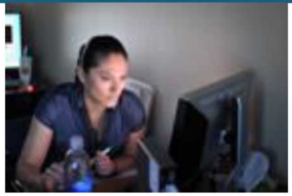
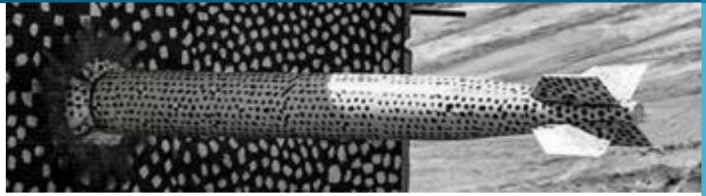
Sandia National Laboratories

SAND2019-6232C

# Calculation of Parametric Variance using Variance Deconvolution





PRESENTED BY

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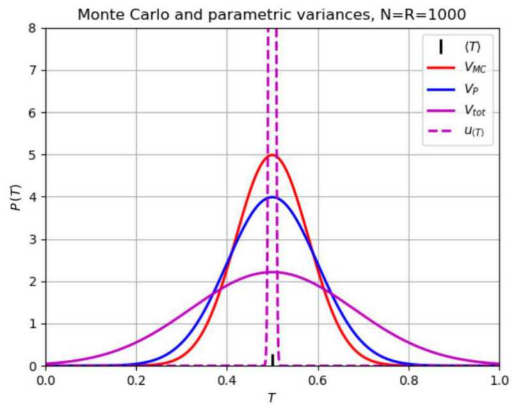
# Introduction

# Overview

Big idea:

- Can compute parametric variance by computing total variance and Monte Carlo variance and taking the difference:

$$V_P = V_{tot} - V_{MC}$$




Two implementations of idea:

- VAriance DEconvolution (VADE) – non-intrusive
- Embedded VAriance DEconvolution (EVADE) – intrusive

Why Care:

- Often want $V_P$, but with Monte Carlo transport solver $V_{MC}$ gets mixed in, need a way to separate, especially for small $N$
- Computational efficiency gains
- Groundwork for stochastic media model yielding variance caused by mixing

# Mean, Variance, and Uncertainty

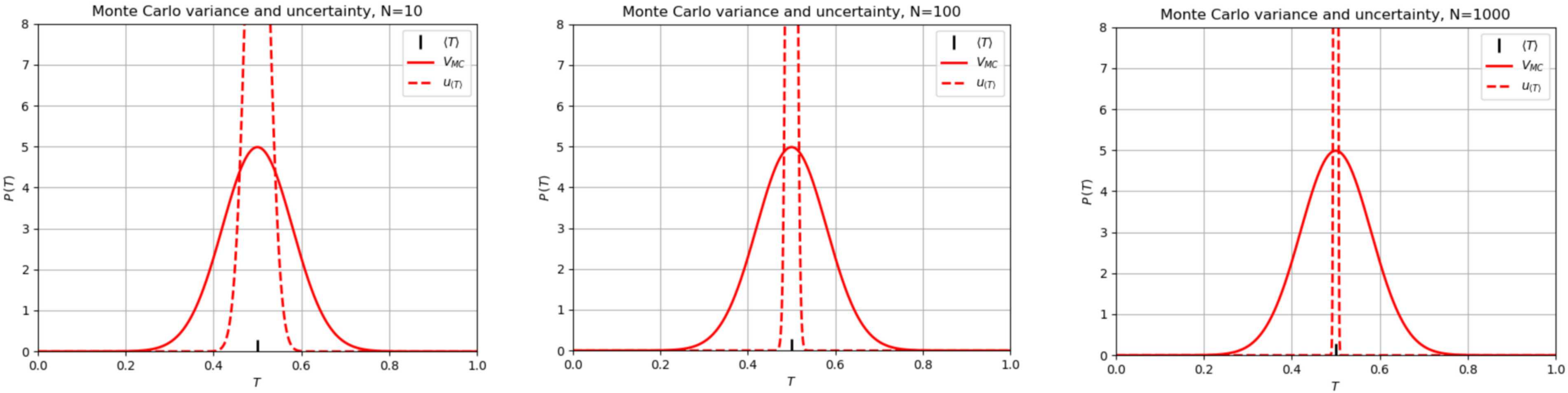
Monte Carlo transport is a stochastic process that yields variance even if problem is deterministic

- "Monte Carlo variance": $V_{MC}$ - constant for problem/solver combo ("variance reduction" reduces this)
- "Monte Carlo uncertainty": $u_{MC}$ - decreases with more samples (histories)

"Monte Carlo statistical uncertainty"
"Standard deviation of the mean"
"Standard error of the mean"

$$u_{MC} = \frac{\sqrt{V_{MC}}}{\sqrt{N}}$$

"Monte Carlo variance"
"Variance of the distribution"
"Leading coefficient of convergence"




Figures demonstrate that $u_{MC}$ shrinks with more histories (10, 100, 1000) whereas $V_{MC}$ is constant for a problem/solver combination

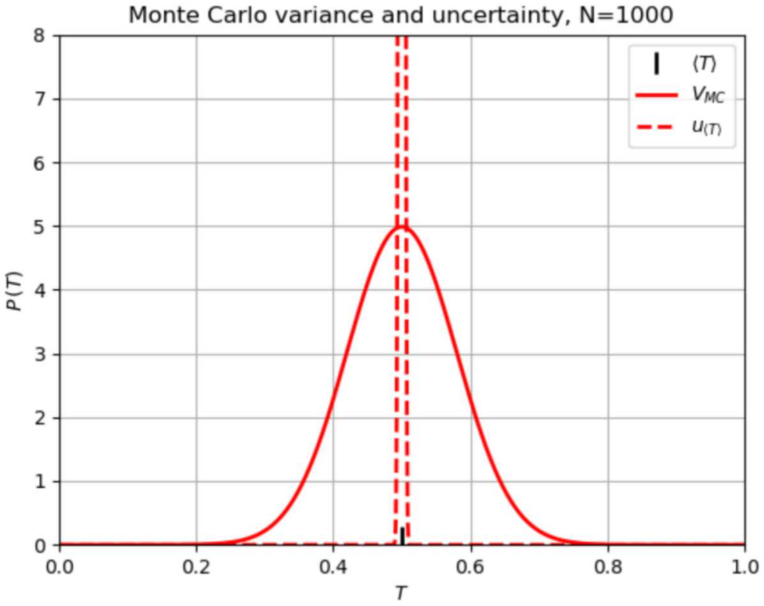
# Monte Carlo, Parametric, and Total Variances

Monte Carlo can be used as transport solver or to resolve the effects of random parameters

- "MC" – Monte Carlo transport solver, "N" – number of histories per realization
- "P" – randomness described by input parameters, "R" – number of randomly sampled realizations

