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# Developing Inverse Capabilities to Support Computational Modeling of Complex Applications

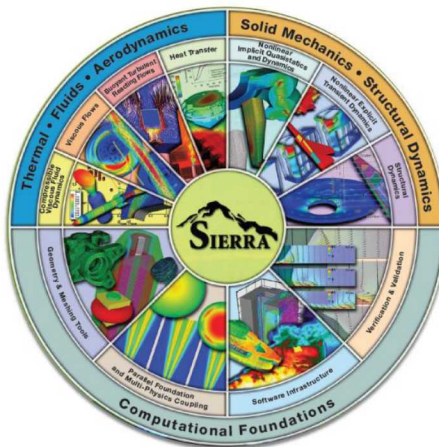
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# Acknowledgments

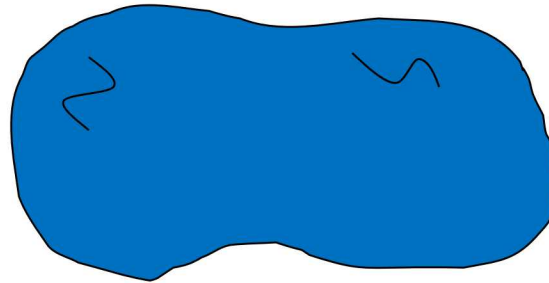
- Sierra Teams
- Denis Ridzal, Drew Kouri, Bart van Bloemen Waanders (1441), Rapid Optimization Library (ROL)
- Wilkins Aquino, Clay Sanders, Duke University



# Inverse Problems: Observing the Unobservable

Suppose we have a “black box” system in the *as-manufactured* state that has only partially known parameters

Question: can we *non-destructively* interrogate the system to “see what is inside”?



Typical quantities of interest:

- Material properties
- Loads
- Boundary conditions
- Residual stresses
- Size/shape/location of inclusions (e.g. composite materials)

Example applications:

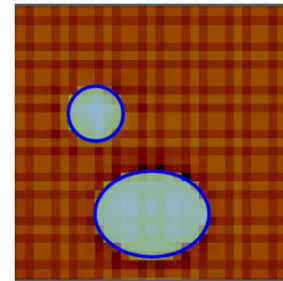
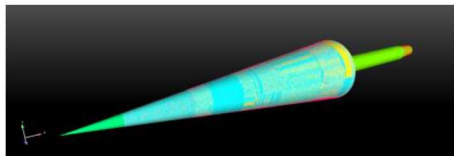
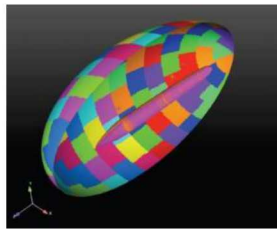
- Seismic imaging
- Medical imaging
- Non-destructive evaluation

# Categories of Inverse Problems

- Imaging
  - Medical ultrasound
  - Seismic exploration
- Calibration of material models
  - Structural material properties, circuits, thermal properties, etc.
- Force reconstruction
  - Sub-structuring for mechanical testing of components
- Optimal Experimental Design
  - Best placement of sensors, test fixture setups
- Shape reconstruction
  - E.g. inverse scattering

# Research Challenges in Inverse Problems: Sandia Applications

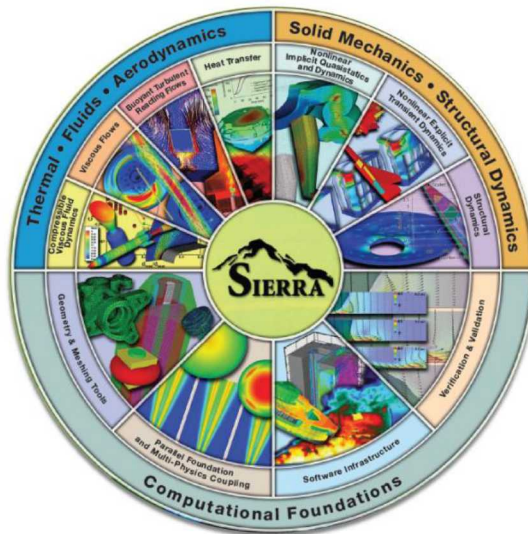
- 3D models required since measurements at several locations
- Large parameter spaces/high-dimensionality of inverse problem
  - Spatially-varying material parameters
  - Temporally-varying functions (boundary conditions, loads, etc)
- Rolling uncertainty quantification into the inverse problem: 2 approaches
  - Stochastic optimization
  - Bayesian methods





