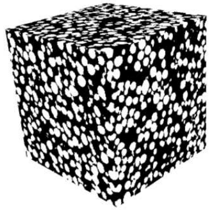
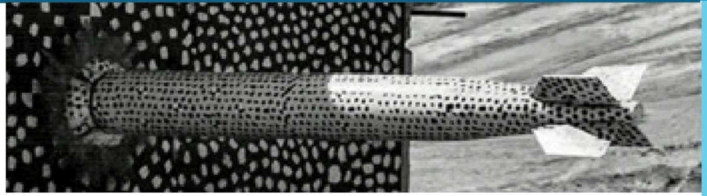
SAND2019-6212C

# Exploring Microstructural Descriptors in Elastomeric Foams Using Digital Image Correlation and Statistical Analysis





PRESENTED BY

Robert Waymel

COAUTHORS

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SEM Annual Conference – Reno, NV – June 6, 2019

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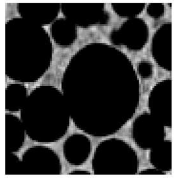
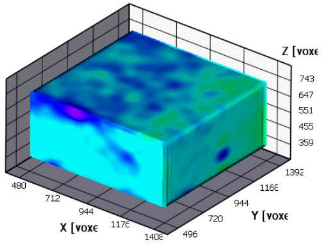
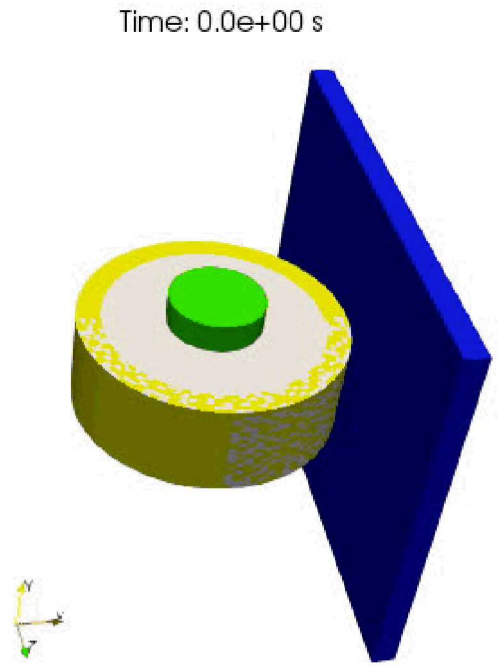
SAND2019-####C

# Motivation – Elastomeric Foams

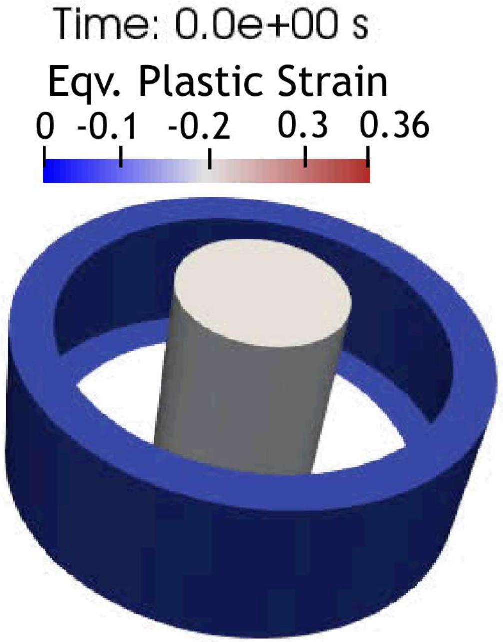
- Elastomeric foams are commonly used in packaging and are capable of absorbing significant quantities of energy
- Current models may be tedious/expensive to calibrate
  - Flex Foam Model* allows 9 constants and 14 user-prescribed functions to be input
- Goal: Reduce number of calibration tests required to satisfactorily qualify a foam or generate a new, physically-motivated constitutive relation
- Approach: Identify microstructural descriptors besides porosity that significantly affect the macroscopic behavior by applying statistical analysis to full-field data sets such as computed tomography (CT) and digital image/volume correlation (DIC/DVC)

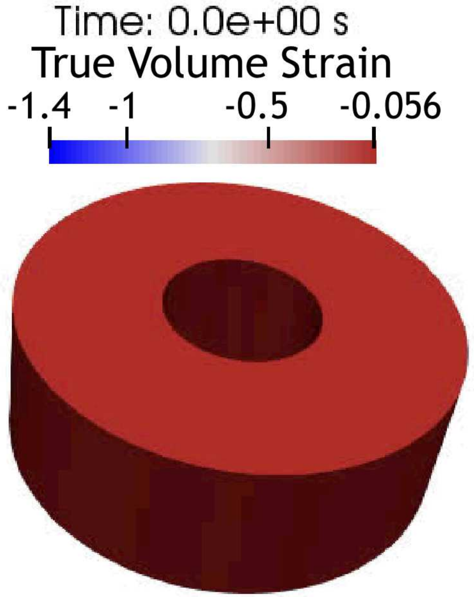





Crash Scenario (~65 mph)



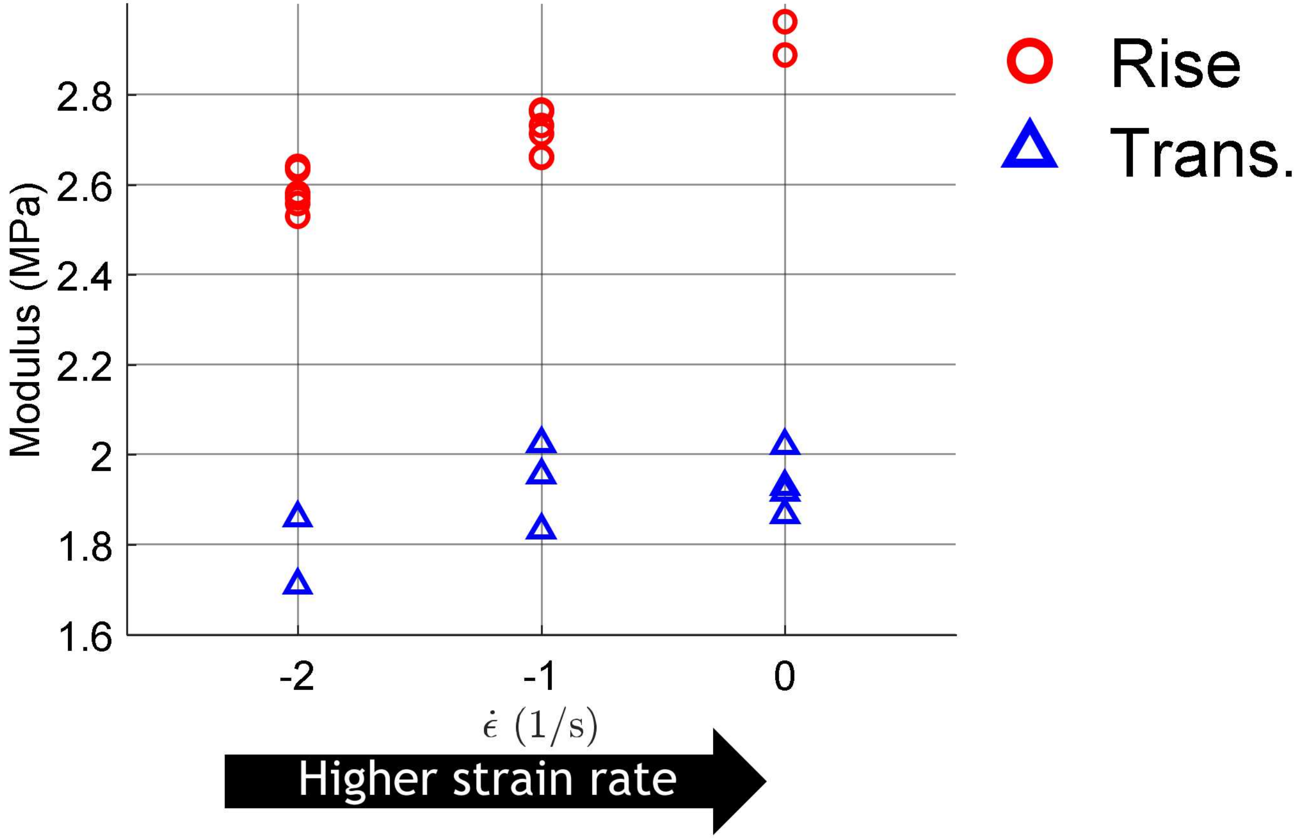




*SAND2018-2433 (available through OSTI)

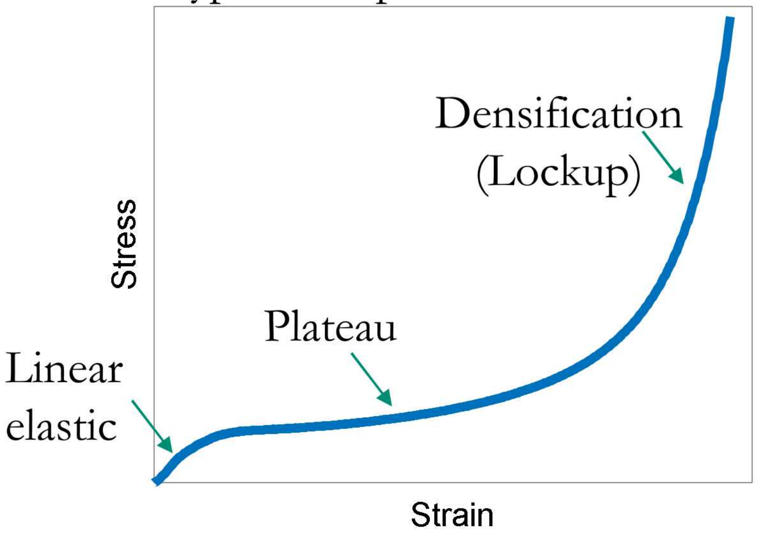
# Background – Polyurethane Foam Behavior

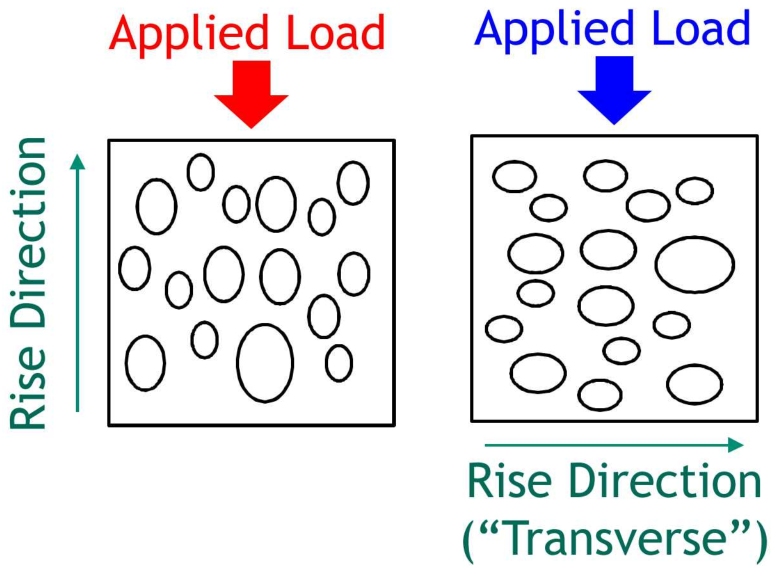
- Three distinct regions in compression
- Manufacturing process produces eccentric pores
- Transverse linear elastic moduli are 30-60% less

Results from 240 kg/m³ (15 lb/ft³) foam




Typical σ-ε profile for PU foam



# Background – Spatial Statistics in Mechanics

<u>*n*-Point Probability Functions</u>

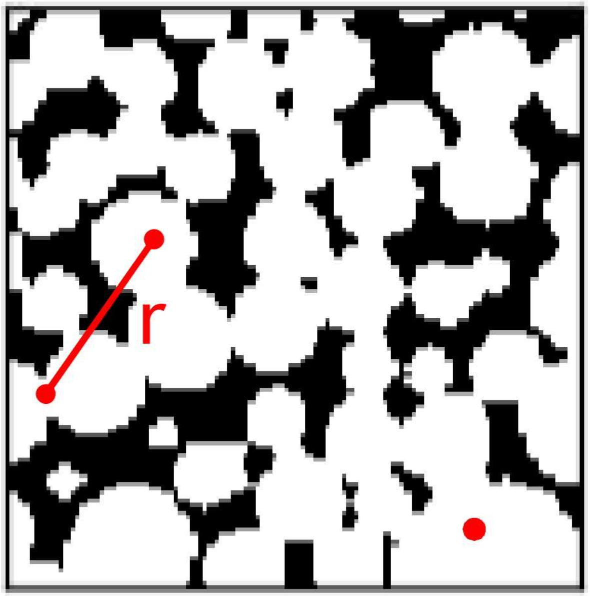
Consider two phase material (foam)

$$S_n(\boldsymbol{x}_1, \boldsymbol{x}_2, \dots \boldsymbol{x}_n) \equiv \langle I(\boldsymbol{x}_1) I(\boldsymbol{x}_2) \dots I(\boldsymbol{x}_n) \rangle$$

Where $I(\boldsymbol{x}) = 1$ in pore phase, 0 in solid phase.

