

# ADJOINT-ENABLED MULTIDIMENSIONAL OPTIMIZATION OF SATELLITE RADIATION SHIELDS



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PRESENTED BY

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Context

Optimization approaches

Derivation of 2D transport sensitivities

Results

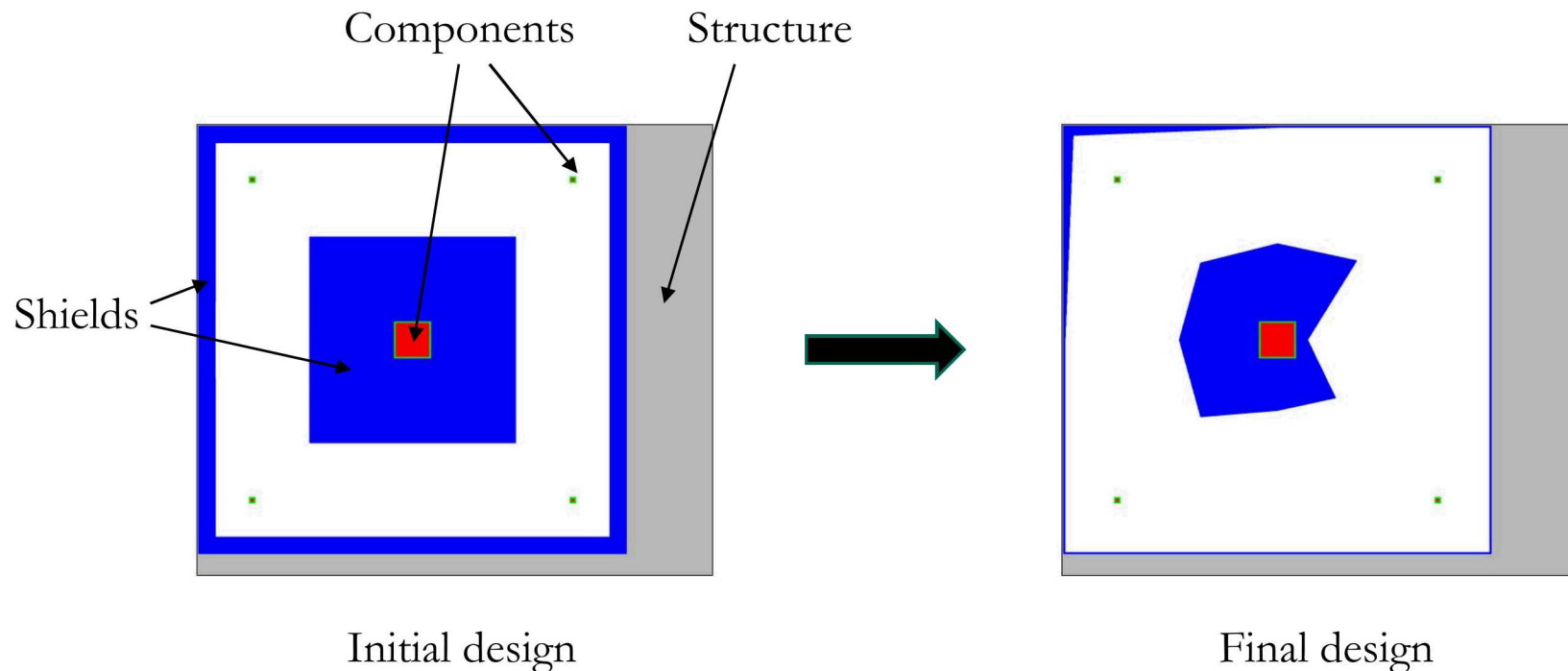
Conclusions

- Increased demand for optimization and UQ
- Increasing interest in advanced materials/manufacturing
- Recent advances in transport community with adjoint-based sensitivities

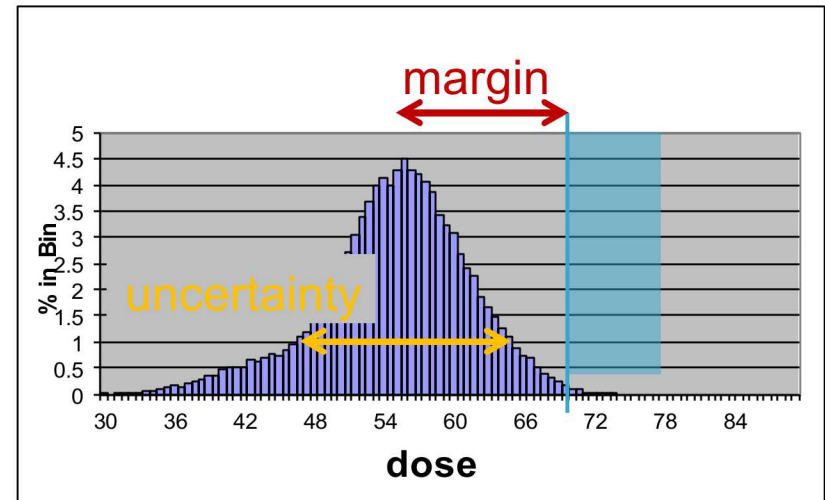
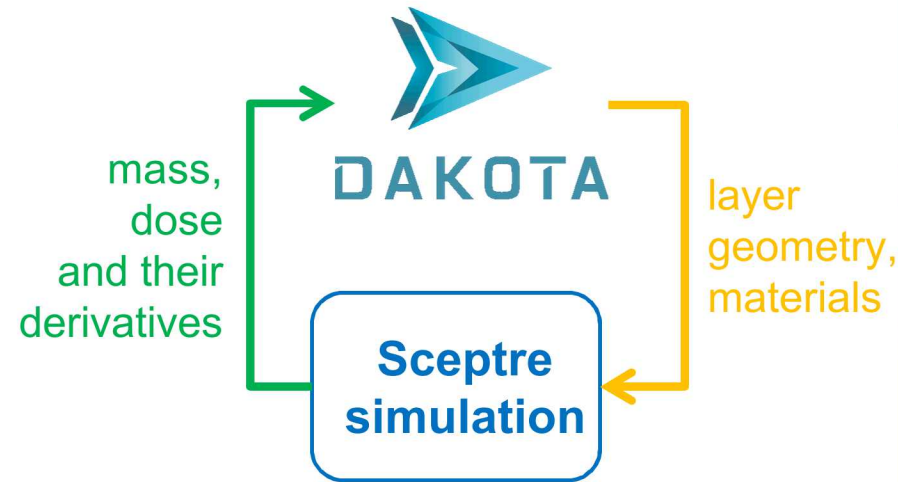
Opportunity: use adjoint-based transport methods to efficiently determine parameter sensitivities, which will be used to drive design optimization and/or UQ calculations in high-dimensional spaces.