If no parametric randomness

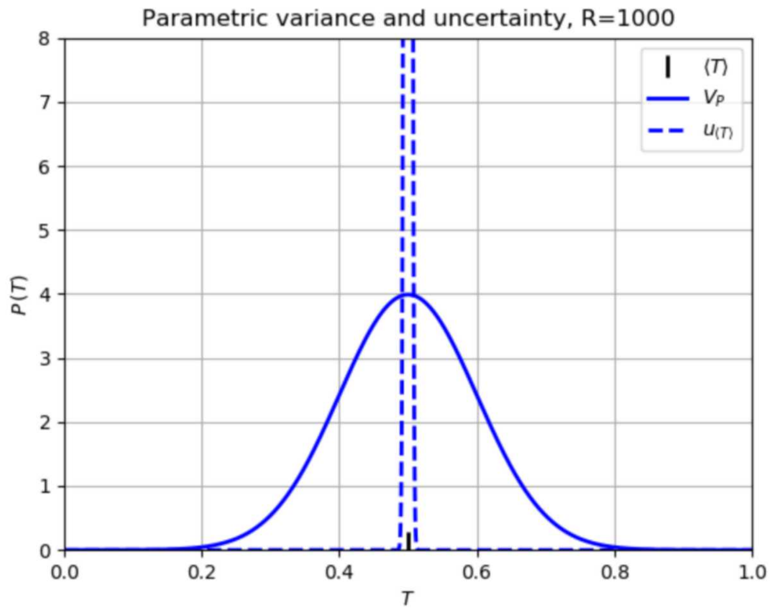
$$u_{MC} = \frac{\sqrt{V_{MC}}}{\sqrt{N}}$$




Monte Carlo variance and uncertainty

If no solver uncertainty

$$u_P = \frac{\sqrt{V_P}}{\sqrt{R}}$$

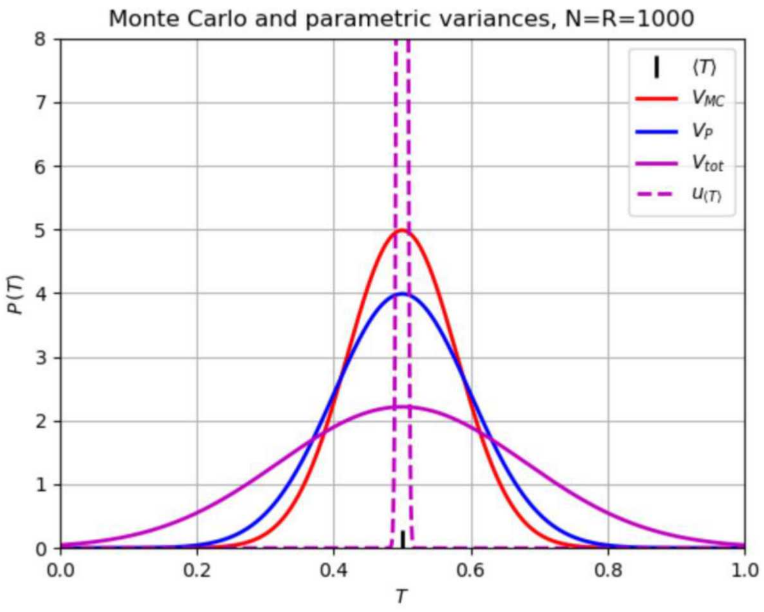



Parametric variance and uncertainty

If parametric randomness and solver uncertainty

$$u_{tot}^2 = u_{MC}^2 + u_P^2$$

$$V_{tot} = V_{MC} + V_P$$

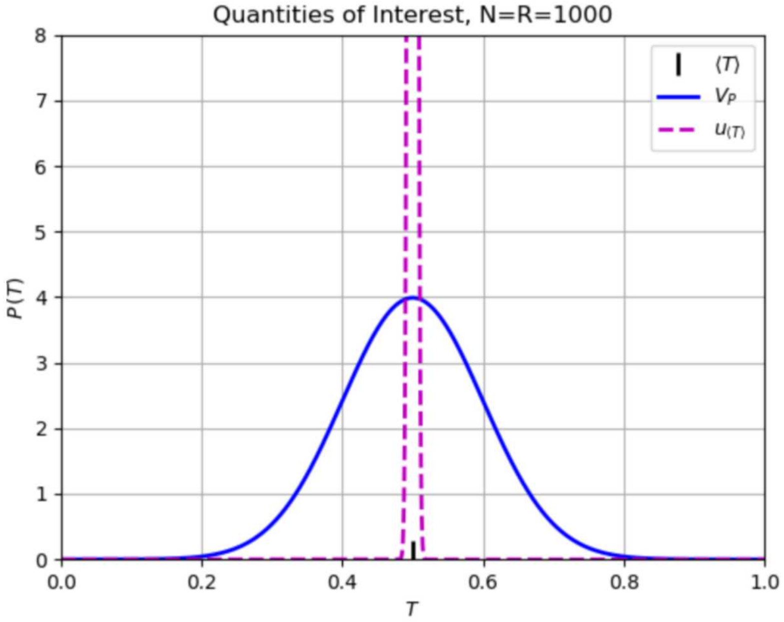



Total variance and uncertainty

# Deconvolution of Variances for Parametric Variance

What we really want:

- $\langle T \rangle \pm u_{tot}$
- $V_P \pm u_{V_P}$




Common quantities of interest

Main idea:

- Solve for $V_{tot}$ and $V_{MC}$ separately and deconvolve to get $V_P$ ($V_P = V_{tot} - V_{MC}$)

Supporting ideas:

- Use batches to get $u_{tot}$ and $u_{V_P}$
- Expand analytic/semi-analytic benchmarks for test problems
- N=1 convenient path to solving $V_{tot}$ (naturally convolves variances)
- N=2 yields accurate estimates and is efficient

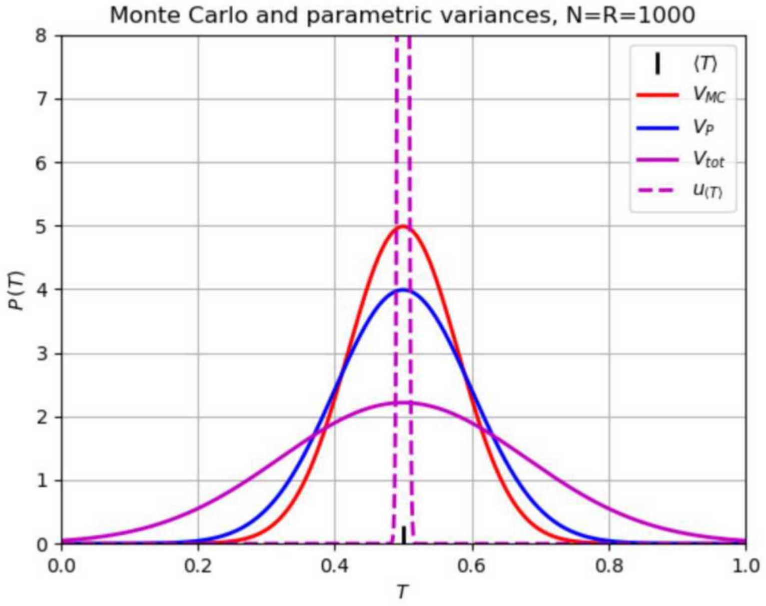



Diagram of $V_P$, $V_{MC}$, $V_{tot}$, and $u_{tot}$

# Related Work (Stochastic Media Applications)

Adams et. al, JQSRT, 1989 (**deterministic solver**)

- Computed $u_{tot}$ ($u_{tot} = u_P$ since deterministic solver)
- Computed $V_P$ ($V_{tot} = V_P$ since deterministic solver)
- Crude estimate for $u_{V_P}$

Donovan et. al, NSE, 2003 (MC solver, $\boldsymbol{N} = \mathbf{1}$)

- Computed $V_{tot}$ and $u_{tot}$ (straightforward since $N = 1$)

Donovan et. al, M&C, 2003 (MC solver, $\boldsymbol{N} \gg \mathbf{1}$)

- Computed $u_{tot}$ using $u_{tot}^2 = u_{MC}^2 + u_P^2$
- To estimate $u_P$, **assumed** $V_P \gg u_{MC}$ (good if $N \gg 1$)
- Computed, **but didn't report** values for $V_{MC}$ and $V_P$