The probability that $n$ points at positions $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, …, $\boldsymbol{x}_n$ are found in the pore phase.

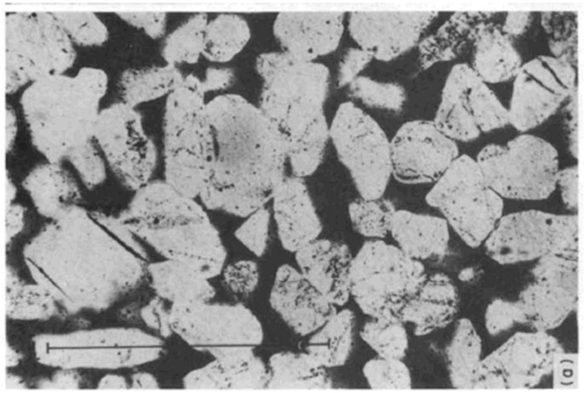
Torquato (2002) *Annu. Rev. Mater. Res.*




$S_1(\boldsymbol{x}_1)$ indicates porosity

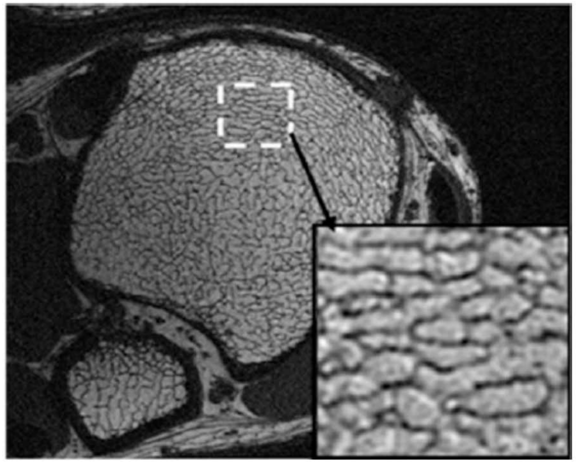
$$S_2(\boldsymbol{r}) = \langle I(\boldsymbol{x}) I(\boldsymbol{x} + \boldsymbol{r}) \rangle$$

Probability that both end points of a randomly located straight line are contained in pore phase.



Predict permeability in Fontainbleau sandstone

Adler et al. (1990) *Int. J. Multiph. Flow*



Determine anisotropy (and health) of tibia

Wald et al. (2007) *Med. Phys.*

# Outline

Motivation and background

Experimental setup and results

Autocorrelation function and anisotropy

Cross-correlating microstructure to strain fluctuations

Conclusions

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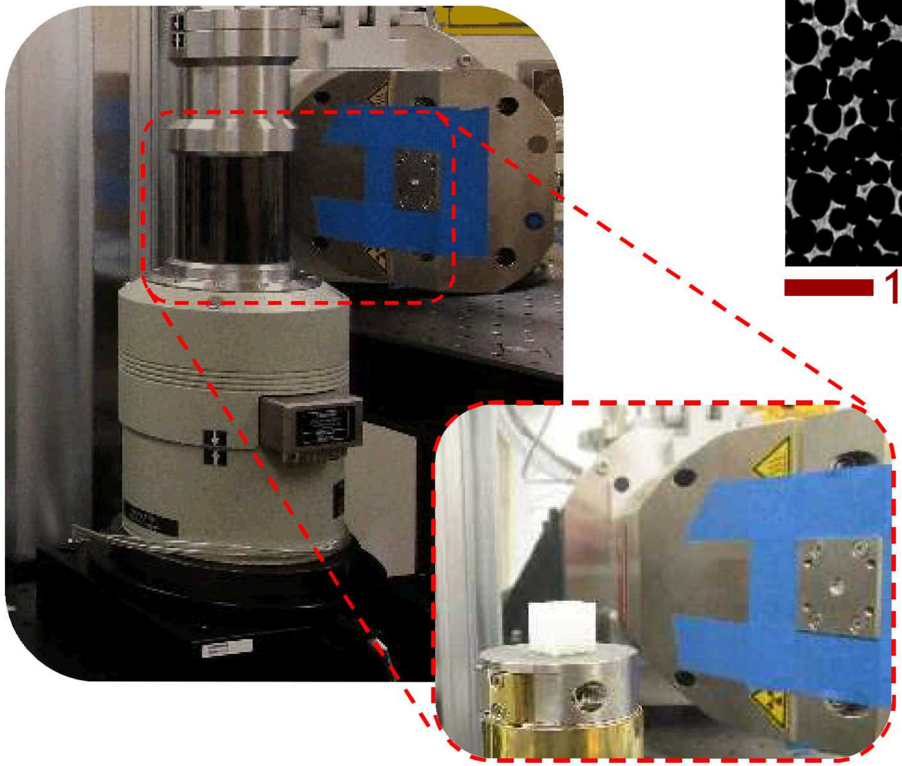
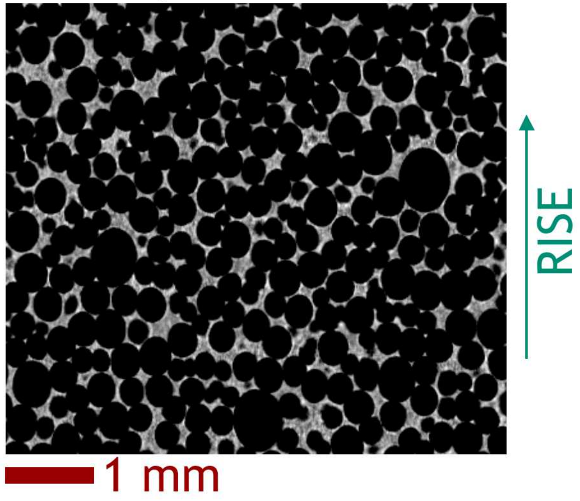
# Experimental Setup

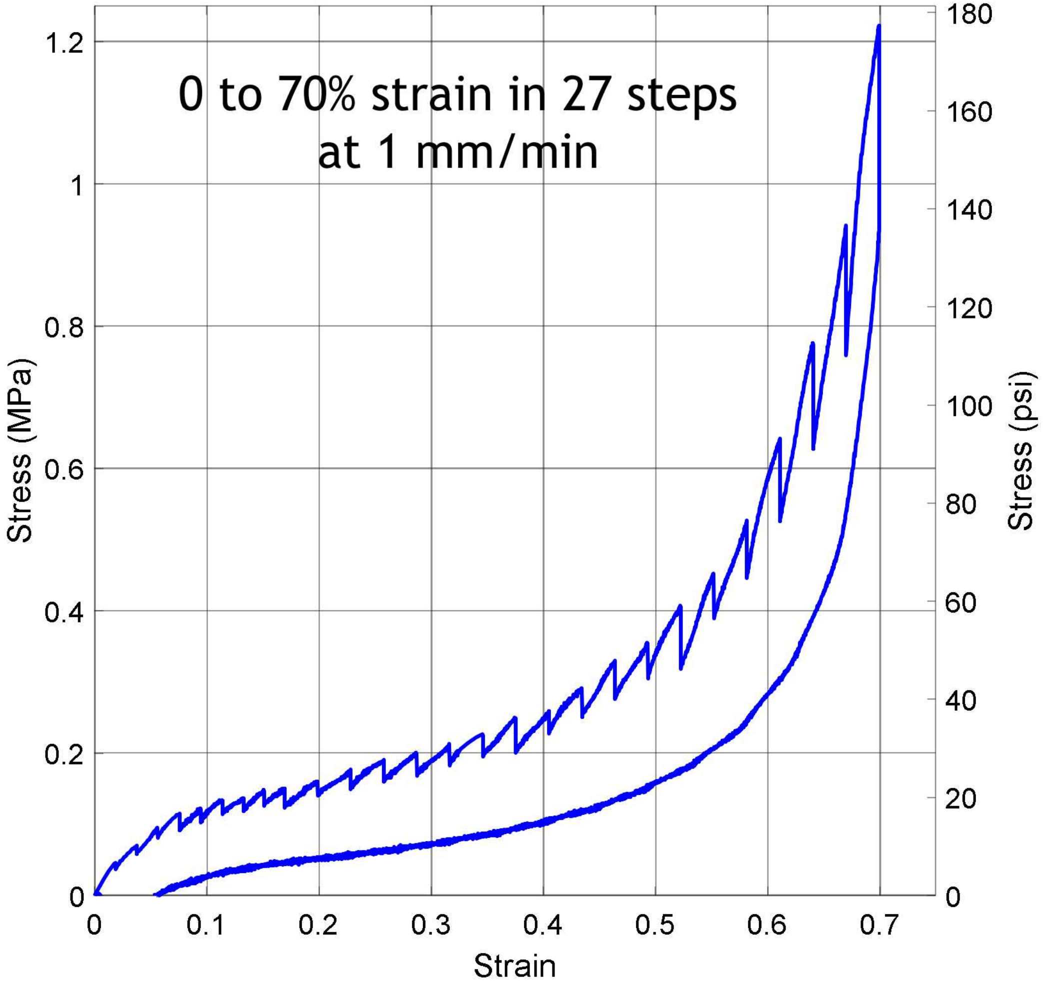
- Compression testing of polyurethane foam
- In-situ CT acquisition (Deben load frame)
- Resolution = $12^3\ \mu m^3/vox$



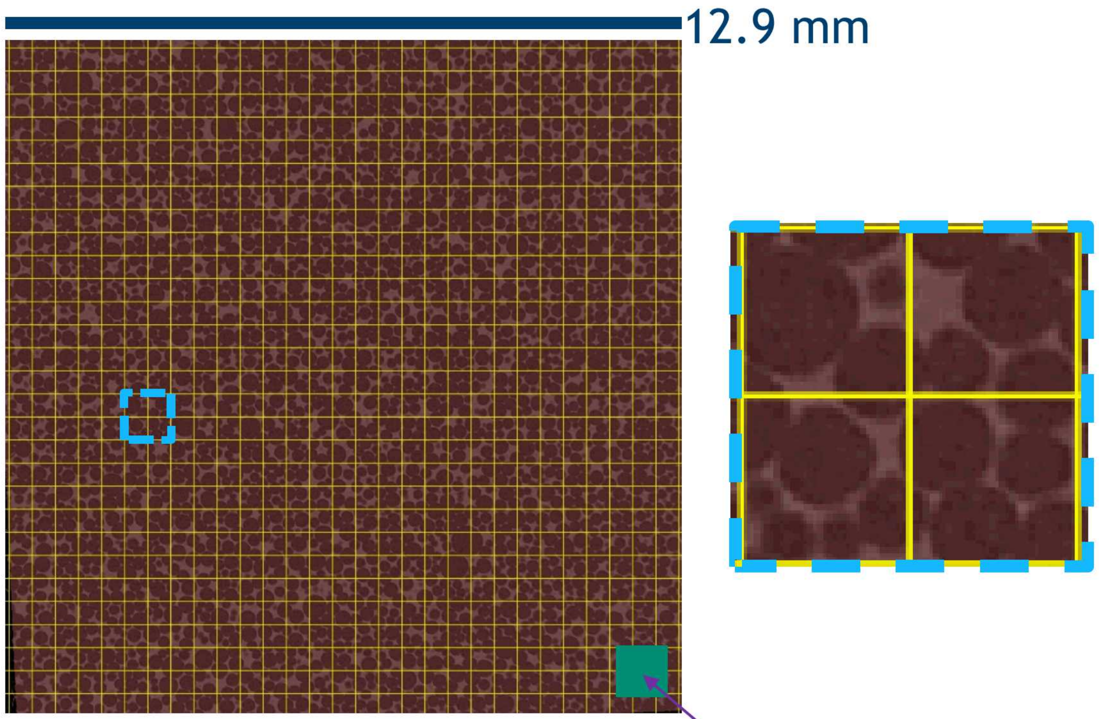
| Polyurethane Foam | |
|---|---|
| Density | 240 kg/m³ (15 lb/ft³) |
| Porosity | 76-81% |
| Dimensions | 12.9×12.9×13.6 mm³ |