- Enable consideration of larger design/trade space
- Provide greater insight to designers

We are interested in satellite electron/proton shielding applications. Mass is at a premium for space missions due to launch costs and/or mass limits. We want to perform design optimization to achieve the required level of protection with the minimum amount of mass. For example, we want to be able to transform the initial design below to a mass-saving one while still meeting the same requirements. This may entail material and/or geometric changes.



- Use **Dakota with Sceptre (deterministic transport solver)** to systematically ask what-if questions: sensitivity, design, uncertainty analyses
- **Optimization:** What component materials, composite material fractions, and shield layer geometries yield the lightest shield meeting strength and dose requirements?
- **Uncertainty Quantification (UQ):** Given variability in manufacturing (mixtures, layer geometry) and state of knowledge (transport cross sections), with what probability will a proposed design meet dose requirements?





### Gradient Descent

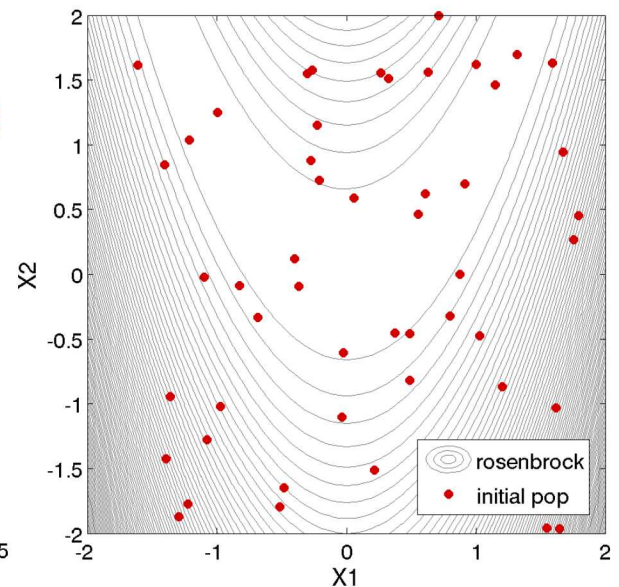
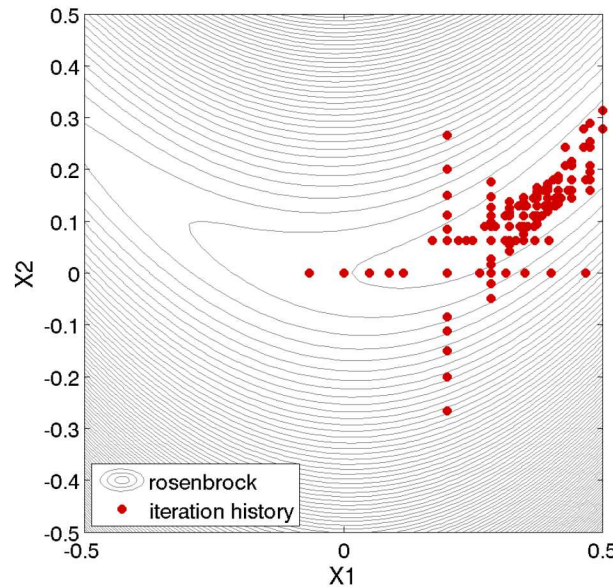
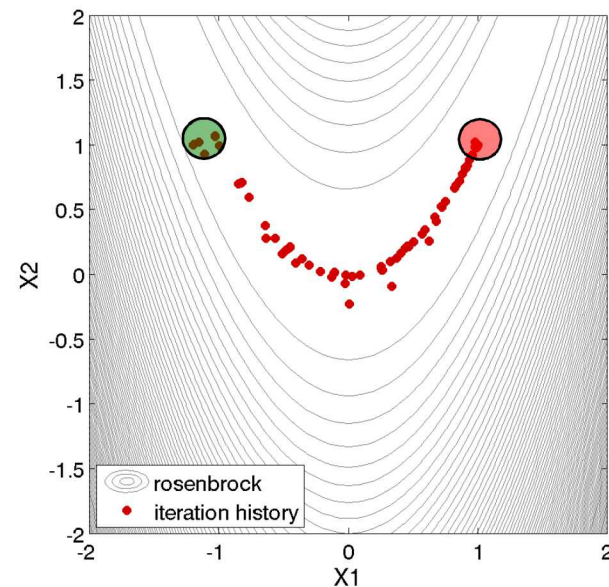
- Looks for improvement based on derivative
- Requires analytic or numerical derivatives (*more soon*)
- Efficient/scalable for smooth problems
- Converges to local extreme

### Derivative-Free Local

- Sampling with bias/rules toward improvement
- Requires only function values
- Good for noisy, unreliable or expensive derivatives
- Converges to local extreme