# Inverse Problem Strategy in Sierra

Finite Element and Optimization Codes operate as independent entities



Objective function,  
derivative operators



Next iterate of  
design variables



RAPID OPTIMIZATION LIBRARY

Sierra Mechanics – massively parallel  
multiphysics simulation

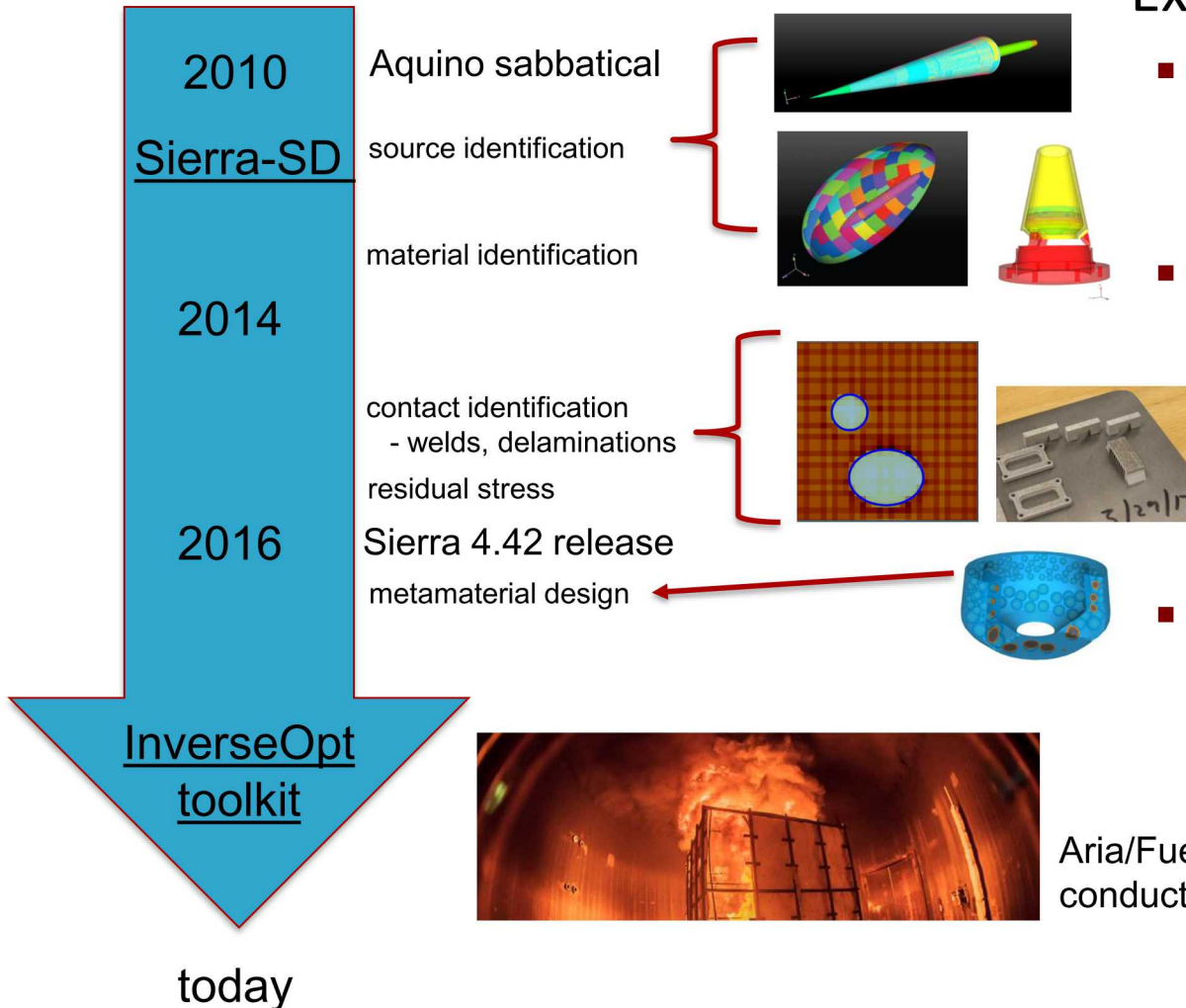
Gradient-based optimization  
(adjoint methods used to compute  
gradients)