Greyscale Image



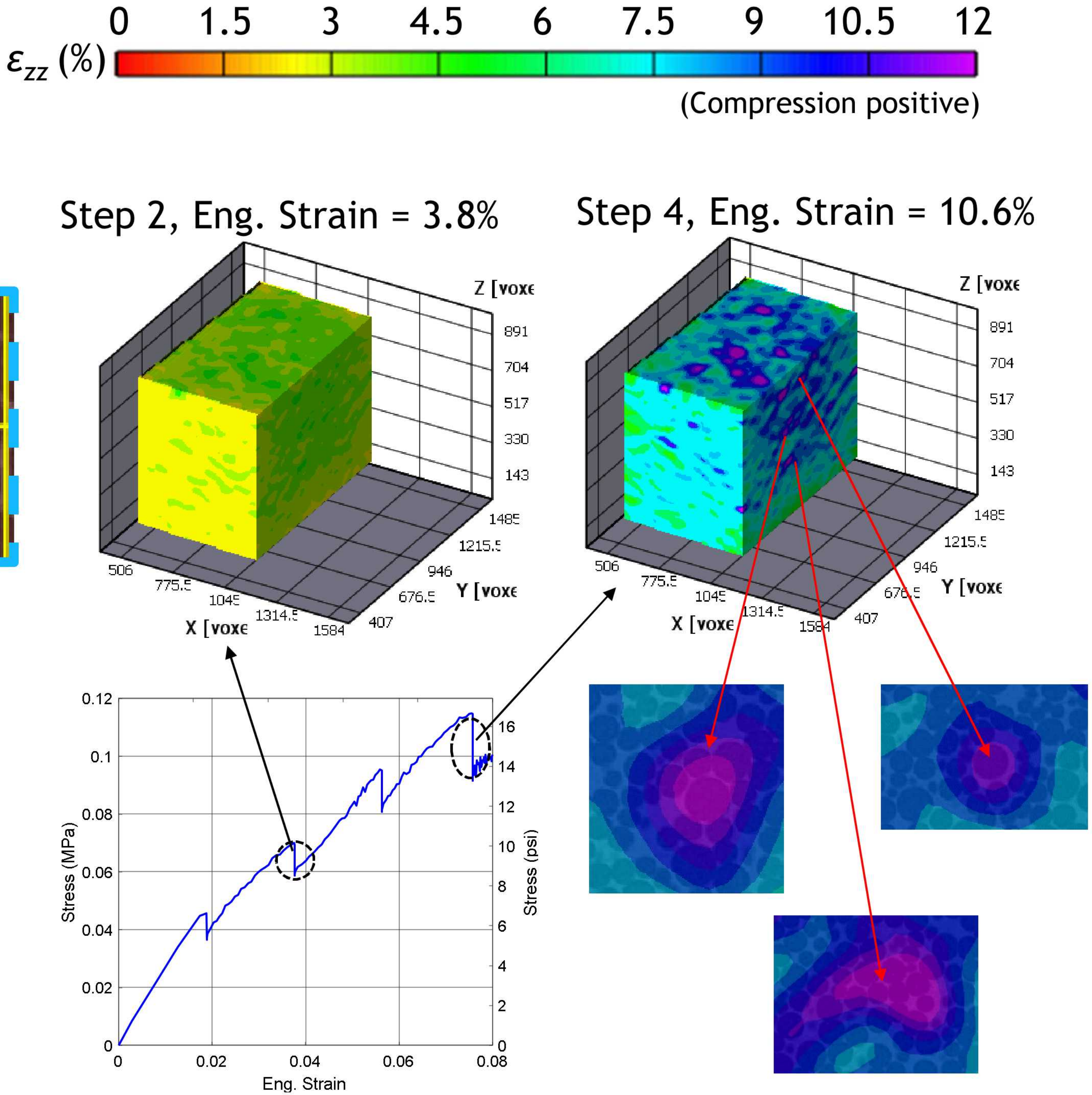



# Digital Volume Correlation




- 3-5 voxels/feature
  - pore diameter: 200-500 μm
- 3-5 features/subset
- Correlated Solutions Vic-Volume

| Resolution | Subset | Step | Filter | VSG |
|---|---|---|---|---|
| $12^3$ μm$^3$/vox | 35 vox | 11 vox | 5 vox | 1 mm$^3$ |



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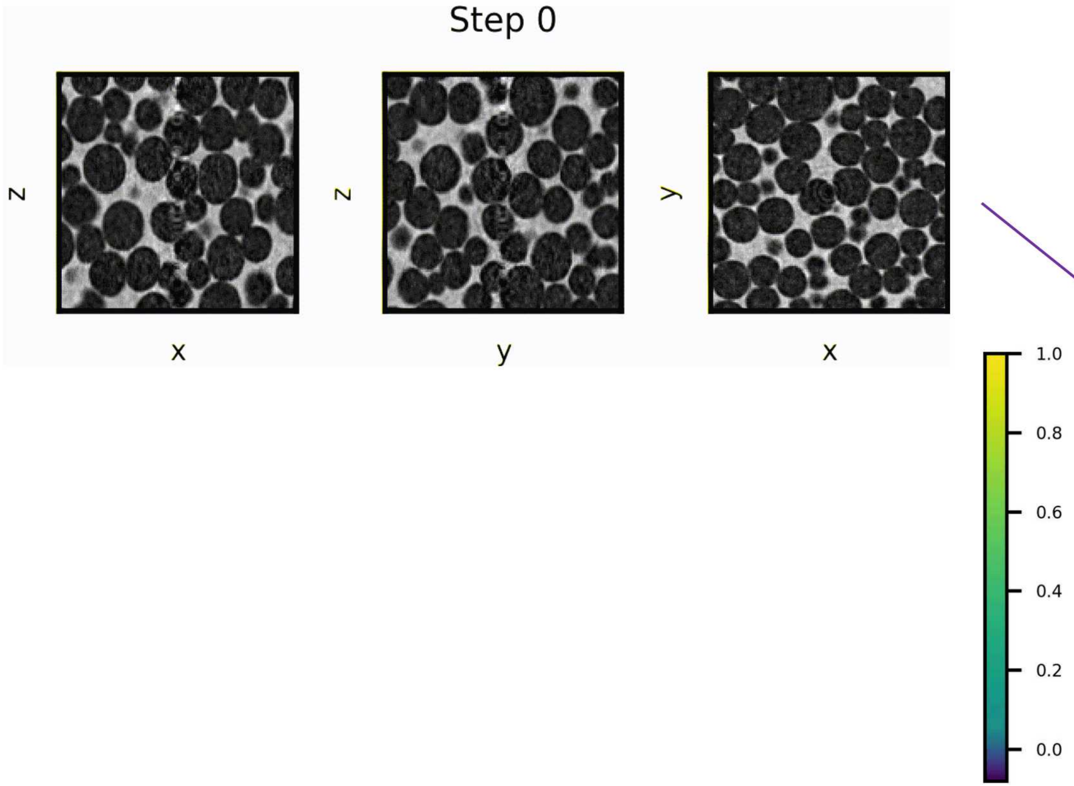
# Autocorrelation Function to Measure Deformation Induced Anisotropy




Autocorrelation function: a measure of how correlated intensity is as a function of spatial separation. No thresholding required!

$I$: grayscale intensity

① → $ACF(\mathbf{r})$ → ②

$$ACF(\mathbf{r}) = \frac{\overbrace{\langle I(\mathbf{x}+\mathbf{r})I(\mathbf{x})\rangle}^{S_2(\boldsymbol{r})} - \overline{I^2(\mathbf{x})}}{\sigma^2}$$

Average grayscale intensity: $\overline{I^2(\mathbf{x})}$

Variance of grayscale intensity: $\sigma^2$

③

Moment of inertia tensor

$$\mathbf{I} = \sum_{k=1}^{N} m_k ((\mathbf{r}_k \cdot \mathbf{r}_k)\mathbf{E} - \mathbf{r}_k \otimes \mathbf{r}_k),$$

where $\mathbf{E}$ is the identity tensor

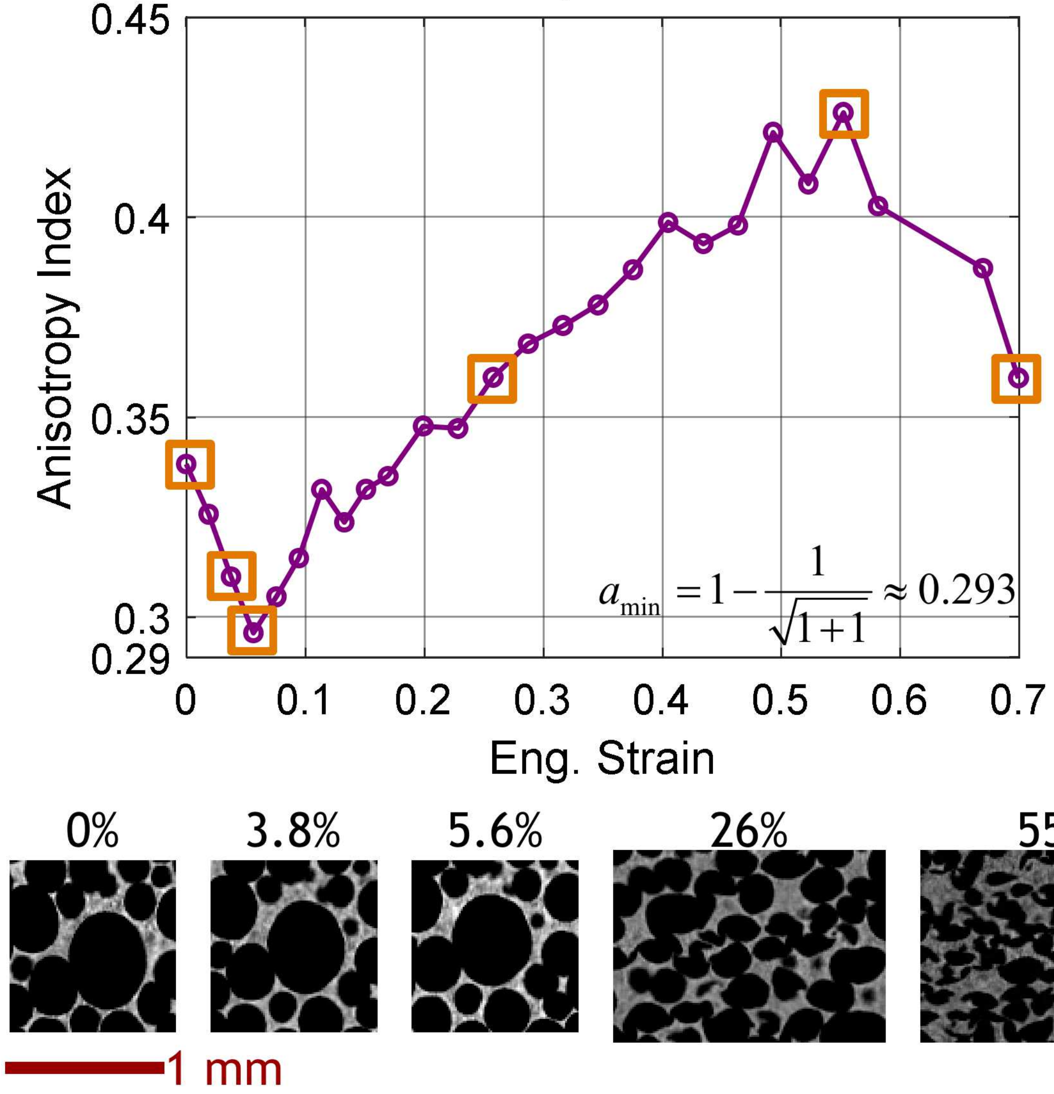
$$\mathbf{E} = \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{e}_3 \otimes \mathbf{e}_3$$