### Derivative-Free Global

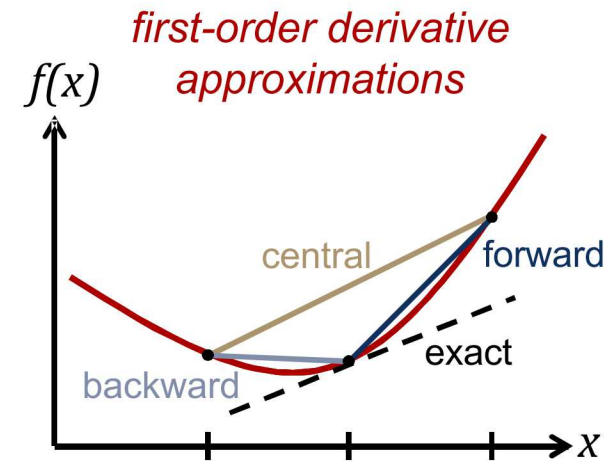
- Broad exploration with selective exploitation
- Requires only function values
- Typically computationally intensive
- Converges to global extreme



- Akin to Newton's method for root-finding, minimize the objective by going “downhill” based on the **gradient of the objective function**:

$$\nabla f_x(x) = \left[ \frac{\partial f(x)}{\partial x_1}, \dots, \frac{\partial f(x)}{\partial x_N} \right]$$

- Most simulations don't calculate derivatives
- Dakota **approximates gradients** (and Hessians if needed) by running the simulation at  $x \pm \Delta x$  as needed



First-order Forward Difference	Second-order Central Difference
$\frac{\partial f}{\partial x} \approx \frac{f(x + \Delta x) - f(x)}{\Delta x}$	$\frac{\partial f}{\partial x} \approx \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$
responses numerical_gradients forward fd_step_size 1.0e-3	responses numerical_gradients central fd_step_size 1.0e-3

- Goal: minimize mass subject to dose constraint, given design variables  $x$ :
  - location  $t_j$  of surface point  $j$
  - fraction  $p_{j,m}$  of material  $m$  in layer  $j$

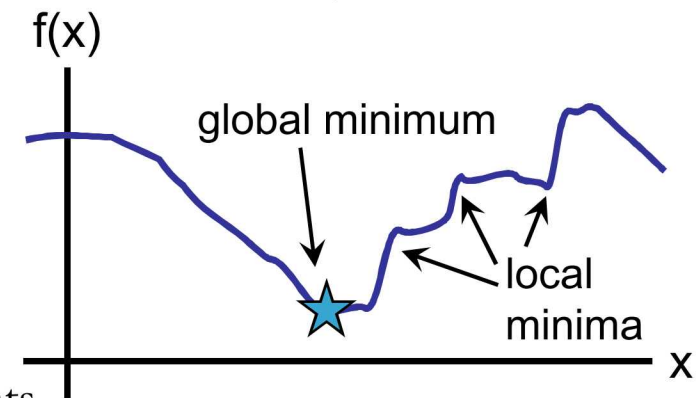
$$\begin{aligned} \min f(x) &= \text{mass}(p, t) \\ \text{s. t. } g(x) &= \text{dose}(p, t) \leq d_{\max} \end{aligned}$$

- We find local minima with Newton-like gradient-based methods; loosely

$$x_{k+1} = x_k - [Hf(x)]^{-1} \nabla_x f(x)$$

where explicit adjoints alleviate finite differencing for  $\nabla_x f(x)$

- Solve the constrained problem with sequential quadratic programming (NPSOL) and  $f(x)$ ,  $g(x)$  adjoints
- Address non-smoothness or multiple minima with multi-start local optimization, trust region surrogate-based optimization instead of pattern search or genetic algorithms...





Problem:

$$\Omega \cdot \nabla \psi + \sigma_t \psi = \int_{E'} dE \int_{4\pi} d\Omega \sigma_s(r, \Omega' \rightarrow \Omega, E' \rightarrow E) \psi(r, \Omega', E') + q, r \in D$$

$$\psi = \psi_b(r, \Omega, E), \{r \in \partial D | \Omega \cdot \vec{n} < 0\}$$

$$R = \int_D dr \int_E dE \int_{4\pi} d\Omega \psi(r, \Omega, E) q^\dagger(r, E)$$

Simplifications/definitions:

$$L\psi + C\psi = S\psi + q$$

$$\int_D dr \int_E dE \int_{4\pi} d\Omega ab \equiv \langle a, b \rangle$$

Lagrangian:

$$\mathcal{L} = \langle \psi, q^\dagger \rangle - \langle \psi^\dagger, L\psi + C\psi - S\psi - q \rangle$$

Derivatives (sensitivities):

$$\frac{d\mathcal{L}}{dp} = \frac{\partial \mathcal{L}}{\partial p} + \frac{\partial \mathcal{L}}{\partial \psi} \frac{\partial \psi}{\partial p}$$

$p$  is any uncertain and/or design parameter. In general it is a vector.

Substituting in the expression for the Lagrangian and carrying out several steps yields:

$$\begin{aligned}
 & \frac{d\mathcal{L}}{dp} \\
 &= \left[ \left\langle \psi, \frac{\partial q^\dagger}{\partial p} \right\rangle + \left\langle \psi^\dagger, \frac{\partial q}{\partial p} \right\rangle - \left\langle \psi^\dagger, \left( \frac{\partial}{\partial p} (L + C - S) \right) \psi \right\rangle \right. \\
 &+ \left. \left\langle \frac{\partial \psi}{\partial p}, q^\dagger - (L^\dagger + C^\dagger - S^\dagger) \psi^\dagger \right\rangle + \left\langle \frac{\partial \psi^\dagger}{\partial p}, q - (L + C - S) \psi \right\rangle \right] \\
 &+ \left[ \left\langle I, q^\dagger - (L^\dagger + C^\dagger - S^\dagger) \psi^\dagger \right\rangle + \left\langle \frac{\partial \psi^\dagger}{\partial p}, q - (L + C - S) \psi \right\rangle \right] \frac{\partial \psi}{\partial p}
 \end{aligned}$$

The derivatives of output quantities (e.g.  $\frac{\partial \psi}{\partial p}$ ) are unknown.