# Inverse Capabilities in Sierra

Enabling adjoint-based inversion capabilities across Sierra Mechanics

## ■ Example applications

- Force/material/contact area reconstruction (structural, acoustic, and thermal)
- Viscoelastic material parameter identification
- Thermal flux, conductivity distributions (under development)
- Additive Manufacturing: residual stress, acoustic metamaterials



# Use Cases for Inverse Problems

Adjoint-based inversion enables new use cases for Sierra Apps



Experiment

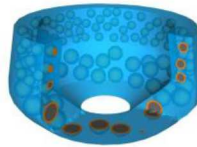


InverseOpt  
Toolkit

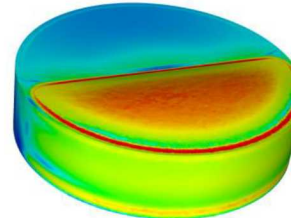
Enables use cases



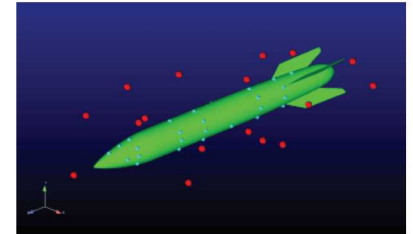
Flux boundary conditions



Thermal/mechanical material  
reconstruction, residual stress,  
metamaterial design



Delamination/weld  
characterization



Ground-based  
acoustic/thermal tests

Source  
Reconstruction

Material/residual  
stress  
Reconstruction

Contact surface  
Reconstruction

Design of  
Experiments

Sierra-SD, Sierra-SM, Sierra-TF  
with embedded sensitivities (adjoints, etc)

**Goal:** enable all Sierra apps to reconstruct forces, materials, contact surfaces, and assist in designing experiments

**Uniqueness:** CompSim-enabled inverse optimization that provides capabilities for the above use cases



# PDE-Constrained Optimization Formulation

Abstract  
optimization  
formulation

$$\underset{\mathbf{u}, \mathbf{p}}{\text{minimize}} \quad J(\mathbf{u}, \mathbf{p})$$

$$\text{subject to} \quad \mathbf{g}(\mathbf{u}, \mathbf{p}) = \mathbf{0}$$

$$\mathcal{L}(\mathbf{u}, \mathbf{p}, \mathbf{w}) := J + \mathbf{w}^T \mathbf{g}$$

Objective function

PDE constraint

Lagrangian

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + \mathbf{g}_u^T \mathbf{w} \\ J_p + \mathbf{g}_p^T \mathbf{w} \\ \mathbf{g} \end{Bmatrix} = \{\mathbf{0}\}$$

First order optimality  
conditions

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & \mathbf{g}_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & \mathbf{g}_p^T \\ \mathbf{g}_u & \mathbf{g}_p & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \delta \mathbf{u} \\ \delta \mathbf{p} \\ \mathbf{w}^* \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ \mathbf{g} \end{Bmatrix}$$