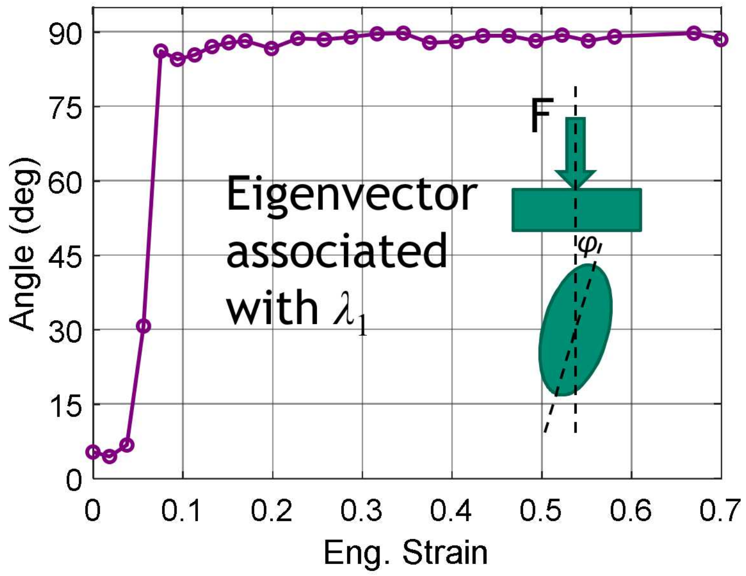
④ Determine eigenvalues

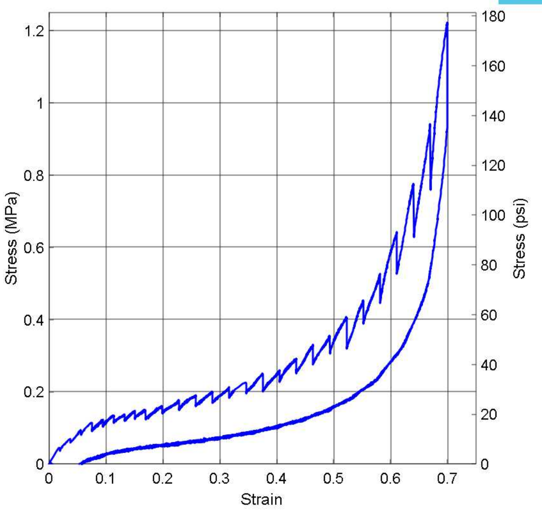
$$a = 1 - \frac{\lambda_1}{\sqrt{\lambda_2^2 + \lambda_3^2}}$$

# Anisotropy Index

$$a = 1 - \frac{\lambda_1}{\sqrt{\lambda_2^2 + \lambda_3^2}}$$




- Minimum in *a* corresponds to steep increase in angle
- Yield has been found to occur at ~4% in a companion study
- Lock-up does not have a defined beginning

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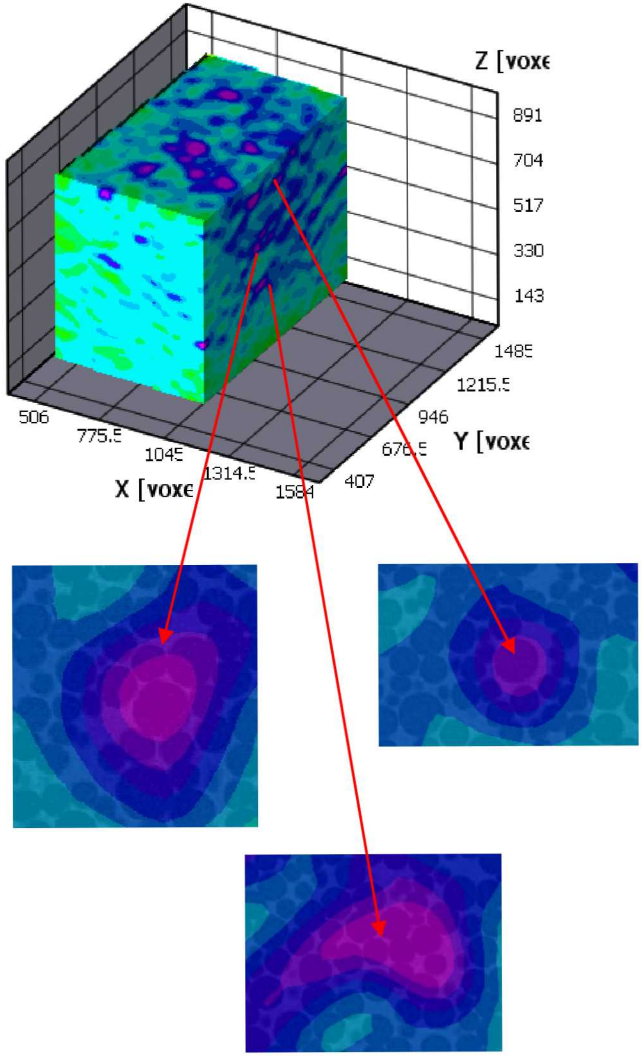
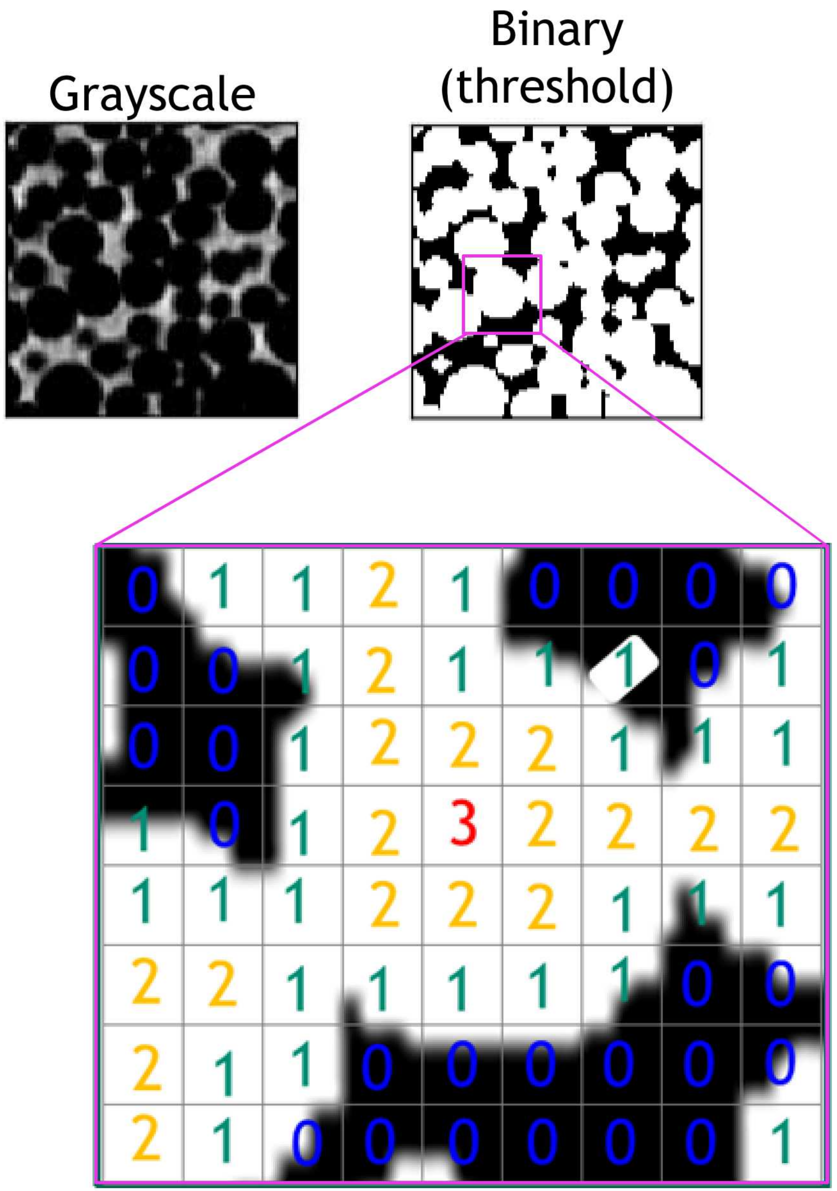
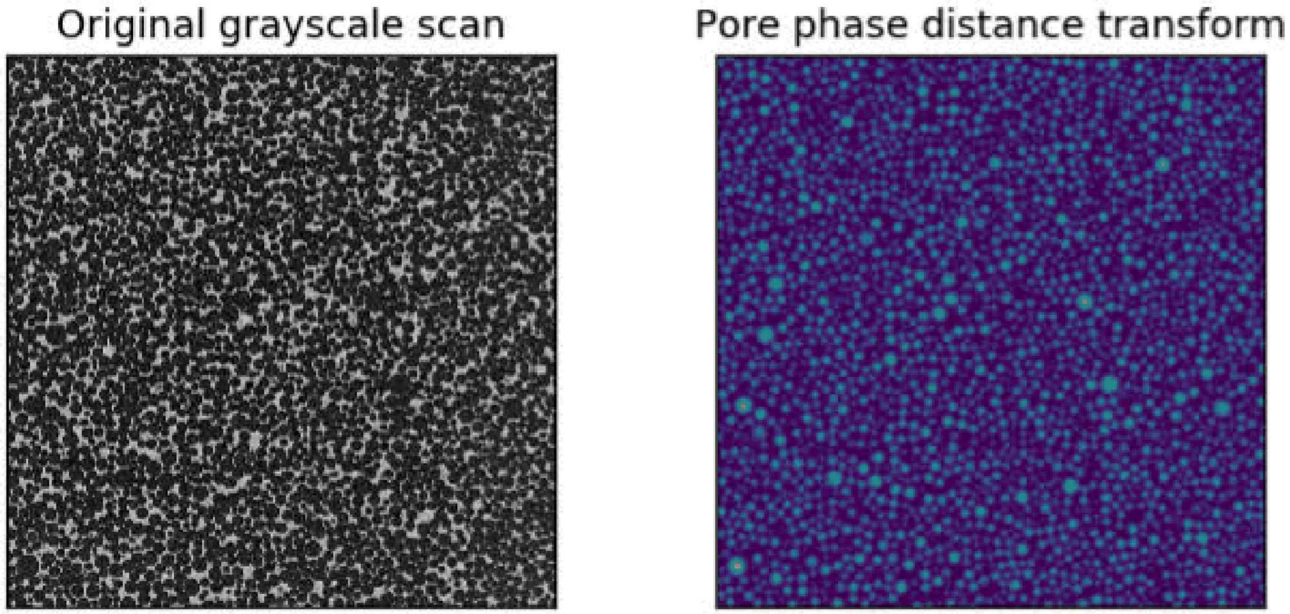


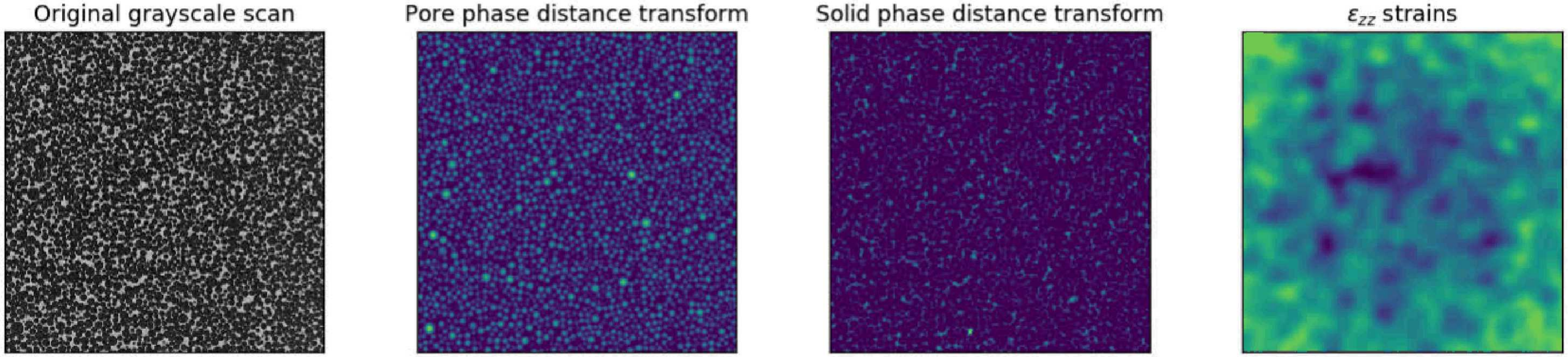
# Distance Transform

- Operation that transforms a binary image into a similar image, but where the pixel values now represent the distance to the closest boundary
- The pore phase distance transform shows a representation of how far a point is from the solid polymer matrix
- A solid phase distance transform is accomplished in a similar manner