If the following equations are satisfied:

$$(L + C - S)\psi = q \quad (\text{forward Boltzmann problem})$$

$$(L^\dagger + C^\dagger - S^\dagger)\psi^\dagger = q^\dagger \quad (\text{adjoint Boltzmann problem})$$

then  $\frac{d\mathcal{L}}{dp}$  reduces to :

$$\frac{d\mathcal{L}}{dp} = \left[ \left\langle \psi, \frac{\partial q^\dagger}{\partial p} \right\rangle + \left\langle \psi^\dagger, \frac{\partial q}{\partial p} \right\rangle - \left\langle \psi^\dagger, \left( \frac{\partial}{\partial p} (L + C - S) \right) \psi \right\rangle \right] = \frac{dR}{dp}$$

The sensitivities of *any*  $R$  to *any*  $p$  are obtained by various inner products involving the forward solution  $\psi$ , the adjoint solution  $\psi^\dagger$ , and derivatives of *input* (i.e. known) quantities.

## WHY WE CARE ABOUT ADJOINT SENSITIVITIES



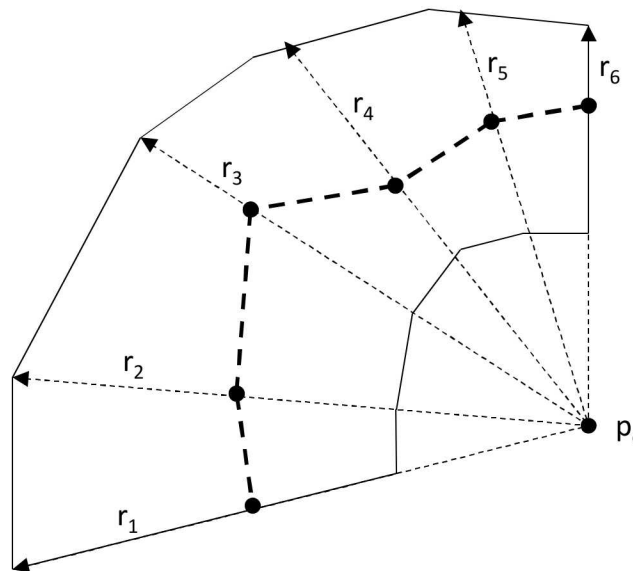
Solver behavior for some 1D optimization runs

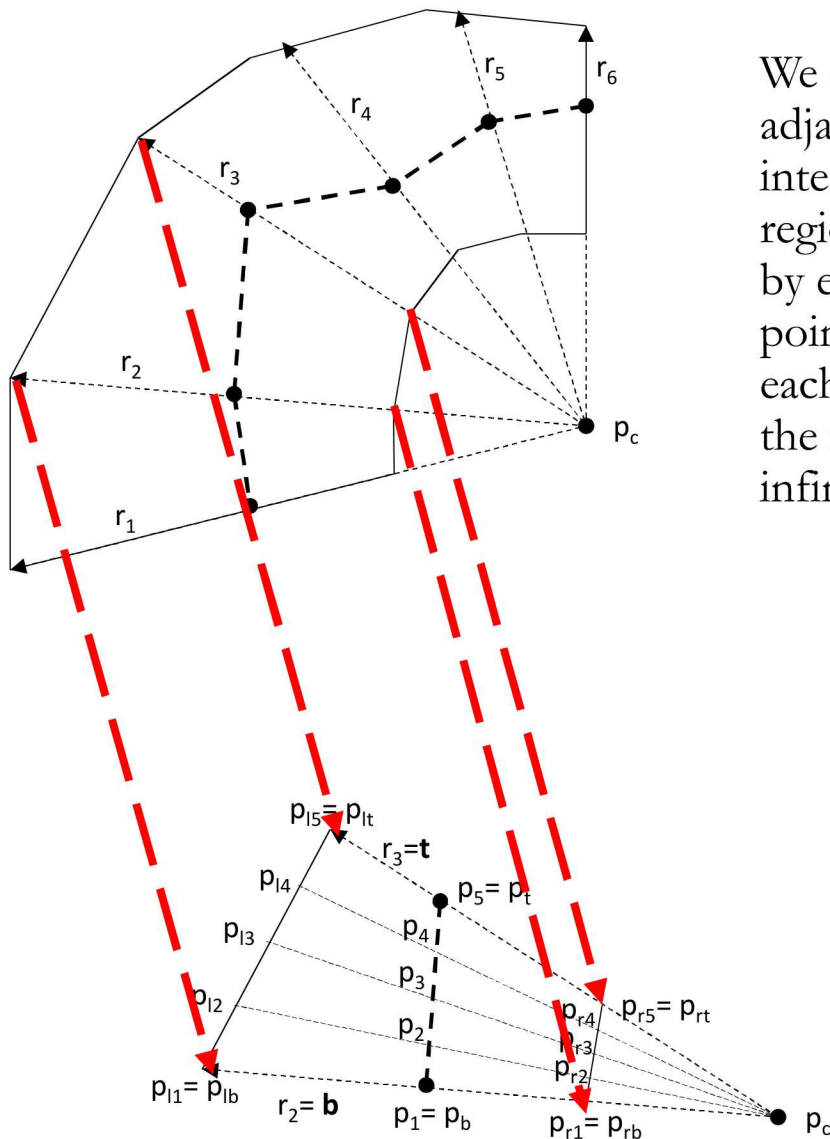
Problem	Design Parameters	Transport solves for finite differences	Transport solves for adjoint approach
1	1	8	8
2	1	4	4
3	2	33	22
4	3	20	10
5	5	84	44
6	6	133	34
7	12	377	60
8	92	-	26
9	184	-	30

*Adjoint-based approach scales much better than finite-difference approach for large number of parameters ( $2 < p+1$ )*



We want to know the effect of geometric changes in satellite electron/proton shields. We postulate a variable-thickness shield surrounding an electronic component or package (a portion of the geometry is depicted below). The shield surface is parametrized by some number of “control” points, each lying some distance along unique rays emanating from a “central” point (perhaps within the package to be protected). The points are connected by line segments to define the surface. We need to drive the sensitivity of the dose to the component to changes in the location of each point.