Newton iteration

$$\mathbf{W} \Delta \mathbf{p} = -\hat{J}',$$

$$\mathbf{W} = \mathbf{g}_p^T \mathbf{g}_u^{-T} (\mathcal{L}_{uu} \mathbf{g}_u^{-1} \mathbf{g}_p - \mathcal{L}_{up}) - \mathcal{L}_{pu} \mathbf{g}_u^{-1} \mathbf{g}_p + \mathcal{L}_{pp}$$

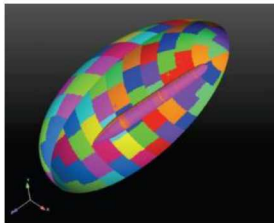
Hessian calculation

# Example: Source Reconstruction

- Goal: reconstruct structural, thermal, and/or acoustic energy sources that produce the given accelerometer/temperature/microphone measurements
- Large parameter space – time histories for pressure functions
- Sensor placement – design of experiments

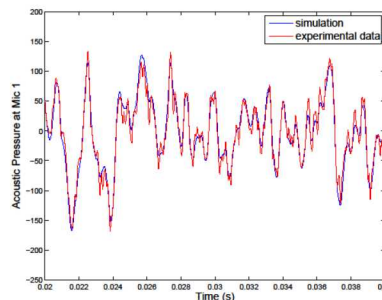
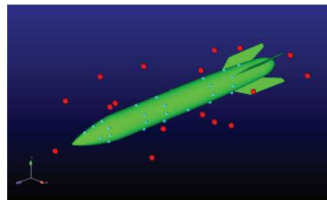
## Structural loads

Attachment forces from accelerometer measurements



## Acoustic loads

- Pressure distributions from microphone measurements



## Thermal flux loads

Flux distributions from temperature measurements

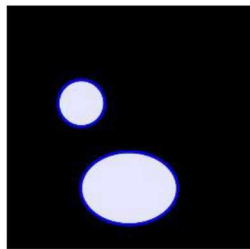


Fire flux boundary conditions

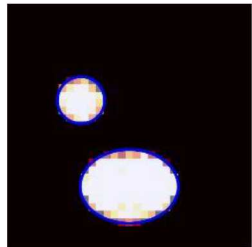
# Example: Partially Connected Surfaces

- Partially-bonded plates/cylinders – can we invert for the bonded/debonded regions?
- Large parameter space – number of FEM modes on surfaces

Frequency-domain pressure load at 2000Hz



Target debonded regions



Detected debonded regions  
(Thresholded plot)

Partially bonded  
plates



Target Debonded Region

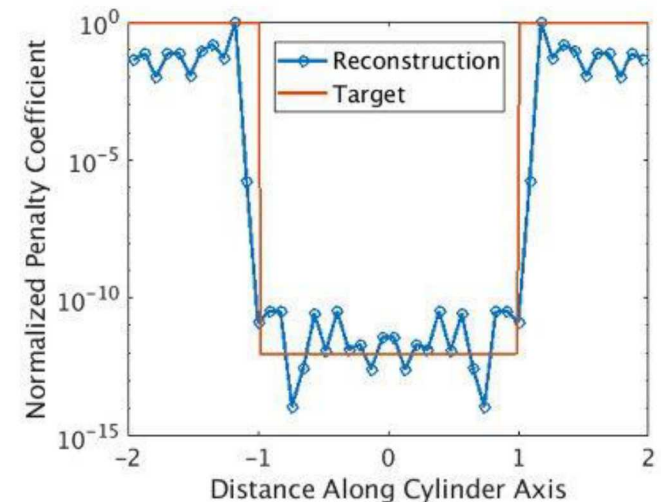


Reconstructed Density Field ( $\beta_h$ )



Detected Debonded Region (Thresholded plot)