# Cross-correlation between microstructure and DVC strains




Cross-correlation between two fields $I_1$ and $I_2$: $C(\mathbf{r}) = \langle I_1(\mathbf{x}+\mathbf{r}) I_2(\mathbf{x}) \rangle$

Where:

$$I_1(\mathbf{x}) = (I - \overline{I})/\sigma_I$$

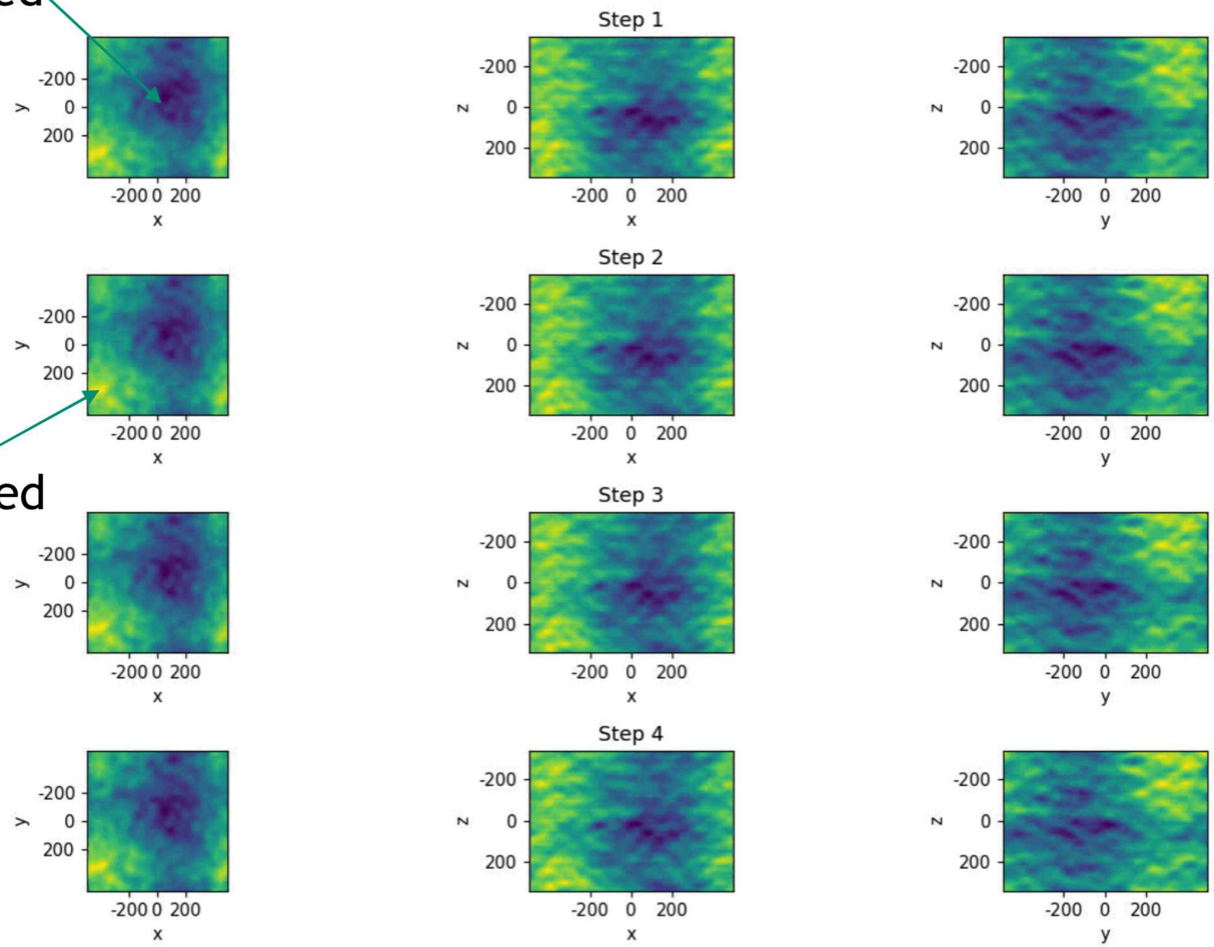
$I$ is either pore or solid phase distance transform

$$I_2(\mathbf{x}) = (\epsilon_{zz} - \overline{\epsilon_{zz}})/\sigma_\epsilon$$

# Cross-correlation between microstructure and DVC strains

Blue anti correlated

Pore distance transform and e_zz strains:



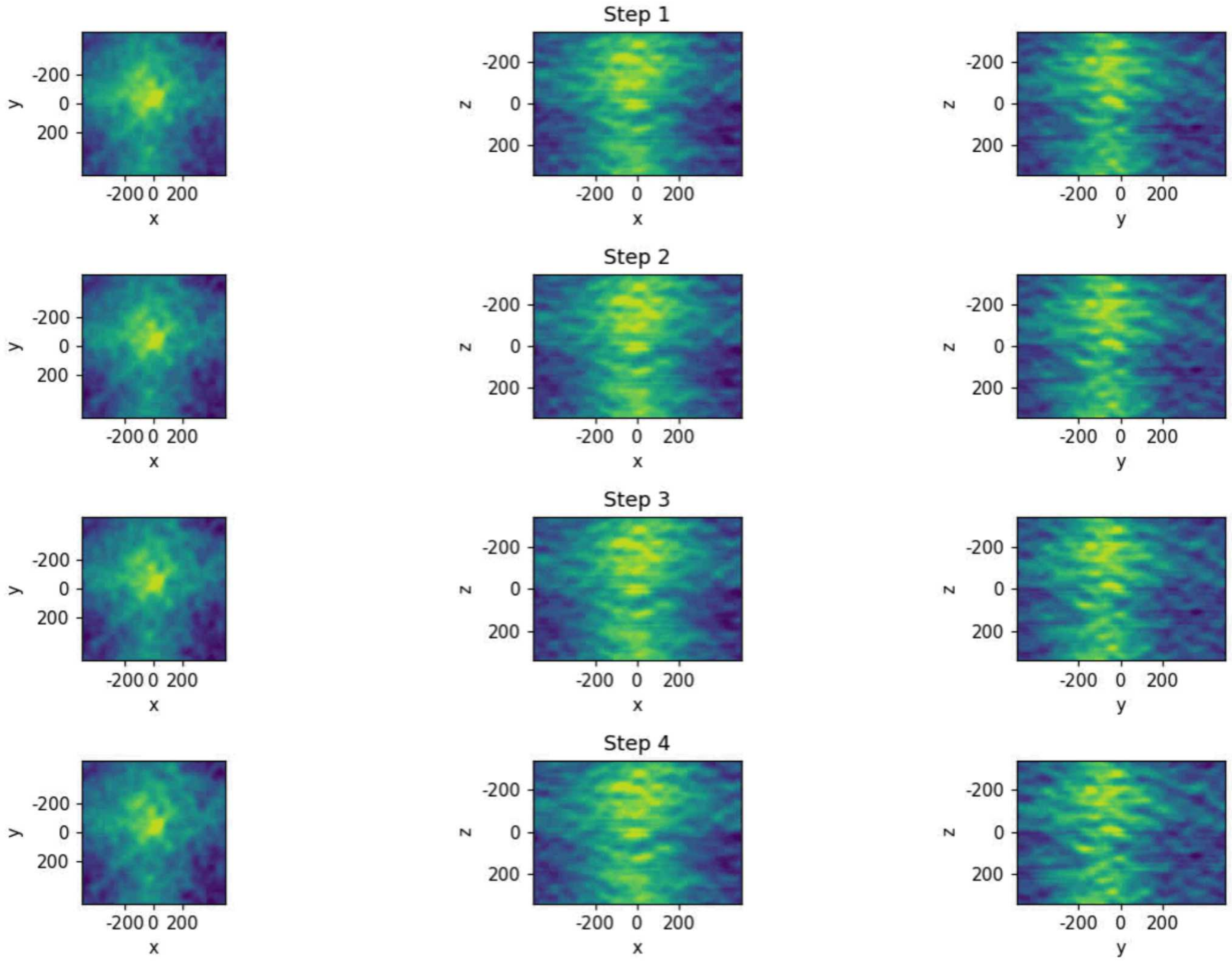

Yellow correlated

Larger compressive stress are correlated with bigger pores

More negative (blue) → more negative strain (greater compressive strain)

# Cross-correlation between microstructure and DVC strains

Solid phase distance transform and e_zz strains:




Thicker struts → more positive (less negative) strains

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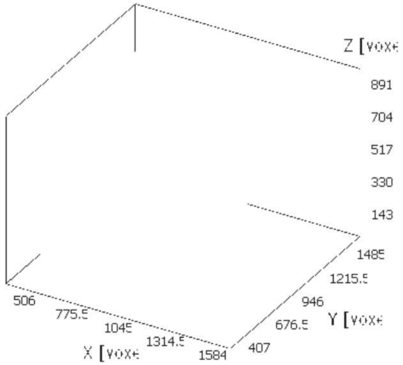
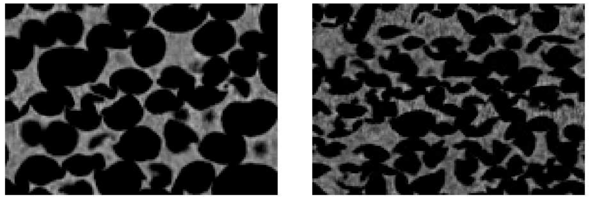
# Conclusions

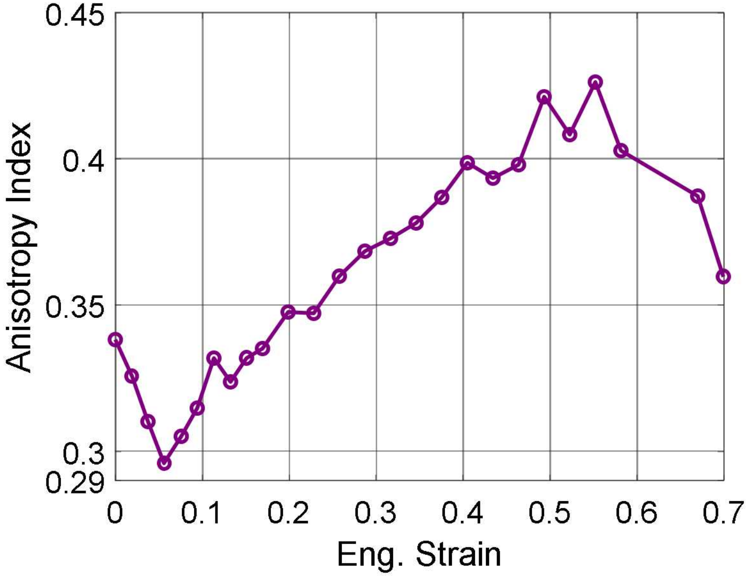
- Generated full field microstructural data using X-ray CT and volumetric strain information with DVC