Larmier et. al, JQSRT, 2018 (MC solver)

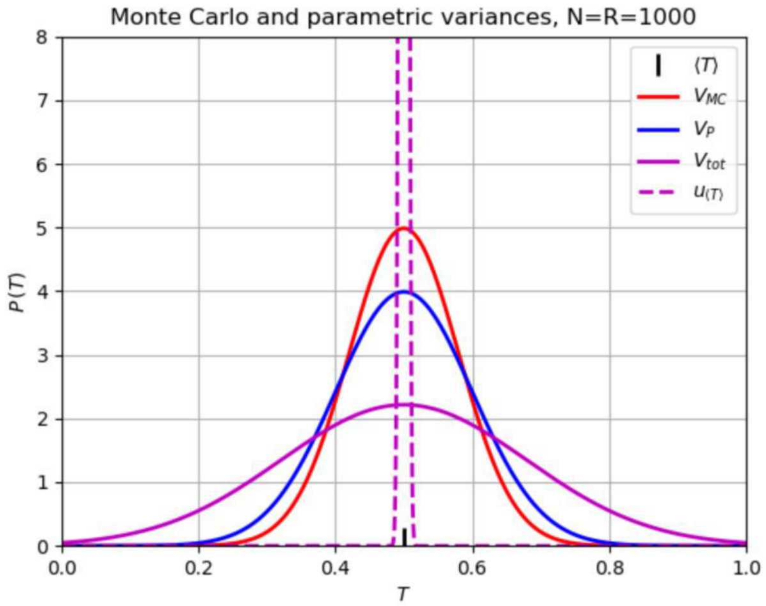
- Approaches same as Donovan's

What we want:

- $\langle T \rangle \pm u_{tot}$
- $V_P \pm u_{V_P}$

New:

- Compute $V_P$ w/ MC, no assumption, small $N$
- Rigorously compute $u_{tot}$ and $u_{V_P}$



# Role of this Work

Goal: Improved methods for uncertainty quantification and stochastic media with Monte Carlo transport

| | Acronyms | Random Variables (r.v.)/ Stochastic Media (s.m.) | | Takeaway(s) |
|---|---|---|---|---|
| Olson, ANS, 2017 | | r.v. | (s.m.) | • Characterize MC/parametric convergence<br>• Analytic benchmark |
| Olson, ANS, 2018 | Optimal-Cost Monte Carlo (OCMC) | (r.v.) | s.m. | • Optimize MC convergence for mean |
| | | | | |
| Vu, ANS, 2019 | Conditional Point Sampling (CoPS) | | s.m. | • New algorithm accurate in 1D |
| **Olson, ANS, 2019** | **Embedded Variance Deconvolution (EVADE)** | **r.v.** | **(s.m.)** | • **Method for computing parametric variance**<br>• **Expand benchmarks** |
| | | | | |
| Vu, M&C, 2019 | | | s.m. | • CoPS can use EVADE to compute parametric variance |
| Olson, M&C, 2019 | | | s.m. | • CoPS accurate for mean in multi-D |

# Variance Deconvolution Implementations

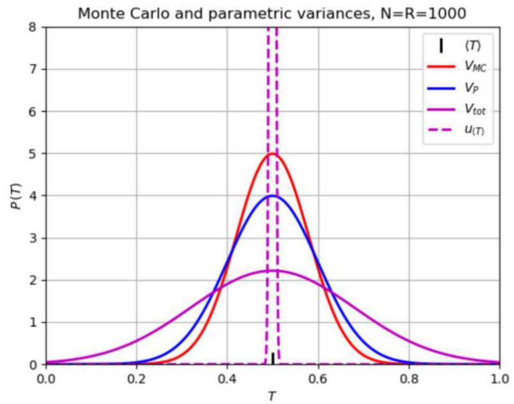
# Computing Total Variance

Use (or only tally) one history per realization ($N = 1$)

This convolves the effect of random variables and the Monte Carlo transport process

Compute (sample) total variance on results:

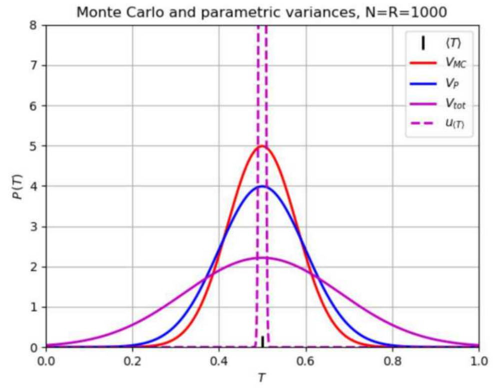
$$V_{tot} \approx \frac{R_{tot}}{R_{tot} - 1}\left(\langle T^2 \rangle - \langle T \rangle^2\right)$$




Mathematical justification:

- Propagation of parametric and Monte Carlo uncertainties on mean: $u_{tot}^2 = u_p^2 + u_{MC}^2$
- Relationship of parametric and Monte Carlo variances: $V_{tot} = V_p + V_{MC}$
- Combine to derive $u_{tot}^2 = \left(V_{tot} - \left(1 - \frac{1}{N}\right) V_{MC}\right)/R$
- When one history per realization ($N = 1$), $u_{tot}^2 = V_{tot}/R$
- Therefore $V_{tot}$ is leading coefficient of uncertainty estimate that can be computed with moments of $N = 1$ transport computations

# Computing Monte Carlo Variance



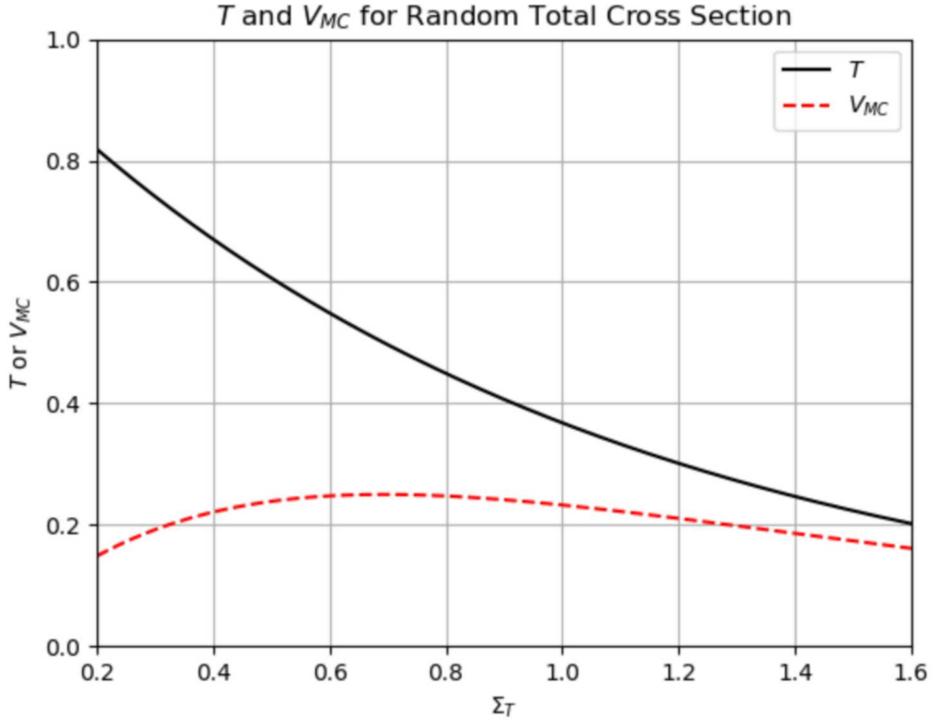

Monte Carlo variance for a realization $r$ computed

$$V_{MC,r} \approx \frac{N_{MC}}{N_{MC}-1}\left(\left\langle T^2\right\rangle_{N_{MC,r}} - \langle T\rangle^2_{N_{MC,r}}\right)$$