Partially bonded  
cylinders



Line plot through  
delaminated area

# Example: Material Parameter Extraction

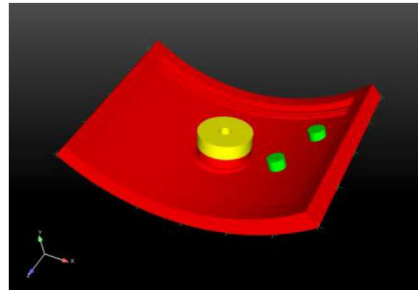


Problem: in-situ material parameters often unknown

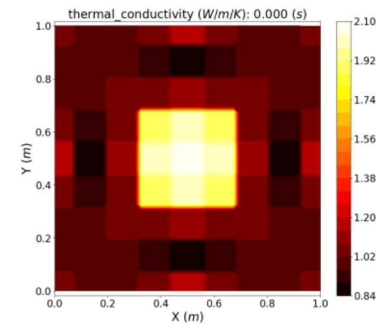
- Parameters not measurable without destroying structure
- Interrogate material with mechanical/thermal inputs
- Measure response, infer missing in-situ properties
- Large parameter space – spatially-varying parameters



Viscoelastic material and joint stiffness extraction using Sierra-SD



Orthotropic material extraction for composite panel using Sierra-SD



Reconstructed thermal conductivity from Sierra-TF (Fuego)



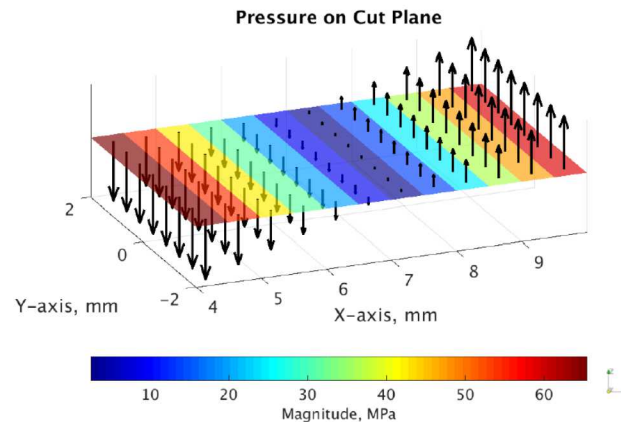
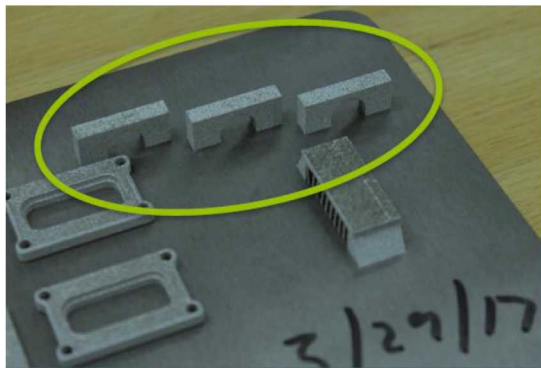
# Example: Residual Stress Extraction



Problem: additively manufactured parts suffer from large residual stresses

- Compromises part integrity
- FEM modeling needs stresses for initial conditions
- Stress is not a measurable quantity
- Large parameter space – spatially-varying stress fields

Goal: estimate residual stress fields from measured displacement data (digital image correlation)