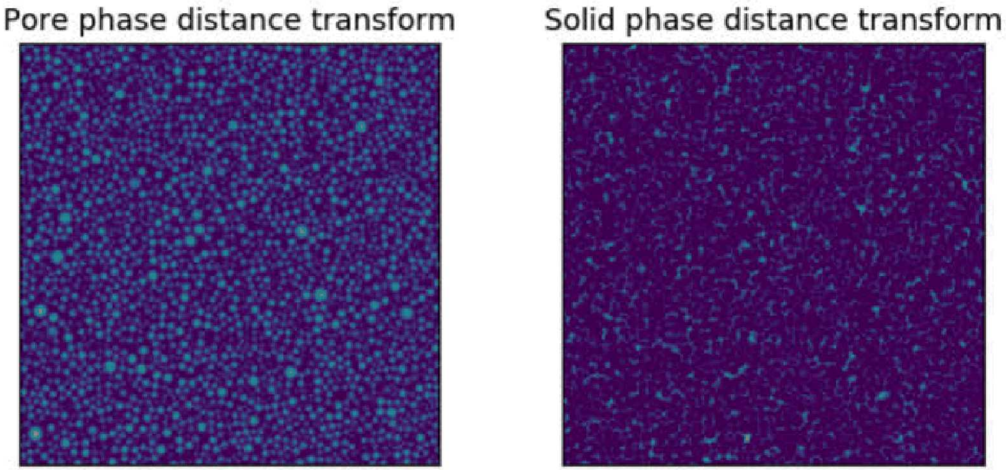


- Quantified the compression induced anisotropy using the autocorrelation function

- Applied the cross-correlation function between the distance transform and the DVC strains
  - Observed large strains in the vicinity of large pores and less strain in thicker struts

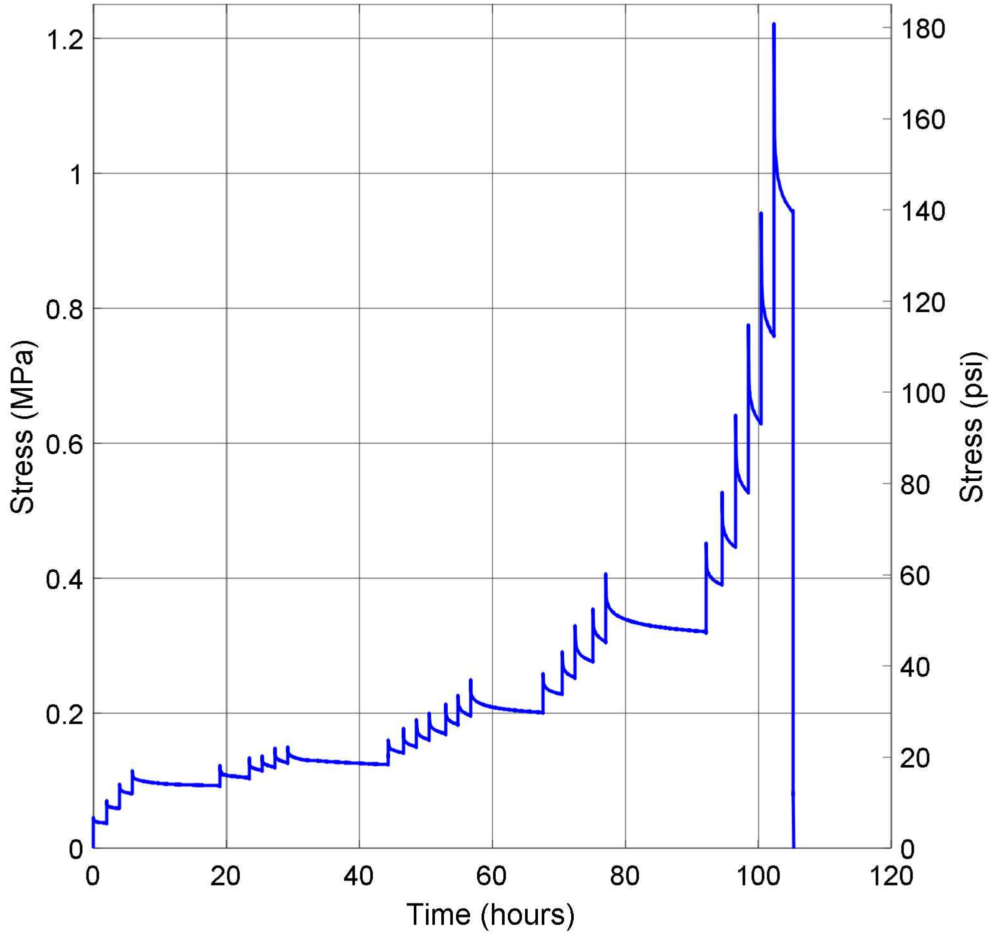


# Extra Slides

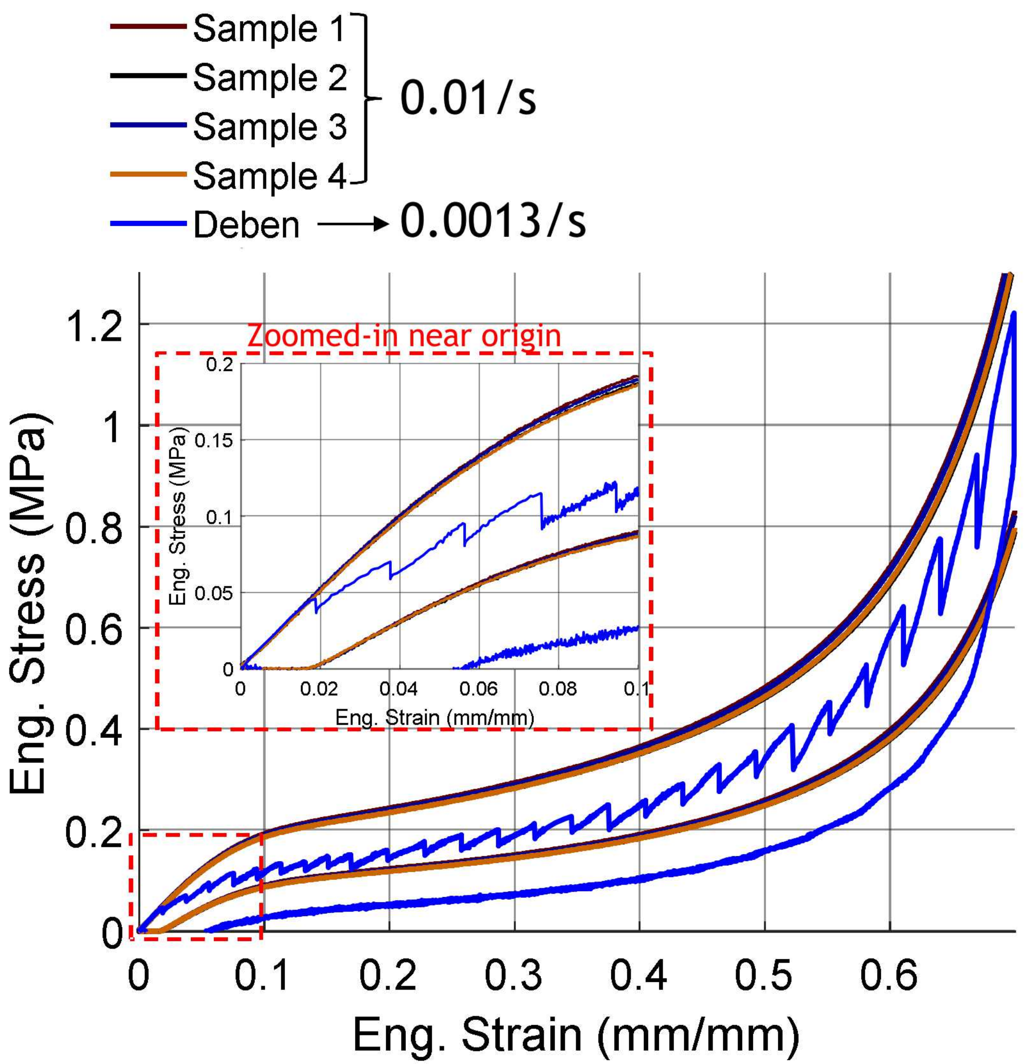
# Stress Relaxation

Stress-time Profile of 15 pcf Compression Experiment

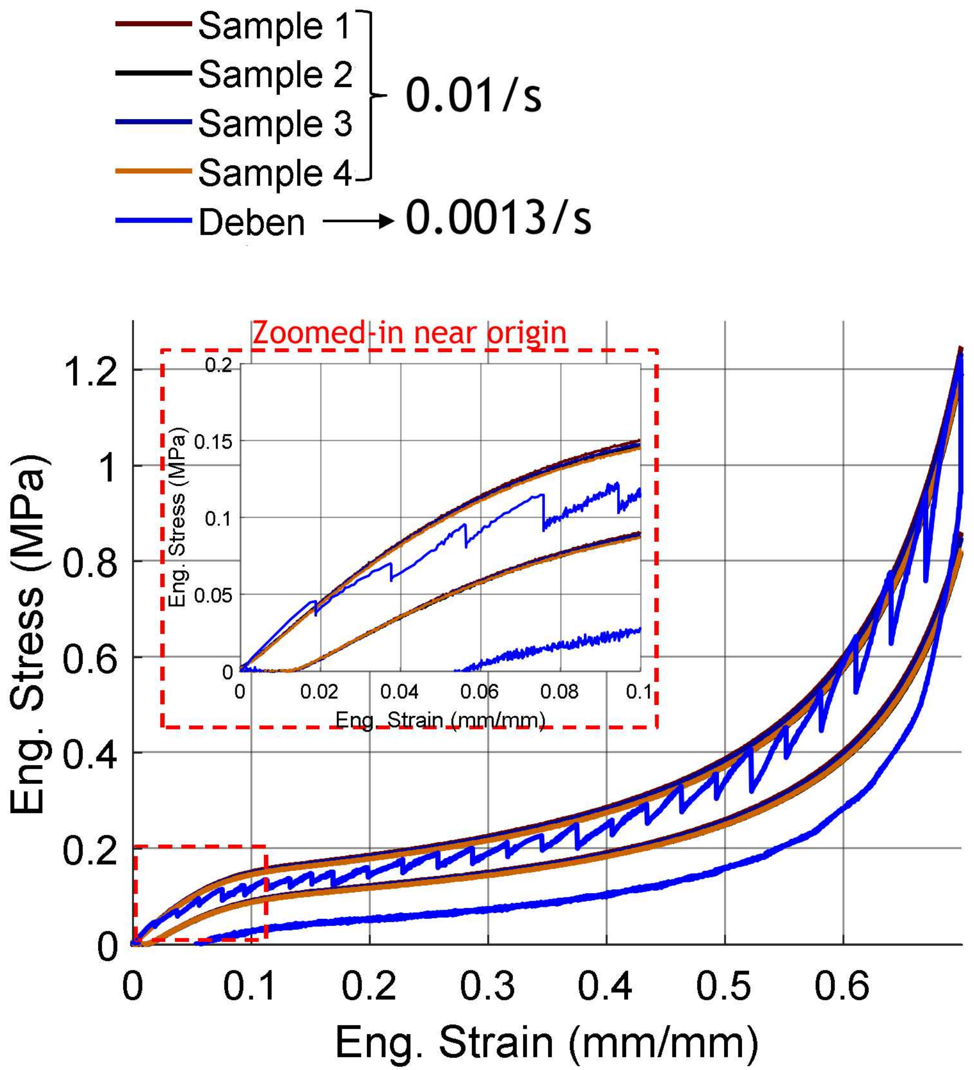


# Stress Relaxation (cont.)

Pristine Deben Sample Compared with Pristine Samples Hydraulic Load Frame



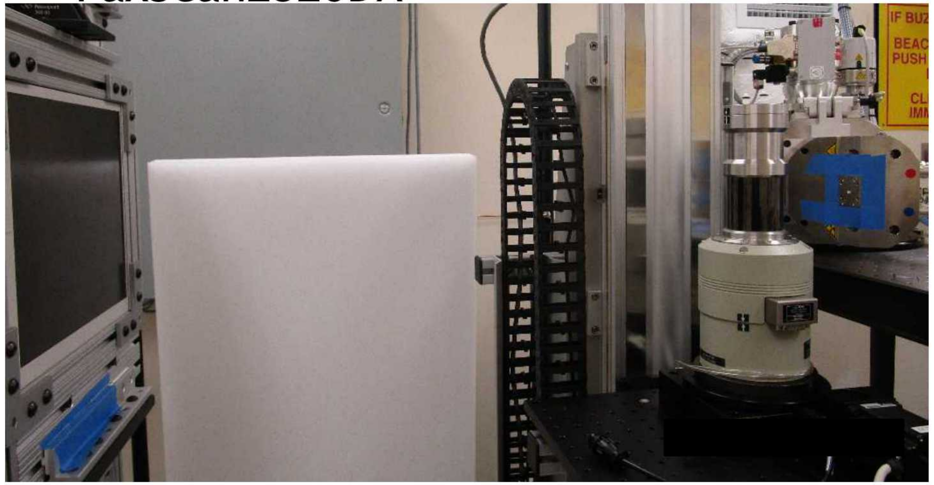

Pristine Deben Sample Compared with Pre-cycled Hydraulic Load Frame Samples



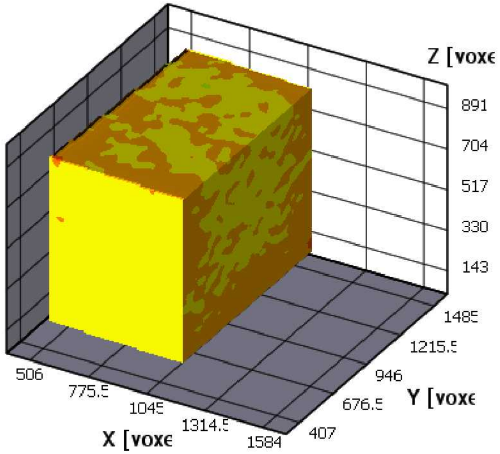
# Additional DVC Steps and X-ray Bay Image