We examine the region between two adjacent control points on both sides of the interface (bottom figure). We subdivide the region into several other subregions defined by equidistant points between the control points. After writing various equations for each subregion, we will examine the limit as the number of subregions increases to infinity.

The collision term for one region is given by:

$$\begin{aligned}
 C\psi = & \sum_n [H(x' - x'_{p_{ln}}) - H(x' - x'_{p_n})] [H(y' - y'_{p_n}) - H(y' - y'_{p_{n+1}})] \sum_m p_{ml} (\sigma_{t,g,m,l} - \sigma_{\delta,gg,m,l}) \psi_g \\
 & + \sum_n [H(x' - x'_{p_n}) - H(x' - x'_{p_{rn}})] [H(y' - y'_{p_n}) - H(y' - y'_{p_{n+1}})] \sum_m p_{mr} (\sigma_{t,g,m,r} - \sigma_{\delta,gg,m,r}) \psi_g
 \end{aligned}$$

The derivative of the collision term with respect to movement of the bottom control point is:

$$\begin{aligned}
 \frac{\partial C}{\partial h_b} = & \sum_n \delta(x' - x'_{p_n}) \nabla_{nc,b} \vec{p}_n [H(y' - y'_{p_n}) - H(y' - y'_{p_{n+1}})] \sum_m p_{ml} (\sigma_{t,g,m,l} - \sigma_{\delta,gg,m,l}) \\
 & - \sum_n \delta(x' - x'_{p_n}) \nabla_{nc,b} \vec{p}_n [H(y' - y'_{p_n}) - H(y' - y'_{p_{n+1}})] \sum_m p_{mr} (\sigma_{t,g,m,r} - \sigma_{\delta,gg,m,r})
 \end{aligned}$$

The portion of the response sensitivity related to the collision term is given by:

$$\begin{aligned}
 \frac{\partial R}{\partial h_b} &= -\langle \psi_g^\dagger, \left( \frac{\partial C}{\partial h_b} \right) \psi_{g'} \rangle \\
 &= -\int_V dV \sum_g \int_{4\pi} d\Omega \psi_g^\dagger(r, \Omega) \sum_n \delta(x' - x'_{p_n}) \nabla_{nc,b} \vec{p}_n \left[ H(y' - y'_{p_n}) \right. \\
 &\quad \left. - H(y' - y'_{p_{n+1}}) \right] \sum_m p_{ml} (\sigma_{t,g,m,l} - \sigma_{\delta,gg,m,l}) \psi_g(r, \Omega) \\
 &\quad + \int_V dV \sum_g \int_{4\pi} d\Omega \psi_g^\dagger(r, \Omega) \sum_n \delta(x' - x'_{p_n}) \nabla_{nc,b} \vec{p}_n \left[ H(y' - y'_{p_n}) \right. \\
 &\quad \left. - H(y' - y'_{p_{n+1}}) \right] \sum_m p_{mr} (\sigma_{t,g,m,r} - \sigma_{\delta,gg,m,r}) \psi_g(r, \Omega)
 \end{aligned}$$

## EXAMPLE: COLLISION TERM (CONTINUED)

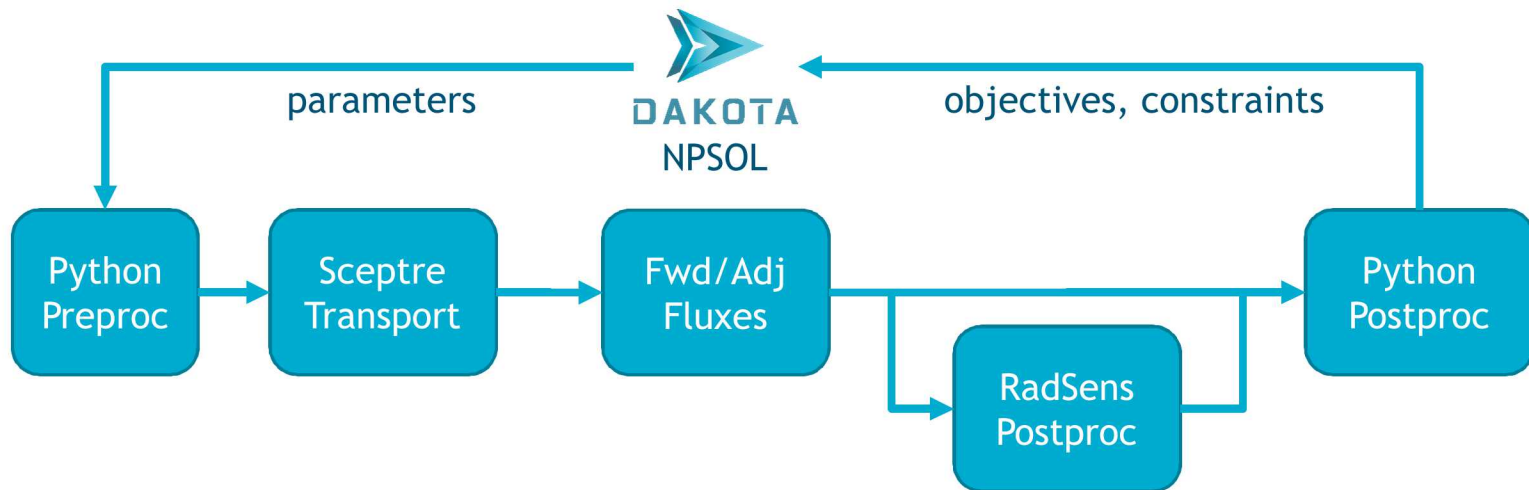


We take the limit as the number of subregions goes to infinity to obtain the portion of the response sensitivity related to the collision term:

$$\begin{aligned}
 & \lim_{N \rightarrow \infty} \frac{\partial R}{\partial h_b} \\
 &= - \sum_g \sum_m p_{ml} (\sigma_{t,g,m,l} - \sigma_{\delta,gg,m,l}) \int_{4\pi} d\Omega \int_{\partial D_l} ds \vec{i}' \cdot \vec{n} \nabla_{nc,b} \vec{p}_n \psi_g^\dagger(s, \Omega) \psi_g(s, \Omega) \\
 &+ \sum_g \sum_m p_{mr} (\sigma_{t,g,m,r} - \sigma_{\delta,gg,m,r}) \int_{4\pi} d\Omega \int_{\partial D_r} ds \vec{i}' \cdot \vec{n} \nabla_{nc,b} \vec{p}_n \psi_g^\dagger(s, \Omega) \psi_g(s, \Omega)
 \end{aligned}$$

Analogous expressions are derived for the scattering and adjoint source terms to obtain the full sensitivity of the response to movement of the control point for one region. Movement of a point may involve more than one region.





- Dakota NPSOL optimization drives the process
- Python tools translate between Dakota and Sceptre
- Sceptre deterministic transport produces forward and adjoint angular flux fields
- Material and geometric sensitivities are post-processed from these fields

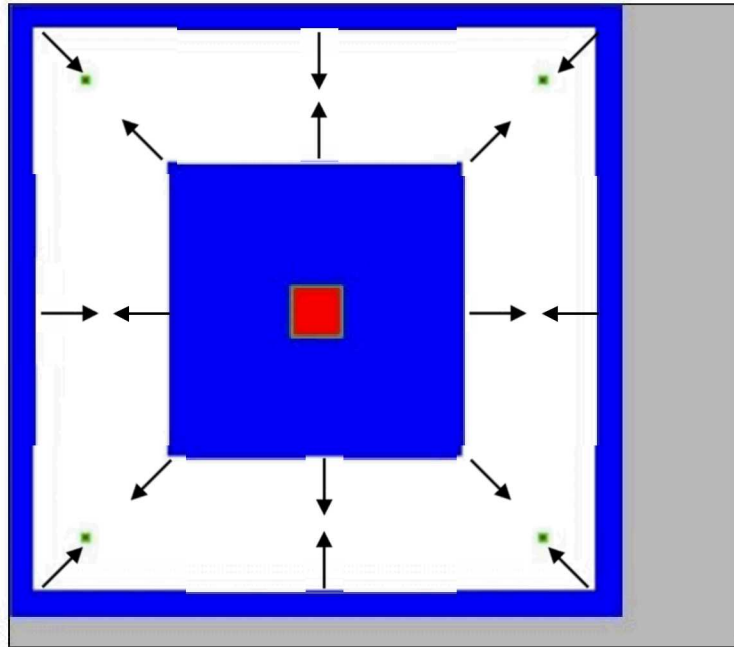
## APPLICATION: ELECTRON AND/OR PROTON SATELLITE SHIELDING



We are interested in satellite shielding applications. The problems we will study are:

- 2000 km circular equatorial orbits (arbitrarily chosen to demonstrate optimization)
- Proton and/or electron environments as defined by the AP8 and AE8 models in Spenvis
- Various components to be protected to various levels
- Multiple shielding regions of arbitrary geometry and/or materials

# APPLICATION: ELECTRON AND/OR PROTON SATELLITE SHIELDING



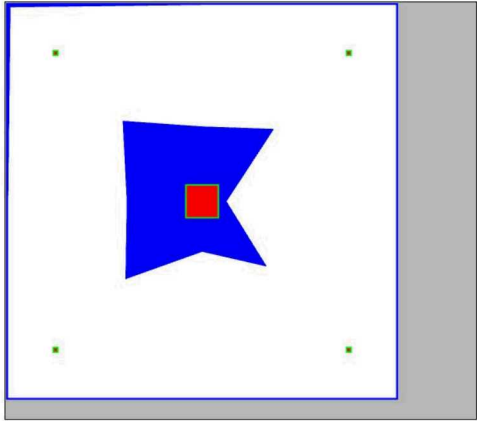
Initial design

We want to protect components in a satellite. The electron/proton flux is isotropic, but asymmetric aluminum structure produces an asymmetric internal environment. The thick region on the right is an approximation to the satellite structure. The region on the bottom represents other instruments. There are four components in the corners, and a larger fifth component in the middle. Nominal shields are in blue. The location and allowed movement of control points is represented by arrows.