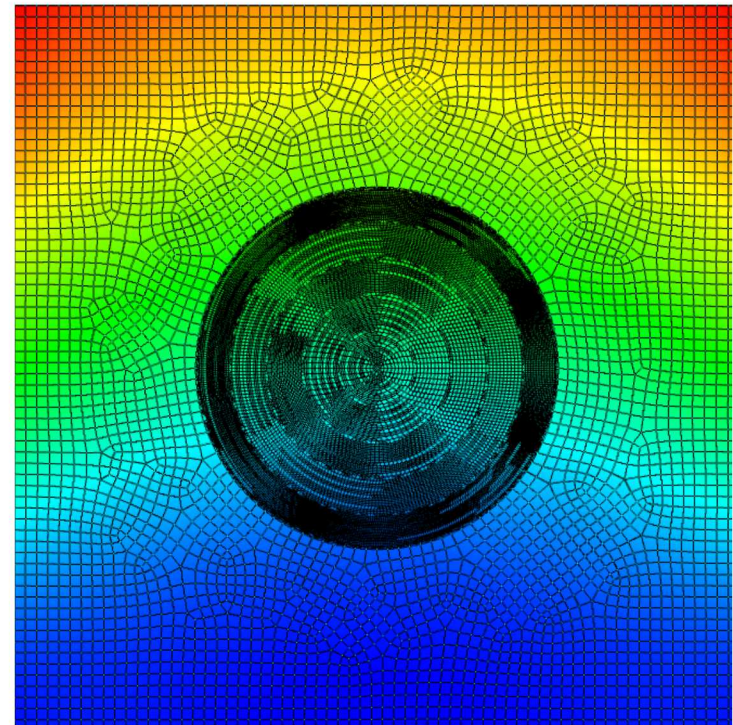
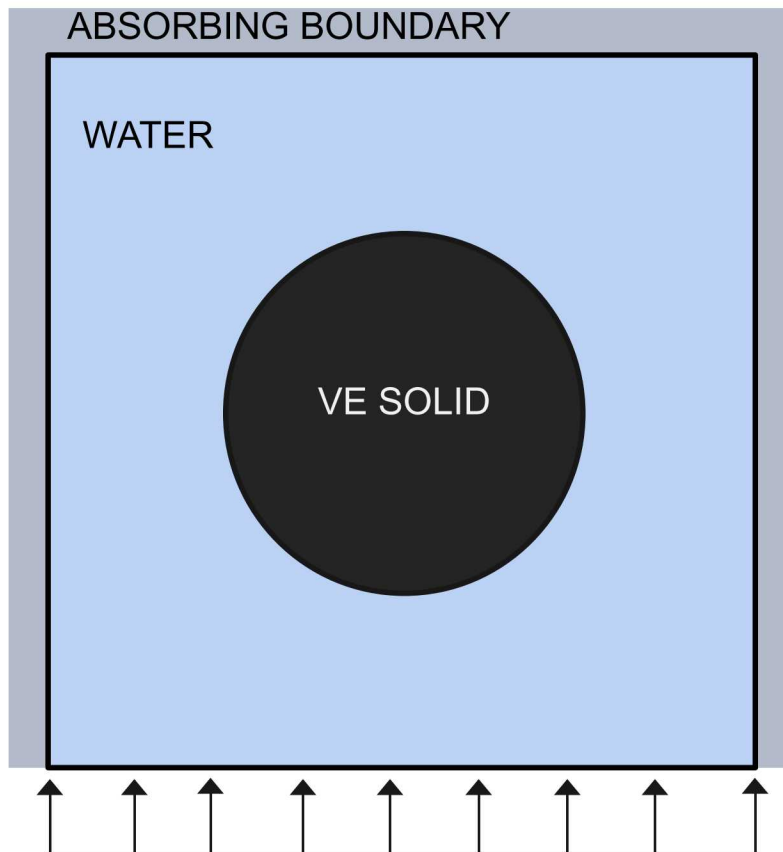
Predicted traction field from  
inverse solution (Sierra-SD)



Reconstructed stress  
field from  
inverse solution  
(Sierra-SD)

# Inverse Problems: *Acoustic Cloaking*

- 2-D fluid region with circular VE solid inclusion
- Inclusion consists of concentric rings w/ distinct material properties
- Periodic acoustic load applied to end
- Match forward problem pressure distribution by adjusting **VE material parameters**