Varian PaxScan2520DX

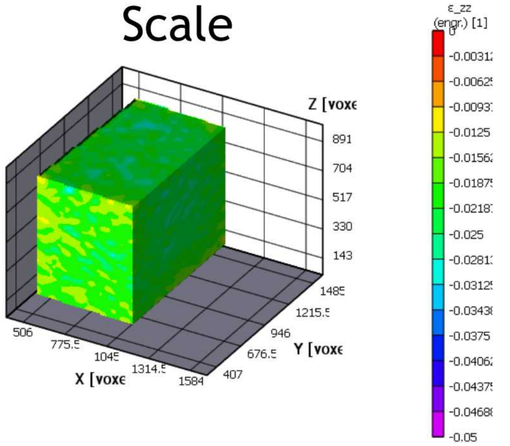
Xray Worx 250kV



Step 1

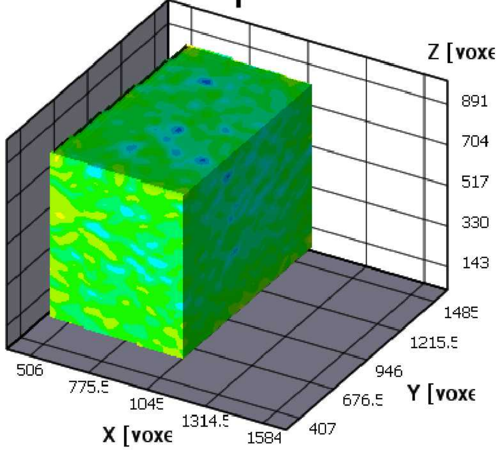



Step 1 Alternate Scale




| CT Acquisition Settings | |
|---|---|
| Resolution | $12^3$ $um^3$/vox |
| Voltage | 180 kV |
| Current | 72 µA |
| Power | 13 W |
| Magnification | 10.6 |

Step 3

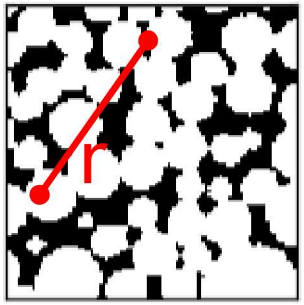


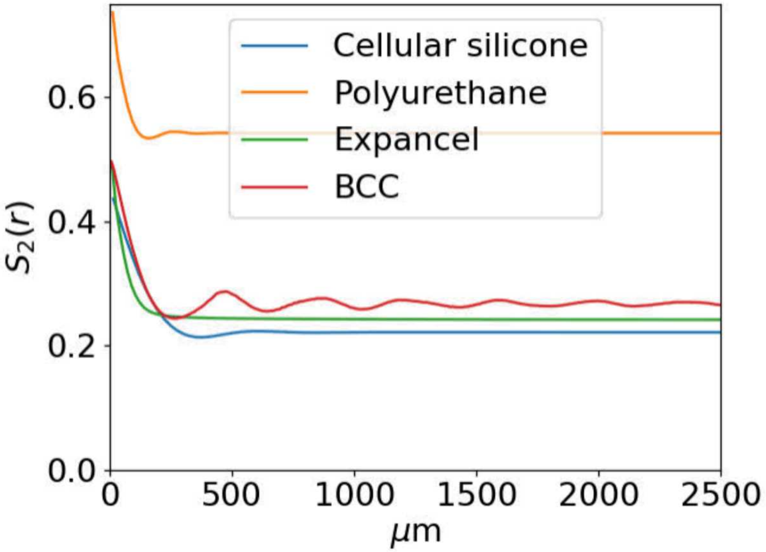
# Spatial correlations: how material is spatially distributed

- Two-point correlation function: $S_2(\mathbf{r}) = \langle I(\mathbf{x}) I(\mathbf{x}+\mathbf{r}) \rangle$

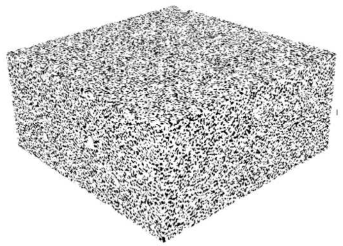
  where $I(\mathbf{r}) = 1$ in pore phase, 0 in solid phase
- Fast to compute via 3D FFTs: $F(\mathbf{k}) = \mathrm{FFT}(I(\mathbf{r}))$

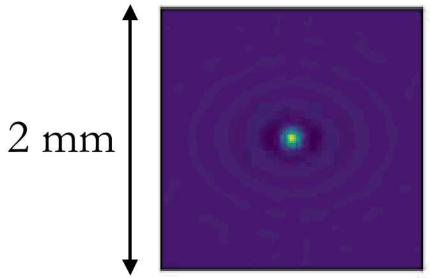
$$S_2(\mathbf{r}) = \mathrm{FFT}^{-1}(F(\mathbf{k}) F^*(\mathbf{k}))$$

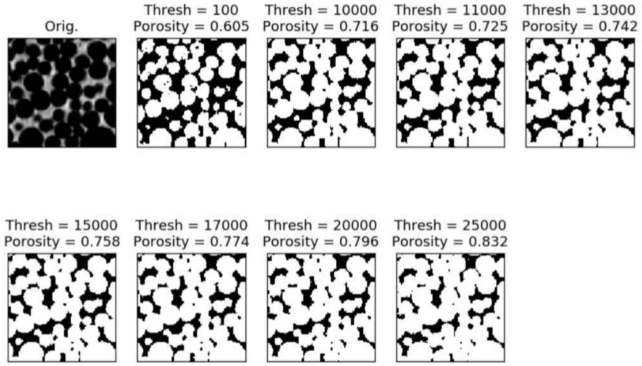



Polyurethane





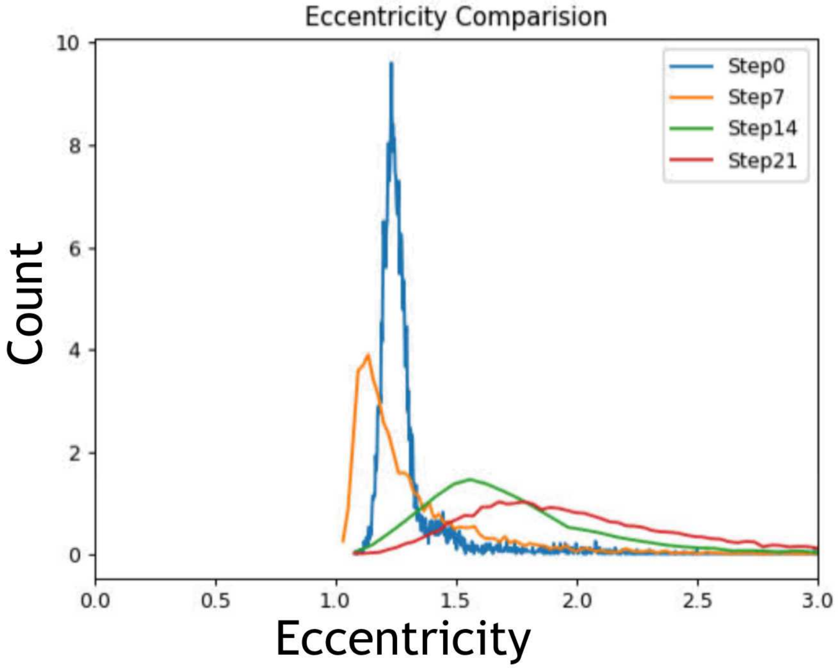

Intuition:
- Characteristic length scale
- Anisotropy

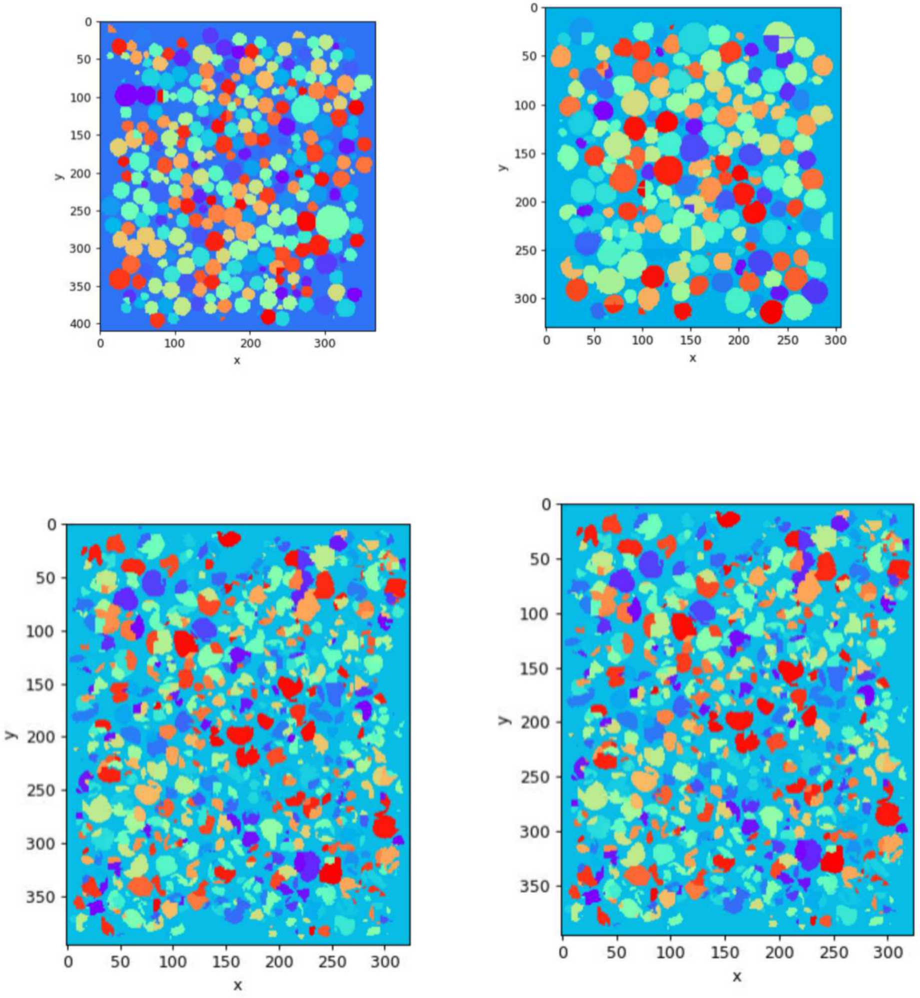
**More importantly: a complete, objective description of microstructure**

# Individual Pore Segmentations

Reconstructed image was thresholded

Result similar to anisotropy index



# Level set plot and all moment of inertia components

$$ACF(\mathbf{r}) = \frac{\langle I(\mathbf{x}+\mathbf{r})I(\mathbf{x})\rangle - \overline{I^2(\mathbf{x})}}{\sigma^2}$$

*I:* CT greyscale intensity

⟨⟩: denotes spatial average (i.e. over all locations **x**)

