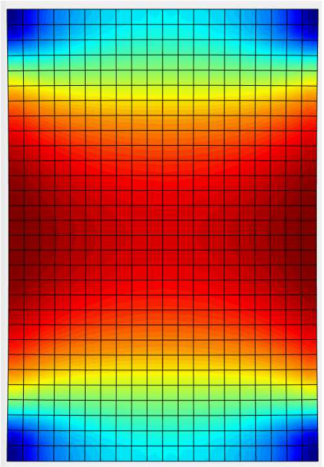
Future Work

Optimization

- Better parameterization for intermediate density values
- More appropriate objective functions/constraints



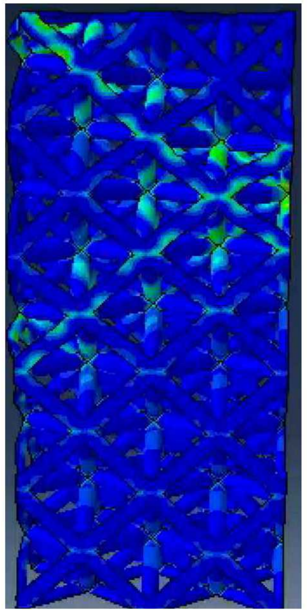


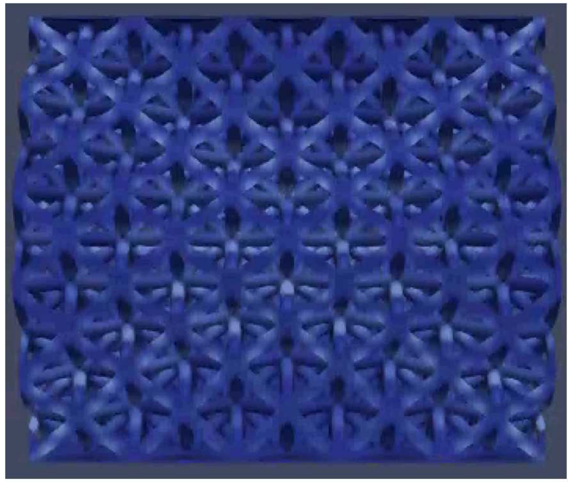
Model Extensions

- Pressure dependent plasticity models
- Full microstrain tensor

Calibration through DNS

$$f(\boldsymbol{\phi}) = F(\hat{\boldsymbol{u}}^1(\boldsymbol{\phi}), \ldots, \hat{\boldsymbol{u}}^n(\boldsymbol{\phi})) = \sum_{k=1}^{n} \sum_{i=1}^{n_n} \|\bar{\boldsymbol{d}}_i^k - \boldsymbol{d}_i^k\|^2$$





Transient Analysis

- Account for dispersive effects
- Coupling with plasticity