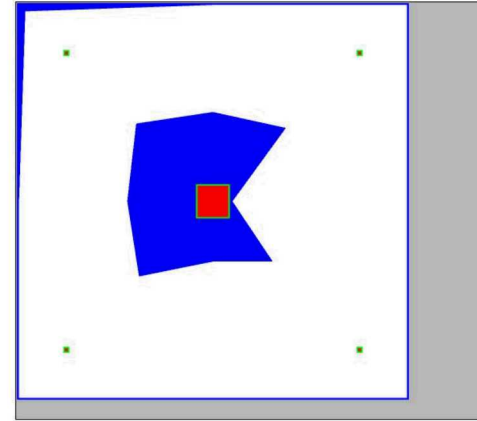
# EXAMPLE: PROTON SHIELDING WITH POLYETHYLENE

REQUIREMENTS: 100 KRAD/YR AT CORNERS, 40 KRAD/YR AT CENTER

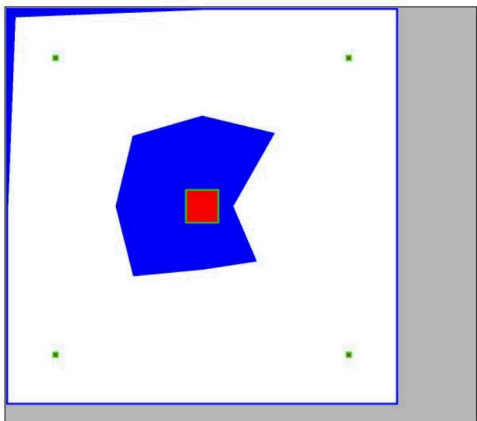
(The initial design is oversized and requires 225 g/cm)



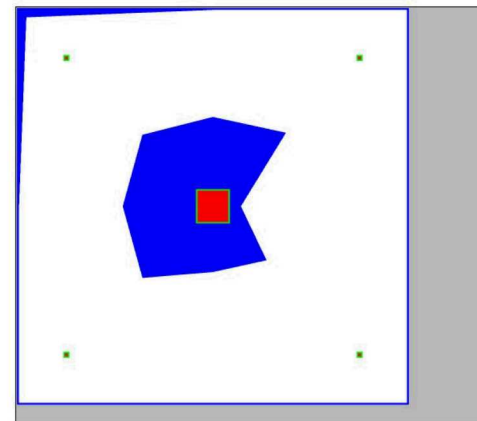
Second design iteration (71 g/cm)



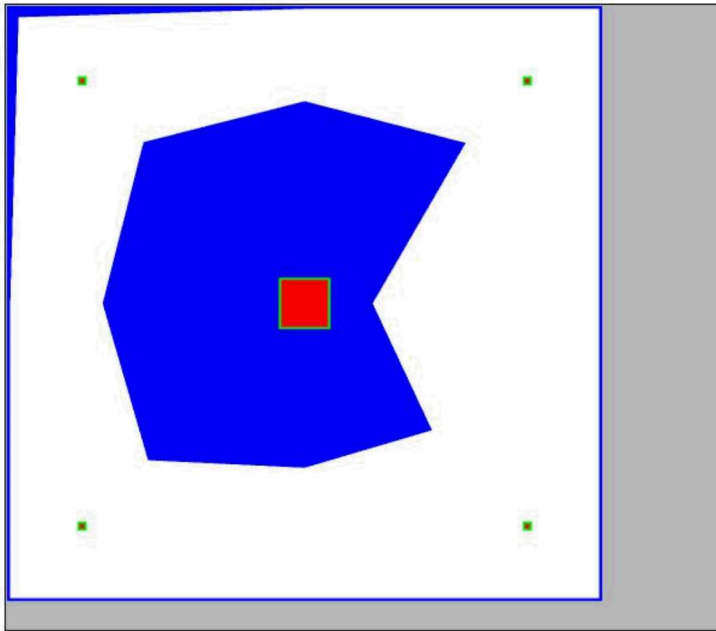
Third design iteration (81 g/cm)



Fourth design iteration (81 g/cm)