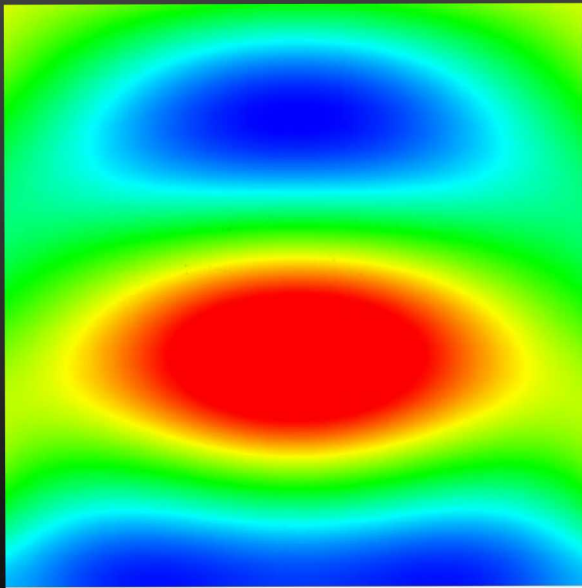
**Left:** Model Set up

**Right:** Forward problem pressure distribution (500 Hz loading) in model with 50 layers

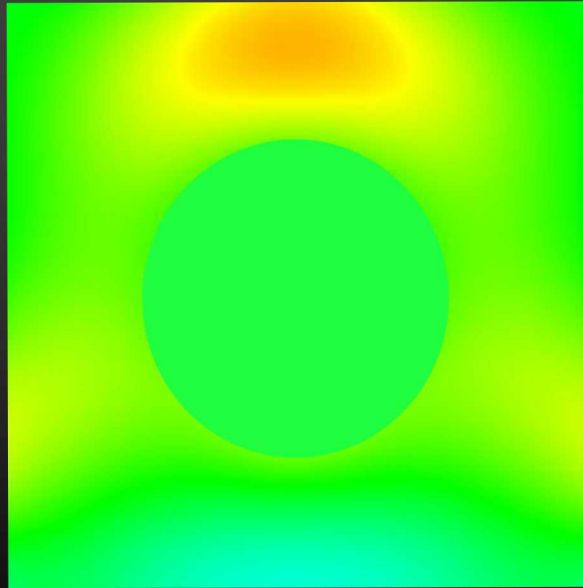
# Acoustic Cloaking

- Optimized VE foams allow recovery of desired pressure distribution

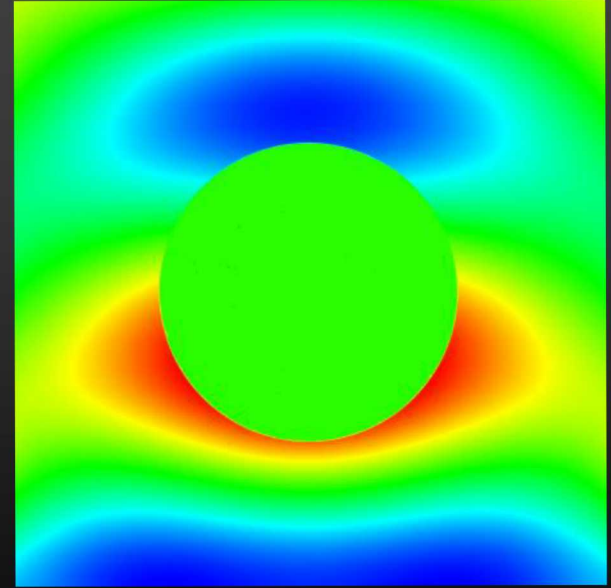
*Forward*



*Initial Guess*



*Optimized*



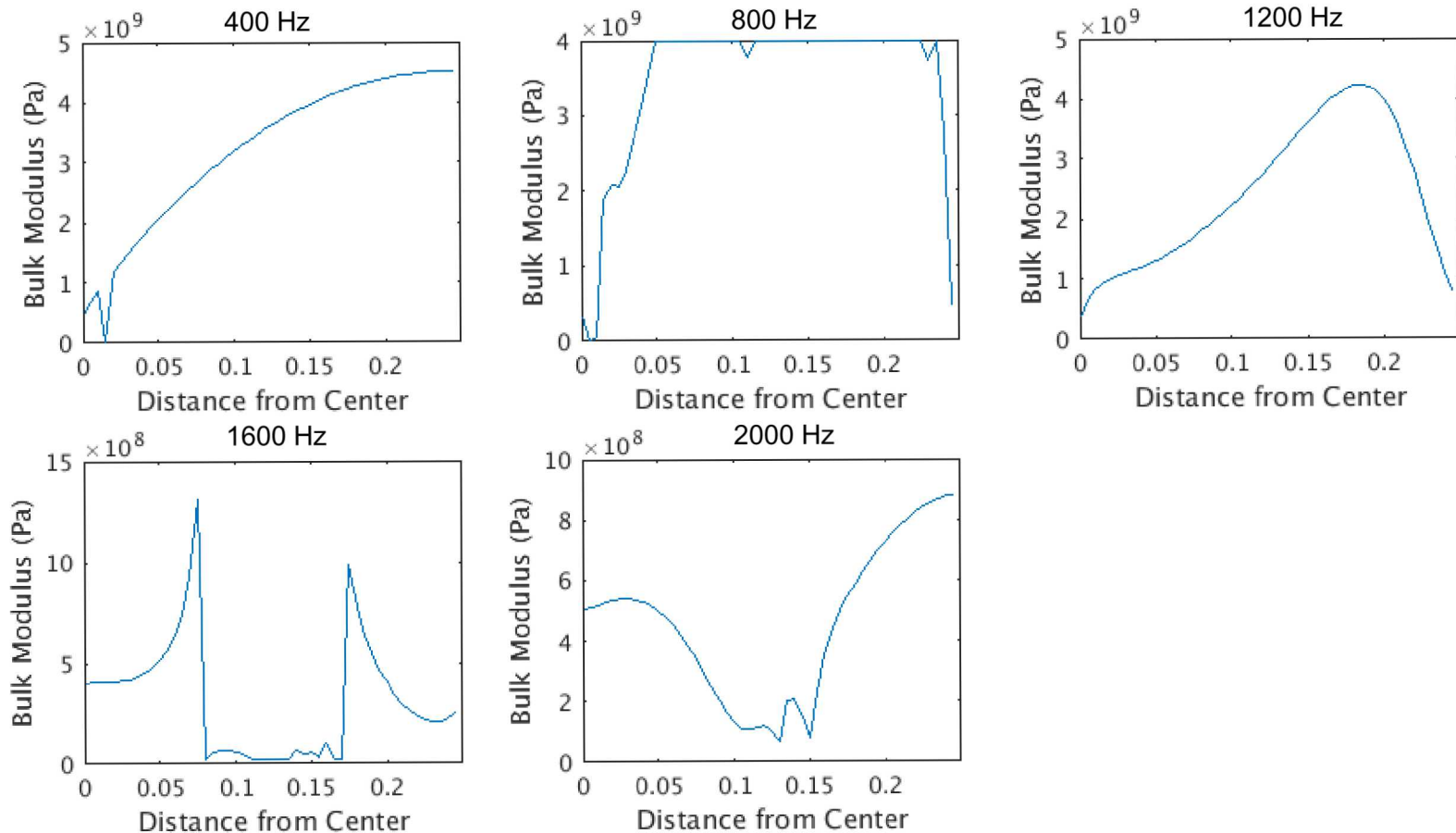
**Left:** Target acoustic pressure distribution, from forward problem

**Center:** Acoustic pressure distribution with initial material guess (2000 Hz Loading)

**Right:** Pressure distribution after convergence to optimized design

# Acoustic Cloaking Results: Bulk Modulus

Bulk modulus sensitive to frequency, and varies nontrivially along disk radius



**Figures:** Real component of bulk modulus along radius, for various frequency



# Conclusions

- Adjoint-based optimization enables inversions with **large parameter spaces** and/or **high dimensionality** of interest to Sandia
- Leveraging Sandia software components
  - Sierra Mechanics for massively parallel multiphysics forward simulations
  - Rapid Optimization Library (ROL) for gradient-based optimization
- Application spaces at Sandia are broad and continuing to grow