Monte Carlo variance is different for different problems, i.e., for different values of the random variables (see figure to right)

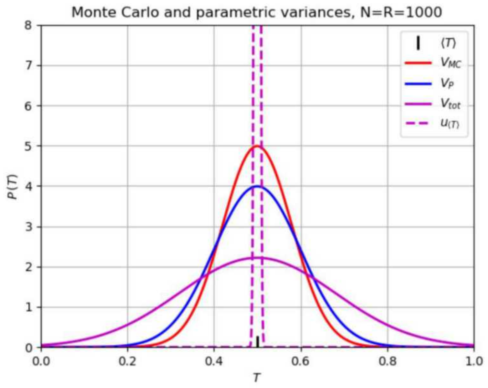
Must compute the average Monte Carlo variance over the uncertainty space:

$$V_{MC} \approx \frac{1}{R_{MC}}\sum_{r=1}^{R_{MC}} V_{MC,r}$$




Transmittance and Monte Carlo variance of transmittance (analytically solved) as a function of random total cross section for a single-material attenuation problem. Demonstrates $V_{MC}$ varying as a function of random variable.

# Non-intrusive Implementation: VADE




VAriance DEconvolution (VADE) – non-intrusive implementation

- (Can be implemented with existing transport solver and external scripting)

Task 1: Compute $V_{MC}$

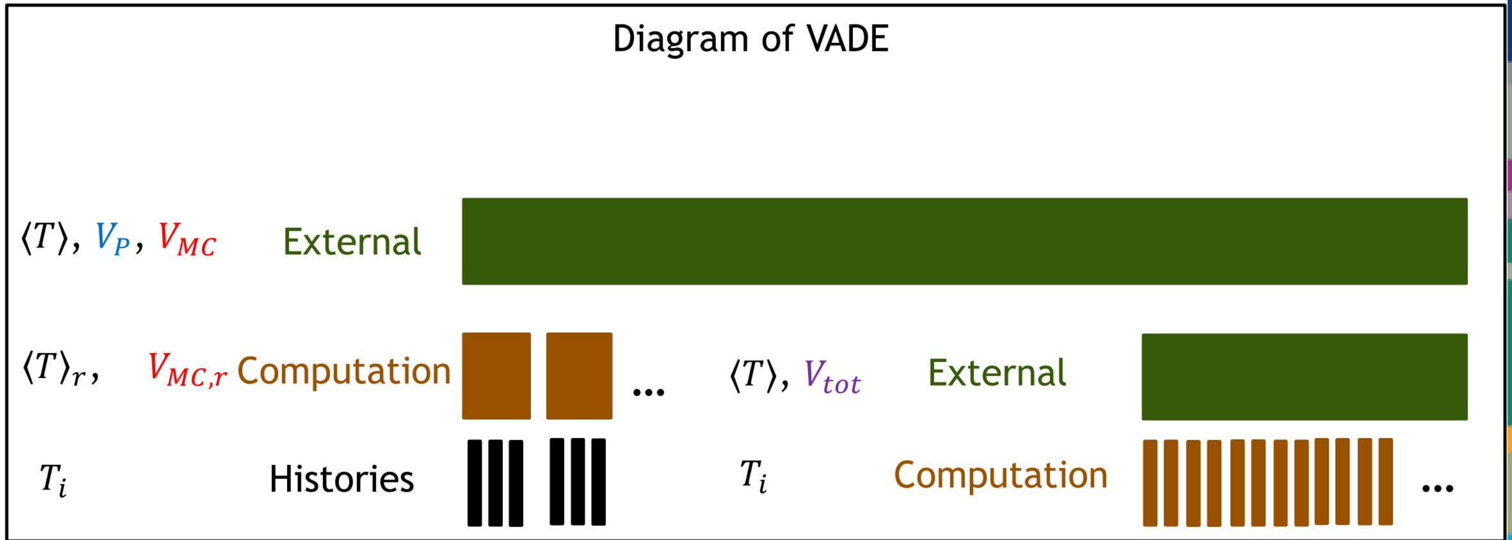
- Compute $V_{MC,r}$ on each of a collection of realizations with N>1
- Take average of $V_{MC,r}$ for $V_{MC}$

Task 2: Compute $V_{tot}$

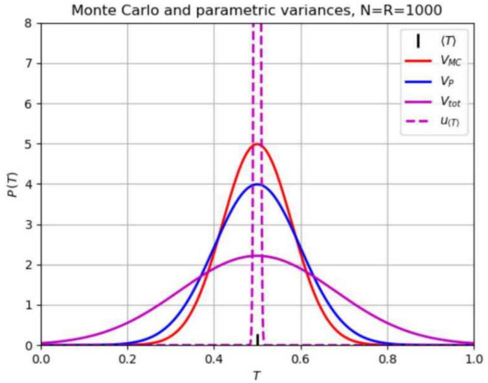
- Compute $T_i$ on each of a collection of realizations with N=1
- Use these results to compute $V_{tot}$

Task 3: Compute $V_P$

- Use variance deconvolution to get $V_P$



# Intrusive Implementation with Batches: EVADE




<u>E</u>mbedded <u>VA</u>riance <u>DE</u>convolution (EVADE) – intrusive implementation

Compute $u_{V_P}$ (and $u_{tot}$) using batches of realizations:

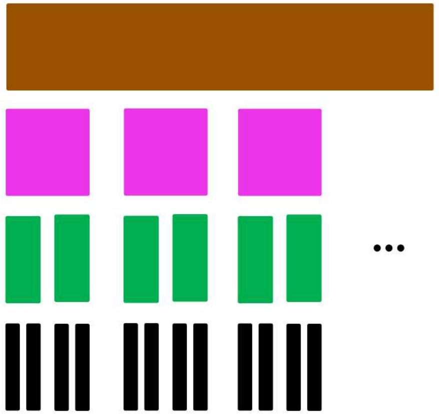
$$u_{V_p} = \frac{1}{B-1}\left(\langle V_p^2 \rangle_B - \langle V_p \rangle_B^2\right); \qquad u_{tot} = \frac{1}{B-1}\left(\langle \langle T \rangle^2 \rangle_B - \langle \langle T \rangle \rangle_B^2\right)$$

Practically:

- Compute $V_{tot}$ for batch tallying one history per realization in batch
- Compute $V_{MC}$ and $\langle T \rangle$ for batch by averaging estimates from each realization in batch
- Compute $V_p$ for that batch ($V_p = V_{tot} - V_{MC}$)
- Estimate $\langle V_p \rangle$ and $\langle T \rangle$ (and $\langle V_{tot} \rangle$ and $\langle V_{MC} \rangle$ if desired) and uncertainties over batches

### Diagram of EVADE

| | |
|---|---|
| $\langle T \rangle$, $\langle V_P \rangle$, $\langle V_{tot} \rangle$, $\langle V_{MC} \rangle$<br>$u_{tot}$, $u_{V_P}$, $u_{V_{tot}}$, $u_{V_{MC}}$ | Computation |
| $\langle T \rangle$, $V_P$, $V_{tot}$, $V_{MC}$ | Batches |
| $\langle T \rangle_r$, $V_{MC,r}$ | Realization |
| $T_i$ | Histories |