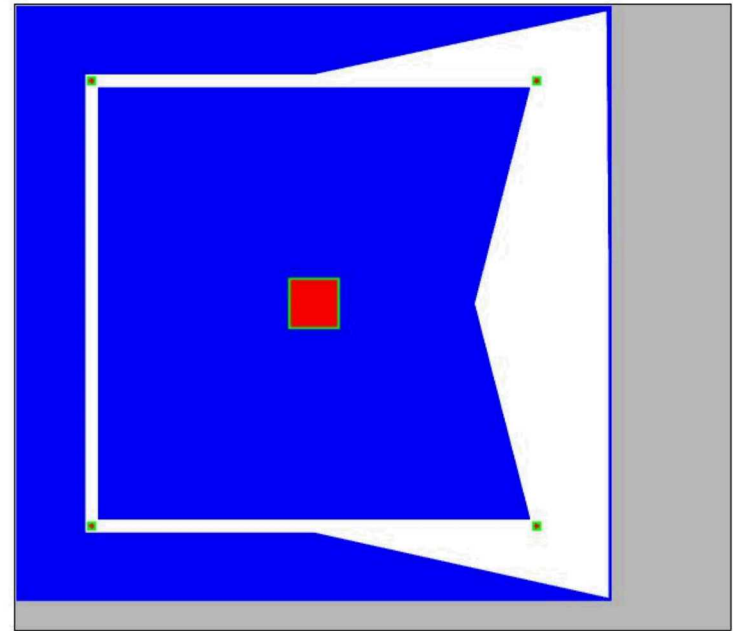


Final design: 82 g/cm



Center dose requirement: 30 krad/yr

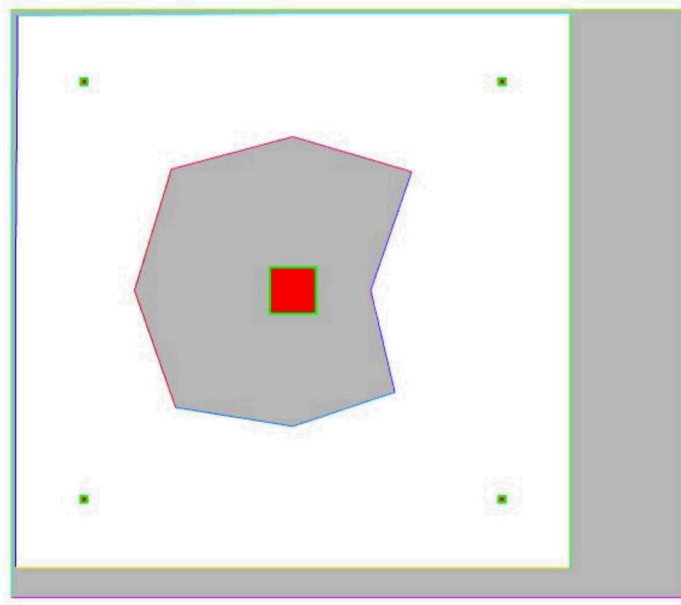
Final design: 174 g/cm



Center dose requirement: 20 krad/yr

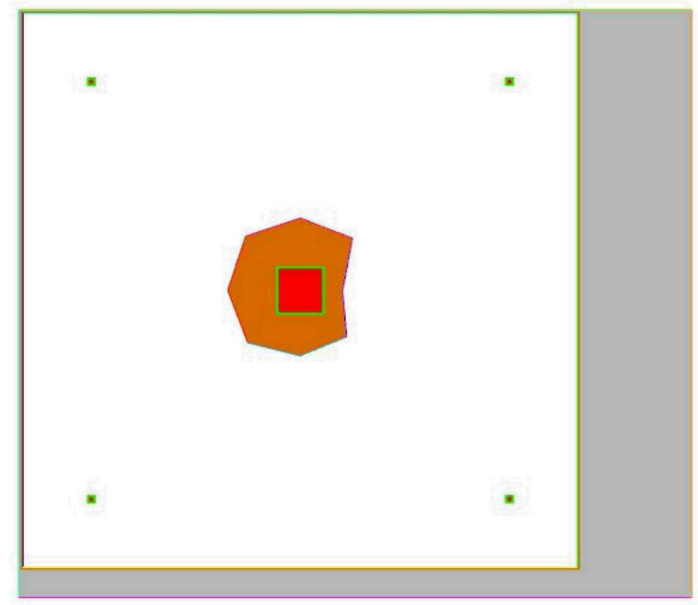
Final design: 450 g/cm





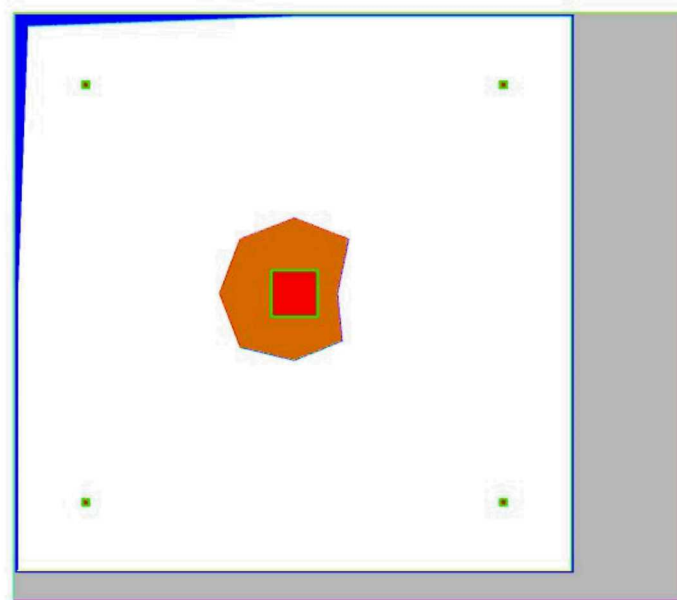
Aluminum shields, 20 krad/yr at center

Final design: 357 g/cm



Copper shields, 20 krad/yr at center

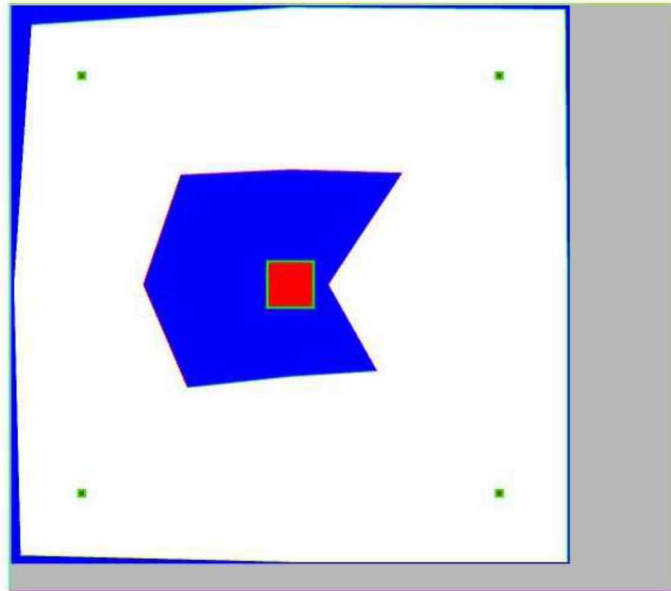
Final design: 289 g/cm



Requirement: 20 krad/yr at center

For this design we arbitrarily allowed for the use of polyethylene, aluminum, copper, molybdenum, and/or tantalum

Final design: Polyethylene outer shield, copper inner shield, 226 g/cm



Requirement: 40 krad/yr at center

For this design we only allowed for the use of polyethylene

Final design: 97 g/cm

- Our previous work demonstrated our ability to compute geometric and material sensitivities in 1D slab geometry and to incorporate them into satellite shield design optimization
- In the current work we derived sensitivities for parametrized 2D geometries and incorporated them into our design optimization tools
- We have demonstrated the use of these tools for a variety of satellite problems:
  - combined environments
  - multiple materials
  - numerous control points
- Future work:
  - More combined environment studies with multiple materials
  - 3D sensitivities/optimization
  - Robust design