‾ (overbar): mean

$\sigma^2$: variance

$$I_{11} = I_{xx} = \sum_{k=1}^{N} m_k \left(y_k^2 + z_k^2\right),$$

$$I_{22} = I_{yy} = \sum_{k=1}^{N} m_k \left(x_k^2 + z_k^2\right),$$

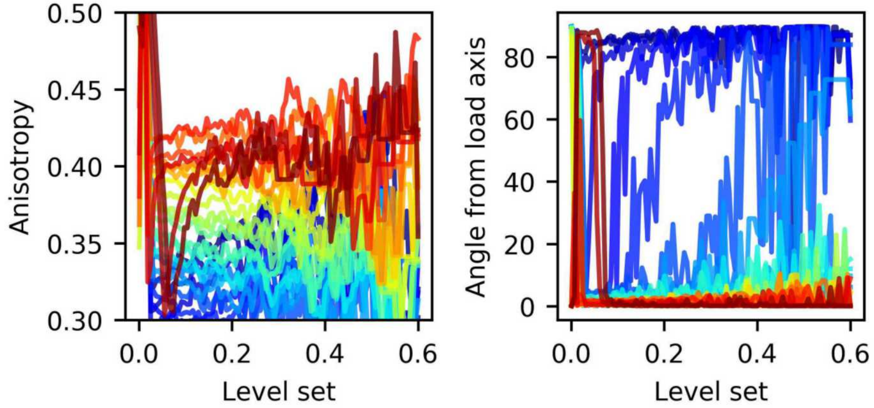
$$I_{33} = I_{zz} = \sum_{k=1}^{N} m_k \left(x_k^2 + y_k^2\right),$$

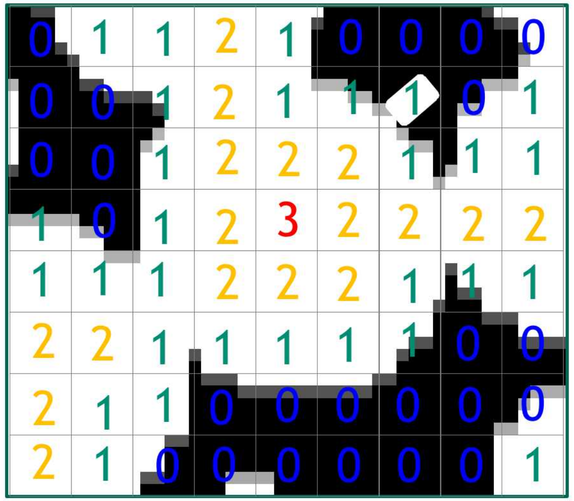
$$I_{12} = I_{21} = I_{xy} = -\sum_{k=1}^{N} m_k x_k y_k,$$

$$I_{13} = I_{31} = I_{xz} = -\sum_{k=1}^{N} m_k x_k z_k,$$

$$I_{23} = I_{32} = I_{yz} = -\sum_{k=1}^{N} m_k y_k z_k.$$

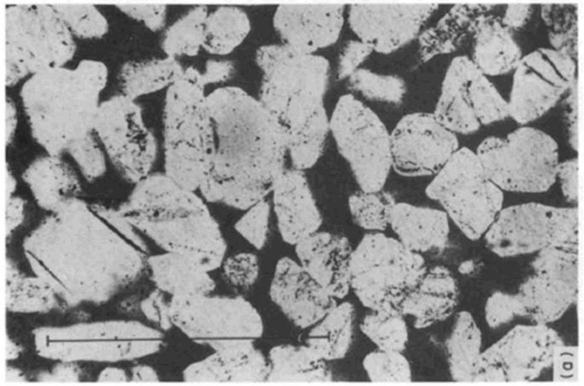
No thresholding required



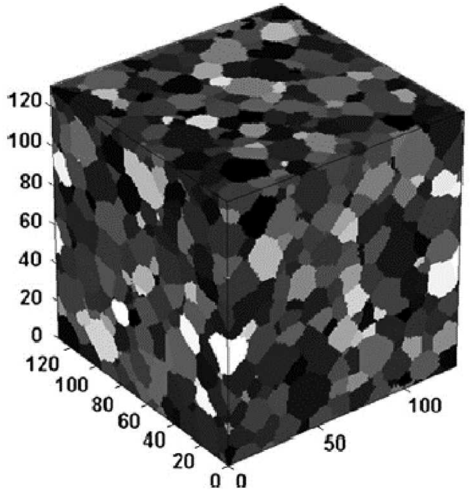
# Background – Spatial Statistics from Full-Field Data

Photograph of Fontainbleau sandstone
Predict permeability based on spatial statistics

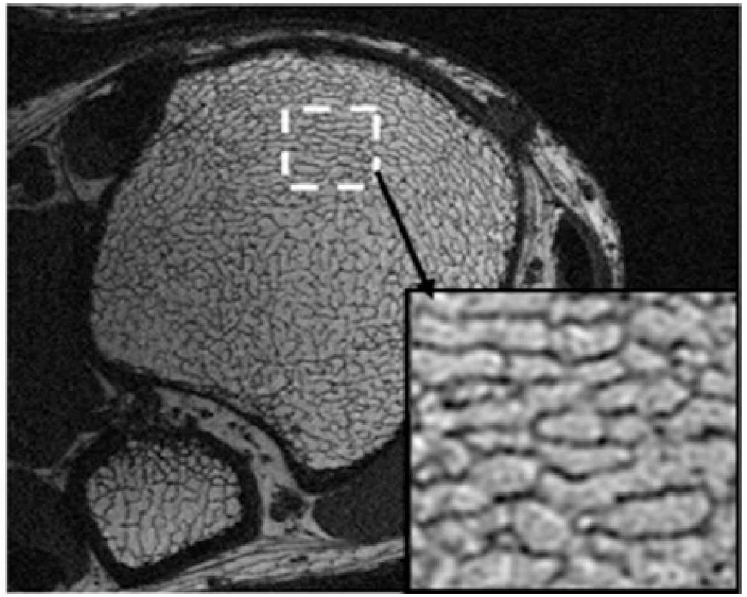



Adler et al. (1990) *Int. J. Multiph. Flow*

Digitally generated grain structure. Ability to recreate structure from statistical values.



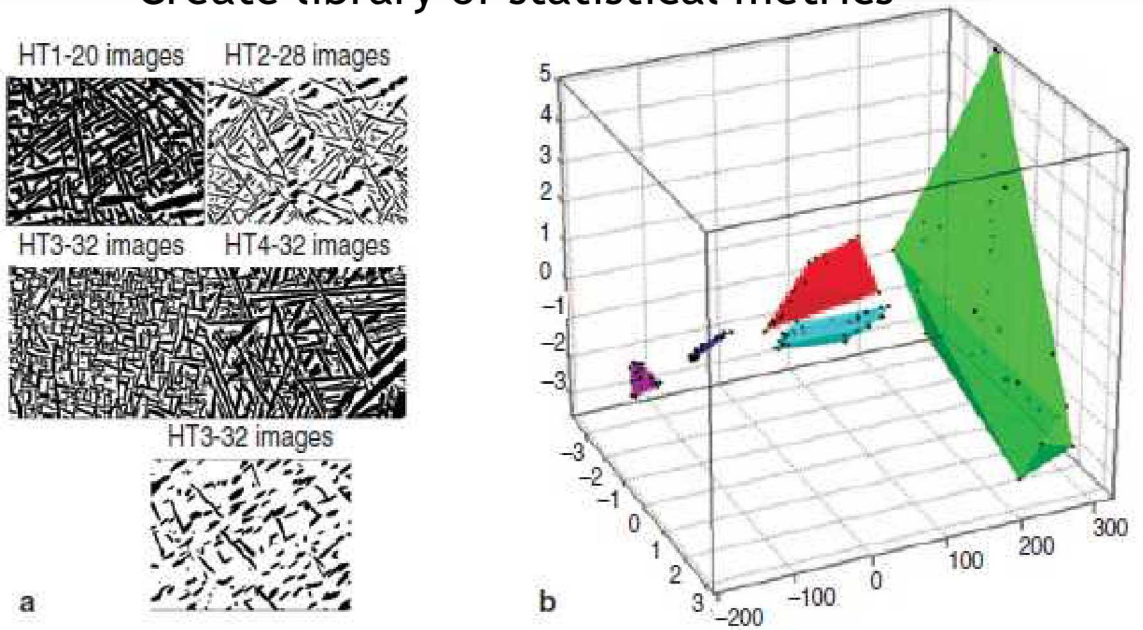

Fullwood et al. (2007) *Acta Mater.*



MRI of tibia; Determine anisotropy (and health) of bone

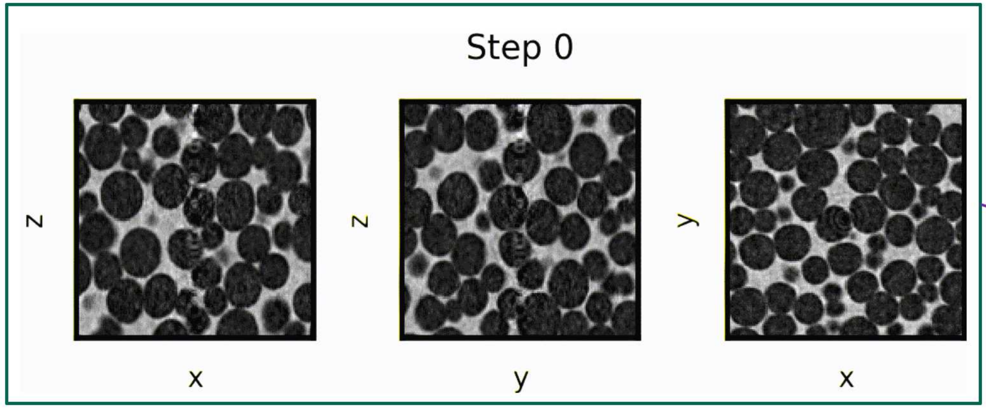
Wald et al. (2007) *Med. Phys.*

Create library of statistical metrics




Kalidindi et al. (2011) *JOM*

# Autocorrelation Function to Measure Deformation Induced Anisotropy

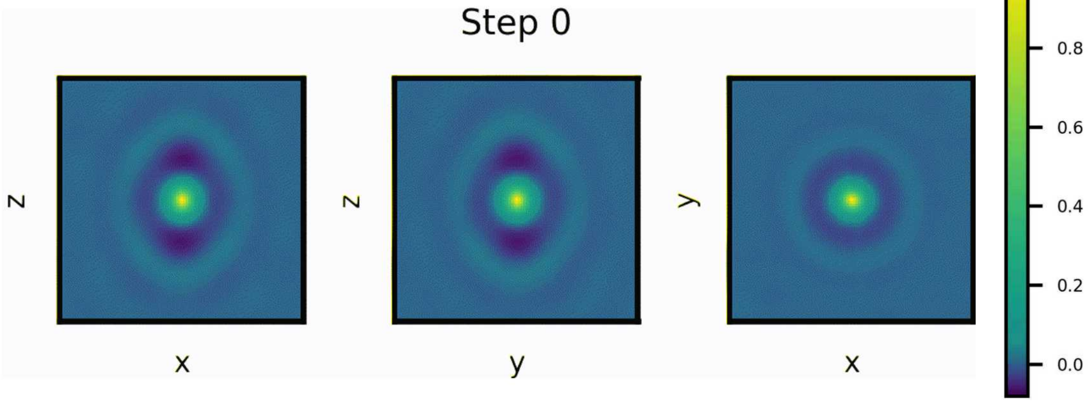



1

Autocorrelation function: a measure of how correlated intensity is as a function of spatial separation

$$ACF(\mathbf{r}) = \frac{\langle I(\mathbf{x}+\mathbf{r})I(\mathbf{x})\rangle - \overline{I^2(\mathbf{x})}}{\sigma^2}$$

2




*I:* CT greyscale intensity

⟨⟩: denotes spatial average (i.e. over all locations **x**)

¯ (overbar): mean

$\sigma^2$: variance

No thresholding required

3

$$\mathbf{I} = \sum_{k=1}^{N} m_k((\mathbf{r}_k \cdot \mathbf{r}_k)\mathbf{E} - \mathbf{r}_k \otimes \mathbf{r}_k),$$

where **E** is the identity tensor

$$\mathbf{E} = \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{e}_3 \otimes \mathbf{e}_3$$

4

$$a = 1 - \frac{\lambda_1}{\sqrt{\lambda_2^2 + \lambda_3^2}}$$