### Special Case Possible Values

If:

- Analog MC
- Leakage tally
- # Hist/realization = 2
- # Realization/batch = 2

Then:

$\langle T \rangle \in [0,1]$, $\langle V_P \rangle \in [-0.5,0.5]$, $\langle V_{tot} \rangle \in [0,0.5]$, $\langle V_{MC} \rangle \in [0,0.5]$

Note: If $\langle V_{tot} \rangle \geq \langle V_{MC} \rangle$ : $\langle V_P \rangle \in [0,0.5]$

$\langle T \rangle \in \{0,0.25,0.5,0.75,1\}$, $V_P \in \{-0.5,-0.25,0,0.25,0.5\}$, $V_{tot} \in \{0,0.5\}$, $V_{MC} \in \{0,0.25,0.5\}$

$\langle T \rangle_r \in \{0,0.5,1\}$, $V_{MC,r} \in \{0,0.5\}$

$T_i \in \{0,1\}$

# Expand Analytic/Semi-Analytic Benchmarks

# Problem Description

Stochastic Transport Equation (attenuation physics only)

$$\mu \frac{\partial \psi(x,\mu,\omega)}{\partial x} + \Sigma_t(x,\omega)\psi(x,\mu,\omega) = 0,$$

$$0 \le x \le L; \quad -1 \le \mu \le 1,$$
$$\psi(0,\mu) = \delta(1-\mu), \mu > 0; \psi(L,\mu) = 0, \mu < 0$$

- $x, \mu, \omega$ – spatial, angular, and stochastic dependence
- $\Sigma_t(x,\omega)$ – total cross section, source of uncertainty
- $\psi(x,\mu,\omega)$ – angular flux, response
- Beam source on "left" boundary, otherwise vacuum BCs
- Quantity of interest: Transmittance

Sources of parametric uncertainty (described on next slides)

- Random total cross sections
- Random material boundaries

# Analytic Benchmark Random Cross Section Problem

Uniformly distributed random cross sections:

$$\Sigma_{t,m}(\omega) = \langle \Sigma_{t,m} \rangle + \widehat{\Sigma_{t,m}}\xi_m(\omega),$$

$$\xi_m(\omega) \in U[-1,1] \qquad \forall m \in \{1,2,\dots,M\}$$

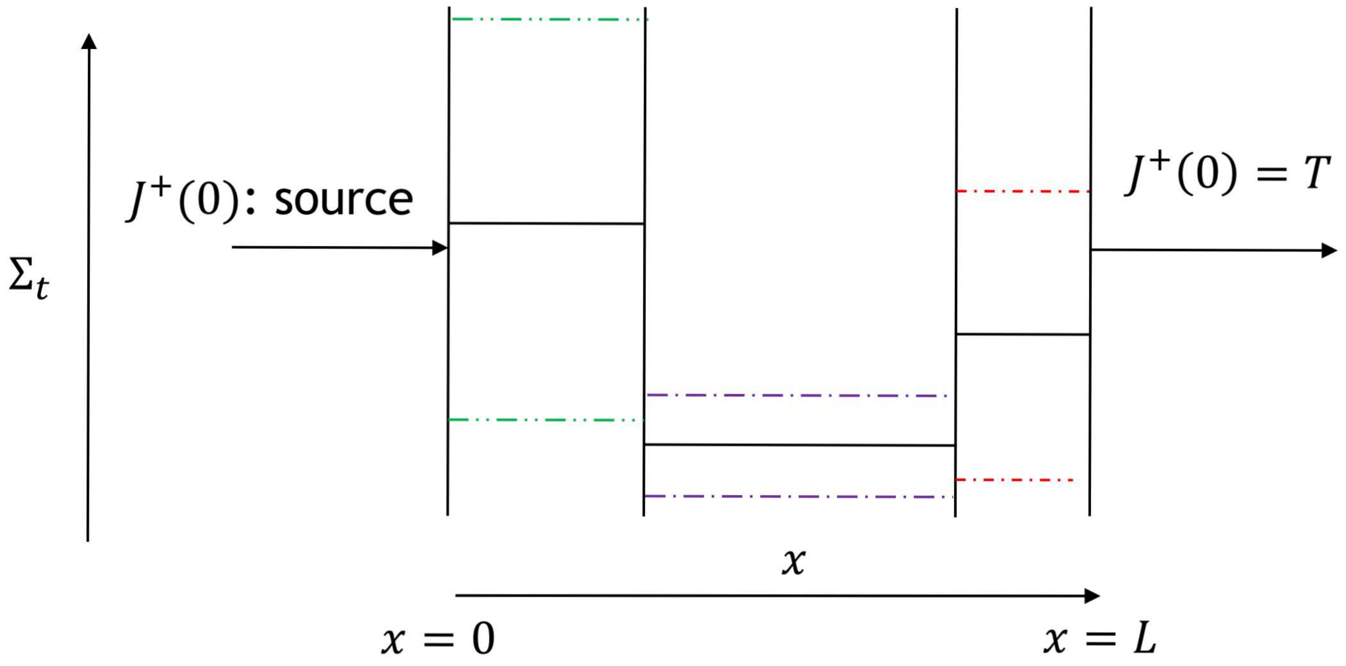
Analytic Transmittance ($T$) for realization $\omega$:

$$T(\omega) = \exp[-\tau(\omega)]; \quad \tau(\omega) = \sum_{m=1}^{M} \Sigma_{t,m}(\omega)\Delta \mathrm{x_m}$$

Analytic Moment of Expectation of Transmittance:

$$\mathbb{E}[T^p] = \int_{-1}^{1} \dots \int_{-1}^{1} T^p(\omega) \left(\frac{1}{2}\right)^M d\xi_1 \dots d\xi_M \ \Rightarrow$$

$$\mathbb{E}[T^p] = \prod_{m=1}^{M} \exp\left[-p\langle \Sigma_{t,m} \rangle \Delta x_m\right] \frac{\sinh\left[p\widehat{\Sigma_{t,m}}\Delta x_m\right]}{p\widehat{\Sigma_{t,m}}\Delta x_m}$$



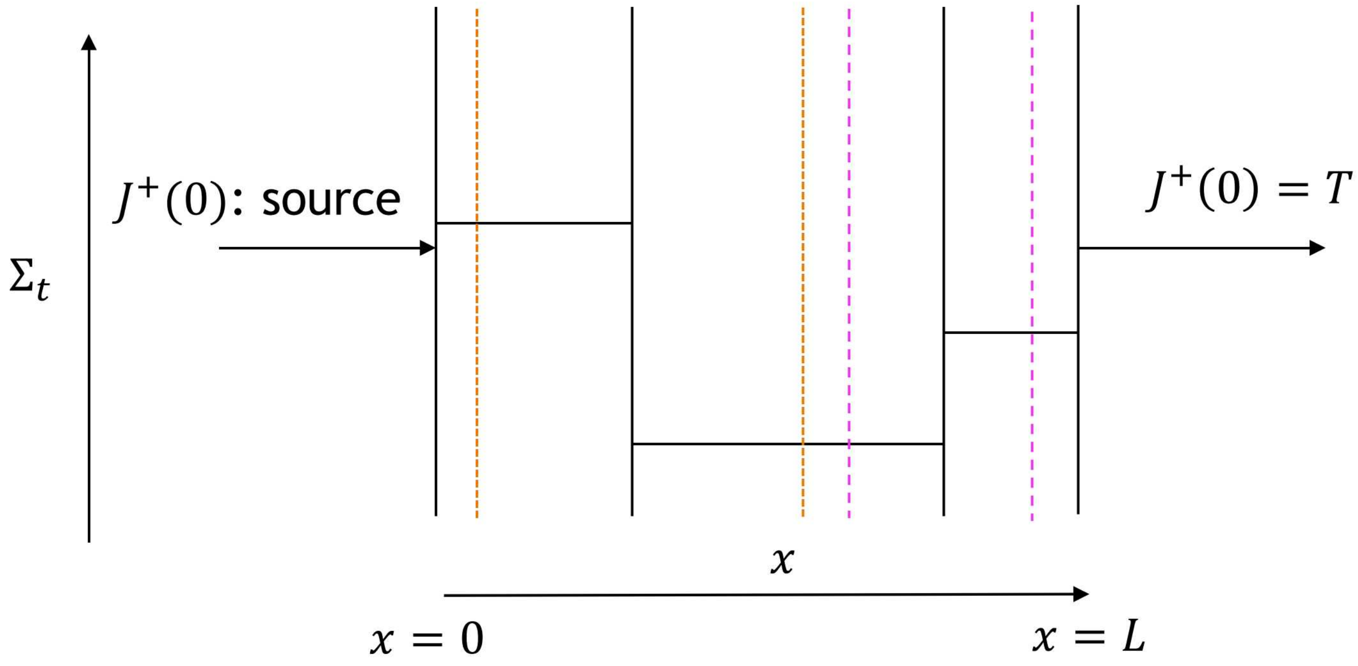
# Analytic Benchmark Random Boundary Location Problem

Uniformly distributed random boundary locations:

$$x_m(\omega) = \langle \mathrm{x}_m \rangle + \widehat{\mathrm{x}_m}\zeta_m(\omega),$$

$$\Delta x_m(\omega) = \begin{cases} x_1(\omega) & if\ m = 1 \\ x_{m(\omega)} - x_{m-1}(\omega) & if\ 1 < m < M \\ L - x_{M-1}(\omega) & if\ m = M \end{cases}$$

$$\zeta_m(\omega) \in U[-1,1] \qquad \forall m \in \{1,2,\dots,M-1\}$$




Analytic Transmittance ($T$) for realization $\omega$:

$$T(\omega) = \exp[-\tau(\omega)]\,; \qquad \tau(\omega) = \sum_{m=1}^{M} \Sigma_{t,m}\Delta \mathrm{x_m}(\omega)$$

Analytic Moment of Expectation of Transmittance:

$$\mathbb{E}[T^p] = \int_{-1}^{1} \dots \int_{-1}^{1} T^p(\omega)\left(\frac{1}{2}\right)^M d\zeta_1 \dots d\zeta_{M-1} \;\Rightarrow$$

$$\mathbb{E}[T^p] = \left(\prod_{m=1}^{M} \exp\left[-p\langle\Sigma_{t,m}\rangle(\langle x_m\rangle - \langle x_{m-1}\rangle)\right]\right)\left(\prod_{m=1}^{M-1} \frac{\sinh\left[p\left(\langle\Sigma_{t,m-1}\rangle - \langle\Sigma_{t,m}\rangle\right)\widehat{x_m}\right]}{p\left(\langle\Sigma_{t,m-1}\rangle - \langle\Sigma_{t,m}\rangle\right)\widehat{x_m}}\right)$$

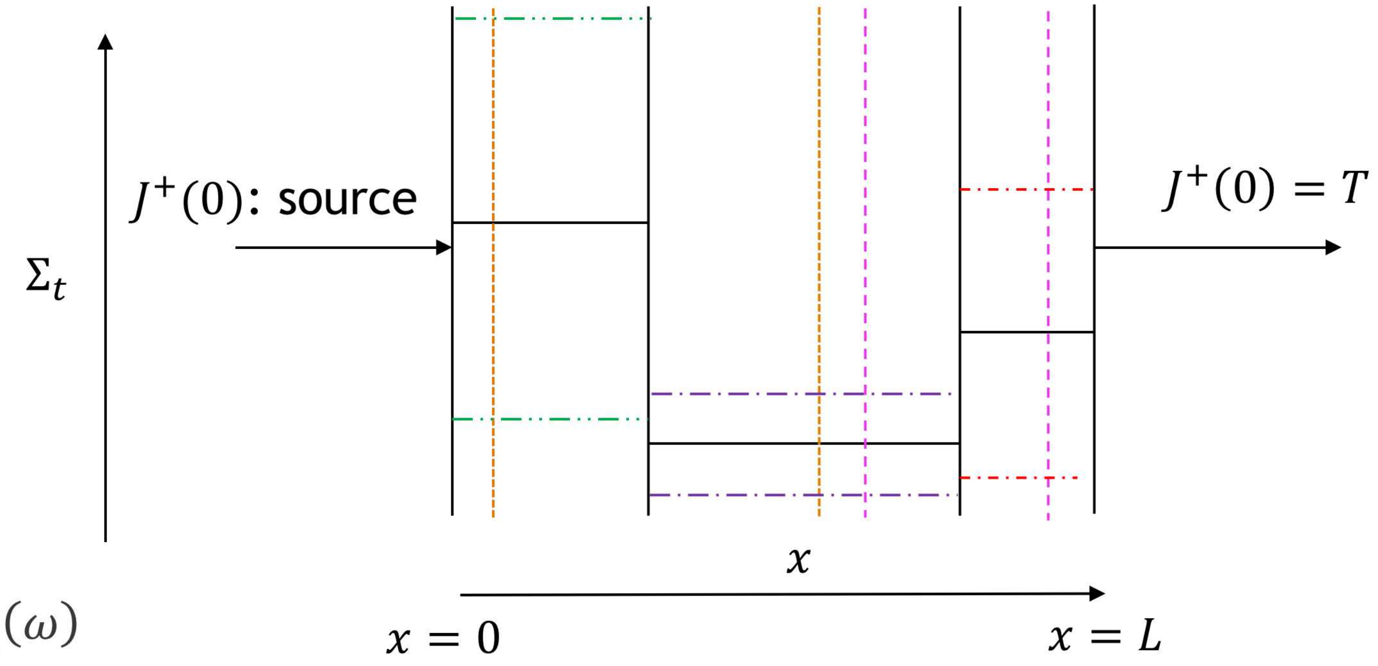
# Semi-Analytic Benchmark for Problem with Both Uncertainty Sources

Uniformly distributed random cross sections and material boundary locations:

$$\Sigma_{t,m}(\omega) = \langle \Sigma_{t,m} \rangle + \widehat{\Sigma_{t,m}} \xi_m(\omega)$$

$$x_m(\omega) = \langle \mathrm{x}_m \rangle + \widehat{\mathrm{x}_m} \zeta_m(\omega)$$

$$\xi_m(\omega), \zeta_m(\omega) \in U[-1,1] \ \ \forall m$$




Analytic Transmittance ($T$) for realization $\omega$:

$$T(\omega) = \exp[-\tau(\omega)]\,; \quad \tau(\omega) = \sum_{m=1}^{M} \Sigma_{t,m}(\omega) \Delta \mathrm{x_m}(\omega)$$

Semi-analytic Moment of Expectation of Transmittance (numerically integrate analytic cross section solution):

$$\mathbb{E}[T^p] = \int_{-1}^{1} \dots \int_{-1}^{1} T^p(\omega) \left(\frac{1}{2}\right)^{2M-1} d\xi_1 \dots d\xi_M d\zeta_1 \dots d\zeta_{M-1} \Rightarrow$$

$$\mathbb{E}[T^p] = \left(\frac{1}{2}\right)^{M-1} \int_{-1}^{1} \cdots \int_{-1}^{1} d\zeta_1 \cdots d\zeta_{M-1} \prod_{m=1}^{M} \exp\left[-p \langle \Sigma_{t,m} \rangle \Delta x_m(\omega)\right] \frac{\sinh\left[p \widehat{\Sigma_{t,m}} \Delta x_m(\omega)\right]}{p \widehat{\Sigma_{t,m}} \Delta x_m(\omega)}$$

# Analytic Benchmark for Total and Monte Carlo Variances

## Parametric variance

- Computed from analytic/semi-analytic moments:

$$V_p = \mathbb{E}[T^2] - \mathbb{E}^2[T]$$

## Total Variance

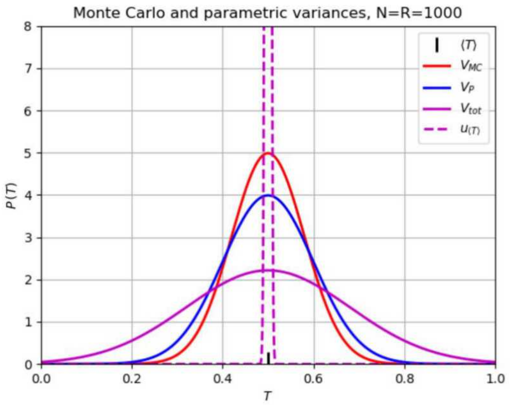
- With analog Monte Carlo and one history per realization, all tallies $T_r$ are either $0$ or $1$ such that all tallied moments are the same: $\langle T \rangle_{R_{tot}} = \langle T^p \rangle_{R_{tot}} \forall p$
- In the limit of many realizations, the Monte Carlo simulation yields the expected value: $\lim_{R_{tot}\to\infty} \langle T \rangle_{R_{tot}} = \mathbb{E}[T]$
- The limit of the Monte Carlo computation thus yields an analytic solution for variance based on analytic/semi-analytic moments:

$$V_{tot} = \lim_{R_{tot}\to\infty} \frac{R_{tot}}{R_{tot}-1}\left(\langle T^2 \rangle - \langle T \rangle^2\right) = \mathbb{E}[T] - \mathbb{E}^2[T]$$

## Monte Carlo Variance

- Using variance deconvolution:

$$V_{MC} = V_{tot} - V_p = \mathbb{E}[T] - \mathbb{E}[T^2]$$



# Numerical Results

# Description of Problems

Problem 1: Cross Sections are Random Variables

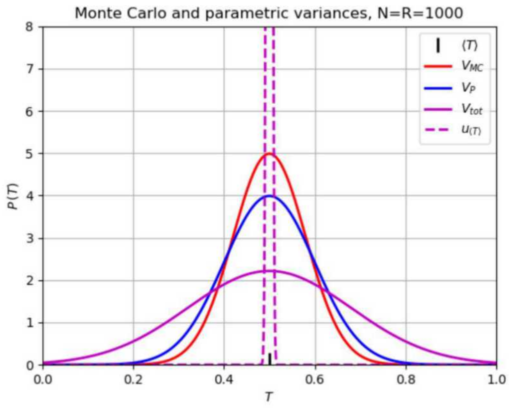
Problem 2: Boundary Locations are Random Variables

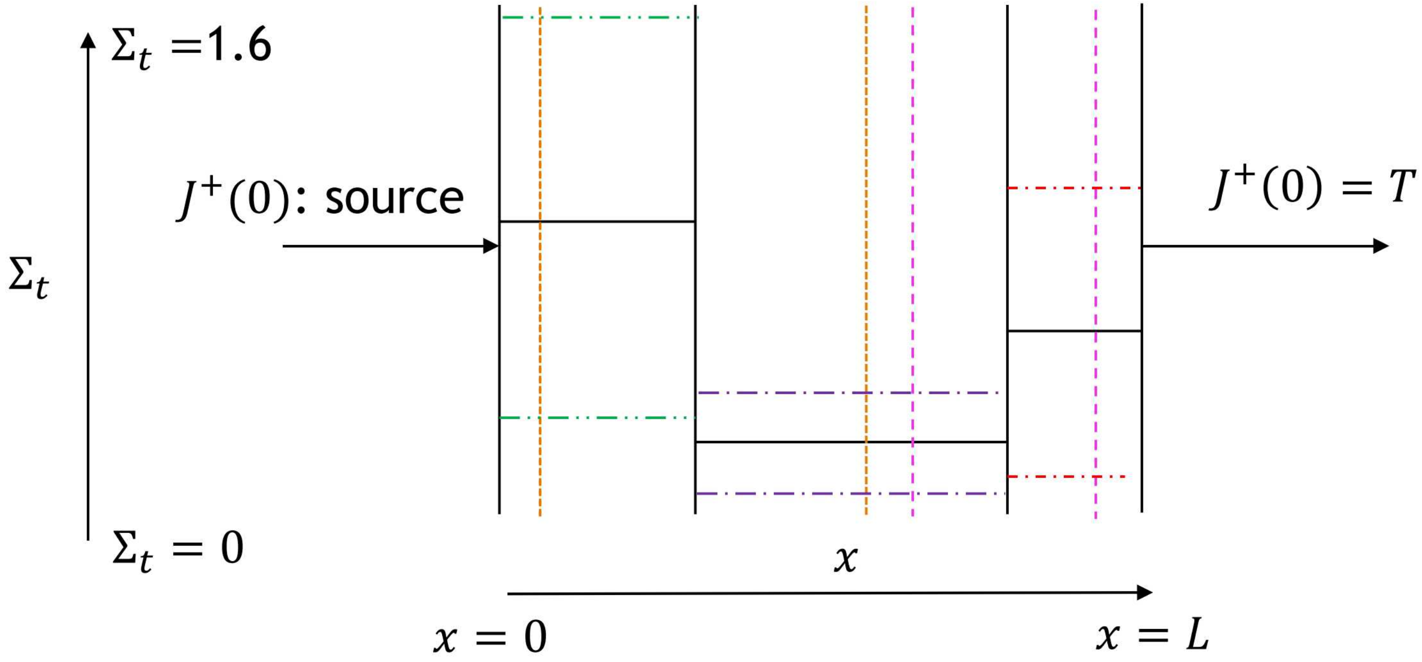
Problem 3: Both quantities are Random Variables

All problems use parameters in table

TABLE I. Stochastic Attenuation Problem Parameters

| | $m = 0$ | $m = 1$ | $m = 2$ | $m = 3$ |
|---|---|---|---|---|
| $\langle \Sigma_{t,m} \rangle$ | - | 0.9 | 0.15 | 0.6 |
| $\widehat{\Sigma}_{t,m}$ | - | 0.7 | 0.12 | 0.5 |
| $\langle x_m \rangle$ | 0.0 | 2.0 | 5.0 | 6.0 |
| $\widehat{x}_m$ | - | 1.75 | 0.95 | - |



# High-Fidelity Solve – Example

VADE and EVADE both accurate for each quantity on each problem type

TABLE II. Higher-Fidelity Solver Parameters

| VADE | | EVADE | |
|---|---|---|---|
| $R_{tot} = 10^8$ | $R_{MC} = 10^4$ | $R_{tot} = R_{MC} = 10^8$ | |
| $N_{tot} = 1$ | $N_{MC} = 10^4$ | $N_{tot} = 1$ | $N_{MC} = 2$ |
| $B$ =N/A | | $B = 5 \times 10^7$ | |
| $N_{TOT} = 2 \times 10^8$ | | $N_{TOT} = 2 \times 10^8$ | |

TABLE III. Higher-Fidelity Simulation Representative Results

| Random Cross Sections | | | |
|---|---|---|---|
| Quantity | Analytic | VADE | EVADE |
| $\langle T \rangle$ | 0.083783 | 0.083981 | 0.083773 ± 0.000020 |
| $V_P$ | 0.005505 | 0.005637 | 0.005509 ± 0.000022 |
| $V_{tot}$ | 0.076763 | 0.076749 | 0.076762 ± 0.000026 |
| $V_{MC}$ | 0.071259 | 0.071112 | 0.071253 ± 0.000018 |
| Random Boundaries | | | |
| Quantity | Analytic | VADE | EVADE |
| $\langle T \rangle$ | 0.078277 | 0.078828 | 0.078314 ± 0.000020 |
| $V_P$ | 0.003731 | 0.002782 | 0.003734 ± 0.000021 |
| $V_{tot}$ | 0.072150 | 0.072140 | 0.072176 ± 0.000025 |
| $V_{MC}$ | 0.068419 | 0.069358 | 0.068442 ± 0.000017 |
| Random Cross Sections and Boundaries | | | |
| Quantity | Semi-An. | VADE | EVADE |
| $\langle T \rangle$ | 0.104428 | 0.103672 | 0.104398 ± 0.000023 |
| $V_P$ | 0.010069 | 0.011029 | 0.010097 ± 0.000024 |
| $V_{tot}$ | 0.093523 | 0.093503 | 0.093532 ± 0.000028 |
| $V_{MC}$ | 0.083454 | 0.082474 | 0.083435 ± 0.000019 |

# Low-Fidelity Solves – Statistical Verification

EVADE uncertainty estimates accurate for each quantity ($\epsilon < u$ ~68% of time)

EVADE ~an order of magnitude more precise than VADE for three quantities (including two target quantities)

VADE ~25% more precise than EVADE for total variance – effect of batching?

TABLE IV. Lower-Fidelity Solver Parameters

| VADE | | EVADE | |
|---|---|---|---|
| $R_{tot} = 10^5$ | $R_{MC} = 10^2$ | $R_{tot} = R_{MC} = 10^5$ | |
| $N_{tot} = 1$ | $N_{MC} = 10^3$ | $N_{tot} = 1$ | $N_{MC} = 2$ |
| $B$ =N/A | | $B = 5 \times 10^4$ | |
| $N_{TOT} = 2 \times 10^5$ | | $N_{TOT} = 2 \times 10^5$ | |

TABLE V. Lower-Fidelity Simulation Ensemble Results

| Random Cross Sections | | | | |
|---|---|---|---|---|
| Quantity | $\langle\epsilon\rangle_{VADE}$ | $\langle\epsilon\rangle_{EVADE}$ | $\langle u\rangle_{EVADE}$ | $\epsilon < u$ |
| $\langle T\rangle$ | 0.00375 | 0.00052 | 0.00064 | 668/1000 |
| $V_P$ | 0.00551 | 0.00054 | 0.00068 | 673/1000 |
| $V_{tot}$ | 0.00052 | 0.00065 | 0.00081 | 681/1000 |
| $V_{MC}$ | 0.00550 | 0.00044 | 0.00055 | 681/1000 |
| **Random Boundaries** | | | | |
| Quantity | $\langle\epsilon\rangle_{VADE}$ | $\langle\epsilon\rangle_{EVADE}$ | $\langle u\rangle_{EVADE}$ | $\epsilon < u$ |
| $\langle T\rangle$ | 0.00333 | 0.00049 | 0.00062 | 658/1000 |
| $V_P$ | 0.00525 | 0.00053 | 0.00065 | 674/1000 |
| $V_{tot}$ | 0.00053 | 0.00065 | 0.00079 | 661/1000 |
| $V_{MC}$ | 0.00521 | 0.00043 | 0.00054 | 682/1000 |
| **Random Cross Sections and Boundaries** | | | | |
| Quantity | $\langle\epsilon\rangle_{VADE}$ | $\langle\epsilon\rangle_{EVADE}$ | $\langle u\rangle_{EVADE}$ | $\epsilon < u$ |
| $\langle T\rangle$ | 0.00468 | 0.00058 | 0.00072 | 681/1000 |
| $V_P$ | 0.00621 | 0.00059 | 0.00077 | 702/1000 |
| $V_{tot}$ | 0.00057 | 0.00069 | 0.00087 | 707/1000 |
| $V_{MC}$ | 0.00620 | 0.00047 | 0.00059 | 685/1000 |

# Conclusions and Future Work

# Conclusions

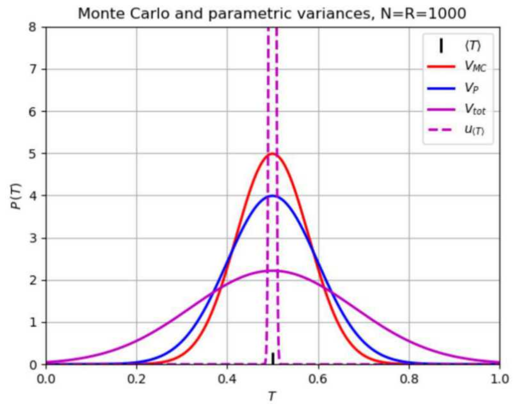
Deconvolution of variance to get $V_P$ with Monte Carlo solver and no $V_P \gg u_{MC}$ assumption

- non-intrusively (VADE), intrusively (EVADE)
- for three r.v. problem types

Use of batches to accurately get $u_{tot}$ and $u_{V_P}$ (in EVADE)

Deconvolution demonstrated w/ N=2 (in EVADE)

- more precise than N>2 (in VADE)
- N=2 optimized?




Analytic benchmarks expanded

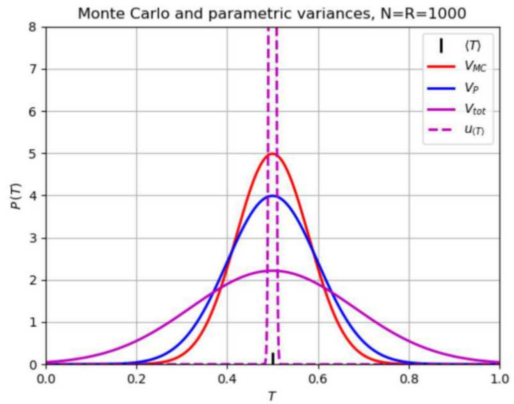
- Random boundary location problem (analytic)
- Random cross section and boundary location problem (semi-analytic)
- $V_p$, $V_{tot}$, and $V_{MC}$ (analytic)

# Future Work

Why $V_{tot}$ more precise in VADE than EVADE (effect of batches?)

Optimal-Cost Monte Carlo (OCMC) analysis for $V_P$

- N=2 optimal?
- N=1 for some realizations, N=2 for others?




Apply to stochastic media problems:

- EVADE with CoPS for $V_P$ (Vu, M&C, 2019)
- Problems with both traditional r.v.s and stochastic media
- $V_{tot}$ with one model and $V_{MC}$ with a different one (VADE)
  - (e.g., $V_{tot}$ using chord length sampling, $V_{MC}$ using realizations)

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Ron Kensek – for sharing with me his idea to "deconvolve" the total variance into the Monte Carlo and parametric variances ($V_{tot} = V_p + V_{MC}$)

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# Questions

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